The ECA-method
for identifying sensible reactions within the IO-context

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Abstract:

The paper introduces a new method to determine "sensitivity" within the input-output-context and makes the most relevant connections of sensitivity visible in an over all graph. The method called ECA (Elasticity Coefficient Analysis) dwells on a 1980 publication of S. Maass and tries -- like the Important Coefficient Analysis (ICA) of Aroche-Reyes -- to derive structures from a certain property of the Leontief inverse. As the core of the method is elasticity -- i.e., questioning the relative reaction of the element b_{ij} of the inverse as a response to a 1% change of the input coefficient a_{ij} -- it describes dynamic potential of certain sectors on possible changes of technical coefficients. In the second part of the paper comparisons are made between the results of MFA, ICA and ECA upon using the same table for analysis. ICA and ECA to some extent show similarities but also some differences. Both together contrast with MFA with respect to the very basis of the approach which enforces different interpretations of the results.

Finally some hypotheses on the differences and similarities are set up which give a perspective for the further use of the different methods.

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1. Introduction

When in 1996 Aroche-Reyes published his paper on „Important Coefficients and structural change“ (Aroche-Reyes 1996) he introduced a new type of formal cutting rule in order to differentiate important/unimportant links resp. sectors and thus to determine intertemporal structural changes. According to his arguments he looked for a simpler solution than MFA or QIOA had to solve the problem of deliberateness in choosing an adequate filter threshold. He thought that the relatively deterministic definition, given by a formula for r_{ij} in a notation form that we will use in the following pages, will do it,

$$r_{ij} = 100/(a_{ij}\ [\ b_{ij} + (b_{ii}/X_i)*X_j)] \qquad \% \qquad (1)$$

where r_{ij} is the “tolerable limit of change” of any input coefficient a_{ij} resulting in a change of no more then 1% in the gross output of sectors *i* or *j*, X_i (or X_j) Thus changes in a_{ij} by more than r_{ij} (%) will cause changes in the related gross outputs by more than 1 %. “Conventionally, ICs are those whose r_{ij} value is not greater than 20% ($r_{ij} \leq 0,2$)” (Aroche Reyes 1996, p. 236).

By defining the reciprocal relationship $IC_{ij} = 1/r_{ij}$ we could get “importance” directly. The corresponding “conventional” filter threshold to differentiate important/unimportant links would then be $IC_f = 5.0$. If $IC_{ij} > 5.0$ then the link *ij* is considered “important” else, “unimportant”. If we multiply the a_{ij} in eq. (1) with the terms within the brackets we get:

$$IC_{ij}(1\%) = a_{ij}*b_{ij}*(1)/100 + \ 100\ b_{ii}X_j/X_i \qquad (2)$$

if we note IC_{ij} in %. The parenthesis (1) is given in order to remind of the 1%-change, which else would disappear. Using $o_{ij} = X_{ij}/X_i$ as output coefficient we can simplify to

$$IC_{ij}(1\%) = a_{ij}*b_{ij}*(1)/100 \ + 100\ o_{ij}*b_{ii} \qquad (3)$$

At the cost of a tiny imprecision which results in small differences at usually the third decimal, we can simplify eq. (1) because the first right hand term vanishes (Schintke 1976):

$$IC_{ij}(1\%) = \ 100\ o_{ij}*b_{ii} \qquad (3a)$$

A close inspection shows that Aroche-Reyes with his escape into IC-Analysis and threshold-fixing did not really solve the problem of deliberateness, but shifted it to the question, *why* 5.0 (~ $r_{ij} = 0.2$) should just be the appropriate filter value, besides the feature of being “conventional”, which explains nothing. The convention may well have practical use but lacks any theoretical reason so far.

Moreover, a closer comparison with MFA shows that both types of analysis do not unveil the same structure, nor even the same *type* of structure, a fact which Aroche-Reyes might have overlooked in the first moment. While MFA – according to its name – measures economic *flows* between sectors, ICA actually measures a kind of *elasticity*, i.e., a degree of sensitive reaction of changes of X_i with respect to a given 1% change of the input coefficient a_{ij}. This kind of elasticity is a totally different thing compared to an economic *flow* which represents the economic value (price × quantity) of intersectoral deliveries.

The possibility to represent these sensitivity coefficients in a graphical pictures like it is done in MFA, opens up very interesting structural relationships between the graphs. These graphs are not in competition with the MFA-graphs but rather complementary to them as they give us a chance to evaluate certain *flows* of the MFA-graph as more or less “*sensitive*” with respect to changes in the underlying a_{ij}. Thus we get *additional* information on certain qualities of intersectoral flows which are sorted by MFA only with respect to the hierarchy of their *size*. Adding and integrating ICA-information on these flows could thus give us additional possibilities of evaluating IO-structures. These steps of integration could be a task for future research, which however is not started here already.

