SIMULATING TAX POLICIES IN THE SAM FRAMEWORK

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ABSTRACT: Exogenizing tax rates in the SAM framework enables a range of simulations for assessment and comparison of different tax policies, regarding their economy-wide costs. Especially worth considering are the alternatives: direct versus indirect taxes and personal versus corporate income taxes.

The first part of the study is dedicated to methodological problems and solutions, especially focused on allowing prices variability in a SAM-based model. The second part contains simulation results and their implications for the Polish economy.

1. Introduction

SAM framework is widely used for analyzing how income redistribution patterns affect economy's performance. A specific problem in this topic is the cost-benefit analysis of tax policies. Given different fiscal instruments, e.g. PIT, CIT, VAT, excise, import related taxes etc., it is interesting to examine effects of imposing chosen category of tax. It is intuitively obvious that different kinds of taxation trigger different mechanisms and, thus, an element of tax policy is to properly chose among those instruments to achieve certain goals, e.g. to realize additional budget revenues at the lowest cost (greatest benefit) in terms of GDP change, to decrease corporations' tax burdens saving budget revenues at the same time etc. An important feature of analyses carried out according to the approach proposed in this paper, is that state budget planning is consistent in the sense of capturing not only direct but also indirect effects of different feedbacks.

The first part of the paper is dedicated to methodological issues. In the second part an empirical study based on the latest SAM for Poland is presented.

2. Background - SAM'2000 for the Polish economy

Methodological discussion, without loss of generality, is more transparent when referring to the actual SAM upon which simulations are based. The structure of the SAM'2000 presented in *table 1* generally reflects the arrangement of national accounts data published by the Polish Central Statistical Office (see Tomaszewicz [2001], CSO publications [1996], [1997]). Domestic institutions are divided into five sectors: non-profit, households, government, financial enterprises and non-financial enterprises. This division is used not only for current, but also for accumulation accounts, which results in the necessity of including net liabilities account in the matrix. The part of SAM'2000 connected with production consists of 15 commodity accounts conforming with NACE classification at the section level[1]. Income taxes account was distinguished in the redistribution block for convenience in designing the simulations. Sums of all the accounts are consistent with national accounts for the year 2000.

[1] The last 2 of the 16 NACE sections were linked due to insignificance of the last section.

Table 1. SAM'2000 (in millions of PLN).

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
GDP creation		1 Compensation of employees	0	0	0	0	0	0	0	0	0	0	0	835	0	0	0	0	0	0	0
		2 Taxes from producers	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		3 Gross operating surplus	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		4 Taxes on products	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Redistribution & consumption		5 Property income	0	0	0	0	0	0	618	12075	27595	28446	57778	8936	0	0	0	0	0	0	0
		6 Income taxes	0	0	0	0	0	0	0	32590	0	3128	14232	0	0	0	0	0	0	0	0
	Institutions - current	7 Non-profit institutions	0	0	3598	0	968	0	0	3460	856	130	242	1061	0	0	0	0	0	0	0
		8 Households	299571	0	167541	0	59193	0	0	0	121906	14854	0	10631	0	0	0	0	0	0	0
		9 Government	0	10001	4193	82749	4139	49950	0	119601	0	2268	0	1382	0	0	0	0	0	0	0
		10 Financial enterprises	0	0	83	0	46730	0	0	8051	8540	0	1823	0	0	0	0	0	0	0	0
		11 Non-financial enterprises	0	0	117156	0	9221	0	0	0	0	2700	0	0	0	0	0	0	0	0	0
		12 Rest of the world	926	0	0	0	15197	0	504	1973	252	0	0	0	0	0	0	0	0	0	0
Accumulation	Institutions - accum.	13 Non-profit institutions	0	0	0	0	0	0	2260	0	0	0	0	0	0	0	0	0	0	0	0
		14 Households	0	0	0	0	0	0	0	55426	0	0	0	0	0	0	0	0	0	0	0
		15 Government	0	0	0	0	0	0	0	0	8821	0	0	0	0	0	0	0	0	0	21147
		16 Financial enterprises	0	0	0	0	0	0	0	0	0	13701	0	0	0	0	0	0	0	0	0
		17 Non-financial enterprises	0	0	0	0	0	0	0	0	0	0	55002	0	0	0	5043	0	0	209	55118
		18 Rest of the world	0	0	0	0	0	0	0	0	0	0	0	43352	0	0	0	0	0	0	0
		19 Net liabilities	0	0	0	0	0	0	0	0	0	0	0	0	1904	27247	0	3971	0	43143	0
Production (NACE sections)		20 (A) Agriculture, forestry etc.	0	0	0	0	0	0	0	19462	0	0	0	3096	12	828	-3	-1	897	0	0
		21 (B) Fishing	0	0	0	0	0	0	0	171	0	0	0	42	0	0	0	0	0	0	0
		22 (C) Mining and quarrying	0	0	0	0	0	0	0	3863	0	0	0	5140	13	49	0	0	54	0	0
		23 (D) Manufacturing	0	0	0	0	0	0	0	190917	2188	0	0	153789	56	6382	3001	6050	44823	0	0
		24 (E) Electricity, gas, water	0	0	0	0	0	0	0	17310	0	0	0	517	0	1	0	0	1	0	0
		25 (F) Construction	0	0	0	0	0	0	0	20862	0	0	0	6363	183	15955	18993	1405	49290	0	0
		26 (G) Whole. and retail trade	0	0	0	0	0	0	0	76756	613	0	0	742	30	1985	840	1694	12766	0	0
		27 (H) Hotels and restaurants	0	0	0	0	0	0	0	7872	0	0	0	3857	0	0	0	0	0	0	0
		28 (I) Transport, storage and com.	0	0	0	0	0	0	641	29204	585	0	0	16818	3	150	62	125	943	0	0
		29 (J) Financing and insurance	0	0	0	0	0	0	0	10674	0	0	0	3939	0	0	0	0	0	0	0
		30 (K) Dwellings, business services	0	0	0	0	0	0	0	32041	5047	0	0	5864	37	2715	2032	457	6474	0	0
		31 (L) Government services	0	0	0	0	0	0	0	0	44798	0	0	0	0	0	0	0	0	0	0
		32 (M) Education	0	0	0	0	0	0	506	4930	26758	0	0	0	0	8	0	0	9	0	0
		33 (N) Health	0	0	0	0	0	0	468	2736	24431	0	0	0	0	1	0	0	1	0	0
		34 (O+P) Other services	0	0	0	0	0	0	5318	23722	1893	0	0	1341	22	105	0	0	114	0	0
		TOTAL	**300497**	**10001**	**292571**	**82749**	**135448**	**49950**	**10315**	**673696**	**274283**	**65227**	**129077**	**267705**	**2260**	**55426**	**29968**	**13701**	**115372**	**43352**	**76265**

