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The Decomposition by Factors and Partial Derivatives in Direct and Indirect Requirements:
With Attention to Applications

by

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ABSTRACT

Gim and Kim (1998) proposed the general relation between the notion of direct and indirect requirements of commodity i to support a unit of final demand of commodity j and that to produce a unit of gross output of commodity j in the open static input-output model. Recently, Gim (2000) also showed that the elements of the Leontief inverse can be decomposed into four different parts (the final demand, the direct effect, the technical indirect effect and the interrelated indirect effect); that the elements of the total requirements matrix for a unit of final demand can be decomposed into three different parts (the direct effect, the technical indirect effect and the interrelated indirect effect); and that the elements of the total requirements matrix for a unit of gross output can be decomposed into two different parts (the direct effect and the technical indirect effect).

In this paper, we examine and show that the findings of the decomposition by factors drawn from Gim's 2000 paper can also be expressed as the partial derivatives with respect to the final demand and gross output. Using this method, we obtained the same results as the major findings of Gim (2000). Through a practical illustration, we interpret the significance and economic meanings of the components of the partial derivative decompositions. Some useful applications are also presented for the effectiveness and validity of the decomposition by different factors in direct and indirect requirements. Therefore, we hope and expect that these results can be applied in many different areas: especially in environmental and energy input-output models, intensity analysis of resources, multiplier and impact analysis.

Table of Contents

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1. Introduction

Gim and Kim (1998) proposed the general relation between the notion of direct and indirect input requirements of commodity i to support a unit of final demand of commodity j, γ^{f}_{ij}, and that to produce a unit of gross output of commodity j, γ^{g}_{ij}, in the open static input-output model. As it is well known, each element of the Leontief inverse, c_{ij}, represents the direct and indirect output requirements to support a unit of final demand. Therefore, we know that there exists three different types of direct and indirect requirements that have the cumulatively interrelated dependence among elements. It is a matter of course that the identical meaning cannot exist between three notions of direct and indirect requirements.

Recently, Gim (2000) also showed that the elements of the Leontief inverse C can be decomposed into four different parts (the final demand, the direct effect, the technical indirect effect and the interrelated indirect effect); that the elements of the total input requirements matrix for a unit of final demand Γ^{f} can be decomposed into three different parts (the direct effect, the technical indirect effect and the interrelated indirect effect); and that the elements of the total input requirements matrix for a unit of gross output Γ^{g} can be decomposed into two different parts (the direct effect and the technical indirect effect).

On the basis of the research findings mentioned above, the study objectives of this paper are as follows. (1) We examine and show that the results of the decomposition by factors can also be expressed as the partial derivatives with respect to the final demand and gross output. Using this method, we are assured that the same results as the major findings of Gim (2000) can be obtained. Through a practical illustration, moreover, we interpret the significance and economic meanings of the components of the partial derivative decompositions. (2) Some useful applications, such as the Hawkins-Simon conditions, energy intensity, estimating the total pollution generation, and multiplier analysis, are also presented to demonstrate the effectiveness and validity of the decomposition by different factors in direct and indirect requirements. This research is mainly focused on various applications to interindustry analysis by using the general relation, the decomposition by factors and partial derivatives. Therefore, we hope and expect that these results can be applied in many different areas.

This article has five main sections. Section 1 is an introduction to this paper. Section 2 explains briefly the general relation between γ^f_{ij} and γ^g_{ij} and introduces the decomposition by factors for γ^f_{ij}, γ^g_{ij} and c_{ij}. Section 3 expresses the findings of the decomposition as the partial derivatives associated with different variables. Section 4 presents some practical applications to show the usefulness of the decomposition by different factors in the total input (or output) requirements matrix. Section 5 contains a brief summary of this paper and concluding remarks.