In this broader context a new type of analysis is introduced here which I call ECA for *E*lasticity *C*oefficient *A*nalysis. It is rather similar to Aroche's ICA but – as we will see – also transgresses the scope of the ICA and seems even possible to assimilate it.

The next paragraphs are dedicated to the definition of the Elasticity Coefficient (EC) as developed by S. Maaß in 1980 (Maaß 1980, for a short summary see also Holub & Schnabl 1994 pp.399-417, both references in German only!). This is followed by the a comparative analysis of *two* – to be introduced – types of EC and the structure depicted by them in parallel to MFA and ICA. Finally the paper tries to summarise the findings and thus come to some conclusions upon the scope of ECA, ICA and MFA as well as potential benefits of an integration of these analyses.

2. Definition of ECs

In his profound work (Maaß 1980) Maaß startet with the simple question "How much will an elemente b_{ij} of the Leontief-inverse **B** change, if the corresponding element *ij* of the input coefficient matrix **A**, a_{ij}, will change by 1%. This applies the typical elasticity definition

$$\varepsilon_{y,x} = dy/dy*x/y \tag{4}$$

very well known from standard microeconomic textbooks where *x* is the exogenous variable and *y* the dependent or endogenous variable, reacting to a small change d*x* in *x*.

Maaß starts with the elasticity of a *single* Element b_{ij} of the Leontief inverse **B**, with respect to a single change da_{kl} in a certain input coefficient a_{kl}, in terms of eq. (4):

$$\varepsilon_{\mathbf{b}ij/\mathbf{a}kl} = db_{ij}/da_{kl}*a_{kl}/b_{ij} \tag{5}$$

If we define for practical reasons:

$$da_{kl} = a_{kl}\,(1+p)$$

with *p* as the relative change in a given Element a_{kl}, we come – after several steps in the chain of derivation steps to the expression (cf Holub & Schnabl 1994, pp. 402ff):

$$\varepsilon_{\mathbf{b}ij/\mathbf{a}kl}\,(p) = b_{ik}a_{kl}b_{lj}/\,[b_{ij*}(1-p*a_{kl}*b_{lk}] \tag{6}$$

The derivations assume *infinitesimal* small changes, but actually changes of 1% (p = 0.01) up to 20% changes (p = 0.2) are estimated by Maaß. This leads to certain errors in the practical results but this is accepted in order to not facing the necessity of still more complicated derivations. Moreover, $\varepsilon_{\mathbf{b}ij/\mathbf{a}kl}$ is depending on *p*, which however is a monotone dependency and – as practical calculations show – causes differences only at the 4^{th} decimal for a 12 x 12 table. For each entry *k,l* in the **A**-matrix we get n^2 elasticities *ij* of the Inverse. Thus we get n^2*n^2 elasticities, which is just too much in order to handle. One could take the arithmetic mean of all n^2 or some other function in order to catch the information. Maaß decided instead, to take the *maximum* of all $\varepsilon_{\mathbf{b}ij/\mathbf{a}kl}$ for a given position *k,l* and to call this maximum a *measure of importance*. Thus we have:

$$EC_{k,l}\,(p) = \max_{ij}\,(\varepsilon_{\mathbf{b}ij/\mathbf{a}kl}) \qquad k,l = 1,...,n \tag{7}$$

Maaß' calculation showed that the maximum element is attained, if $k = i$ and $l = j$ because then the main diagonal element of the inverse is involved twice and since the main diagonal element is usually the biggest one in a row or column this turns into maximum of the $\varepsilon_{\mathbf{b}ij/\mathbf{a}kl}$. .Thus we get

$$EC_{k,l}\,(p) = b_{kk}a_{kl}b_{ll}/[b_{kl}*(1\text{-}p*a_{kl}*b_{lk})] \tag{8}$$

As an alternative approach Maaß designs a similar elasticity which however, answers the question, "how big is the relative change of gross output X_i of sector *i*, if a certain input coefficient a_{kl} is changed by 1 %" which is given by eq. (9), where Y_j defines the final demand of sector *j* :

$$\varepsilon_{\mathbf{X}i/\mathbf{a}kl}(p) = dX_i/da_{kl}*a_{kl}/X_i = b_{ik}a_{kl}\Sigma_j\, b_{lj}*Y_j/\,[(1\text{-}p*a_{kl}*b_{lk})X_i] \tag{9}$$