Table 1. Continued.

			20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	**TOTAL**
GDP creation	1	Compensation of employees	4633	81	9587	79491	9186	21606	31196	3902	21471	12023	22315	29274	25139	18078	11680	**300497**
	2	Taxes from producers	711	6	-52	3515	924	959	770	166	933	1365	127	0	7	32	538	**10001**
	3	Gross operating surplus	15649	24	3856	53598	9025	30810	81059	3533	17581	452	56665	2327	1255	2578	14159	**292571**
	4	Taxes on products	819	-4	957	62335	670	6632	4667	1095	787	22	4813	0	-25	-167	148	**82749**
Redistribution & consumption	5	Property income	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**135448**
	6	Income taxes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**49950**
	7	Non-profit institutions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**10315**
	8	Households	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**673696**
	9	Government	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**274283**
	10	Financial enterprises	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**65227**
	11	Non-financial enterprises	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**129077**
	12	Rest of the world	6892	94	21084	188183	282	3193	600	0	13960	8112	5927	0	0	0	526	**267705**
Accumulation	13	Non-profit institutions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**2260**
	14	Households	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**55426**
	15	Government	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**29968**
	16	Financial enterprises	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**13701**
	17	Non-financial enterprises	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**115372**
	18	Rest of the world	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**43352**
	19	Net liabilities	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**76265**
Production (NACE sections)	20	(A) Agriculture, forestry etc	13431	10	46	24347	51	452	5060	313	170	25	367	263	48	527	154	**69555**
	21	(B) Fishing	12	36	0	165	1	1	7	9	4	5	7	4	1	8	4	**477**
	22	(C) Mining and quarrying	1230	3	2087	13896	8622	3579	2382	159	1156	19	1822	160	71	247	555	**45107**
	23	(D) Manufacturing	13412	120	3518	155187	7125	34703	37131	1944	14538	2463	14241	2974	1213	3249	4172	**703196**
	24	(E) Electricity, gas, water	1022	4	499	9118	3630	1172	3686	504	2006	649	5517	867	456	564	1861	**49384**
	25	(F) Construction	617	2	379	4676	759	9716	1670	223	995	59	2124	2132	653	471	2650	**140177**
	26	(G) Whole. and retail trade	6346	60	1409	56454	3658	12164	22844	1293	6303	2346	8136	2290	799	1383	2772	**223683**
	27	(H) Hotels and restaurants	84	0	11	917	28	173	589	443	1515	513	222	46	119	130	294	**16813**
	28	(I) Transport, storage and com.	1263	21	645	13815	2343	3311	9989	1255	15874	2568	3599	1048	578	633	2157	**107630**
	29	(J) Financing and insurance	1250	4	69	8593	383	1083	5310	197	1953	11221	704	336	165	247	554	**46682**
	30	(K) Dwellings, business services	1603	9	788	25112	1929	9770	13981	1034	5136	4362	28492	2243	2074	1787	3137	**156124**
	31	(L) Government services	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**44798**
	32	(M) Education	13	0	7	164	11	15	50	8	50	86	47	18	33	7	57	**32777**
	33	(N) Health	91	1	23	416	32	75	630	35	667	182	92	249	14	440	80	**30664**
	34	(O+P) Other services	477	6	194	3214	725	763	2062	700	2531	210	907	567	177	450	5484	**50982**
		TOTAL	**69555**	**477**	**45107**	**703196**	**49384**	**140177**	**223683**	**16813**	**107630**	**46682**	**156124**	**44798**	**32777**	**30664**	**50982**	

Institutions - accum. Institutions - current

However, to compile the parts connected with intermediate and final use in the matrix, biproportional projection was necessary together with some additional assumptions. This projection was based on the latest available unpublished data concerning creation and use of products and services in the year 1998.

3. Methodology

The study is generally based on SAM multiplier model, described by the relation (see e.g. Pyatt, Round [1985]):

$$\mathbf{x} = \left(\mathbf{I} - \mathbf{A}_s\right)^{-1}\mathbf{y} = \mathbf{M}_s\mathbf{y} \qquad (1)$$

where **x** - vector of total revenues/expenditures in endogenous accounts, **y** – vector of exogenous inflows into endogenous accounts, $\mathbf{A}_s$ – matrix of shares of transactions in total expenditures from individual accounts, $\mathbf{M}_s$ – matrix of SAM multipliers.

