Consistency of the supply-driven model: A typological approach

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Abstract. This paper examines the consistency the Ghosh supply-driven model (SM), stressing on the question of the treatment of quantities and prices. The complete typology of all supply-driven models is examined, exploring two versions: in each one again two sub-versions are considered, one with physical data and prices, the other with value data and index prices; for each model, the primal as well as the dual are solved. The first model (SM1) considers a single input by agent along with input prices instead of a single output by sector along with output prices as in the Leontief demand-driven model (DM). The interpretation of SM1 as the economic dual of the DM is incorrect: the meaning of the variables is different between both models; the interpretation of SM1 to describe the functioning of a centrally planned economy is also wrong and the interpretation of agents as cooperatives is incorrect. The second version of the supply-driven model (SM2) is presented by switching back to the more traditional hypotheses of DM: single output by sector along with output prices. SM2 offers solutions of limited interest, being incapable to separate quantities and prices.

1 Introduction

The Leontief model (1936, 1986) assumes that all inputs are bought by producers in fixed proportions; it is qualified as "demand-driven". The Ghosh model (1958) assumes that, in an input-output framework, each commodity is sold to each sector in fixed proportions; it is qualified as "supply-driven". If the Leontief model is well accepted, the Ghosh model has its critics and its supporters [1]: the quantity Ghosh model seems not plausible (Oosterhaven, 1988, 1989), (Dietzenbacher, 1989), the price version of the model could seem more acceptable (Oosterhaven, 1996), while for Dietzenbacher (1997) the model can be interpreted as a price model. On the other hand, the applied studies to compare the stability of technical or allocation coefficients are not fully concluding in favor of one model or to the other (Bon 1986, 2000), (Mesnard, 1997). However, two remarks can be done.

First, in the literature (see for example, (Dietzenbacher, 1997)), the models are explored in a special configuration: data are in value, that is, in currency units (e.g. in dollar, in euro, etc.), while the prices considered are index prices [2]. It is a pity and gives a limited validity to the results, while it could be economically more correct to consider also data in physical quantities instead of data in value (that is, true data), and simple prices instead of index prices.

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[1] See Helsmstadter and Richtering (1982); Chen and Rose (1986, 1991); Deman (1988, 1991); Oosterhaven (1988, 1989, 1996); Davar (1989); Dietzenbacher (1989, 1997); Gruver (1989); Miller (1989); Rose and Allison (1989); Sonis and Hewings (1992); de Mesnard (1997).

[2] Index prices are ratios of prices, **always** used when the data are computed in money terms instead of physical terms, as it is the case in the national accounting systems. Note that Davar (1989) uses the term "latent prices".

Second, the nature of prices could change between the Leontief model and the Ghosh model. The demand-driven model [3] uses the ordinary concept of price, the price of sector's output, also called "output price". An output price is the traditional "production price" in the Classical sense (Ricardo, Marx, Sraffa), that is the price of commodities sold by sellers and affecting a whole row of the exchange table; for a review, see Seton (1993). The supply-driven model is presented in the literature with two concepts of price: with the output prices as for the demand-driven model but also with a more exotic concept, the price of sector's input, called "input price". An input price is a price controlled by the buyer, affecting a whole column of the exchange table; this concept could seem strange, but it is introduced to be sure that all possible cases will be explored when developing the complete typology of the possible supply-driven models, recalling the demand-driven model in annex.

Instead of focusing the discussion on how the model can be deduced from an economic reasoning (cost minimization, revenue maximization, etc., as in Oosterhaven, 1989) or on how the axioms that serve as basis for the model can be established, the starting point will be the accounting identities: the solution that can be deduced from them will be computed and compared. Considering the two main versions of the supply driven model (input prices, output prices) and their four sub-versions (primal in quantities / dual in prices, data in quantities and prices / data in value and index prices), each of the eight cases will be discussed. For each model, the coherency will be discussed. The model with input prices will be presented first, then the model with output prices. This will allow showing that the supply-driven model with input prices lacks transformation process, even if it is a sort of mathematical dual of the

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[3] See the Annex to be reminded about the Leontief model.

Leontief model, while the supply-driven model with output prices has not this drawback but offers solutions of limited interest.