This elasticity is rather similar to the "important coefficient" question of Schintke (Schintke 1988), which Aroche-Reyes took for his cutting procedure, but eq. (9) seems not to be very similar to eq. (2). Again Maaß defines the *maximum* value of all $\varepsilon_{\mathbf{X}i/\mathbf{a}kl}(p)$ as a *measure of importance* for a certain input coefficient a_{kl} for the gross output X_i and calls ist $EC^*_{k,l}$ (p). To make the point in the difference here: while $EC_{k,l}$ (p) is an importance measure for the input coefficient a_{kl} with respect to the Leontief inverse, *not implying* any influence of final demand Y, $EC^*_{k,l}$ (p).refers directly to a change in X_i, implying the *final demand* structure but no *changes* in final demand.

As Maaß could show, again $\varepsilon_{\mathbf{X}i/\mathbf{a}kl}(p)$ is maximum if $k = i$ so that the final formula for calculating $EC^*_{k,l}$ is given by eq. (10)

$$EC^*_{k,l}\,(p).= \; b_{kk}a_{kl}\Sigma_j\, b_{lj}*Y_j/\,[(1\text{-}p*a_{kl}*b_{lk})X_k] \tag{10}$$

Maaß could also show that

$$EC^*_{k,l}\,(p) \;\leq\; EC_{k,l}\,(p) \tag{11}$$

This is due to the fact that the elasticities $\varepsilon_{\mathbf{X}i/\mathbf{a}kl}(p)$ are a kind of arithmetic *mean* of the elasticities $\varepsilon_{\mathbf{X}i/\mathbf{a}kl}(p)$. It can be pointed out that the basic logic of $EC^*_{k,l}$ (p) is well in line with the ICA as used by Aroche-Reyes and also with the so called "current" structure approach of the MFA which all dwell on the incorporation of (factual) final demand, while $EC_{k,l}$ (p) restricts itself to information given by the input coefficients alone and thus is similar to the so called "*standard* structure" the MFA develops by *ignoring* the influence of final demand. These similarities will be used in comparing the different results of our analysis.

3. Empirical Results

We are now going to calculate the measures of importance for the $EC^*_{k,l}$ (p) approach as well as $EC_{k,l}(p)$. This delivers two matrices of (maximum) elasticity coefficients, for each *k* and *l* for the German IO-table of dimension 12 for the year 1995.

Table 1: Elasticity coefficients EC (Germany 1995)

	Agr	Enr	Chm	Met	Mch	ELM	WDP	Fod	Cns	Trd	Srv	Gov
Agr	0,076	0,077	0,275	0,037	0,017	0,064	0,880	1,006	0,041	0,107	0,550	0,324
Enr	0,605	0,243	0,937	0,931	0,340	0,347	0,673	0,523	0,084	0,690	0,669	0,565
Chm	0,847	0,587	0,273	0,718	0,734	0,796	0,838	0,540	0,912	0,734	0,557	0,590
Met	0,674	0,537	0,664	0,481	0,928	0,942	0,518	0,078	0,706	0,336	0,176	0,070
Mch	0,716	0,872	0,686	0,633	0,197	0,821	0,528	0,431	0,780	0,835	0,548	0,711
ELM	0,416	0,771	0,566	0,546	0,907	0,097	0,760	0,598	0,822	0,482	0,729	0,692
WDP	0,487	0,204	0,686	0,308	0,473	0,623	0,199	0,660	0,771	0,676	0,885	0,473
Fod	0,976	0,079	0,600	0,052	0,082	0,084	0,174	0,163	0,035	0,334	0,959	0,608
Cns	0,661	0,828	0,383	0,452	0,324	0,296	0,452	0,266	0,025	0,493	0,952	0,686
Trd	0,730	0,650	0,804	0,846	0,735	0,735	0,819	0,757	0,686	0,083	0,696	0,695
Srv	0,502	0,792	0,780	0,687	0,739	0,751	0,767	0,717	0,688	0,922	0,356	0,884
Gov	0,582	0,694	0,590	0,535	0,526	0,400	0,381	0,550	0,413	0,474	0,945	0,128

Table 2: Elasticity coefficients EC* (Germany 1995)