In all presented analyses, only the rest of the world accounts are assumed exogenous. Such choice is justified by the intention to capture a wide range of economical linkages among domestic institutions (see e.g. Tarp, Roland-Holst, Rand [2002]). Especially important for the topic of tax policies is endogenization of government accounts, implying that feedback connections of government sector with the rest of domestic sectors are also modeled.

Approach to the use of the model given by *formula 1* presented in this work is different from typical multiplier analysis. It can be summarized in the two following rules (see Tomaszewicz, Boratyński [2002]):

- exogenous inflows are unchanged over subsequent simulations,
- matrix $\mathbf{A}_s$ is subject to changes according to assumptions of a given simulation.

It is thus analyzed how economic equilibrium is affected by a change of economy structural properties while external conditions are established. Solving *formula 1* with a new $\mathbf{A}_s$ yields new **x** and, thus, new equilibrium, as well as the new multiplier matrix.

In particular, such an approach is useful for analyzing the revenue side of state budget, which is rarely raised in typical multiplier analyses.

Income taxes

The proposed approach can be applied to examine effects of a change in income tax burdens. Consider expenditures in households current account (column 8 in SAM'2000). Regarding budget constraint, an increase in tax payment (row 6, column 8) forces a decrease of the same amount in other outflows. Actual behavior of the households sector, which in this case means the decision on which expenditures to cut, is a subject of assumptions[2]. According to these assumptions, new shares of outflows in household account can be calculated, thus yielding a new $\mathbf{A}_s$ matrix, which in the discussed example differs from the original by only one – household – column. Although shifts in household outflows do not violate their budget constraints, they do cause disequilibrium of the system. A new equilibrium is obtained by solving *formula 1* with a new $\mathbf{A}_s$. A study of results can encompass comparison of new equilibrium with the original SAM (i.e. totals of accounts, individual transaction values etc.) as well as comparison of multipliers.

This procedure can be in a sense treated as exogenizing income tax rate. For simplicity income tax rate will hereafter stand for the share of income tax payment in total outflows from current account of household or enterprise sectors, which does not in fact conform with the actual meaning of this notion. Transforming it to actual tax rates would require additional

[2] This behavior can also be subject of modeling which is characteristic of CGE methodology.

data and computations, due to the existence of progressive tax scales, complexity of dispensation system and heterogeneity of aggregate institutional sectors[3].

Compared to analyses in which exogenous inflows are changed, modifying internal structures satisfies budget constraints, e.g. an increase in government expenditures is only possible when greater revenues from taxes are provided. This property was underlined by Tomaszewicz & Boratyński [2002]. Also Rose, Hanson & Li [2001] bring up the problem of closure rules in extended input-output and CGE models. They generally simulate an increase in government transfers assuming that either there is budget spending tradeoff or taxes are raised or deficit is increased. Such an approach avoids for example what Rose et al. [2001] name pure budget expansion. Therefore, any change of any transaction, impacts of which is simulated, requires certain tradeoff, being then closer to a real-world situation.

As an implication of government's endogenous character, the structure of budget expenditures is fixed rather than their values. This corresponds with an assumption of budget's flexibility in the sense of spending every unit of additional revenues (that unit is divided proportionally, according to the existing budget spending mix).

It should be stressed that changes of macroeconomic categories in a new equilibrium, compared to the original one generally result from the same multiplier mechanisms as observed in typical multiplier analysis. The difference amounts to, firstly, treating government as endogenous, allowing thus expenditure-income feedbacks to be present in this sector and, secondly, observing rather net effects of a number of multipliers than a single multiplier effect.

Indirect taxes

Simulating changes of indirect tax rates is methodologically more sophisticated as it requires repealing of the assumption on price constancy. In general, the procedure is similar to the one for income taxes, meaning that initially an increase of any of the indirect taxes must be accompanied by adequate growth of the share of payments from product accounts into indirect taxes account (row 4 in *table 1*). However, in this case tradeoff on other expenditures in product accounts is not necessary, since producers usually compensate higher tax with price increase. Changes of relative prices of different products in the economy affect structures of intermediate and final consumption in nominal expression, which is the case in SAM tables. Writing down new $\mathbf{A}_s$ matrix then, firstly requires computation of new prices. The computation is done outside of the main model, using the input-output cost formula, based on input-output matrix table excluded from SAM. Bardazzi & Grassini [1991] and Grassini [1997] give formulations of prices taking into account the existence of product taxes, like VAT, excise etc.

Concepts presented in the cited works require adjustment in order to conform with the input-output data available in Poland. In the input-output table included in SAM'2000, both intermediate and final consumption are valuated at prices of final users, excluding trade and transport margin[4]. Expenditures on commodities, thus, include all kinds of taxes paid on products (VAT, excise, duties, import related taxes, etc.) minus subsidies. An exception to this rule is for intermediate flows, in which the deductible VAT is not counted

[3] E.g. household sector in the Polish national accounts contains information on both households and small family enterprises. A part of an expenditure recorded as a single transaction in SAM is therefore paid from household disposable income (after taxation), another part is paid by enterprise as its cost (before taxation), causing additional confusion in calculation of tax rates.

[4] Trade and transport margins for a given good are recorded as intermediate consumption of trade and transport services. It is therefore equal to producer's price plus taxes on products.