2. The General Relation and the Decomposition by Factors

In the open static input-output model, Gim and Kim (1998) proposed the general relation between the notion of direct and indirect input requirements of commodity i to support a unit of final demand of commodity j, γ^f_{ij}, and that to produce a unit of gross output of commodity j, γ^g_{ij}. The authors generalized the relation between γ^f_{ij} and γ^g_{ij} by dividing the elements into two parts. The two different notions have the following relationships for the diagonal elements:

$$\gamma^f_{ii} = c_{ii} \, \gamma^g_{ii} \tag{2-1}$$

$$\text{or} \quad \gamma^g_{ii} = 1 - \frac{1}{c_{ii}}, \tag{2-2}$$

and as for the nondiagonal elements:

$$\gamma^f_{ij} = c_{ii}\ \gamma^g_{ij} \tag{2-3}$$

$$\text{or} \quad \gamma^g_{ij} = \frac{c_{ij}}{c_{ii}}\ ,\ i \neq j\ , \tag{2-4}$$

where c_{ii} is the diagonal element of the Leontief inverse. c_{ii} represents the direct and indirect output requirements of that commodity itself to support a unit of final demand of commodity i.

Since the two notions of direct and indirect input requirements mentioned above are not the same, it would be interesting to find out what constitutes the γ^g_{ij} and γ^f_{ij} and what causes the two notions to be different. Thus, Gim (2000) also showed that γ^g_{ij} consists of two fundamental components, namely the direct effect, and the technical indirect effect which comes from only the purely technical relation between inputs and output. Moreover, they showed that the cumulative indirect effect can be decomposed into two different parts, the technical indirect effect, and the interrelated indirect effect which only means the interrelated interdependence effect except for the technical indirect effect from the total cumulative indirect effect; that γ^f_{ij} can be decomposed into three different parts: the direct effect, the technical indirect effect, and the interrelated indirect effect; and that c_{ij} can be decomposed into four different parts: the final demand itself, the direct effect, the technical indirect effect, and the interrelated indirect effect.

Denoting I, A, T, and R as the identity matrix, the direct effect, the technical indirect effect, and the interrelated indirect effect; and letting their elements be δ_{ij}, a_{ij}, t_{ij}, and r_{ij}, respectively; where Γ^g is the total input requirements matrix for a unit of gross output, Γ^f is the total input requirements matrix for a unit of final demand, and C is the Leontief inverse. Then the elements of Γ^g, Γ^f and C can be written as:

$$\Gamma^g = A + T \tag{2-5}$$

$$\text{or } \gamma^g_{ij} = a_{ij} + t_{ij}\ ; \tag{2-6}$$

$$\Gamma^f = A + T + R \tag{2-7}$$

$$\text{or } \gamma^f_{ij} = a_{ij} + t_{ij} + r_{ij}\ ; \tag{2-8}$$

$$C = I + A + T + R \tag{2-9}$$

$$\text{or } c_{ij} = \delta_{ij} + a_{ij} + t_{ij} + r_{ij}\,. \tag{2-10}$$

3. The Decomposition by Partial Derivatives

The major results obtained in section 2 can also be expressed as the concept of partial derivatives. The matrix Γ^g is written as

$$\Gamma^g = (\gamma^g_{ij})_{(n \times n)}. \tag{3-1}$$

Let us consider the following relation:

$$v = \Gamma^g u, \tag{3-2}$$

where u is the gross output vector given by the exogenous change and v the total input requirements vector for gross output. Substituting (2-5) into (3-2), we get

$$\begin{aligned} v &= (A + T)u \\ &= Au + Tu \\ &= v^A + v^T, \end{aligned} \tag{3-3}$$

by letting

$$v^A = Au, \tag{3-4}$$

$$v^T = Tu. \tag{3-5}$$

From (3-2), we can obtain

$$\gamma^{g}_{ij} = \frac{\partial v_i}{\partial u_j}, \tag{3-6}$$

and similarly, from (3-4) and (3-5),

$$a_{ij} = \frac{\partial v^A_i}{\partial u_j}, \tag{3-7}$$

$$t_{ij} = \frac{\partial v^T_i}{\partial u_j}, \tag{3-8}$$

where the following notations are used:

γ^{g}_{ij}: the amount of change in the i*th* commodity of the total input requirements when a unit is increased in the j*th* commodity of gross output;

a_{ij}: the amount of change in the i*th* commodity of the direct input requirement in conjunction with the direct effect when a unit is increased in the j*th* commodity of gross output;

t_{ij}: the amount of change in the i*th* commodity of the indirect input requirement in conjunction with the technical indirect effect when a unit is increased in the j*th* commodity of gross output.

Also, the matrix Γ^f is given by

$$\Gamma^f = (\gamma^f_{ij})_{(n\times n)}. \tag{3-9}$$

Consider the following relation again:

$$w = \Gamma^f d, \tag{3-10}$$

where w is the total input requirements vector for final demand and d the final demand vector. Substituting (2-7) into (3-10), we have

$$\begin{aligned} w &= (A+T+R)d \\ &= Ad+Td+Rd \\ &= w^A+w^T+w^R, \end{aligned} \tag{3-11}$$

by letting

$$w^A = Ad, \tag{3-12}$$
$$w^T = Td, \tag{3-13}$$
$$w^R = Rd. \tag{3-14}$$

From (3-10), we can obtain

$$\gamma^f_{ij} = \frac{\partial w_i}{\partial d_j}, \tag{3-15}$$

and similarly, from (3-12), (3-13) and (3-14),

$$a_{ij} = \frac{\partial w^A_i}{\partial d_j}, \tag{3-16}$$
$$t_{ij} = \frac{\partial w^T_i}{\partial d_j}, \tag{3-17}$$
$$r_{ij} = \frac{\partial w^R_i}{\partial d_j}, \tag{3-18}$$

where the following notations are adopted:

γ^f_{ij}: the amount of change in the i*th* commodity of the total input requirements when a unit is increased in the j*th* commodity of final demand;

a_{ij}: the amount of change in the i*th* commodity of the direct input requirement in conjunction with the direct effect when a unit is increased in the j*th* commodity of final demand;

t_{ij}: the amount of change in the i*th* commodity of the indirect input requirement in conjunction with the technical indirect effect when a unit is increased in the j*th* commodity of final demand;

r_{ij}: the amount of change in the i*th* commodity of the indirect input requirement in conjunction with the interrelated indirect effect when a unit is increased in the j*th* commodity of final demand.

For the Leontief inverse C, let us assume that the unique solution is given by

$$x = Cd, \tag{3-19}$$

where $C = (c_{ij})_{(n\times n)}$ and x is the gross output vector. Substituting (2-9) into (3-19), we obtain

$$\begin{aligned} x &= (I+A+T+R)d \\ &= Id+Ad+Td+Rd \\ &= x^I+x^A+x^T+x^R, \end{aligned} \tag{3-20}$$

by letting:

$$x^I = Id, \tag{3-21}$$

$$x^A = Ad, \tag{3-22}$$

$$x^T = Td, \tag{3-23}$$

$$x^R = Rd. \tag{3-24}$$

From (3-19), we can obtain

$$c_{ij} = \frac{\partial x_i}{\partial d_j}, \tag{3-25}$$

and similarly, from (3-21), (3-22), (3-23), and (3-24),

$$\delta_{ij} = \frac{\partial x_i^I}{\partial d_j}, \tag{3-26}$$

$$a_{ij} = \frac{\partial x_i^A}{\partial d_j}, \tag{3-27}$$

$$t_{ij} = \frac{\partial x_i^T}{\partial d_j}, \tag{3-28}$$

$$r_{ij} = \frac{\partial x_i^R}{\partial d_j}, \tag{3-29}$$

where the following notations are defined as:

c_{ij}: the amount of change in the i*th* commodity of the total output requirements when a unit is increased in the j*th* commodity of final demand;

$$\delta_{ij} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases};$$

a_{ij}: the amount of change in the i*th* commodity of the direct output requirement in conjunction with the direct effect when a unit is increased in the j*th* commodity of final demand;

t_{ij}: the amount of change in the i*th* commodity of the indirect output requirement in conjunction with the technical indirect effect when a unit is increased in the j*th* commodity of final demand;

r_{ij}: the amount of change in the i*th* commodity of the indirect output requirement in conjunction with the interrelated indirect effect when a unit is increased in the j*th* commodity of final demand.