2 The supply-driven model with input prices

This model is those considered by Oosterhaven (1989, 1996). The supply-driven model with its input prices requires considering a single homogenous input and multiple outputs, while the Leontief demand-driven model assumes that there is a single homogenous output and multiple inputs. Input prices, denoted p_j, are prices that are along the columns of the exchange table, while the familiar prices, that I will call in the following *output prices* by easiness, denoted p_i, are along the rows of the exchange table; output prices are those used in the production-models family (Ricardo-Sraffa, Marx, etc. [4]). While the output price p_i is classically the price of commodity *i* that will be sold to other sectors and to final demand, the input price p_j can be only interpreted as the price to which agent *j* buys every things (and not the price of commodity *j*).

With input prices, the flow $\bar{z}_{ij} \geq 0$ can be only interpreted as the physical flow of commodity *j* bought by agent *j* to agent *i* and not as in the Leontief model, as the flow of commodity *i* sold by agent *i* to agent *j*. Similarly $\bar{x}_i$ must be the total input of agent *j* measured in physical terms and not the total output of agent *i* as in the Leontief model. The term $\bar{f}_i \geq 0$ must be the quantity of a residual $n+1^{th}$ commodity bought by a residual $n+1^{th}$ agent to agents *i* and not as in the Leontief model the quantity of commodity *i* sold to final demand. This leads to introduce the price of final demand, p^f, what is not the price of the commodity "final demand" (final demand is made of many commodities) but it is the price of the agent "final demand". The term $\bar{v}_j \geq 0$ must be only a category of the input commodity *j*, measured in physical terms: the correct word must be "exogenous final supply" of commodity *j* made by a special agent, the "exogenous supplier", and it cannot be the value added of sector *j* measured in physical

[4] See Pasinetti 1977.

terms as in the Leontief model (the concept of added value takes part only in the production sphere while the final demand is validly a category of commodity i in the Leontief model). For each commodity i, the total sold is equal to the total input: $\bar{x}_j = \sum_i \bar{z}_{ij} + \bar{v}_j$. All this is drawn in tables 1 and 2 for both types of models.

Table 1, 2, 3 and 4 here

2.1 Solving the model

All types of prices are such that the accounts of all agents are equilibrated. For input prices, this gives the following accounting equations:

(1) $\sum_j \bar{z}_{ij}\, p_j + \bar{f}_i\, p^f = \bar{x}_i\, p_i$ for all i

for the rows, where p^f is the input price of the final demand, and,

(2) $\sum_i \bar{z}_{ij}\, p_j + \bar{v}_j\, p_j = \bar{x}_j\, p_j \Leftrightarrow \sum_i \bar{z}_{ij} + \bar{v}_j = \bar{x}_j$ for all j

for the columns. One can see immediately that (2) requires homogeneity by columns, while (1) doesn't. It is assumed that each commodity is sold in fixed proportions to each agent.

2.1.1 Data in physical quantities

With data in physical quantities, the allocation coefficients are defined as: $\bar{b}_{ij} = \frac{\bar{z}_{ij}}{\bar{x}_i}$ for all i, j.

By rows (I call this the *primal*), (1) becomes: $\bar{\mathbf{B}}\,\mathbf{p} + \bar{\mathbf{d}}\, p^f = \mathbf{p}$, where $\bar{d}_i = \frac{\bar{f}_i}{\bar{x}_i}$ is the output coefficient toward the residual, what gives $\mathbf{p} = \left(\mathbf{I} - \bar{\mathbf{B}}\right)^{-1} \bar{\mathbf{d}}\, p^f$ if $\left|\mathbf{I} - \bar{\mathbf{B}}\right| \neq \mathbf{0}$: prices are found from the ratios of final demands multiplied by the price of final demand.

By columns (I call this the *dual*), (2) transforms into $\bar{\mathbf{x}}' \, \bar{\mathbf{B}} + \bar{\mathbf{v}}' = \bar{\mathbf{x}}'$, what gives $\bar{\mathbf{x}}' = \bar{\mathbf{v}}' \left(\mathbf{I} - \bar{\mathbf{B}} \right)^{-1}$ if $\left| \mathbf{I} - \bar{\mathbf{B}} \right| \neq \mathbf{0}$: outputs in physical quantities are found from physical quantities of exogenous final supply.

