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**ENERGY INPUT-OUTPUT ECONOMICS:  
WHAT'S THE MATTER ?**

Abstract: The purpose of the paper is to show that integrating energetic resources into input-output analysis does not need any a priori reformulation of this analysis. In their classical textbook, Miller and Blair consider that such an integration needs using «hybrid units» in order to satisfy some «energy conservation conditions». We show that these conditions are specifically defined and founded on a particular case, inspired by empirical data of the U.S economy.

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Energetic resources are inputs of the productive processes. In the first part of this paper we show that the use of such resources can be integrated into input-output analysis without any basic change in that analysis. In the second part, we are concerned with the treatment of the same question propounded by Miller and Blair in their classical textbook [Miller-Blair (1985), chap.6, Energy Input-Output Analysis, pp.200-35]. The main part of their argument is to consider that it is necessary to use «hybrid units» in order to satisfy some «energy conservation conditions». We show that these conditions are specifically defined and founded on a particular case, inspired by empirical data of the U.S economy.

We consider a four-sector economy: three sectors are energy sectors, namely crude oil, refined petroleum and electric power. The fourth sector, autos, is the only nonenergy sector.

Producing energetic goods does not need nonenergetic goods while producing nonenergetic goods needs energetic goods. An algebraic version of this paper could be developed. For our present purpose, it will be more convenient to use numerical data. So we shall start our argument using the following interindustry exchanges table:

	Crude Oil	Refined Petroleum	Electric Power	Autos	Final Demand	Total Output
Crude Oil	0	20	10	0	10	40
Refined Petroleum	1	3	0	1	15	20
Electric Power	2.5	1.25	1.25	2.5	12.5	20
Autos	0	0	0	0	20	20

At the beginning of their book, Miller and Blair note that «in accounting for transactions between and among all sectors, it is possible in principle to record all exchanges *either in physical or in monetary terms*» [Miller-Blair (1985), p.7. Our italics]. Noting that there are «enormous measurement problems» when physical measures are used, they finally conclude that «for these and other reasons, then, *accounts are generally kept in monetary terms*» » [Miller-Blair (1985), p.7. Our italics].

There is only a slight difference between our numerical data and those used by Miller and Blair [op. cit., Example 2, pp.204-5. Compare with their matrices  $Z^*$  and  $Y^*$ , p.205]. But it is essential to note that, in our text, goods are measured in their own physical terms. This and the slight difference we introduce into numerical data will suffice for the main part of our argument.

## 1. Energy Requirement and Energy Conservation

### 1.1. The Analysis in Physical Terms

The technological matrix  $A^*$  associated with our interindustry exchanges table is the following:

$$A^* = \begin{bmatrix} 0 & 1 & .5 & 0 \\ .025 & .15 & 0 & .05 \\ .0625 & .0625 & .0625 & .125 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The Leontief inverse matrix is:

$$[I - A^*]^{-1} = \begin{bmatrix} 1.068 & 1.298 & .57 & .136 \\ .031 & 1.214 & .017 & .063 \\ .073 & .168 & 1.106 & .147 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The matrix  $\alpha$  of total energy coefficients is:

$$\alpha = \begin{bmatrix} 1.068 & 1.298 & .57 & .136 \\ .031 & 1.214 & .017 & .063 \\ .073 & .168 & 1.106 & .147 \end{bmatrix}$$

The coefficients of matrix  $\alpha$  show the direct and indirect energy requirement necessary to produce each product: they can be interpreted as the energy value of each product, just as we can compute the labour value  $L[I - A]^{-1}$  of each product in the traditional input-output model.

Furthermore, globally, energy is conserved: for each type of energy, total demand is equal to total production. To the final demand:

$$Y^* = \begin{bmatrix} 10 \\ 15 \\ 12.5 \\ 20 \end{bmatrix}$$

is associated a total demand of each product equal to:  $[I-A^*]^{-1}Y^*$  (in order to take account of intermediate consumptions).

Numerically:

$$[I-A^*]^{-1} \cdot Y^* = [I-A^*]^{-1} \cdot \begin{bmatrix} 10 \\ 15 \\ 12.5 \\ 20 \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \\ 20 \\ 20 \end{bmatrix}$$

This result is equal to vector  $X^*$  of productions of each product. Thus energy is conserved just as goods are «conserved» in traditional input-output analysis: the production of each good is divided between intermediate and final consumption.

## 1.2. Equilibrium Price Determination

Price determination in this version of the input-output model is the same as in the traditional one. We can distinguish two polar cases, the first with a margin of profit equal to zero, the second with a positive margin of profit.

### - Equilibrium Prices with Zero Margin of Profit

From  $P=wL[I-A]^{-1}$ , with  $L=[1 \ 2 \ 3 \ 5]$  and  $w=1$ , we deduce:

$$P=[1.351 \ 4.23 \ 3.92 \ 5.702]$$

Thus, the input-output table of this economy writes:

	Crude oil	Ref. petr.	Elec. power	Autos	Fin. dem.	Total Output
Crude oil	0	27.0157	13.5079	0	13.5079	54.0314
Ref. petr.	4.2303	12.6911	0	4.2303	63.4555	84.6073
Elec. power	9.8010	4.9005	4.9005	9.8010	49.0052	78.4084
Autos	0	0	0	0	114.0314	114.0314
Wages	40	40	60	100		
Total	54.0313	84.6073	78.4084	114.0313		