Table 2. Schematic input-output table with taxes.

$q_{11}p_1(1+s_1)$ $+m_{11}p_1^{(m)}(1+s_1^{(m)})$	$q_{12}p_1(1+s_1)$ $+m_{12}p_1^{(m)}(1+s_1^{(m)})$	$q_{13}p_1(1+s_1)(1+t_1)$ $+m_{13}p_1^{(m)}(1+s_1^{(m)})(1+t_1)$	$f_{11}p_1(1+s_1)(1+t_1)$ $+f_{11}^{(m)}p_1^{(m)}(1+s_1^{(m)})(1+t_1)$	$f_{12}p_1(1+s_1)(1+t_1)$ $+f_{12}^{(m)}p_1^{(m)}(1+s_1^{(m)})(1+t_1)$	E_1p_1	G_1
$q_{21}p_2(1+s_2)$ $+m_{21}p_2^{(m)}(1+s_2^{(m)})$	$q_{22}p_2(1+s_2)$ $+m_{22}p_2^{(m)}(1+s_2^{(m)})$	$q_{23}p_2(1+s_2)(1+t_2)$ $+m_{23}p_2^{(m)}(1+s_2^{(m)})(1+t_2)$	$f_{21}p_2(1+s_2)(1+t_2)$ $+f_{21}^{(m)}p_2^{(m)}(1+s_2^{(m)})(1+t_2)$	$f_{22}p_2(1+s_2)(1+t_2)$ $+f_{22}^{(m)}p_2^{(m)}(1+s_2^{(m)})(1+t_2)$	E_2p_2	G_2
$q_{31}p_3(1+s_3)$ $+m_{31}p_3^{(m)}(1+s_3^{(m)})$	$q_{32}p_3(1+s_3)$ $+m_{32}p_3^{(m)}(1+s_3^{(m)})$	$q_{33}p_3(1+s_3)$ $+m_{33}p_3^{(m)}(1+s_3^{(m)})$	$f_{31}p_3(1+s_3)$ $+f_{31}^{(m)}p_3^{(m)}(1+s_3^{(m)})$	$f_{32}p_3(1+s_3)$ $+f_{32}^{(m)}p_3^{(m)}(1+s_3^{(m)})$	E_3p_3	G_3
V_1	V_2	V_3				
Q_1p_1	Q_2p_2	Q_3p_3				
$(Q_1-E_1)p_1s_1$	$(Q_2-E_2)p_2s_2$	$(Q_3-E_3)p_3s_3$				
$M_1p_1^{(m)}$	$M_2p_2^{(m)}$	$M_3p_3^{(m)}$				
$M_1p_1^{(m)}s_1^{(m)}$	$M_2p_2^{(m)}s_2^{(m)}$	$M_3p_3^{(m)}s_3^{(m)}$				
T_1	T_2	$T_3=0$				
G_1	G_2	G_3				

where:

q_{ij} - intermediate consumption of domestic commodities, f_{ik} - final consumption (category *k*) of domestic commodity *i*, E_i - export of domestic commodity *i*, p_i - basic price of domestic commodity *i*, s_i - effective rate of all taxes (except VAT) minus subsidies on domestic commodity *i*, m_{ij} - intermediate consumption of imported commodities, $f_{ik}^{(m)}$ - final consumption (category *k*) of imported commodity *i*, $p_i^{(m)}$ - basic price of imported commodity *i*, $s_i^{(m)}$ - effective rate of all taxes (except VAT) minus subsidies on imported commodity *i*, t_i - effective VAT rate on commodity *i* (both domestic and imported), V_j - value added connected with commodity *j*, Q_j - global output of commodity *j*, M_j - global imports of commodity *j*, T_j - total VAT paid connected with commodity *j*, G_j - total supply (domestic and imported) of commodity *j* in final user's prices. Below or right of double lines, partial or final totals are presented.

Table 3. Composition of product taxes minus subsidies in SAM'2000 (in millions of PLN).

		20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	**TOTAL**
4	**Taxes on products.....**	**819**	**-4**	**957**	**62 335**	**670**	**6 632**	**4 667**	**1 095**	**787**	**22**	**4 813**	**0**	**-25**	**-167**	**148**	**82 749**
4a	.....Domestic (excl. VAT).....	608	1	-80	9 338	5	-1	0	0	-367	0	-30	0	0	0	-9	**9 464**
4b	.....Imported (excl. VAT).....	-57	-5	-55	23 183	-11	-49	-52	-60	-2 002	0	-943	0	-25	-167	-915	**18 841**
4c	.....VAT.....	268	0	1 092	29 814	676	6 683	4 720	1 155	3 156	22	5 786	0	0	0	1 072	**54 444**

(see Bardazzi & Grassini [1991] for discussion of various cases in which VAT is not deductible). Import is not separated from domestic output for any of those transactions.

Following Grassini [1997], in *table 2* a schematic input-output table is presented. The table, as an example which can be easily generalized, represents the case of three commodities, two categories of domestic final use and exports. In an ideal situation, information on all of the elements amounting to a final or an intermediate expenditure is provided in the form of separate tables. Otherwise, as in the case of the Polish statistics, simplifying assumptions are necessary to extract actual tax payments. *Table 2* represents these assumptions for tax rates, that is:

- all tax rates are uniform across rows,
- the only possible situation in which VAT is not deductible (apart from final consumption where VAT is not deductible by definition) concerns intermediate consumption of sectors fully exempted from VAT.