Therefore, by using the concept of partial derivatives with respect to the final demand and gross output, the definitions of γ^{g}_{ij}, γ^{f}_{ij}, and c_{ij} can be distinguished more clearly.

4. Applications

The results obtained in sections 2 and 3 can be easily used in many different applications. Among them, we will present four examples below.

First, we can apply the notion of γ^{g}_{ii}, which means the direct and indirect input requirements of that commodity itself to produce a unit of gross output of commodity i, to grasp the economic interpretation of the Hawkins-Simon (H-S) conditions (Hawkins and Simon, 1949, p. 248; Yan, 1968, p. 36; Jeong, 1982; Fujita, 1991). Its interpretation is well known to mean "to produce one unit of a commodity, the direct and indirect input requirements of that commodity itself must not exceed one unit," which is precisely the definition of γ^{g}_{ii}. From (2-6), we have

$$\gamma^{g}_{ii} = a_{ii} + t_{ii}, \tag{4-1}$$

where a_{ii} represents the direct requirement and t_{ii} the technical indirect requirement. Therefore, by using the concept of γ^{g}_{ii}, we can see whether the input-output tables constructed by the different research objectives hold the economic interpretation of the H-S conditions.

Second, we can apply the total (direct and indirect) input requirements matrix for a unit of gross output, Γ^g, to calculate the energy intensity (Gim, 1998). The energy intensity or the total energy requirement can be defined as the measure of the direct and indirect energy required to produce a unit of each sector's gross output. Namely, the energy intensity is the concept measured in physical units for a unit of gross output. Nevertheless, until now, the Leontief inverse matrix of the traditional input-output model has been used to compute the energy intensity for a unit of each sector's gross output. As it is well known, each element of the Leontief inverse, c_{ij}, represents the direct and indirect output requirements to support a unit of final demand. We cannot put the identical meaning between the notion of the direct and indirect energy required to support a unit of final demand and that required to produce a unit of gross output in energy input-output models expressed in hybrid units. Therefore, by using the total input requirements matrix for a unit of gross output Γ^g, we can get more meaningful values which coincide with the definition of energy intensity.

Third, we can also apply the matrix Γ^g for a unit of gross output to estimate the total pollution generation when it is only associated with interindustry technical relations (Gim, 2000). So far, the same is the case for the energy intensity mentioned above. The Leontief inverse has been usefully used in estimating the total pollution generation resulting from economic activities. Therefore, we can get three different concepts of the total pollution generation: one from the Leontief inverse and the other two from the general relation between Γ^g for a unit of gross output and Γ^f for a unit of final demand. Moreover, in section 2, we showed that Γ^g consists of the direct effect, and the technical indirect effect which indicates the purely technical relation between inputs and output. Γ^g can be a useful total input requirements matrix to estimate the total pollution generation as an alternative method distinguished from the traditional Leontief inverse. From three different total amounts of pollution generation, we can get the best-fitting pollution generation which coincides with the study objectives.