2.1.2 Data in value

With data in value, the "economic allocation coefficients" are $b_{ij} = \dfrac{\bar{z}_{ij} \, p_j}{\bar{x}_i \, p_i}$ for all i, j. Considering the most simple type of price index [5], index input prices are defined as the ratio of the prices of the current date t to those of the date 0: $\pi_j = \dfrac{p_j^t}{p_j^0}$ for all j, where p_j^0 and p_j^t stand for the input prices of commodity j at dates 0 and t.

By rows (1) writes:

(3) $$\sum_j z_{ij} \, \pi_j + f_i \, \pi^f = x_i \, \pi_i$$

that is, if $d_i = \dfrac{f_i}{x_i}$ for all i:

(4) $$\mathbf{B} \, \boldsymbol{\pi} + \mathbf{d} \, \pi^f = \boldsymbol{\pi}$$

and index prices $\boldsymbol{\pi}$ are found from the index price of final demand π^f: $\boldsymbol{\pi} = (\mathbf{I} - \mathbf{B})^{-1} \, \mathbf{d} \, \pi^f$ [6].

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[5] Laspeyres or Paasche are more sophisticated, taking into account not only the evolution of true prices but also of physical quantities by introducing weights; see in (Fisher and Shell, 1997) a complete theory of production price indexes. Some authors (e.g., Davar, 1989; Oosterhaven, 1996; Dietzenbacher, 1997) have touched upon the plausibility of index prices.

[6] Equation (4) corresponds to equation (6) of Oosterhaven (1996); the only difference is that Oosterhaven considers many categories of output while, for simplicity, one category -- final demand -- has been introduced in this paper; remember that this equation does not provides prices but only index price.

By columns, (2) changes into:

(5) $$\sum_i z_{ij} + v_j = x_j$$

which implies that:

(6) $$\mathbf{x}' \, \mathbf{B} + \mathbf{v}' = \mathbf{x}'$$

an output in money terms -- a value -- is found from the value final supply: $\mathbf{x}' = \mathbf{v}' \, (\mathbf{I} - \mathbf{B})^{-1}$.

2.2 Comments

The above developments retrieve Oosterhaven's result (1988, 1989, 1996): this supply-driven model in all its versions (either with data in physical terms and input prices or data in value and input index prices) is a sort of mathematical dual of the demand driven model in all its versions (either with data in physical terms and output prices or with data in value and output index prices). It is not the true mathematical dual as the prices are input prices, not output prices, and the price of labor must be uniform among sectors in the demand-driven model, i.e., $p_i^v = p^v$ for all i [7]; see in annex the Leontief model. In other words, the model in price corresponds to the model in quantity and vice versa: mathematically, the model works fine, however it is not coherent.

2.2.1 The meaning of "supply-driven"

Supply-driven means what? In the Leontief model, at the first round, a sector *j*, forced by the final demand f_j of commodity *j*, sells the equal quantity $x_j = f_j$ of commodity *j* he has in

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[7] In the Leontief model (or in production models), there are many industrial sectors *j* that are making each one a unique output commodity *j* (each sector produces one and only one commodity); to do it, they buy different input commodities *i* to the sectors in fixed proportion defined by the technical coefficients.

stock, earns the money and then spends all to buy a quantity z_{ij} of each input and a quantity v_j of labor following technical coefficients a_{ij} and l_j. In the supply-driven model, at the first round, the "exogenous supplier" forces each agents *i* to accept the exogenous final supply v_i of commodity *i*. So agent *i* is obliged to pay what it costs to the exogenous supplier and then, in order to rebuild its fortune, to sell a quantity z_{ij} of each commodity *j* and a quantity f_i of "residual commodity" he has in stock, following the allocation coefficients b_{ij} and d_i , for a total amount x_i equal to v_i. In other words, an agent is forced to buy by other agents, and this implies that at his turn he must force all the agents to buy...