### - Equilibrium Prices with Positive Margin of Profit

From  $P=(1+\lambda)wL[I-(1+\lambda)A]^{-1}$ , with  $\lambda=0.2$  and  $w=1$ , we deduce:

$$[I-1.2\cdot A]^{-1} = \begin{bmatrix} 1.104 & 1.681 & .716 & .208 \\ .040 & 1.281 & .026 & .081 \\ .093 & .240 & 1.141 & .186 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and:

$$P = [1.756 \ 5.957 \ 5.031 \ 7.112]$$

Thus, the input-output table of this economy writes:

	Crude oil	Ref. petr.	Elec. power	Autos	Fin. dem.	Total Output
Crude oil	0	35.1205	17.5602	0	17.5602	70.2409
Ref. petr.	5.9567	17.8702	0	5.9567	89.3515	119.1353
Elec. power	12.5773	6.2887	6.2887	12.5773	62.8867	100.6187
Autos	0	0	0	0	142.2409	142.2409
Wages	40	40	60	100		
Profits	11.7068	19.8559	16.7698	23.7068		
Total	70.2408	119.1353	100.6187	142.2408		

## 2. «Hybrid Units» and Energy Conservation Conditions

2.1. Throughout chapter 6 of their book, Miller and Blair insist on the fact that an energy conservation condition should be satisfied: «This condition will be a fundamental determinant in assessing whether or not a particular energy input-output model formulation [...] *accurately* depicts the energy flows in the economy» [Miller and Blair (1985), p.201. Our italics].

According to Miller and Blair, there have been two generations of energy input-output models. The earlier formulation of energy input-output models, while still «widely applied in the literature [...], has several serious shortcomings; any attempt at resolving them will have only limited success. *The fundamental difficulty is a violation of consistency among energy transactions (the energy conservation condition discussed in Chapter 6) except under very specific conditions (namely, uniform interindustry prices)*» [Miller and Blair (1985), p.222. Our italics].

In the «hybrid-units approach», energy flows are measured in «physical» units, British thermal units (Btus), for example, rather than in dollars, and nonenergy flows are measured in

dollars. According to Miller and Blair, this new approach does not suffer from the limitations of the earlier one: they «show that the hybrid-units approach yields energy coefficients that conform to a fundamental definition of energy conservation conditions» [Miller and Blair (1985), p.217].

The definition they give to the energy conservation condition is the following: «In computing the energy intensity of a product, we will distinguish between primary energy sectors (e.g., crude oil or coal mining) and secondary energy sectors (e.g., refined petroleum or electricity). The latter receive primary energy as an input and convert it into secondary energy forms. Hence, if we compute both the total amount of primary energy required to produce an industry's output and the total amount of secondary energy required to produce that same output, they must be equal, net of any energy lost in converting energy from primary to secondary energy forms, for example, electric power production from coal. Different technologies, of course, have different energy conversion efficiencies. Hence, our energy input-output formulation should include the condition that *the total primary energy intensity of a product should equal the total secondary energy intensity of the product plus the amount of energy lost in energy conversion*. We refer to this condition as an *energy conservation condition*» [Miller and Blair (1985), p.201. Our italics].

Miller and Blair offer a numerical example of an economy with one primary energy sector and two secondary energy sectors. The total energy requirement matrix of such an economy is the following:

	Crude Oil	Refined Petroleum	Electric Power	Autos	Final Demand	Total Output
Crude Oil	0	20	20	0	0	40
Refined Petroleum	1	3	0	1	15	20
Electric Power	2.5	1.25	1.25	2.5	12.5	20
Autos	0	0	0	0	20	20

From the above table, Miller and Blair deduce the technological matrix  $A^*$  of the model with hybrid units integrating energetic resources into the input-output model:

$$A^* = \begin{bmatrix} 0 & 1 & 1 & 0 \\ .025 & .15 & 0 & .05 \\ .0625 & .0625 & .0625 & .125 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The first three rows of  $A^*$  define the direct energy requirement matrix.

The Leontief inverse writes:

$$[I-A^*]^{-1} = \begin{bmatrix} 1.109 & 1.391 & 1.183 & .217 \\ .033 & 1.217 & .035 & .065 \\ .076 & .174 & 1.148 & .152 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The first three rows of  $[I-A^*]^{-1}$  define the total (direct and indirect) energy requirement matrix:

$$\alpha = \begin{bmatrix} 1.109 & 1.391 & 1.183 & .217 \\ .033 & 1.217 & .035 & .065 \\ .076 & .174 & 1.148 & .152 \end{bmatrix}$$

Energy is conserved in the Miller and Blair sense: the energy content of the primary good is equal to the total energy contained in the other two energy goods (for example:  $1.391 = 1.217 + .174$ ). The first column is a particular case: one unit of the primary good is needed anyway, thus:  $.109 = .033 + .076$ ).

2.2. In our sense, the definition of energy conservation condition offered by Miller and Blair has no *a priori* raison d'être. It needs a particular structure of the economy, the one in which an energetic good is entirely used for intermediate consumption. In other words, it is not used for final demand (for example, it is not exported).

Such a particular structure of the economy is precisely the one used by Miller and Blair in their numerical example: crude oil is not exported. It corresponds indeed to the structure of the American economy, but it does not constitute the general case.

The numerical data we have used in 1. correspond effectively to the general case, in which crude oil can be exported. And, as we have seen, energy is conserved, in the sense used in physics, a property we deduce from the general structure of the input-output model. According to us, it is not necessary to begin the presentation of Energy Input-Output Analysis by using a specific definition of energy conservation, as Miller and Blair do in chapter 6 of their book.

#### References:

MILLER Ronald E. - BLAIR Peter D. [1985], *Input-Output Analysis: Foundations and Extensions*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey