A CRITIQUE OF POST-SRAFFIAN APPROACHES TO EXHAUSTIBLE RESOURCES

by

Christian Bidard* and Guido Erreygers**

Paper to be presented at the 14th International Conference on Input-Output Techniques

Université du Québec à Montréal, Canada

10-15 October 2002

Abstract. Several post-Sraffian attempts have been made to build a theory of exhaustible resources of classical inspiration. We examine the formalisations successively elaborated by Parrinello, Schefold and Kurz & Salvadori, and show that all of them suffer from logical inconsistencies. We also respond to the critiques addressed to our own approach, based on the study of a simple model with one commodity and one exhaustible resource, called the corn-guano model.

Keywords: exhaustible resources, Sraffa, intertemporal analysis

JEL Classification Code: Q30, D57

* University of Paris X-Nanterre, Department of Economics, 200 Avenue de la République, F-92001 Nanterre.

** University of Antwerp, Faculty of Applied Economics, Prinsstraat 13, B-2000 Antwerpen.

1. INTRODUCTION

About twenty years ago Parrinello (1983) wondered if a classical theory of exhaustible resources is conceivable. The standard post-Sraffian formalisation, which may be considered as the modern version of the classical theory of prices, assumes that in equilibrium the current prices of goods remain the same over time. These prices are generally interpreted as 'long-term' prices; except when gravitation is introduced explicitly, there are no real dynamics. In the theory of exhaustible resources, however, the simplest form of Hotelling's (1931) rule states that in equilibrium the current value of an exhaustible resource rises at a rate equal to the rate of interest. The introduction of exhaustible resources in post-Sraffian models thus leads to a qualitative jump.

Parrinello's answer to his own question was optimistic, and his intuition has proved correct: today at least four different post-Sraffian treatments of exhaustible resources are available. The difficulty is that any two of them are contradictory. In a recent symposium (*Metroeconomica*, 2001) the solution proposed by ourselves has been criticised by Parrinello, Schefold, and Kurz & Salvadori. In this paper we briefly examine the three alternative formulations, and show that they are logically indefensible. We also reply to their critiques of our approach, which is based on the study of the corn-guano model, viz. a model with one reproducible commodity and one exhaustible resource. Section 2 is devoted to a brief presentation of our model and its results. Then we return to the chronological order: In section 3 we deal with the model of Parrinello, in section 4 with that of Schefold, and in section 5 with that of Kurz & Salvadori. We scrutinise this last model more thoroughly than the other two, because in spite of the fact that it has been presented as a theory with 'classical features' (Kurz & Salvadori, 1997), we believe it constitutes the most dangerous tool against the classical approach, from a conceptual point of view.

2. THE CORN-GUANO MODEL

The corn-guano model has been conceived as a methodological tool: its analytical simplicity allows us to shed light on the original economic features linked to the introduction of exhaustible resources. However, its structure is rich enough to initiate the reader to the study of the dynamics of models with exhaustible resources. There is only one produced commodity, called corn, and one exhaustible resource, called guano. Corn can be produced in a one-period time either by means of the "guano method"

$$a_1\,(corn) \oplus l_1\,(labour) \oplus 1\,(guano) \quad \rightarrow \quad 1\,(corn) \tag{1}$$

or by means of the "backstop method" (which will be necessarily used after exhaustion of the stock of guano)

$$a_2\,(corn) \oplus l_2\,(labour) \quad \rightarrow \quad 1\,(corn) \tag{2}$$

The quantities of guano not used up to date t are available at date t+1:

$$1\,(guano) \quad \rightarrow \quad 1\,(guano) \tag{3}$$

These three production processes admit constant returns. During any period, the operated methods yield the same rate of profit whereas the non-operated method(s) do not yield more. For simplicity, it is assumed that the date T when the stock of guano becomes exhausted is known. The underlying hypotheses may be that the initial stock and the demand for corn are exogenously given and that the guano method is continuously used until exhaustion.[1]

In the first version of the model (Bidard & Erreygers, 2001a), corn is chosen as numéraire. The rate of profit is exogenously given, and is such that

$$(1+r)<1/a_1, \quad (1+r)<1/a_2 \tag{4}$$

(these inequalities are required in order that processes (1) and (2) can sustain that rate of profit: the idea comes from the Ricardian corn model). As long as guano is not exhausted, the preservation process (3) is operated, therefore the price of guano, or royalty rises at a rate equal to the rate of profit (Hotelling's rule). Since the speed of evolution of the royalty is determined, it suffices to know its level at some date. A simple economic argument is:

> At the moment of exhaustion (…) we expect the backstop method to be used alongside the guano-method. Only by fluke would the then remaining supply of guano be sufficient to satisfy the whole demand for corn by means of the guano process: normally the remaining quantity will be too low, and the backstop process must be operated to fill the gap. (Bidard & Erreygers, 2001a: 249)

The co-existence of the two processes in the period of exhaustion requires that they are equally costly at that time. This condition determines the royalty at the date T of exhaustion, when it is equal to the differential rent between the two processes to produce corn. Thanks to the Hotelling rule, the royalties in the preceding periods can be calculated by backward induction. The last unknown, viz. the real wage, is then obtained. As a consequence of the increasing royalties, the real wage decreases as time passes, a phenomenon at variance with the behaviour of Ricardian models without exhaustible resources. One may alternatively

[1] *Ex post*, one must check that the last assumption is consistent with the analysis of prices, i.e. one must check that, up to date T, the guano method is cheaper that than the backstop method.

assume that the real wage is given and show that the current rate of profit declines up to the exhaustion period.

In a second version of the model (Bidard & Erreygers, 2001b), the numéraire is a given combination of d units of corn and f units of labour, with both d and f positive (by contrast, in our first version we assumed $d = 1$ and $f = 0$). For period t, which begins at date t and ends at date t+1, the price system is such that:

$$p(t+1) \leq (1+r)\left[a_1 p(t) + z(t) + l_1 w(t)\right] \qquad \left[y_1(t)\right] \tag{5}$$

$$p(t+1) \leq (1+r)\left[a_2 p(t) + l_2 w(t)\right] \qquad \left[y_2(t)\right] \tag{6}$$

$$z(t+1) = (1+r)z(t) \tag{7}$$

$$dp(t) + fw(t) = 1 \tag{8}$$

where $p(t)$ is the price of corn, $z(t)$ the royalty of guano, $w(t)$ the wage, and $y_1(t)$ and $y_2(t)$ the activity levels of the two corn production processes, all at date t. During the exhaustion period T, these two processes are operated simultaneously. After the exhaustion of guano, only the second process can be operated, and expressions (5) and (7) become irrelevant.

Since the relative price of corn and labour changes with time, the profitability of a given process now depends on the composition of the numéraire, in the same way as the profitability of an international firm depends on whether it is calculated in dollars or euros. The dynamics depend on the numéraire and become complex. The study of system (5)-(8) shows that the price equations admit one degree of freedom, say the level of $p(T)$. Once $p(T)$ is known, the system can be solved by means of backward induction. This procedure, however, leads to a negative price $p(T-\tau)$ for τ great enough, except for one specific choice of $p(T)$. That choice defines what we have called the 'natural path'. In fact, there exists an infinite number of paths with positive prices for T periods, but for large T all these paths are close to the natural path. Comparable complications occur in multisector models (several produced commodities) even when the numéraire is made of a unique commodity, but their resolution is similar when the formulae obtained for the corn-guano model are conveniently re-interpreted.

3. PARRINELLO

Exhaustible resources would not present a major obstacle to post-Sraffian theory if their exhaustion were of no concern to economic agents. This would be the case if they foresaw that the supply of these resources would be forever sufficient to cover demand (e.g. because the resource becomes obsolete after a certain date). When exhaustion is not an issue, there is no reason why the price of exhaustible resources should change over time. Even stronger: their price would be zero because they are 'free goods'.

So the interesting case arises when economic agents do worry about exhaustion. Two points of view may be adopted here. Either one assumes that agents acknowledge that exhaustion will be on the agenda some time in the future, but do not have a clue about the date at which exhaustion will occur. Or, alternatively, one assumes that agents know the date of exhaustion exactly. Let us designate these hypotheses as those of 'imperfect' and 'perfect foresight', respectively. There is no doubt that the hypothesis of imperfect foresight is more realistic. From a theoretical point of view, however, it has the disadvantage of making the determination of prices subject to uncertainty and fragile hypotheses on expectations. The hypothesis of perfect foresight, on the other hand, is certainly heroic, but it allows us to calculate prices with certainty.

Following Hotelling (1931) and a large part of the literature on exhaustible resources, we have assumed perfect foresight in our corn-guano model. Parrinello (2001), however, explicitly rejects this hypothesis and assumes that the date of exhaustion is unknown. In order to close the model he must come up with an alternative assumption. The trouble is that Parrinello's assumption – the *rank condition* – is of a purely mathematical character, and may be in conflict with other assumptions of the model.

Parrinello's oil-corn model is very similar to the corn-guano model. As in our model, Parrinello assumes that the rate of profit is given, that corn serves as the numeraire, and that before the exhaustion of guano, two processes are available for the production of corn. (In Parrinello's model the processes can change over time, but this is a non-essential variant.) It is easy to show that, in order to arrive at a determinate solution, in exactly one period two processes must be operated simultaneously while in all others only one process (the cheapest of the two available) is used. If two processes were operated in more than one period, the system of prices would be over-determined[2]; and if in no period two processes were used, it would be under-determined. In our corn-guano model a simple economic criterion is invoked to state that the two-process period must be the period of exhaustion (cf. the quotation in the previous section). Parrinello does not address this economic argument and does not assume that the two-process period is necessarily the period of exhaustion. In his oil-corn model, the period of exhaustion is unknown and the period of coincidence may be any period before exhaustion. Towards the end of the article, Parrinello seems to opt for the solution that the two-process period must be the initial period, on the grounds that "*The future cannot affect the past.*" (Parrinello, 2001: 311). Right, but what matters here is that (expectations about) the future can affect the *present*. The difficulty with Parrinello's procedure is that, except by fluke, the price of the exhaustible resource will be either too high or too low. This 'razor edge' problem was already clearly present in Hotelling's original formulation:

[2] Parrinello (2001: 305) admits that in this case over-determination can be avoided only by fluke.

- If the initial price of the resource is too high, the resource will be priced out of the market before it is exhausted; but since no resource owner wants to end up with an unsold stock of his resource, there will be a downward pressure on the resource price.
- On the other hand, if the initial price is too low, resource owners will realise that they can increase their prices without running the risk of remaining stuck with an unsold stock, and so there will be an upward pressure on the resource price.

Hence both a too high and a too low price are incompatible with the notion of long-term equilibrium, which implies the notion that no agent wants to change her behaviour. Given that the price of an exhaustible resource increases at the rate of interest, its right initial level is the one which ensures that its level at the time of exhaustion equalises the cost of the resource-using process and that of the backstop process. Nothing guarantees that in Parrinello's model the price of the exhaustible resource is at this level.

Independently of these economic considerations, a strong argument against Parrinello's proposal is that it is self-contradictory and time inconsistent. Parrinello rightly stresses the fact that the price of the exhaustible resource cannot be permanently equal to the differential rent between the resource-using method and the backstop method, and his problem is to identify the period when the coincidence occurs. Since the period referred to as "today" moves when time passes, Parrinello's rule (coincidence in the present period) would be wrong tomorrow if it were true today. This contradiction does not occur when the period of coincidence is defined independently of the origin of time as that of exhaustion.

4. SCHEFOLD

The corn-guano model is a theoretical tool that sheds light on the treatment of exhaustible resources in a classical approach. It proceeds by building a bridge between the corn model, which belongs to the Ricardian tradition, and Hotelling's seminal model on exhaustible resources. Like its basic bricks, it is an economic abstraction and its ambition is methodological. Its main feature is to proceed by mixing the simplest characteristics of two models: three equations are sufficient and their treatment is transparent. Since there are substantial differences between the solution of the corn-guano model and that of the standard corn model, these differences can be attributed unambiguously to the presence of an exhaustible resource. For instance, in the corn-guano model the relationship between the wage and the rate of profit is not time invariant, despite the fact that the same production process remains in use as long as the stock of guano is not exhausted. This result is at odds with the 'objective' point of view defended by the classical economists and Sraffa, according to which the knowledge of the operated methods and the real wage suffices to infer the level of the rate of profit.

Once it is acknowledged that the introduction of exhaustible resources leads to qualitatively different results, a second step consists of examining the degree of generality and the robustness of the laws derived from the simple model (for instance: is the exhaustible resource always used continuously until exhaustion?), and of questioning key concepts (how is the notion of rate of profit defined in the presence of changing prices?). This justifies the analysis of more complicated multisector models. In our minds, models of exhaustible resources are simple cases of models characterised by time-varying prices, with the cause of changing relative prices lying in production (as opposed to psychological motives, such as the consumer's impatience). Therefore, the study of the corn-guano model is the first step in the elaboration of a research program of classical inspiration. It is not at all meant as an attempt to describe a "Peruvian" economy. When Schefold criticises our model for its unrealistic features, he is obviously right; yet, since we did not aim at empirical accuracy but at theoretical consistency, at least that part of his critique is ill-oriented.

Let us now turn to Schefold's own theoretical model. Hotelling's rule implies that the price of guano *in situ* increases at a rate equal to the rate of interest. Schefold criticises the lack of distinction between guano *in situ* and guano extracted in our model. We assumed – for reasons of simplicity – that guano *in situ* can be used without further processing or effort in the production of corn. According to Schefold this is nonsense; guano can be used as a fertiliser in the production of corn only after it has been extracted and transformed. Hence he stresses the need to make a distinction between exhaustible resources in the ground and exhaustible resources above the ground. It should be noted that for an unknown reason he shifts terminology and considers the extracted resource ('above the ground') rather than the *in situ* one ('in the ground') to be "the" exhaustible resource of his model – we will not follow this peculiarity and stick to the usual terminology. The issue at stake is whether the distinction makes a significant difference. We will soon show that it does not, but before we do so we have to deal with several disturbing aspects of Schefold's own formalisation.

Schefold starts from the adage that "the classical approach relies on the conception of normal prices and is inseparable from it" (Schefold, 2001: 320), and therefore accepts only with the greatest reluctance the possibility of changing relative prices. In his view, production by means of exhaustible resources is comparable to production by means of lands of different fertility, in the sense that the normal prices of produced commodities "will rise and fall in steps, as in Sraffa's rendering of Ricardo's theory of rent" (*ibid.*). More specifically, Schefold divides time in successive 'long periods' – 'decades' in his terminology – during which prices of produced commodities remain at their normal levels. Normal prices change spasmodically at the instant of time which separates one decade from the other. Schefold does not explain, however, why such changes occur only between two decades. In the theory of rent, a change of normal prices follows an increase in demand, which requires a change in the productive methods, for instance a switch to a less productive type of land. Nothing of the sort happens in

Schefold's model; it is a complete mystery why prices are frozen for long periods of time, and then change suddenly. In his formalisation, the precise length of a decade is an essential characteristic, whereas in the usual classical theory of prices the unit period of time (usually referred to as a 'year') has no incidence.

Schefold's construction becomes even stranger when one realises that a different rule applies to the prices of commodities (including guano extracted) than to the price of the *in situ* resource. Commodity prices remain at their normal levels during each period, but "an essential change in the price of the resource takes place within each period" (*ibid.*). This is reflected in Schefold's system of price equations:[3]

$$(1+r)\mathbf{a}_i\mathbf{p}(t)+w(t)l_i=\mathbf{b}_i\mathbf{p}(t) \qquad i=1,2,\ldots,n-1 \tag{9}$$

$$(1+r)\left[s_n(t)z(t)+\mathbf{a}_n\mathbf{p}(t)\right]+w(t)l_n=p_n(t)+s_n(t+1)z(t+1) \tag{10}$$

Equations (9) refer to the production of commodities other than extracted guano, and (10) to the extraction of guano, both for period t. From these equations it is clear that the prices $\mathbf{p}(t)$ of the produced commodities, including the price $p_n(t)$ of extracted guano, remain fixed during the period, whereas equation (10) expresses that the price of guano *in situ* changes from $z(t)$ at the beginning to $z(t+1)$ at the end of the period.

We do not understand how such an asymmetrical treatment can be justified. *All* prices should be allowed to change within a period, not only the prices of the resources in the ground. It seems to us that system (9)-(10) is more inspired by a nostalgic desire to remain within the confines of the familiar classical formalisation with its constant long-term prices

[3] For clarity, we assume that the technical coefficients for the extraction of guano are independent of time, which is a special case of Schefold's model.

than by economic reasoning. If we correct this *faux pas*, his equations (9)-(10) would look as follows:

$$(1+r)\mathbf{a}_i\mathbf{p}(t)+w(t+1)l_i=\mathbf{b}_i\mathbf{p}(t+1) \qquad i=1,2,\ldots,n-1 \tag{11}$$

$$(1+r)\left[s_n(t)z(t)+\mathbf{a}_n\mathbf{p}(t)\right]+w(t+1)l_n=p_n(t+1)+s_n(t+1)z(t+1) \tag{12}$$

But that is not all: equation (12) itself is intriguing. It is the price equation associated with the production of one unit of guano in period *t*, with $s_n(t)$ and $s_n(t+1)$ the available stocks of *in situ* guano at the beginning and at the end of the period. (We follow here Schefold's simple assumption that one unit of the resource is extracted every period, until it is exhausted.) Let q_n be the amount of *in situ* guano used to produce one unit of extracted guano, i.e. it is the amount by which the stock of guano diminishes in each period *t*: $q_n=s_n(t)-s_n(t+1)$. This means that in each period *t*, $1/q_n$ units of extracted guano are produced per unit of *in situ* guano used in the extraction industry. Before exhaustion sets in, at the beginning of any period any owner of a unit of *in situ* guano must choose between making this unit available for extraction now, or letting it rest *in situ* and extracting it in the future. Since some owners choose to extract and others to wait, both opportunities are equally profitable, hence these two equations must be satisfied at the competitive equilibrium:

$$(1+r)\left[z(t)+\frac{\mathbf{a}_n}{q_n}\mathbf{p}(t)\right]+\frac{l_n}{q_n}w(t+1)=\frac{1}{q_n}p_n(t+1) \tag{13}$$

$$(1+r)z(t)=z(t+1) \tag{14}$$

Schefold's equation (12) describes the aggregate outcome of these two activities, obtained after multiplying each of them by its appropriate 'activity level'. In other terms, it comes from a linear combination of the two underlying equations: if we multiply (13) by q_n, i.e. the amount of *in situ* guano used by the extraction industry, and (14) by $s_n(t) - q_n$, i.e. the amount of *in situ* guano left untouched, and add the equations side by side, we arrive at Schefold's result. From a formal point of view, Schefold's presentation obscures the fact that it is derived from a competitive equal profitability criterion. Instead, Schefold gives the impression that the *in situ* resource is exploited by a monopolist, albeit one who obtains only the normal rate of profit. As a monopolist, the agent compares the whole value of the inputs (present stock of guano *in situ* and other inputs) with the whole value of the joint output (remaining stock *in situ* and one unit of produced guano), hence equation (10). With such an approach the derivation of the Hotelling rule becomes problematic. This is clearly visible from the shaky foundation which Schefold gives for the rule:

> It is now easy to see how this solution may be extended further back. The owner of the mine can always expect his stock of ore to appreciate at a rate equal to the rate of interest since the price of the substitute is above the current price of the commodity produced, so that the price of the latter must go up if he withholds production, whereas an even steeper rise is incompatible with competition. (*ibid.*: 322-323)

Having clarified these issues, we can return to Schefold's critique concerning the distinction between resources in and resources above the ground. A simple modification of the corn-guano model will suffice. As in the original corn-guano model, we assume two processes are available to produce corn: the first, or guano-process, uses a_1 units of corn, l_1 units of labour and 1 unit of guano of extracted guano to produce 1 unit of corn; the second, or backstop process, uses a_2 units of corn and l_2 units of labour to produce 1 unit of corn. We introduce a third process, which uses 1 unit of guano *in situ* and l_3 units of labour to produce 1 unit of extracted guano.[4] And finally we have a fourth process which describes the

[4] It would not make a big difference if we assumed that this process also needs a positive input of corn. Observe moreover that we assume that the technology in the extraction industry is constant and that one unit of *in situ* guano is transformed into one unit of extracted guano.

conservation of guano *in situ*. Hence we have a model with two produced commodities (corn and extracted guano), one exhaustible resource (guano *in situ*), and one type of labour. The numéraire consists exclusively of corn ($p_1 = 1$) and, by assumption, the rate of profit is given.

Let us suppose that guano will be used without interruption from the beginning until it is exhausted.[5] Assume that all agents know that the supply of *in situ* guano will be exhausted in period T. This implies – and the point seems to have been missed by Schefold – that the supply of extracted guano will be exhausted in period T+1. The main findings are the following:

- From period T+2 onwards, only the backstop process is used. The wage will be equal to its long-term level determined by the backstop process.
- In period T+1, the backstop process is used alongside the guano-process, which uses the remaining amount of extracted guano. The wage will be equal to its long-term level determined by the backstop process, and the price of extracted guano will be equal to the difference in costs between the two corn-production processes.
- In period T, the guano-process is used for the production of corn, and the guano extraction process is used for the excavation of the remaining amount of *in situ* guano. Given the price of extracted guano at time T+1, there are two processes and three unknowns to be determined (viz. the price $p_2(T)$ of extracted guano, the royalty $z(T)$ of *in situ* guano, and the wage $w(T)$).
- From the beginning until period T-1, three processes are in operation: the guano-process for the production of corn, the guano extraction process for the production of guano above the ground, and the conservation process for the remaining quantities of *in situ* guano. Given the prices at time t+1, there are three equations to determine the three unknowns at

[5] It is not excluded that more complicated patterns can occur, but these are not explored here.

time t, viz. the price $p_2(t)$ of extracted guano, the royalty $z(t)$ of guano *in situ*, and the wage $w(t)$.

Taken together, these conditions mean that the price equations have one degree of freedom, with the understanding that from time T+1 onwards the price system is perfectly determined. Hence the model comes close to our second corn-guano model, viz. the model with the mixed corn-labour numéraire (Bidard & Erreygers, 2001b). This conclusion is not surprising, as our second model aims at maintaining the simple structure of the first while pointing at the complications linked to multisector models. Since Schefold's distinction amounts to introducing extracted guano as a second produced commodity, besides corn, the parallel with a two-commodity economy is expected. As in the second corn-guano model, there exists a 'natural guano-path'; on this path the price of extracted guano rises steadily according to the formula:

$$p_2(t) = p_2^* + (1+r)^{t-T+1}(p_2^{**} - p_2^*) \tag{15}$$

where p_2^* is the price of extracted guano as determined by the guano-process for corn and the extraction process for guano assuming the royalty to be zero, and p_2^{**} the price of extracted guano at date T+1 (p_2^* and p_2^{**} are constant scalars; see Bidard & Erreygers, 2001, sections 2 and 3, for details). Therefore, our second corn-guano model shows that the price of extracted guano may follow a law directly derived from the Hotelling rule, as pointed out by Schefold.[6] But it may also deviate from this pattern, which Schefold did not notice.

To sum up, we reject Schefold's critiques and his model for two reasons:

---

[6] It follows the Hotelling rule exactly if the extraction of guano requires time only, i.e. if no labour or commodity inputs are used.

- From a methodological point of view, we maintain that our original corn-guano model is the core of what might become the 'Sraffian' theory of exhaustible resources. This model can be adapted, alongside the paths explored in our second model, to take into account distinctions and refinements which we dropped on purpose from the original model. Schefold's distinction between guano *in situ* and produced guano falls within this type of complication.
- From an analytical point of view, Schefold's alternative model relies on weird assumptions on price changes and competition, and can be criticised on several points. It is worth mentioning that if the prices of produced commodities (including the produced guano) are stable for a decade while that of guano *in situ* changes, numerous opportunities for arbitrage are open. For instance, it is profitable to buy a produced commodity at the end of a decade and sell it at the beginning of the next, after the price increase. Schefold's implicit thesis is that a competitive economy cannot adapt itself smoothly to the presence of exhaustible resources and suffers a dramatic crisis at the end of every decade. The definition of a decade, which is essential for the determination of the ensuing chaotic dynamics, remains unclear.

5. KURZ AND SALVADORI

5.1. A conceptual switch

Since many years Kurz and Salvadori have developed a theory of exhaustible resources of Sraffian inspiration (Salvadori, 1987; Kurz & Salvadori, 1995, 1997, 2000, 2001). To clarify our critiques and ease the comparison with other models, let us introduce some simplifications in their formalisation: one exhaustible resource, called oil, is used for the production of the nth commodity, called electricity, up to the exhaustion date T; apart from the switch to the backstop method, there is no choice of technique; all methods are of the single-production type. Their simplified system of prices is:

$$\forall t \quad (1+r)\mathbf{a}_i\mathbf{p}(t)+w(t+1)l_i = p_i(t+1) \quad (1 \le i \le n-1) \tag{16}$$

$$\forall t \le T \quad (1+r)(\mathbf{a}_n\mathbf{p}(t)+q_n z(t))+w(t+1)l_n = p_n(t+1) \tag{17}$$

$$\forall t \le T \quad (1+r)z(t) = z(t+1) \tag{18}$$

$$\forall t \ge T \quad (1+r)\mathbf{a}'_n\mathbf{p}(t)+w(t+1)l'_n = p_n(t+1) \tag{19}$$

$$Tq_n < s(0) < (T+1)q_n \tag{20}$$

Relation (16) is Kurz & Salvadori's price equation associated with the first n-1 commodities; relation (17) is the price equation for electricity produced by oil (by convention, one unit of electricity is produced in every period, and this requires q_n units of oil as well as other reproducible inputs $\mathbf{a}_n$; relation (18) is the Hotelling rule; relation (19) is the price equation for electricity when the backstop method is operated; finally, inequalities (20) imply that the initial stock $s(0)$ of oil is exhausted in period T. The system contains no equation similar to our numéraire equation (8).

Though Kurz & Salvadori have maintained the same system of equations throughout their works, they have modified its economic interpretation significantly. Since the origin of the difficulty has no relationship with the theory of exhaustible resources, let us eliminate oil from the above system ($q_n = 0, T = \infty$). The system, now reduced to (16)-(17), is written in matricial form as:

$$(1+r)\mathbf{Ap}(t)+w(t+1)\mathbf{l} = \mathbf{p}(t+1) \tag{21}$$

If the wage is advanced and incorporated into the input matrix, then $w(t+1)=0$ and the system becomes:

$$(1+r)\mathbf{Ap}(t)=\mathbf{p}(t+1) \tag{22}$$

The question is: do equations (21) or (22) mean that the rate of profit is equal to r? This was the interpretation in chapter 12 of *Theory of Production* (Kurz & Salvadori, 1995), where the economic comments on these relations have been copied-and-pasted from those concerning the Sraffian equations studied in the previous chapters. There is, however, an important formal difference with the standard Sraffian equations: in relations (21) and (22), the price vector depends on t and is not the same for inputs and outputs, whereas it is independent of the date in the usual formalisation of classical theory. Although chapter 12 is entirely devoted to time-dependent prices, Kurz & Salvadori maintain the traditional interpretation.

The truth, however, is that the magnitude r in equations (21) and (22) cannot be interpreted as a rate of profit without further precautions or qualifications.[7] It deserves to be noticed that the standard properties of the rate of profit are lost:

- In the standard case (prices independent of t), the rate of profit is defined once the wage is known. For instance, if the wage is incorporated in the input matrix, the rate of profit R is the unique scalar defined by the conditions $(1+R)\mathbf{Ap}=\mathbf{p}, \quad \mathbf{p}>\mathbf{0}$. Equation (22), however, does not allow us to calculate the level of r.
- In the standard case, the rate of profit admits an upper bound R. There is no upper limit for the scalar r in equations (21) or (22).

[7] This might explain why Schefold was tempted to write down the same price vector on both sides of his price equations: Schefold is conscious of the difficulty of measuring profits when prices are changing, but we disagree with his answer.

5.2. Nominal rates

Kurz & Salvadori have become aware of these formal anomalies in their later works (Kurz & Salvadori, 1997, 2000, 2001). Yet they do not analyse the nature of the problem and only respond to it by means of a purely nominalist answer. Their original mistake has been to evade any reflection on the concept of profit, which could start from the simple observation that profit results from the comparison of two values, those of investments at date t and of receipts at date t+1. But before these values can be compared, they have to be made comparable. This is accomplished by means of an intertemporal standard of value. In the standard Sraffian theory, where prices are identical from period to period, the choice of the standard of value does not matter. If relative prices change over time, however, the measured rate of profit depends upon the standard, and the question is to select the relevant standard.

From 1997 onwards, Kurz & Salvadori refer to the magnitude r in equations (16)-(22) as the "nominal" rate of profit (NRP), though they never explain the reason for their move from a real to a nominal interpretation or the meaning of the new concept. What is the economic content of the NRP? Has the notion something to do with monetary illusion or inflation?[8] What are the classical grounds for the uniformity of the NRP across industries? None of these questions is evoked and the NRP remains a name without explicit meaning, even if it serves as a basis of a theory "with classical features".

Kurz & Salvadori's paper (2000) on exhaustible resources illustrates the dramatic consequences of that choice. Let us assume that the real wage is given, i.e. we refer to equation (22). A remarkable effect of a change in the NRP, as opposed to the rate of profit in Sraffian theory, is that the prices $p(t+1)$ are modified proportionally, i.e. the relative prices at

[8] 'Inflation' is referred to by Salvadori (1998: 328) in a similar context concerning a non-monetary endogenous growth model.

each date are not affected. Therefore the influence of the NRP can be analysed in two times: one can first consider a given level r_0 of the NRP and examine the properties of the associated prices; then, for an arbitrary level or a sequence of arbitrary levels, the properties derive from those established in the particular case, once combined with an homothetic deformation of prices. The choice $r_0 = 0$ being the simplest, equation (22) is reduced to

$$\mathbf{A}\mathbf{p}(t) = \mathbf{p}(t+1) \tag{23}$$

By multiplying both members of (21) by scalar $(1+r)^{-t-1}$, and after an obvious change of variables, this equation is similarly reduced to

$$\mathbf{A}\pi(t) + \omega(t) = \pi(t+1) \tag{24}$$

It is consistent with Kurz & Salvadori's approach to consider (24) as the basic equation of the "nominal labour theory of value".

Equations (23) and (24), according to whether labour is taken into account explicitly or not, are well known in the economic literature: they express that the value of the product is equal to that of inputs, that is, they are the formal expression of Walras's 'no profit, no loss' equilibrium condition, under the constant returns hypothesis: "Thus, in a state of equilibrium in production, entrepreneurs make neither profit nor loss." (Walras 1954 [1874], § 188, p. 225).

5.3. The machinery

Kurz & Salvadori's analytical machinery to enrich classical theory works as follows. Take a Walrasian or post-Walrasian result concerning production; the equilibrium prices satisfy equations (23) or (24). Then introduce a positive NRP to return to equations (21) or (22): an original 'post-Sraffian' result is obtained. The strategy is so efficient that the conclusion needs no more than a few lines of explanation, but most post-Sraffians would object against the transfer if they were aware of it. A useful precaution is to hide the idea behind harassing algebraic calculations, which consist in re-establishing for the nominal theory of labour value the results already known for the general equilibrium theory.[9]

As a first example, let us introduce the possibility of choice of technique in an intertemporal model with finite horizon *H*. A well known neoclassical result is the equivalence between efficient choices (i.e., it is not possible to increase the surplus available at each date) and competitive choices (i.e., the selected methods are profit maximising for some adequate sequence of price vectors satisfying (23) or (24)).[10] With no effort, we conclude that an efficient chronicle is also supported by a system of intertemporal prices with an arbitrary given 'nominal rate of profit'. This applies in particular to a model with exhaustible resources of the type (16)-(20) with choice of technique, provided that its horizon H remains finite ($H > T$).

This equivalence is lost for models with infinite horizon, because competitive economies may accumulate too much capital permanently. The seminal study on this topic is Malinvaud (1953). Using a 'pedestrian' method which consists in pushing the horizon H up to infinity, Malinvaud establishes that efficient chronicles are sustained by competitive prices which moreover satisfy a cost-minimisation condition (such prices are now called Malinvaud prices; the second condition is met if the more familiar transversality condition holds).

---

[9] Whether or not Kurz & Salvadori apply this strategy consciously is not the question: from an objective point of view, Kurz & Salvadori's result (2000) is a transfer from the general equilibrium theory. The translation of Malinvaud's elegant geometrical proof into algebraic terms is at best unaesthetical and useless.

[10] More precisely, it is a near-equivalence: in a competitive economy, zero-priced inputs may be spoilt without incidence on the profit, therefore the system may be inefficient. For details, see Koopmans (1957, Essay 1).

As a simple example reveals, it has never been Malinvaud's intention to show that efficient chronicles are sustained by competitive prices without qualification. Consider indeed a neoclassical production function $Q_{t+1} = F(K_t, L_t)$, with $L_t = 1$ at any date. The model is therefore reduced to $Q_{t+1} = F(K_t, 1) = f(K_t)$. Consider *any* feasible sequence of surpluses, which is generated by some sequence of invested capitals $K_0, K_1, ..., K_t, ...$ Such a sequence is sustained by the competitive interest rates $r_t = f'(K_t)$. This shows that any feasible sequence of surpluses, be it efficient or not, is sustained by competitive prices (though not by Malinvaud prices).

Kurz & Salvadori (2000) apply Malinvaud's general result to a model with exhaustible resources and infinite horizon. Why not? But they misunderstand the nature of the problem and only show that efficient chronicles of surpluses are sustained by competitive prices with an arbitrary NRP. This is to prove nothing, since the result holds for any feasible chronicle.[11]

5.4. The critique of the corn-guano model

In their last contribution, Kurz & Salvadori (2001) severely criticise the corn-guano model. They begin by an analysis of the corn-guano model on the assumption that the real wage is given. This case has been examined in our study (Bidard & Erreygers, 2001a: 251-2), and our conclusions summarise its essential properties. Kurz & Salvadori write down correctly the price equations – system (1) in their paper – but then they lose contact with the ground and start to drift. The trouble begins when they state: "The sequence of nominal rates of profit $\{r_t\}$ is assumed to be given." (Kurz & Salvadori, 2001: 284). It has escaped their attention that 'nominal' rates of profit make sense only if a numéraire is specified. Now in this model with

[11] For mathematicians, the fact that something is missing in Kurz & Salvadori's translation of Malinvaud's proof is apparent: Malinvaud's proof requires a fixed point theorem whereas Kurz & Salvadori's proof does not use its algebraic equivalent, viz. the fundamental theorem of linear programming.

exactly one produced commodity and a real wage specified in terms of that commodity, there is one simple and obvious candidate to be numéraire: the single commodity of the economy, i.e. corn. As we explained in Bidard & Erreygers (2001a: 246) we assume that the numéraire remains the same in time. Hence it appears to be natural to take one unit of corn as the standard of prices (and profits!), so that at any date the price of corn is equal to one:

$$p(0) = 1, \quad p(1) = 1, \quad p(2) = 1, \quad \ldots \tag{25}$$

As soon as (25) is considered alongside their system of price equations (1), *all* unknowns are perfectly determined. No sequence of nominal rates of profit can be given, since there is simply no room for it. Observe moreover that if α units of corn are taken as the standard of value (α > 0), i.e.

$$\alpha p(0) = 1, \quad \alpha p(1) = 1, \quad \alpha p(2) = 1, \quad \ldots \tag{26}$$

the same solution is obtained for the rates of profit, whereas the price of corn and the royalty are multiplied by 1/α.

Kurz & Salvadori, however, jump in the water before they have learned to swim. Believing that a sequence of 'nominal' rates of profit must be specified to solve the model, they discover that any sequence of numbers can be fitted in, and that there remains one degree of freedom. In their opinion: "This means that there is room for a further equation fixing the numéraire." (Kurz & Salvadori, 2001: 285). Such a statement is at least misleading, since the (unique!) numéraire is precisely a concept which is absent from Kurz & Salvadori's framework. Instead, we have a series of implicit changing numéraires, so defined that they yield the desired rate of profit. This is how it goes. Suppose that $\{r(0), r(1), r(2), \ldots\}$ is the series of profit rates obtained by taking one unit of corn as the numéraire, and let $\{r'(0), r'(1), r'(2), ...\}$ be an arbitrary series of nonnegative rates of profit. Let us adopt as numéraire at date *t* the amount $\alpha(t)$ of corn, i.e. we have:

$$\alpha(0)p(0) = 1, \quad \alpha(1)p(1) = 1, \quad \alpha(2)p(2) = 1, \quad ... \tag{27}$$

If we choose the coefficients $\alpha(t)$ such that:

$$\alpha(t+1) = \frac{1+r(t)}{1+r'(t)}\alpha(t), \qquad t = 0,1,2,... \tag{28}$$

then we obtain the series $\{r'(0), r'(1), r'(2), ...\}$ as the 'nominal rates of profit' of the Kurz-Salvadori type. Observe that the coefficients $\alpha(t)$ are defined up to a scalar only; if we multiply all of them by the constant β (β > 0), i.e. if we replace (27) by:

$$\beta\alpha(0)p(0) = 1, \quad \beta\alpha(1)p(1) = 1, \quad \beta\alpha(2)p(2) = 1, \quad ... \tag{29}$$

then the same sequence of 'rates of profit' is obtained. (This degree of freedom is the one mentioned by Kurz & Salvadori in their already quoted sentence : "There is room for a further equation fixing the numéraire.")

So Kurz & Salvadori's main message is that, for a given intertemporal trajectory, it is possible, by means of a judicious choice of numéraires, to obtain an arbitrary series of profit rates as a solution to the corn-guano model.[12] The point was mentioned in our paper, but we rejected the underlying manipulation explicitly:

> Obviously, prices can change if the theorist makes a change in the numéraire when time passes. Changes of this type are neutralized by adopting the same numéraire at all times, as we do in this paper. (Bidard & Erreygers, 2001a: 246)

It is evident that if one constantly changes the definition of what constitutes a 'metre', then the nominal size of table also constantly changes, even though the object itself remains the same. We can only guess why Kurz & Salvadori chose to spend a large part of their paper on this insignificant question: putting the cart before the oxen – specifying the rates of profit before defining the numéraire – is not the way to make progress.

A clear illustration of their belief about the influence of the numéraire is given by the following quotation:

> We maintain that, whenever the choice of the numéraire seems to affect the objective properties of the economic system under consideration, then there is something wrong with the theory of model: the objective properties of the economic system must be totally independent of the numéraire adopted by the theorist. Hence the choice of a particular numéraire may be useful or not, but it cannot be right or wrong. (Kurz & Salvadori, 2001: 285)

To begin with, it is almost incredible that this statement comes in the middle of the section in which they act as magicians and – surprise! – show that any sequence of profit rates may be produced as a solution of the corn-guano model. Though they do not reveal the secret of their trick, the audience knows that all that is involved is a sequence of changing numéraires. Ignoring the whistles from the public, they continue as if nothing happened: Since the phenomenon they point at (arbitrary rates of profit) depends on their changing constantly the numéraire, the logical consequence of their claim is that, as economists of classical inspiration, they are not interested in the 'objective properties of the economic system'. Moreover, their statement is false. By restricting ourselves to the case of a constant numéraire, we have shown (Bidard & Erreygers 2001a, 2001b) that it makes a structural difference

[12] As shown by condition (28), it is not difficult to obtain a series of only *negative* rates of profit.

whether the numéraire consists of corn only, or of corn and labour. The reason for this is that the numéraire acts as the *standard of value* of investors. They calculate the profitability of their investments by comparing the number of units of the numéraire their inputs are worth with the number of units of the numéraire their outputs will be worth. If they 'sacrifice' x units of the numéraire today and obtain y units of the numéraire in one year's time, their rate of return is equal to $(y - x)/x$.

Since the core of the debate concerns the problem of measurement more than that of exhaustible resources, we wonder why Kurz & Salvadori have not gone a step further in their analysis and hesitate to apply their findings to the pure corn model itself, with advanced wage. They could have shown that the rate of profit for a given period is unbounded and that the sequence of the rates of profit is itself undefined. Such results would open the way to a drastic revision of the whole Ricardian approach, inspired by the newest advances of post-Sraffian methodology.

5.5. An appraisal

Our analysis has clarified Kurz & Salvadori's analytical strategy. It is time to return to the concepts and the articulation between classical and neoclassical theories.

Kurz & Salvadori's basic view, as it is reconstructed from their analytical practice, uses the double interpretation of equation (23), both as a post-Sraffian equation with a zero nominal rate of profit and as a Walrasian no-profit condition. But zero profits are not no profits, since the word 'profit' does not have the same meaning in both theories! In classical theory, as well as in most economic theories and the accounting practice, profit is the income accruing to capital; in Walras's specific sense, however, profit (or, more precisely, 'pure' profit) is the income accruing to the entrepreneur *after* the remuneration of capital (and labour). Walras's statement on the absence of pure profits at equilibrium is therefore compatible with positive profits in the usual sense. The Walrasian theory is not a labour theory of value, be it nominal or not. More generally, when r is positive, the nominal rate of profit has no relationship with a rate of profit.

We do not suggest that Kurz & Salvadori do not know this basic distinction. But, as a matter of fact, the incautious and ill-considered introduction of the notion of nominal rate of profit leads them to ignore it and to assimilate two different concepts. In their hands, the nominal rate of profit becomes a Trojan horse to introduce neoclassical theory within post-Sraffian theory.

We are not opposed to a comparison between theories provided that it is based on a well-defined methodology and on a sound understanding of the concepts and results specific to each of them. Kurz & Salvadori's approach to the theory of exhaustible resources does not satisfy this criterion and proceeds from a conceptual confusion which cannot help economic analysis in general and is especially detrimental to classical theory.

6. CONCLUSION

Building a theory of exhaustible resources is a challenge to the modern approach to classical theory. In a competitive framework, the current price of an exhaustible resource rises at a rate equal to the rate of interest and, therefore, the structure of relative prices is changing. We have examined several attempts to deal with this problem. Parrinello fails to define the current royalty in a consistent way. Schefold assumes that the prices of the produced commodities, including the extracted resource, change every period, whereas the price of the *in situ* resource is stable for long periods, then is adjusted suddenly between two 'decades'. This evolution leaves open the opportunity of arbitrage and depends crucially on the missing definition of the decade. Kurz & Salvadori's reading of their equations has been significantly modified after 1995, but their recent interpretation is ultimately based on a confusion between the notion of pure profit in Walras's sense and that of profit.

Our own approach starts from a simple corn-guano model, with one commodity (corn) and one exhaustible resource (guano). In a first version, corn is chosen as numéraire; in a second version of the model, the use of a mixed numéraire (corn and labour) illustrates the difficulties linked to the fact that the measure of the rate of profit depends on the numéraire when the relative prices evolve with time. The same model also serves as a proxy for multisector models. Under the hypotheses explicitly retained (mainly, competition and perfect foresight), the critiques formulated against this approach have been shown to be unconvincing: the level of royalty is determined by the competitiveness hypothesis; the distinction between the *in situ* resource and the extracted resource is inessential, and the basic model is easily adapted to take it into account, if necessary; the fact that the rate of profit changes with time and depends on the (fixed) numéraire does not mean that the sequence of rates is arbitrary. At this stage of the debate, our approach appears to be the only solid construction to the competitive theory of exhaustible resources inspired by the classical ways of thought.

REFERENCES

Bidard, C. & G. Erreygers (2001a), "The corn-guano model", *Metroeconomica*, **52**, pp. 243-253.

Bidard, C. & G. Erreygers (2001b), "Further reflections on the corn-guano model", *Metroeconomica*, **52**, pp. 254-268.

Hotelling, H. (1931), "The economics of exhaustible resources", *Journal of Political Economy*, **39**, pp. 137-175.

Koopmans, T.C. (1957), *Three Essays on the State of Economic Science*, New York, McGraw Hill.

Kurz, H.D. & N. Salvadori (1995), *Theory of Production. A Long Period Analysis*, Cambridge, Cambridge University Press.

Kurz, H.D. & N. Salvadori (1997), "Exhaustible resources in a dynamic input-output model with 'classical' features", *Economic Systems Research*, **9**, pp. 235-251.

Kurz, H.D. & N. Salvadori (2000), "Economic dynamics in a simple model with exhaustible resources and a given real wage rate", *Structural Change and Economic Dynamics*, **11**, pp. 167-179.

Kurz, H.D. & N. Salvadori (2001), "Classical economics and the problem of exhaustible resources", *Metroeconomica*, **52**, pp. 282-296.

Malinvaud, E. (1953), "Capital accumulation and the efficient allocation of resources", *Econometrica*, **21**, pp. 233-268.

Parrinello, S. (1983), "Exhaustible natural resources and the classical method of long-period equilibrium", in: J. Kregel (Ed.), *Distribution, Effective Demand and International Economic Relations*, London, Macmillan, pp. 186-199.

Parrinello, S. (2001), "The price of exhaustible resources", *Metroeconomica*, **52**, pp. 301-315.

Salvadori, N. (1987), “Les ressources naturelles rares dans la théorie de Sraffa”, in: C. Bidard (Ed.), *La Rente. Actualité de l’Approche Classique*, Paris, Economica, pp. 161-176.

Salvadori, N. (1998), “A linear multisector model of ‘endogenous’ growth and the problem of capital”, *Metroeconomica*, **49**, pp. 319-335.

Schefold, B. (2001), “Critique of the corn-guano model”, *Metroeconomica*, **52**, pp. 316-328.

Sraffa, P. (1960), *Production of Commodities by Means of Commodities*, Cambridge, Cambridge University Press.

Walras, L. (1954 [1874]), *Elements of Pure Economics, or The Theory of Social Wealth*, Homewood, Ill., Richard D. Irwin; London, George Allen & Unwin.