Is it a behavior only valid in a centrally planned economy, as suggested by Ghosh? Two arguments can be stressed. First, the true historical centrally planned economies, USSR mainly, were not driven like that. The plan was computed following the Leontief model for about ten thousand of goods, not following the Ghosh model, even if the quantities computed to solve the model at equilibrium had to be imperatively respected by all productive sectors. Second, if we are not in a centrally planned economy, if it is not difficult to understand why agents *i* must sell all commodities -- to earn money --, one cannot understand why an agent *j* is allowed to buy only the commodity *j* and why it is passive regarding the forced supply of agents *i*. Remember that in the Leontief story, the technical coefficients define the production function of each sector *j*; everybody understands why a sector *j* must buy all commodities as inputs, including its own commodity *j*: to *produce*). The only explanation comes if the agents *j* are types of cooperatives, that engage themselves to buy all the production of some small producers (e.g., of wine, fruits, meat, etc.) *i*, but at their own price p_j. However, in square input-output models, the supplying agents in row must be the same than the demanding agents in column what is not the case in this explanation. How the same agents can be cooperatives and small producers at the same time? Or how an agent can be a cooperative of wine and at the same time a seller of wine, but also of fruits and meats? In the real world, cooperatives buy all the wine of the small producers and then sell this wine, nothing else.

2.2.2 The missing transformation process

Agents j cannot be, as in the Leontief model, productive sectors, producing and selling a unique commodity as output to many other sectors and buying multiple commodities as inputs; the passage from multiple inputs to a unique output being naturally the productive process. The supply-driven model with input prices is not productive, so no productive process is attached to it. Is another type of process attached to it? This eventual process could be called a transformation process, a more vague term to qualify a non-productive process. At least, a transformation process must exist, if a productive process doesn't.

However, what does mean exactly a unique homogenous input and multiple outputs? To whom each agent sells the commodities is defined by a fixed proportion, the allocation coefficient. Each agent j must sell all commodities to be allowed to buy a single commodity j: an agent j buys only commodity j and commodity j is bought only by agent j. There is **nothing** to explain the passage from a unique input to multiple outputs: one cannot understand how it is possible for an agent to buy a unique input (as wheat) then to be able to sell multiple outputs (e.g., wheat, but also textile, steel, automobiles, services, etc.). From where do they come from, these multiple commodities sold by agents? Who produce them? The model provides no answer: there is no transformation process. Consequently, in this version with input prices, the Ghosh model completely lacks theoretical coherency. This is why one must see how the alternative model, the supply-driven model with output prices works.

3 The supply-driven model with (ordinary) output prices

Now the more familiar categories are considered: productive sectors, final demand $\bar{f}_i$ of commodity i, value added $\bar{v}_j$ of sector j, flow $\bar{z}_{ij}$ of commodity i sold by sector i to sector j, output price p_i of commodity i, price p_j^v of labor in sector j, etc.? The idea is to build a supply-driven model on the same principles than the driven model defined in annex, that is, on the traditional economic framework of an exchange productive economy. This is what has been done by Dietzenbacher (1997), when he exposed a vindication of the supply-driven model in terms of a price model, and by most of the authors.

The accounting identities are:

(7) $$\sum_j \bar{z}_{ij}\, p_i + \bar{f}_i\, p_i = \bar{x}_i\, p_i \Leftrightarrow \sum_j \bar{z}_{ij} + \bar{f}_i = \bar{x}_i \text{ for all } i$$

(8) $$\sum_i \bar{z}_{ij}\, p_i + \bar{v}_j\, p_j^v = \bar{x}_j\, p_j \text{ for all } j$$

This second version of the model is conceptually the most simple, but it will be shown that it does not go on to be the dual of the Leontief model and becomes poor regarding the solutions that it provides.

3.1 *Data in physical quantities*

In physical terms, the allocation coefficients $\bar{b}_{ij} = \dfrac{\bar{z}_{ij}}{\bar{x}_i}$ are assumed to be stable for all i, j; in matrix form, this can be written $\bar{\mathbf{B}} = \langle \bar{\mathbf{x}} \rangle^{-1}\, \bar{\mathbf{Z}}$.

By rows, if $\bar{d}_i = \dfrac{\bar{f}_i}{\bar{x}_i}$ for all i, (7) writes :

(9) $$\bar{\mathbf{B}}\, \mathbf{s} + \bar{\mathbf{d}} = \mathbf{s}$$

This equation is an identity that never provides prices as it might be expected to do at first glance.

By columns, (8) is transformed into $\sum_i \bar{b}_{ij}\, \bar{x}_i\, p_i + \bar{v}_j\, p_j^v = \bar{x}_j\, p_j$ for all i, that is:

(10) $\mathbf{x}'\, \bar{\mathbf{B}} + \mathbf{v}' = \mathbf{x}'$

and only value outputs are found, $\mathbf{x}' = \mathbf{v}' \left(\mathbf{I} - \bar{\mathbf{B}}\right)^{-1}$, even if we are in the physical data model: in the physical data Leontief model, outputs in physical quantities and prices are found (see annex).

3.2 Data in value

When data are computed in value, the stable coefficients are defined as: $b_{ij} = \frac{z_{ij}}{x_i}$ for all i, j, that is $\mathbf{B} = \langle \mathbf{x} \rangle^{-1}\, \mathbf{Z}$. So, $b_{ij} = \frac{\bar{z}_{ij}\, p_i}{\bar{x}_i\, p_i} = \frac{\bar{z}_{ij}}{\bar{x}_i} = \bar{b}_{ij}$ for all i, j. Index output prices are denoted as $\pi_i = \frac{p_i^t}{p_i^0}$ for all i, where p_i^0 and p_i^t stand for the output price of commodity i at dates 0 and t, and $\pi_j^v = \frac{p_j^{vt}}{p_j^{v0}}$ for all j, where p_j^{v0} and p_j^{vt} stand for the price of labor in sector j.

For the primal, formula (7) transforms into:

(11) $\sum_j z_{ij}\, \pi_i + f_i\, \pi_i = x_i\, \pi_i$ for all i

so π_i can be simplified on both sides of this equation: $\sum_j z_{ij} + f_i = x_i$, for all i. So $\mathbf{B}\, \mathbf{s} + \mathbf{d} = \mathbf{s}$, where $d_i = \frac{f_i}{x_i}$ is the coefficient of final demand measured as a ratio of money terms: it is again an identity. Nothing is found from this primal.

For the dual, formula (8) becomes:

(12) $\sum_i z_{ij}\, \pi_i + v_j\, \pi_j^v = x_j\, \pi_j$

which implies

(13) $\quad \tilde{\mathbf{x}}' \, \mathbf{B} + \tilde{\mathbf{v}}' = \tilde{\mathbf{x}}'$

where $\tilde{x}_i = x_i \, \pi_i$ and $\tilde{v}_j = v_j \, \pi_j^v$ are values formed by the product of a value (i.e., a quantity in currency units) by an index price; they could be called "time-lag-values" as they can be expressed as $\tilde{x}_i = x_i \, \pi_i = \bar{x}_i \, p_i^0 \, \dfrac{p_i^t}{p_i^0} = \bar{x}_j \, p_i^t$ and $\tilde{v}_j = v_j \, \pi_j^v = \bar{v}_j \, p_j^{vt}$: the time-lag-value of the output is the value of output measured with a time lag between the physical quantity $\bar{x}_i$ of date 0 and the price of date *t*. Time-lag-values of outputs are found as solution of the dual from time-lag values of value added: $\tilde{\mathbf{x}}' = \tilde{\mathbf{v}}' \, (\mathbf{I} - \mathbf{B})^{-1}$.

3.3 Comments

This model with output prices has a better coherency than the former. Each agent *i* is a sector, that sells one commodity *i*, its homogenous output, and that buys multiple inputs *j*. The transformation process between inputs and output is of the productive type. The only difference between this supply-driven productive model and the demand-driven Leontief model of production lies in the coefficients.

However, this model gives solutions of limited interest. In the physical-data model, the primal gives only an identity whereas only values x_i in money terms are found in the dual, while normally output prices p_i and physical quantities $\bar{x}_i$ would be found. Prices are undetermined: neither (9) nor (10) provide prices. In the value-data model, the primal allows to find nothing while the dual allows only the time-lag-values $\tilde{\mathbf{x}}$ to be determined from the time-lag-value of value added $\tilde{\mathbf{v}}$, but it never allows values $\mathbf{x}$ and index prices $\boldsymbol{\pi}$, or quantities $\bar{\mathbf{x}}$ and prices $\mathbf{p}^t$ of date *t*, to be found separately. All this is disappointing. The reader could ponder about the difference of this result and Dietzenbacher's approach (Dietzenbacher, 1997), vindicating the supply-driven model by reinterpreting it as a price model. First, he based its demonstration only on value data because equilibrium by columns as well as by rows was required. Second, he considers only the dual (1997, p. 632) in its equation (5); this equation is the same than equation (13) when all index prices equal 1, that is for date 0: this is why Dietzenbacher is able to write a supply-driven model with outputs in value (its accounting identity is not (12) but $\sum_i z_{ij} + v_j = x_j$, that is (12) with all index prices equal to 1). There is no mistake here: Dietzenbacher is right in doing it. And, assuming matrix **B** to remain fixed, if **v** varies then the output **x** varies.

However, when index prices are considered, equation (12) is the only correct one: only time-lag values are found by the dual of the output-price supply-driven model, not index prices or outputs separately. Remark that, as there is a dissymmetry between the primal and the dual of this supply-driven model, it is not the mathematical dual of the Leontief model (this one is completely symmetrical: see annex).

4 Conclusion

This paper has examined the consistency of the alternative input-output model, the supply-driven model developed by Ghosh, with emphasis on the question of the treatment of quantities and prices. The complete typology of the supply-driven models is examined. Two versions of the supply-driven model are explored, one with input prices and one with more classical output prices, and in each one, again two versions are considered, one with data in quantities and prices, the other with data in value and index prices; each model has been solved in quantities (primal) and in prices (dual). The following conclusions have been set.

When input prices are considered, even if in both its versions (physical quantities and input prices, or value quantities and input index prices) the Ghosh model is a sort of mathematical dual of the Leontief model (giving prices where the Leontief model gives outputs and vice versa), this duality is only an appearance because the meaning of the variables is completely different between the two categories of the model. The notations x_i or x_j, z_{ij}, f_i and v_j are perhaps the same between both models, nevertheless they cover different economic realities in both models. Agents are not productive sectors but only commercial; $\bar{z}_{ij}$ cannot be the flow of commodity *i* sold by sector *i* to sector *j* but the flow of commodity *j* bought by agent *j* to agent *i*; $\bar{f}_i$ cannot be the final demand of commodity *i* but the demand of residual commodity made by the residual agent to agent *i*; v_j cannot be the added value of sector *j* but the final supply of commodity *j*; the input price p_j is the price of commodity *j*, the homogenous input of agent *j*, while the output price p_i is the price of commodity *i*, the homogenous output of sector *i*. This call into question the approaches that qualify the Ghosh model as the dual of the Leontief model: they are economically abusive even if they are mathematically true. The interpretation of this supply demand-driven model to describe the functioning of a centrally planned economy is also wrong and the interpretation of agents as cooperatives is incorrect.

The alternative version of the supply-driven model reintroduces traditional output prices p_i: the model switches back to the more traditional hypotheses of the demand driven model (considering $\bar{z}_{ij}$ as the flow of commodity *i* from sector *i* to sector *j*, f_i as the final demand, v_i as the value added). In this case, the supply-driven model is not the dual of the demand-driven model: its quantity version cannot be interpreted as a price demand-driven model and conversely. However, this model offers solutions of limited interest, being incapable to separate quantities and prices in all its versions or values and index prices.

The table 5 makes the synthesis of all results, along with the recalling of the demand-driven model in annex. These results are perhaps definitive but as they are very elementary, one can wonder why a so large discussion has been developed before in the literature. Should we abandon the Ghosh model? Probably yes for its input-price version but for its output-price version, each one must decide: if the poor results are sufficient, the model can be used as it has no other drawbacks. Remember that, on applied analyses with true data, technical coefficients are not much more stable than allocation coefficients over time (Bon 1986, 2000), (Mesnard, 1997).

Table 5 here

5 Annex. Remind: the demand driven model

The demand driven model corresponds to one of the most popular in economic science, the Leontief model, but it also falls into a very general category, the linear models of production and exchange. In this category one found some of the most familiar and historic models, Ricardo, Marx and Sraffa models. Consider *n* sectors, producing *n* commodities; each sector produces one and only one commodity and each commodity is produced by one and only one sector; $\bar{z}_{ij} \geq 0$ is the physical quantity sold by sector *i* to sector *j* when *j* produces commodity *j*; $\bar{x}_i$ the output of sector *i*. For each commodity *i*, the total sold is equal to the total output $\bar{x}_i$:

(14) $$\bar{x}_i = \sum_j \bar{z}_{ij} + \bar{f}_i$$

where $\bar{f}_i \geq 0$ denotes the final demand [8]. Commodities *i* have a positive price p_i: these prices are true prices in the usual economic sense of the word. Output prices are such that the model is at equilibrium by rows:

(15) $$\sum_j z_{ij} + f_i = x_i \Leftrightarrow \sum_j \bar{z}_{ij} + \bar{f}_i = \bar{x}_i \text{ for all } i$$

[8] For simplicity, is considered here only one category of final demand. The results could be generalized easily.

by simplifying prices in both sides of the equation, where $z_{ij} = \bar{z}_{ij}\, p_i$ is the value of the flow from i to j and $f_i = \bar{f}_i\, p_i$ is the final demand in value; and the model is at equilibrium by columns:

(16) $$\sum_i z_{ij} + v_j = x_j \Leftrightarrow \sum_i \bar{z}_{ij}\, p_i + \bar{v}_j\, p_j^v = \bar{x}_j\, p_j \text{ for all } j$$

where $v_j = \bar{v}_j\, p_j^v$ is the value added measured in money terms, while $\bar{v}_j \geq 0$ is the amount of labor employed by sector j (all must not be equal to zero at the same time), and p_j^v is the wage rate of sector j (profits are taken as the owner's remuneration). In the demand-driven model, it is assumed that each sector buys each commodity in fixed proportions, but there are two possibilities: data may be defined in physical terms or in value terms.

1) With data in physical quantities, coefficients are defined in physical terms, it is assumed that the ratios $\bar{a}_{ij} = \frac{\bar{z}_{ij}}{\bar{x}_j}$ are stable for all i and j that is in matrix terms: $\bar{\mathbf{A}} = \bar{\mathbf{Z}}\,\langle\bar{\mathbf{x}}\rangle^{-1}$.The economy must be at equilibrium by row and by column.

By rows, the accounting identity (15) implies:

(17) $$\bar{\mathbf{A}}\,\bar{\mathbf{x}} + \bar{\mathbf{f}} = \bar{\mathbf{x}} \Leftrightarrow \left(\mathbf{I} - \bar{\mathbf{A}}\right)\bar{\mathbf{x}} = \bar{\mathbf{f}}$$

This Cramer system has a non trivial solution only if the determinant $\left|\mathbf{I} - \bar{\mathbf{A}}\right|$ is not equal to zero; this solution is in found in physical terms $\bar{\mathbf{x}} = \left(\mathbf{I} - \bar{\mathbf{A}}\right)^{-1}\bar{\mathbf{f}}$.

By columns, the accounting identity (16) becomes:

(18) $$\mathbf{p}'\,\bar{\mathbf{A}} + \mathbf{p}^{v\prime}\,\bar{\mathbf{L}} = \mathbf{p}'$$

so, prices are found as a function of the input coefficients of labor multiplied by the price of labor: $\mathbf{p}' = \mathbf{p}^{v\prime}\,\bar{\mathbf{L}}\left(\mathbf{I} - \bar{\mathbf{A}}\right)^{-1}$.

2) With data in value, coefficients are defined as a ratio of values in money terms, that are $z_{ij} = \bar{z}_{ij}\, p_i$ and $x_i = \bar{x}_i\, p_i$. The techno-economic coefficients are the ratio $a_{ij} = \frac{z_{ij}}{x_j}$, assumed to be stable for all i and j.

In the primal, formula (15) is unchanged, that is

(19) $\sum_j z_{ij}\,\pi_i + f_i\,\pi_i = x_i\,\pi_i \Leftrightarrow \sum_j z_{ij} + f_i = x_i$ for all i,

which implies $\mathbf{A\,x} + \mathbf{f} = \mathbf{x}$: outputs, $\mathbf{x}$, in money terms are formed from final demands, $\mathbf{f}$, in money terms: $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{f}$.