Fourth, the matrix Γ^g can be also applied to derive a new concept of multiplier analysis in the input-output model. Until now, the Leontief inverse of the traditional input-output model also has been successfully used to compute input-output multipliers for output, employment and income. Nevertheless, we can get other concepts of interindustry multipliers by using Γ^g. A new concept "input multiplier[1)] for sector j," denoted by μ^I_j, can be defined as the direct and indirect input requirements in all sectors of the economy that are needed to produce a unit of gross output of sector j, which is the sum of the elements in column j of Γ^g, and hence:

$$\begin{aligned} \mu^I_j &= \gamma^g_{1j} + \gamma^g_{2j} + \ldots + \gamma^g_{nj} \\ &= \sum_{i=1}^{n} \gamma^g_{ij}. \end{aligned} \tag{4-2}$$

An input multiplier for sector j is a concept that corresponds to an output multiplier for sector j, μ^o_j, shown as Eq. (4-3), which is the sum of the elements in column j of the Leontief inverse:

$$\begin{aligned} \mu^o_j &= c_{1j} + c_{2j} + \ldots + c_{nj} \\ &= \sum_{i=1}^{n} c_{ij}. \end{aligned} \tag{4-3}$$

Moreover, we can derive a technical impact coefficient for sector j (TIC_j) and a technical sensitivity coefficient for sector i (TSC_i) defined as (4-4) and (4-5) respectively:

$$TIC_j = \sum_{i=1}^{n} \gamma^g_{ij} / \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma^g_{ij}, \tag{4-4}$$

$$TSC_i = \sum_{j=1}^{n} \gamma^g_{ij} / \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma^g_{ij}, \tag{4-5}$$

where n is the number of sectors. These are good indicators explaining the technically backward linkage and technically forward linkage effects respectively. These two coefficients also can not be compared with the impact and sensitivity coefficients drawn from the Leontief inverse, because the two matrixes Γ^g and C have different economic interpretations according to the different initial changes given exogenously. In these new terms, "technical" (or technically) refers to the fact that the matrix Γ^g represents the purely "technical" relation between inputs and output.

Also, sectoral input-output multipliers for employment and income, denoted by μ_j^L and μ_j^Y respectively, can be drawn from the matrix Γ^g, which are entirely distinguished from employment and income multipliers derived from the Leontief inverse. The employment multiplier for sector j, μ_j^L, is defined as the ratio of "input employment effects" (total employment changes) in all sectors of the economy resulting from one unit change for sector j gross output, to the direct employment effect for sector j input, which can be written as

$$\mu_j^L = \frac{\sum_{i=1}^{n} (L_i / Q_i) \gamma_{ij}^g}{L_j / Q_j} = \frac{\sum_{i=1}^{n} \Pi_i \ \gamma_{ij}^g}{\Pi_j}, \tag{4-6}$$

where the following notations are used:

Q_j = total inputs used by sector j;

L_j = amount of employment in sector j;

Π_i = direct employment coefficient of sector i;

Π_j = direct employment coefficient of sector j.

The income multiplier for sector j, μ_j^Y, is also defined as the ratio of "input income effects" (total income changes) in all sectors of the economy resulting from one unit change for sector j gross output, to the direct income effect for sector j input, which can be given by

$$\mu_j^Y = \frac{\sum_{i=1}^{n} (Y_i / Q_i) \gamma_{ij}^g}{Y_j / Q_j} = \frac{\sum_{i=1}^{n} h_i \ \gamma_{ij}^g}{h_j}, \tag{4-7}$$

where the following notations are adopted:

Y_i = income earned by sector i;

h_i = direct income coefficient of sector i;

h_j = direct income coefficient of sector j.

Of course, the values of multipliers for employment and income drawn from Γ^g cannot be compared directly with the values of the same name multipliers derived from the Leontief inverse respectively, because their initial, exogenous changes are different. Γ^g is the concept for a unit of gross output, but C is the concept for a unit of final demand. The two different types of multipliers drawn from Γ^g and C separately do not have the same economic interpretations. Therefore, we can appropriately choose one of them or both of them according to the analysis objectives studied.