Moreover, since import tables are unavailable, as well as tables for taxes, it is assumed that proportion of imported and domestic commodities in each intermediate and final purchase is identical with the proportion of global output and import for a given commodity, i.e. (compare with *Table 2*):

$$m_{ij} = q_{ij} \frac{M_i}{(Q_i - E_i)} \qquad (2)$$

and, for final expenditures:

$$f_{ik}^{(m)} = f_{ik} \frac{M_i}{(Q_i - E_i)} \qquad (3)$$

Finally, for consistency with *table 2*, single row of indirect taxes in SAM'2000 (row 4) needs to be decomposed into three categories: indirect taxes minus subsidies on domestic goods (excluding VAT), indirect taxes minus subsidies on imported goods (excluding VAT) and VAT (see *table 3*). Regarding the above conditions and assuming both domestic and import basic prices to be initially equal to 1, it is possible to decompose all transactions in the input-output table into elements, according to the model from *table 2*.

Next, price equation can be formulated. Let "∘" stand for element by element matrix multiplication operator and "^" denote a diagonal matrix of elements of any vector. The cost equation, thus, can be written as:

$$\mathbf{p} = \left[(\mathbf{U} + \mathbf{T}') \circ \mathbf{A}'\right](\mathbf{I} + \hat{\mathbf{s}})\mathbf{p} + \left[(\mathbf{U} + \mathbf{T}') \circ \mathbf{B}'\right](\mathbf{I} + \hat{\mathbf{s}}^{(m)})\mathbf{p}^{(m)} + \mathbf{v} \qquad (4)$$

where: $\mathbf{p} = \left[p_i\right]_{n\times 1}$, $\mathbf{s} = \left[s_i\right]_{n\times 1}$, $\mathbf{p}^{(m)} = \left[p_i^{(m)}\right]_{n\times 1}$, $\mathbf{s}^{(m)} = \left[s_i^{(m)}\right]_{n\times 1}$, $\mathbf{v} = \left[V_i / Q_i\right]_{n\times 1}$, $\mathbf{A} = \left[a_{ij} = q_{ij} / Q_j\right]_{n\times n}$, $\mathbf{B} = \left[b_{ij} = m_{ij} / Q_j = a_{ij} M_i / (Q_i - E_i)\right]_{n\times n}$, $\mathbf{U} = \left[1\right]_{n\times n}$, $\mathbf{T}$ is the $n\times n$ matrix of t_i or *0* elements depending on whether they correspond with a flow containing VAT (t_i is placed then) or not (*0* is placed), n denotes dimension of the input-output matrix.

Solving *equation 2* for **p** yields:

$$\mathbf{p} = \left\{\mathbf{I} - \left[(\mathbf{U} + \mathbf{T}') \circ \mathbf{A}'\right](\mathbf{I} + \hat{\mathbf{s}})\right\}^{-1} \left\{\left[(\mathbf{U} + \mathbf{T}') \circ \mathbf{B}'\right](\mathbf{I} + \hat{\mathbf{s}}^{(m)})\mathbf{p}^{(m)} + \mathbf{v}\right\} \qquad (5)$$

Imposing changes of tax rates appearing in *formula 5* leads to obtaining a new vector of basic prices **p**. Following the scheme in *table 2*, input-output table can be then rewritten in new prices, which ends the first part of the procedure for indirect taxes.

In the next stage of the procedure, new SAM is computed. SAM by definition is a current-price model, therefore all transactions in the new equilibrium should be expressed in

new prices resulting from the price equation (*formula 5*). It is done through incorporating input-output table expressed in new prices into the wider structure of SAM. Such operation in fact means closing input-output model through accounting rules of distribution and redistribution. Regarding these rules, the input-output table in new prices, however balanced itself, is not valid in the sense of global equilibrium. This fact becomes obvious when taken into account that in the input-output table expressed in new prices, final demand in real terms is identical as originally for all institutions, while after tax changes government receives relatively more income than other sectors, which should have reflection in final demand.

A valid new equilibrium, represented by a fully balanced SAM, can be obtained by solving the system with the use of *formula 1*. Compared to the procedure for income taxes, both $\mathbf{A}_s$ matrix and vector of exogenous inflows $\mathbf{y}$ undergo modification. In $\mathbf{A}_s$, new structures of outflows in commodity accounts are obtained basing on the input-output table in new prices (*table 2*). For final expenditures in current and accumulation accounts of institutions, new proportions are established as resulting from *table 2* in such way that share of final expenditures in total outflows remains constant in all accounts. Other structures, describing distribution and redistribution rules, as well as savings, do not undergo changes. Expenditure composition and technology of production are constant in real terms.

As a rule of the proposed approach, external conditions should remain unaffected. Therefore, real exports are assumed to be constant, implying that individual purchases are multiplied by basic price indices. At the same time, the rest of the world's capital transfers (row 17 & 19, column 18 in *table 1*), being equal to foreign trade deficit, are decreased by the amount of nominal growth of exports, which completes the modification of exogenous vector $\boldsymbol{y}$.

It must be stressed that the presented approaches, both for income and indirect taxes, aim at simulating only of what can be named pure redistribution effects. For this reason, apart from the assumed adjustment of tax rates and changes that usually automatically result from them (like price changes) or are forced by budget constraint (see the case of households at PIT rate increase), all other conditions are assumed to be established. Among these conditions there can be mentioned the consumption and investment behavior of institutional sectors, structure of value added in individual production sectors, rules of primary and secondary income distribution etc. For example it is assumed that households, as well as other sectors, do not revise their consumption preferences due to price changes. In other words, the analysis resolves itself into answering the question of how economy is affected by changes in income redistribution patterns, while other patterns of behavior and structural properties of the economy remain unchanged.

The final stage of the procedure for simulating changes in indirect taxes is computing aggregate weighted deflators for chosen macrocategories. These deflators, mainly connected with final expenditures, provide comparability of the new equilibrium resulting from *formula 1* with the equilibrium described by the original SAM. The following formulas are generalized case of *table 2* (symbol "~" denotes original tax rates before changes are applied for simulation, as well as other values from the original equilibrium; basic prices for domestic and imported commodities are initially assumed 1, n is the number of commodities distinguished). Denote:

$$h_{ik} = f_{ik}(1+s_i)(1+t_i) + f_{ik}^{(m)}(1+s_i^{(m)})(1+t_i), \qquad (6)$$

$$\widetilde{h}_{ik} = \widetilde{f}_{ik}\widetilde{p}_i(1+\widetilde{s}_i)(1+\widetilde{t}_i) + \widetilde{f}_{ik}^{(m)}\widetilde{p}_i^{(m)}(1+\widetilde{s}_i^{(m)})(1+\widetilde{t}_i), \qquad (7)$$

The deflator of single purchase of final commodity h_{ij} is given as:

$$\pi_i = \frac{(\widetilde{Q}_i - \widetilde{E}_i)\, p_i (1+s_i)(1+t_i) + \widetilde{M}_i\, p_i^{(m)} (1+s_i^{(m)})(1+t_i)}{(\widetilde{Q}_i - \widetilde{E}_i)(1+\widetilde{s}_i)(1+\widetilde{t}_i) + \widetilde{M}_i (1+\widetilde{s}_i^{(m)})(1+\widetilde{t}_i)} \qquad (8)$$

Thus, weighted deflator for total final expenditures from institution account *k* is:

$$\pi_k^{(F)} = \frac{\sum_{i=1}^{n} \widetilde{h}_{ik} \pi_i}{\sum_{i=1}^{n} \widetilde{h}_{ik}} \qquad (9)$$

and for expenditures of a group of institutions:

$$\pi^{(G)} = \frac{\sum_{k \in G} \sum_{i=1}^{n} h_{ik}}{\sum_{k \in G} \sum_{i=1}^{n} \frac{h_{ik}}{\pi_i}} \qquad (10)$$

where $G \in \{1,2,...,K\}$, K being the total number of final demand categories, excluding exports. Further, the GDP deflator can be expressed as:

$$p^{(GDP)} = \frac{\sum_{k=1}^{K} \sum_{i=1}^{n} h_{ik} + \sum_{i=1}^{n} \left[E_i p_i - M_i p_i^{(m)} \right]}{\sum_{k=1}^{K} \sum_{i=1}^{n} \frac{h_{ik}}{\pi_i} + \sum_{i=1}^{n} \left[E_i - M_i \right]} \qquad (11)$$

Finally, deflator of global domestic output at basic prices is:

$$\pi^{(Q)} = \frac{\sum_{i=1}^{n} Q_i p_i}{\sum_{i=1}^{n} Q_i} \qquad (12)$$

Using Paasche-type rather than Laspeyres-type indices in the *Formulas 10-12* provides that deflating an aggregated category (i.e. total final consumption) yields exactly the same result as summing up the deflated elements of this category in the new equilibrium (i.e. deflated consumption of households, government and non-profit institutions altogether give the deflated final consumption).

4. Empirical results

The background of the empirical part of this study is the situation in the Polish public finance, where actual budget revenues in the year 2001 proved much lower than forecasted, which resulted in a drastic growth of deficit and forced government to search for additional income. For this reason all simulation variants generally assume increase of effective tax rates[5], that is:

variant A – increase of PIT at the cost of household consumption,

[5] The increase of effective tax rates need not necessarily require the increase of nominal rates but, as more probable, eliminating various tax exemptions.

variant B – increase of PIT at the cost of household savings,
variant C – increase of CIT for non-financial enterprises at the cost of their investment,
variant D – imposing uniform import tax for all commodities.

Variants A and B are examples of two extreme assumptions concerning households behavior. In a sense, such extreme assumptions may help to evaluate boundaries of the system reaction to a higher tax. Variants A, B and C are simulated using the procedure with constant prices, while in variant D the procedure with variable prices is used.

For comparability, in all variants tax rates are raised according to the assumption that government wishes to receive additional 5 billion of PLN[6] (nominally), as a result of the tax increase. Furthermore, it is assumed that government in their accounting do not consider the existence of indirect effects, meaning that if, for example, total expenditures in an institution account equal 100 billion and expenditure on tax equal 10 billion, giving tax rate at 0.1, the government would decide to establish this rate at the level of 0.15 to gain their aims.

Table 4. Changes of tax revenues – planned and actual. Variants A, B & C.

(millions of PLN)	**variant A**		**variant B**		**variant C**	
	direct (planned)	direct + indirect	direct (planned)	direct + indirect	direct (planned)	direct + indirect
Total taxes	**5000**	**5353**	**5000**	**5102**	**5000**	**5134**
.....Taxes from producers	0	2	0	-7	0	-1
.....Taxes on products	0	-18	0	-143	0	-124
.....PIT	5000	5302	5000	5230	0	220
.....CIT (non-financial)	0	34	0	-4	5000	5011
.....CIT (financial)	0	32	0	26	0	28

Table 4 contains results of simulations A, B and C. The first glance at the results reveals regularity, according to which an increase of income tax rates actually causes the economy to work out some additional revenue for the government, over the planned 5000. This fact can be technically explained with reference to general properties of multiplier mechanisms, which are in fact decisive for the results. Multipliers usually tend to be higher for sectors with relatively low leakages, in this case being mainly connected with imports (see Zienkowski & Żółkiewski [2001]). Since government is a purchaser mainly of public services which are fully domestic, not surprisingly redistribution of income towards the state budget results in positive net multiplier effects. Another regularity in all simulations is that growth of income taxes is accompanied by a decrease in revenues from product taxes.

Relatively the most profitable for the budget is variant A, in which households, charged higher tax, reduce their consumption. There exists, however, a possibility, that taxing household sector rather than enterprise sector proves less profitable to the government if households decide to compensate the higher tax by limiting savings.

The criteria for tax policy evaluation are usually composed of several factors, not only the total amount of tax revenues. *Table 5* shows absolute and percentage changes of the main macrocategories in different variants of simulation.

In variant A, GDP and output record growth, which is driven mainly by expansion of government services, education and health sectors, financed from the state budget. Also,

[6] PLN is the symbol of the Polish national currency – the Polish Zloty.

Table 5. Changes of chosen macrocategories. Variants A, B & C.

	variant A		variant B		variant C	
(millions of PLN)	absolute	%	absolute	%	absolute	%
Consumption	**1020**	**0.18%**	**5004**	**0.90%**	**5324**	**0.96%**
.....Non-profit inst	43	0.62%	29	0.42%	33	0.48%
.....Households	-1497	-0.34%	2697	0.61%	2969	0.67%
.....Government	2474	2.33%	2278	2.14%	2322	2.18%
Accumulation	**940**	**0.53%**	**-4393**	**-2.46%**	**-4272**	**-2.39%**
.....Non-profit inst	2	0.62%	1	0.42%	2	0.48%
.....Households	227	0.80%	-2385	-8.46%	190	0.67%
.....Government	233	0.94%	-365	-1.47%	213	0.85%
.....Financial enterprises	101	1.03%	80	0.82%	86	0.89%
.....Non-financial enterprises	377	0.33%	-1725	-1.50%	-4763	-4.13%
Export	0	0.00%	0	0.00%	0	0.00%
Import	-162	-0.07%	-123	-0.05%	-135	-0.05%
GDP	**2121**	**0.31%**	**734**	**0.11%**	**1187**	**0.17%**
Labor income	1627	0.54%	1171	0.39%	1346	0.45%
Gross operating surplus	511	0.17%	-287	-0.10%	-35	-0.01%
Global output	**3120**	**0.23%**	**386**	**0.03%**	**1293**	**0.09%**
.....(A) Agriculture, forestry etc	-95	-0.15%	36	0.06%	100	0.16%
.....(B) Fishing	-2	-0.48%	1	0.28%	1	0.31%
.....(C) Mining and quarrying	-1	0.00%	-5	-0.02%	10	0.04%
.....(D) Manufacturing	-435	-0.10%	-272	-0.06%	-427	-0.09%
.....(E) Electricity, gas, water	-3	-0.01%	140	0.29%	168	0.35%
.....(F) Construction	493	0.38%	-2143	-1.64%	-1523	-1.17%
.....(G) Wholesale and retail trade	585	0.27%	28	0.01%	7	0.00%
.....(H) Hotels and restaurants	-16	-0.10%	53	0.34%	60	0.38%
.....(I) Transport, storage	49	0.05%	178	0.19%	210	0.23%
.....(J) Financing and insurance	-11	-0.03%	72	0.19%	86	0.22%
.....(K) Dwellings, business services.	289	0.20%	-69	-0.05%	154	0.11%
.....(L) Government services	1042	2.33%	960	2.14%	978	2.18%
.....(M) Education	611	1.86%	607	1.85%	622	1.90%
.....(N) Health	581	1.89%	559	1.81%	573	1.86%
.....(O+P) Other services	33	0.07%	239	0.48%	275	0.55%

global output of construction and trade increase significantly. In other sectors, mostly slight declines of output are observed.

The differences in effects observed for variant B and C result from investment patterns characteristic of households and enterprises. In the sense of GDP change, it is more profitable if tax is paid at the cost of enterprise rather than household investment. An interesting fact is that the decline of household accumulation is followed by a similar decline in accumulation of enterprises. As far as activities are concerned, in both variants the cost of additional tax is borne by manufacturing and construction sectors, unlike in variant A, in which burdens are divided more evenly. Construction sector loses the most (comparatively more in the case of PIT increase). Other sectors generally benefit from the tax change in both variants, the benefits being little greater in variant C.

Recovery of household consumption in variant A and its considerable increase in variants B and C show that government, either through financing public services or through direct transfers plays significant role in determining incomes of the household sector. In variant A, households are charged initially with the 5000 tax which is totally paid at the cost of consumption. However, they make up for the lost consumption which eventually becomes only 1497 lower than before taxation. As a comparison, accumulation of non-financial enterprises initially decreased by 5000 in variant C, still remains 4763 lower than before

imposing the tax. Hence, for enterprise sector the negative consequences of income tax increase seem to be more permanent than for the household sector.

Assumptions taken in variant D refer to the idea of introducing import tax in Poland, which was discussed as a possible element of budget plan for the year 2002. Compared to the actually planned rate of 5% on all imported commodities, in the simulation the tax rate amounts approximately to 2%, conforming thus with the assumption that direct effects of such tax policy should give 5000 of excess revenues to the budget. As is presented in *table 6*, regarding indirect effects, total tax revenues increase by 5450 in real terms, which is the highest among all analyzed variants. To convert nominal taxes into real, the price index of government consumption was used (calculated according to *formula 9*).