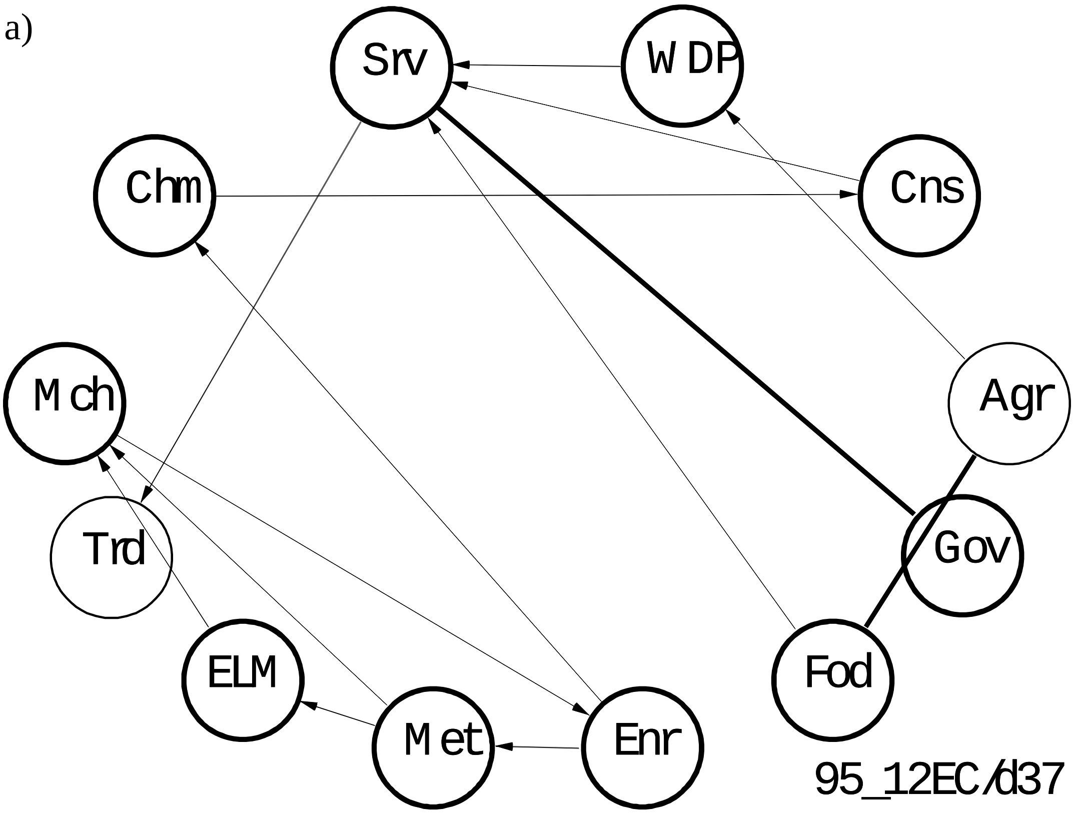
	Agr	Enr	Chm	Met	Mch	ELM	WDP	Fod	Cns	Trd	Srv	Gov
Agr	0,076	0,000	0,006	0,000	0,000	0,001	0,046	0,510	0,001	0,003	0,081	0,020
Enr	0,013	0,243	0,231	0,100	0,037	0,021	0,040	0,026	0,006	0,068	0,090	0,060
Chm	0,016	0,008	0,273	0,012	0,063	0,043	0,038	0,019	0,152	0,043	0,048	0,043
Met	0,007	0,009	0,029	0,481	0,319	0,167	0,010	0,001	0,071	0,010	0,007	0,002
Mch	0,003	0,010	0,013	0,004	0,197	0,019	0,004	0,004	0,026	0,024	0,013	0,023
ELM	0,002	0,012	0,014	0,005	0,107	0,097	0,016	0,010	0,058	0,012	0,049	0,043
WDP	0,003	0,001	0,029	0,002	0,021	0,019	0,199	0,021	0,065	0,040	0,138	0,030
Fod	0,028	0,000	0,012	0,000	0,001	0,001	0,001	0,163	0,000	0,007	0,113	0,031
Cns	0,003	0,012	0,007	0,003	0,007	0,003	0,005	0,003	0,025	0,014	0,111	0,045
Trd	0,009	0,008	0,044	0,021	0,063	0,033	0,031	0,034	0,046	0,083	0,058	0,054
Srv	0,004	0,015	0,046	0,013	0,066	0,036	0,027	0,027	0,049	0,096	0,356	0,138
Gov	0,001	0,002	0,004	0,001	0,005	0,002	0,001	0,003	0,003	0,004	0,032	0,128

If we compare element by element of tables 1 and 2 we see that equation (11) holds. Equality is only attained for the main diagonal entries.