5. Conclusions

In 1998, Gim and Kim proposed the general relation between the notion of direct and indirect input requirements of commodity i to support a unit of final demand of commodity j, γ^f_{ij}, and that to produce a unit of gross output of commodity j, γ^g_{ij}, in the open static input-output model. Recently, Gim (2000) also showed that the elements of the total input requirements matrix for a unit of gross output Γ^g consists of two fundamental components, the direct effect, and the technical indirect effect which comes from only the purely technical relation between inputs and output. Moreover, the author showed that the cumulative indirect effect can be decomposed into two different parts, the technical indirect effect and the interrelated indirect effect; that the elements of the total input requirements matrix for a unit of final demand Γ^f can be decomposed into three different parts (the direct effect, the technical indirect effect and the interrelated indirect effect); and that the elements of the Leontief inverse C can be decomposed into four different parts (the final demand, the direct effect, the technical indirect effect and the interrelated indirect effect).

On the basis of the research findings mentioned above, the study objectives of this paper are as follows. (1) We examined and showed that the results of the decomposition by factors can be also expressed as the partial derivatives with respect to the final demand and gross output. By using this method, we are assured that the same results as the major findings of Gim (2000) can be obtained. (2) Some useful applications were also presented to demonstrate the effectiveness and validity of the decomposition by different factors in direct and indirect requirements.

Taking the partial derivatives on (3-2), (3-10), and (3-19), and by the economic interpretations of the components of the partial derivative decompositions, the definitions of γ^g_{ij}, γ^f_{ij}, and c_{ij} become more clear. By expressing γ^g_{ij}, γ^f_{ij}, and c_{ij} as the partial derivatives, we have the results (3-6), (3-15), and (3-25) respectively, where γ^g_{ij} is the amount of change in the i*th* commodity of the total input requirements when a unit is increased in the j*th* commodity of gross output.

The research findings found in sections 2 and 3 can be applied in many different areas such as: the Hawkins-Simon conditions, energy intensity, estimating the total pollution generation and multiplier analysis. First, the notion of γ^g_{ii} can be applied to see whether the input-output tables constructed by the different objectives hold the economic interpretation of the Hawkins-Simon conditions. Second, by using the total input requirements matrix for a unit of gross output Γ^g, we can get more meaningful values which coincide with the definition of energy intensity (or the total energy requirement), which is defined as the measure of the direct and indirect energy required to produce a unit of each sector's gross output. Third, the matrix Γ^g can be a useful total input requirements matrix to estimate the total pollution generation for the exogenous change of gross output, which is accepted as an alternative estimation method distinguished from the total pollution generation computed from the traditional Leontief inverse. Fourth, the notion of Γ^g can be also applied to derive a new concept in input-output multipliers different from the existing multipliers drawn from the Leontief inverse. A new notion "input multiplier" for sector j, denoted by μ^I_j and shown as Eq. (4-2), can be defined as the direct and indirect input requirements in all sectors of the economy that are needed to produce a unit of gross output of sector j, which is the sum of the elements in column j of the matrix Γ^g. Also, new input-output multipliers for employment and income, denoted by μ^L_j and μ^Y_j respectively, can be drawn from Γ^g. These multipliers are entirely distinguished from the present employment and income multipliers derived from the Leontief inverse. The employment multiplier for sector j, μ^L_j, shown as Eq. (4-6), is defined as the ratio of "input employment effects" (total employment changes) in all sectors of the economy from one unit change for sector j gross output, to the direct employment effect for sector j input. The income multiplier for sector j, μ_j^Y, shown as Eq. (4-7), is also defined as the ratio of "input income effects" (total income changes) in all sectors of the economy resulting from one unit change for sector j gross output, to the direct income effect for sector j input.

Hopefully, we expect that these findings and results will be used in many different applications: especially in environmental and energy input-output models, intensity analysis of resources, and multiplier and impact analysis for one unit of output.

Note

1) The term "input multiplier" used in this section is entirely different from the same term "input (or supply) multiplier" drawn from supply-side input-output models. (Miller and Blair, 1985, p. 321)

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