Table 6. Changes of tax revenues – planned and actual. Variant D.

	variant D			
(millions of PLN)	direct nominal	direct + indirect nominal	price	direct + indirect real
Total taxes	**5000**	**5739**	**0,19%**	**5450**
.....Taxes from producers	0	-4	0,19%	-24
.....Taxes on products	5000	5465	0,19%	5293
.....PIT	0	237	0,19%	173
.....CIT (non-financial)	0	11	0,19%	-17
.....CIT (financial)	0	31	0,19%	25

In the case of import tax, redistribution mechanisms are supported by relative price changes. Public services, like government services, education and health care, not being a subject of imports and having little material costs, demonstrate the lowest increase of prices (see *table 7*). Thus, prices of government consumption expenditures remain relatively low, partly explaining why in variant D budget profit is greater than in variants with constant prices. However, it must be kept in mind that exports are assumed to be unchanged in real terms. If constant-price exports are reduced due to cost increase, all the results should be revised down.

It can be seen in *table 7* that unlike income taxes, import tax is practically neutral as regards private consumption. Also the reaction of global output is distinctively weaker than in variants A, B and C. In this case a substantial cost is borne by non-financial enterprises and their investment expenditures. On the one hand it is caused by the fact that price growth is relatively high for accumulation, on the other hand, enterprises do not benefit, opposite to households, from the increase of government transfers. Moreover, in variant D there is a large negative impact on operating surplus. A general rule which can be deducted from the results is that solutions, which head towards burdening private consumption rather than investment, are favorable.

A natural extension of the proposed methods, not undertaken in the framework of this paper, is analyzing taxes not separately but also in various combinations, which can further lead to searching for optimal composition of tax-policy mix.

5. Concluding remarks

SAM multiplier model used according to the proposed approach can be considered a tool for accounting at the national economy level. Its essential feature is that both direct and indirect effects are estimated in planning of government income and expenditure. The problems of practical application in general consist of at least three issues. Firstly, such model does not actually imitate the behavior of economic system but rather reveals some of its structural properties, answering the question how a tax policy affects equilibrium assuming that institutions do not change their behavior, preferences, etc. Secondly, some of the final results, as presented in *tables 4-7*, might be overestimated, for the solution of SAM multiplier model

Table 7. Changes of chosen macrocategories. Variant D.

	variant D				
(millions of PLN)	absolute nominal	% nominal	price	absolute real	% real
Consumption	**5823**	**1.05%**	**0.60%**	**2468**	**0.45%**
.....Non-profit inst	37	0.53%	0.29%	17	0.24%
.....Households	3203	0.73%	0.71%	80	0.02%
.....Government	2583	2.43%	0.19%	2371	2.23%
Accumulation	**-216**	**-0.12%**	**0.66%**	**-1380**	**-0.77%**
.....Non-profit inst	2	0.53%	0.57%	0	-0.03%
.....Households	205	0.73%	0.59%	37	0.13%
.....Government	1	0.00%	0.53%	-131	-0.53%
.....Financial enterprises	96	0.98%	0.81%	16	0.17%
.....Non-financial enterprises	-519	-0.45%	0.69%	-1302	-1.13%
Export	1020	0.51%	0.51%	0	0.00%
Import	-149	-0.06%	0.00%	-149	-0.06%
GDP	**6777**	**0.99%**	**0.81%**	**1237**	**0.18%**
Labor income	1328	0.44%	0.71%	-797	-0.27%
Gross operating surplus	-12	0.00%	0.69%	-2006	-0.69%
Global output	**7045**	**0.51%**	**0.41%**	**1420**	**0.10%**
.....(A) Agriculture, forestry etc	285	0.46%	0.48%	-12	-0.02%
.....(B) Fishing	2	0.59%	0.55%	0	0.04%
.....(C) Mining and quarrying	76	0.33%	0.35%	-4	-0.02%
.....(D) Manufacturing	2144	0.47%	0.55%	-341	-0.08%
.....(E) Electricity, gas, water	283	0.58%	0.51%	38	0.08%
.....(F) Construction	14	0.01%	0.44%	-558	-0.43%
.....(G) Wholesale and retail trade	604	0.28%	0.33%	-112	-0.05%
.....(H) Hotels and restaurants	55	0.35%	0.31%	6	0.04%
.....(I) Transport, storage	386	0.42%	0.38%	34	0.04%
.....(J) Financing and insurance	155	0.40%	0.39%	6	0.02%
.....(K) Dwellings, business services.	482	0.33%	0.25%	115	0.08%
.....(L) Government services	1077	2.40%	0.17%	999	2.23%
.....(M) Education	636	1.94%	0.11%	600	1.83%
.....(N) Health	629	2.04%	0.21%	563	1.83%
.....(O+P) Other services	214	0.43%	0.25%	86	0.17%

passes over the delays in the action of multiplier mechanisms. Thirdly, a problem of topicality of data appears. In the Polish statistics, a SAM for a given year is usually available with one-two year delay, while for budget planning, an anticipated SAM is actually necessary.

In spite of the constraints, comparative analyses of tax policy impacts on different institutions and production sectors can provide valuable information, even if they are based on historic SAMs, the more so because structural properties of an economy are not subject to large changes in short period.

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