Table 3: Important coefficients IC (Germany 1995)

	Agr	Enr	Chm	Met	Mch	ELM	WDP	Fod	Cns	Trd	Srv	Gov
Agr	7,61	0,03	0,63	0,01	0,03	0,07	4,61	51,03	0,07	0,26	8,12	1,98
Enr	1,26	24,30	23,08	9,99	3,74	2,11	4,01	2,64	0,60	6,77	9,03	6,00
Chm	1,62	0,78	27,30	1,25	6,31	4,28	3,84	1,90	15,19	4,31	4,77	4,31
Met	0,68	0,91	2,95	48,14	31,85	16,74	1,01	0,10	7,09	0,97	0,73	0,23
Mch	0,30	0,98	1,28	0,43	19,71	1,87	0,39	0,36	2,60	2,36	1,31	2,26
ELM	0,16	1,19	1,39	0,48	10,72	9,73	1,61	0,95	5,82	1,22	4,85	4,32
WDP	0,31	0,13	2,85	0,22	2,07	1,93	19,86	2,07	6,47	4,02	13,77	3,01
Fod	2,83	0,02	1,15	0,01	0,11	0,06	0,13	16,28	0,04	0,66	11,26	3,08
Cns	0,32	1,22	0,69	0,33	0,72	0,34	0,48	0,28	2,46	1,36	11,07	4,47
Trd	0,91	0,78	4,40	2,07	6,34	3,34	3,09	3,42	4,57	8,32	5,80	5,42
Srv	0,40	1,50	4,61	1,27	6,59	3,61	2,68	2,71	4,87	9,62	35,58	13,78
Gov	0,08	0,19	0,41	0,12	0,49	0,16	0,11	0,26	0,27	0,37	3,21	12,83

a)



b)

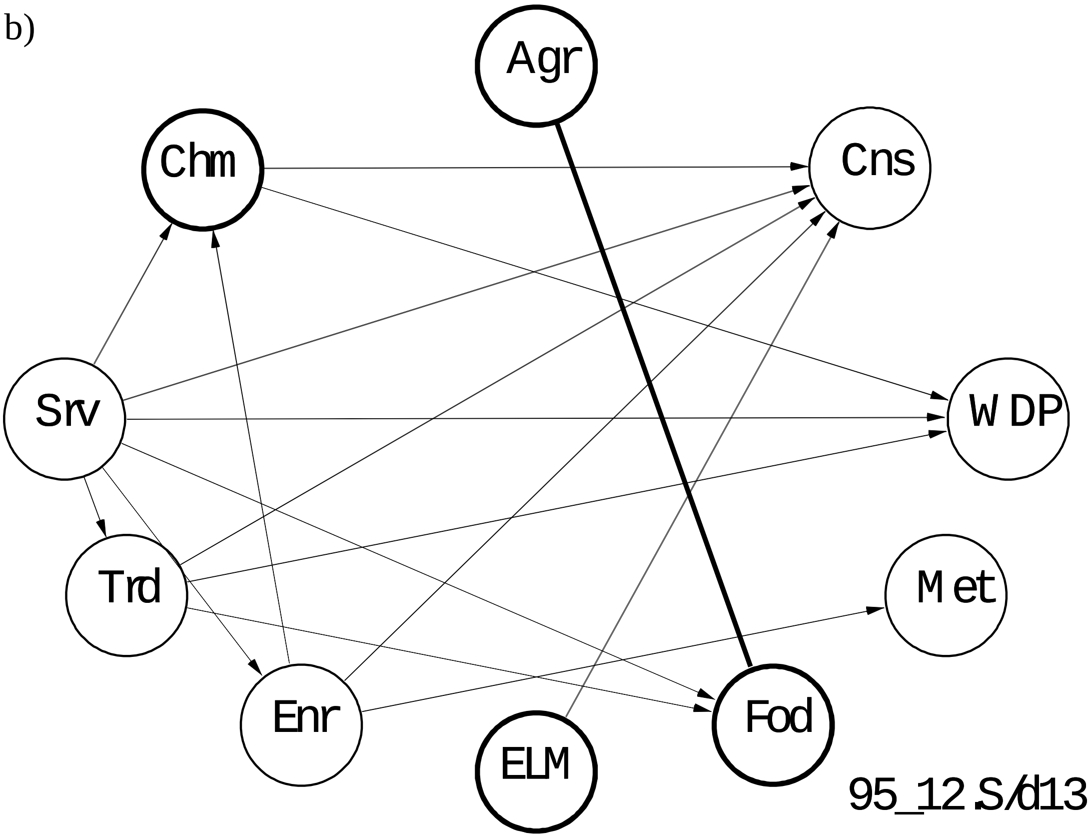


Fig.1 a,b: Comparison of the ECA-Graph (EC) with the Standard MFA-Result

3.1 Comparison of EC- and Standard-MFA-structure

If we compare table 1 and 2, we see that – as expected – that both matrices are quite different. It is however still more interesting to translate tables 1 and 2 into graphs, interpreting the entries as effects from the row-sector to the column-sector. At this point we have to "solve" the problem of choosing an appropriate filter level. Like in ICA this is in the end a deliberate choice which can be made by taking some *average* which in the end delivers a reasonable pattern which is close to the endogenised structure of the MFA with respect to the number of depicted sectors.

If we compare now the EC-graph (Fig. 1a) with the corresponding MFA-*Standard* Structure (Fig. 1b) we see that they are as well different as similar to some extent. There is an *overlapping* set of 10 (!) sectors (out of possible 12) appearing in both graphs which consists of the sectors (in alphabetical order):

Agriculture (*Agr*)
Chemicals (*Chm*)
Construction (*Cns*)
Electrical Machinery (*ELM*)
Energy (*Enr*)
Food/Beverages (*Fod*)
Metal Products (*Met*)
Trade (*Trd*)
Services (*Srv*)
Wood/Paper (*WDP*)

Only the sectors Machinery (*Mch*) and Government services (*Gov*) are not contained in the MFA *standard* structure but show up in the "relevant sectors" group within the EC-context. While the *sectors* selected by both strategies (despite their different approach) are almost identical, their *connections* differ a lot: The only *bilateral* link common in the structures is *Agr=Fod* (Agriculture=Food). Additionally the EC-structure shows *Srv=Gov* (Services=Government) as a bilateral connection which is rather plausible with respect to the growth potential of service industries. I.e., Market services' growth *stimulates* government services growth and vice versa.

However, there is some open discussion upon the use of *indirect* connections within ICA (see Gosh & Roy 1998). MFA (in both forms) dwells on the use of direct as well as indirect flows. A first argument is that ICA – by its use of the Leontief-multipliers (cf. eq. 3 or 3a) – already depicts indirect connections and therefore adding additional "QIOA"-like steps of finding connections would mean to "square the square". The counter argument would say that the table of IC (cf. table 3) mirrors sector-by-sector connections in each entry which of course could be linked into a chain. The synthesis of both arguments could be to calculate the matrices IC^2, IC^3, etc. analogously to MFA check them like layers for still "relevant" connections and than adding these remaining links analogously to the MFA steps. Of course the test for relevance should always use the *same* filter (for ICA: $IC_f = 5.0$) and since the matrix potencies vanish rather quickly, there would only add a few additional links. We leave this discussion to further research and – for the moment – stay with the *only direct* links given in table 3 resp. table 1.

3.2 Comparison of EC*- and ICA-structure

For the EC^* matrix (table 2) we choose the filter of $EC_f^* = 0.05$. This filter value EC_f^* is well in line with the ICA's "conventional" filter value : $IC_f = 5.0$, since ICA multiplies with 100 (100 * 0.05 = 5.0, (cf eq. (1)) in order to get percentage values.

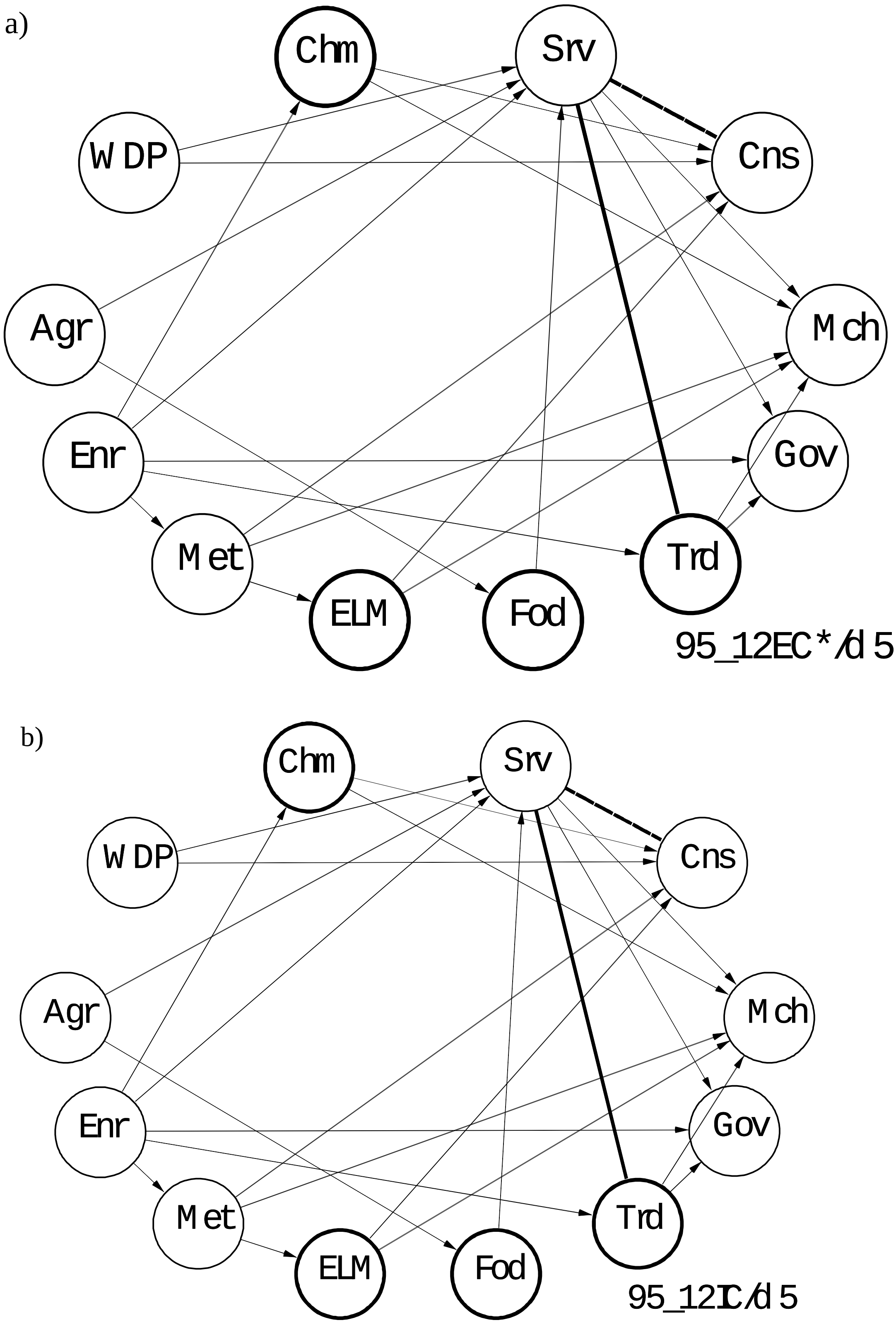


Fig.2 a,b: Comparison of the EC*- and ICA-Result

If we compare the results obtained in fig. 2b (= ICA-structure at $IC_f = 5.0$) and fig. 2a (EC^*-structure at the *corresponding* filter level EC_f^*, we see – astonishingly – that both structures are *identical*. If we get suspicious now and check the corresponding *tables*, we see that both tables are also identical besides the fact that table 3 is 100 times the corresponding entry of table 2. This fact is obviously due to the "percentage view" of ICA in eq. (1). The very outcome is that Schintke's measure of importance in the ICA delivers the *same result* as Maaß' output-elasticity EC*. We can unify the formulas of eq. (3) and eq. (10) if we replace $\sum_j b_{lj} * Y_j = X_l$ and analogously to the step from eq. (3) to (3a) $o_{kl} = a_{kl} * X_l / X_k$. After these transformation we get eq. (12)

$$EC^*_{k,l}(p) .= b_{kk} a_{kl} X_l / [(1-p*a_{kl}*b_{lk}) X_k] = b_{kk} o_{kl} / [(1-p*a_{kl}*b_{lk})] \quad (12)$$

If we realise that $p < 1$ and the denominator practically being identical with 1 we have

$$EC^*_{k,l} = b_{kk} o_{kl} \quad (13)$$

Only the multiplication with 100 establishes a difference with eq. (3a). This means that we also can simply interpret ICA as an *elasticity analysis* which relates output changes and changes of input-coefficients and thus measures *sensibility* of connections (and not flows !)

3.3 Comparison of Y-dependent structures EC* and current MFA

Both, EC* and IC depend on the factual **y**-vector of final demand as does also the "current structure MFA". Therefore it is also interesting to compare the result of this type of analysis which is given in fig. 3 against the structures given in fig. 2 a) and 2 b). It should be emphasised that the EC^* structure of fig. 2a only depicts the so-called *direct* connections. Thus by a possible addition of (a few) *indirect* connections the EC^* structure could be a bit more complex.

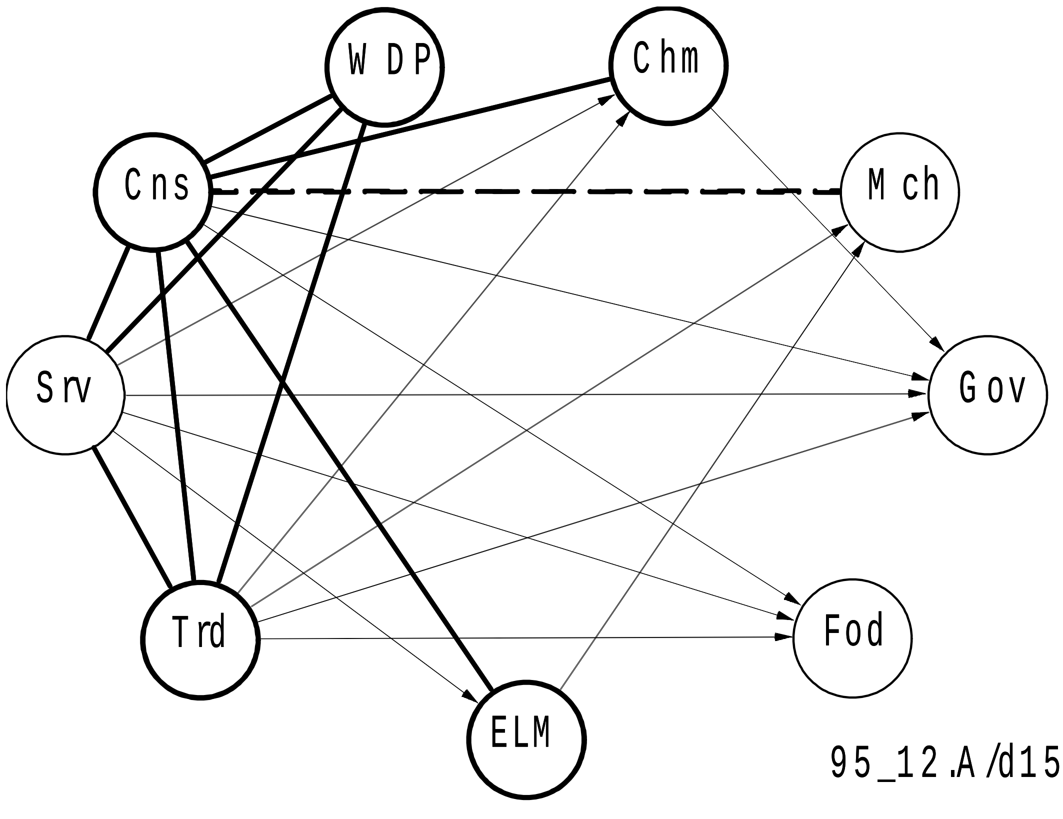


Fig. 3: The "current" structure (including final demand effects) of MFA

The overlap of *sectors* is 9 of 12 possible. These sectors represent 75% of the relevant EC*-sectors. There are only 3 sectors which are *not* represented in the current MFA structure: These are the sectors

Agr (Agriculture)
Enr (Energy)
Met (Metals)

It seems to be not "by chance" that just three "old" branches of *primary* production do not show up here, because the markets mirror a relatively diminished "appreciation for this type. This is another "proof" that both structures are inherently different, due to their differing definition of how to set them up: MFA-structures depict *economic flows*, (like water flowing in a river system) based on the hierarchy of their relative size while EC/IC structures depict *elasticities*, i.e. a pattern of sensibility of reaction (comparable to the speed acceleration of the water e.g., in a cataract). Both structures could be used to complement one another.

This however can only be the task of further research, which I want to stimulate with these argument, at least I hope for a stimulation of discussions upon the question what the similarities and differences between ICA, ECA and MFA are and – if differences exist, as should already be shown above – how we can proceed in joining them again for a higher benefit of knowledge.

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Appendix:

Table 5: Symbols and sector names

No	Symbol	Sector-names
1	*Agr*	Agriculture
2	*Enr*	Energy
3	*Chm*	Chemicals
4	*Met*	Metals
5	*Mch*	Machinery
6	*ELM*	Electrical Machinery
7	*WDP*	Wood/Paper
8	*Fod*	Food/Beverages
9	*Cns*	Construction
10	*Trd*	Trade
11	*Srv*	Services
12	*Gov*	Government