In the dual, equation (16) transforms into:

(20) $\sum_i z_{ij}\,\pi_i + v_j\,\pi_j^v = x_j\,\pi_j$

which implies that $\boldsymbol{\pi}'\,\mathbf{A} + \boldsymbol{\pi}^{v\prime}\,\mathbf{L} = \boldsymbol{\pi}'$. The index prices, $\boldsymbol{\pi}$, are formed from $\mathbf{L}$, the value-added coefficients $l_j = \frac{v_j}{x_j}$ (computed in money terms): $\boldsymbol{\pi}' = \boldsymbol{\pi}^{v\prime}\,\mathbf{L}\,(\mathbf{I} - \mathbf{A})^{-1}$.

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	Sector 1	...	Sector j	...	Sector n	Exogenous final demand	Total output
Sector 1	$\bar{z}_{11}$, p_1		$\bar{z}_{1j}$, p_1	...	$\bar{z}_{1n}$, p_1	$\bar{f}_1$, p_1	$\bar{x}_1$, p_1
...	...		...		...	...	...
Sector i	$\bar{z}_{i1}$, p_i	...	$\bar{z}_{ij}$, p_i	...	$\bar{z}_{in}$, p_i	$\bar{f}_i$, p_i	$\bar{x}_i$, p_i
...	...		...		...	...	...
Sector n	$\bar{z}_{n1}$, p_n		$\bar{z}_{nj}$, p_n	...	$\bar{z}_{nn}$, p_n	$\bar{f}_n$, p_n	$\bar{x}_n$, p_n
Value added	$\bar{v}_1$, p_1^v	...	$\bar{v}_j$, p_j^v	...	$\bar{v}_n$, p_n^v		

Table 1. The ordinary physical I-O table with output prices
(homogeneity by rows and columns)

	Agent 1	...	Agent j	...	Agent n	Residual agent
Agent 1	$\bar{z}_{11}$, p_1		$\bar{z}_{1j}$, p_j	...	$\bar{z}_{1n}$, p_n	$\bar{f}_1$, p^f
...	...		...		...	...
Agent i	$\bar{z}_{i1}$, p_1	...	$\bar{z}_{ij}$, p_j	...	$\bar{z}_{in}$, p_n	$\bar{f}_i$, p^f
...	...		...		...	...
Agent n	$\bar{z}_{n1}$, p_1		$\bar{z}_{nj}$, p_j	...	$\bar{z}_{nn}$, p_n	$\bar{f}_n$, p^f
Exogenous final supply	$\bar{v}_1$, p_1	...	$\bar{v}_j$, p_j	...	$\bar{v}_n$, p_n	
Total input	$\bar{x}_1$, p_1	...	$\bar{x}_j$, p_j	...	$\bar{x}_n$, p_n	

Table 2. The physical I-O table with input prices
(homogeneity by rows and columns)

	Sector 1	...	Sector j	...	Sector n	Exogenous final demand	Total output
sector 1	$z_{11},\ \pi_1$		$z_{1j},\ \pi_1$	...	$z_{1n},\ \pi_1$	$f_1,\ \pi_1$	$x_1,\ \pi_1$
...	...		...		...	...	...
sector i	$z_{i1},\ \pi_i$	...	$z_{ij},\ \pi_i$	...	$z_{in},\ \pi_i$	$f_i,\ \pi_i$	$x_i,\ \pi_i$
...	...		...		...	...	...
sector n	$z_{n1},\ \pi_n$		$z_{nj},\ \pi_n$	...	$z_{nn},\ \pi_n$	$f_n,\ \pi_n$	$x_n,\ \pi_n$
Value added	$v_1,\ \pi_1^v$	...	$v_j,\ \pi_j^v$	...	$v_n,\ \pi_n^v$		
Total output	$x_1,\ \pi_1$	...	$x_j,\ \pi_j$	...	$x_n,\ \pi_n$		

Table 3. The ordinary value I-O table with output prices
(homogeneity by rows)

	Agent 1	...	Agent j	...	Agent n	Residual agent	Total input
Agent 1	$z_{11},\ \pi_1$		$z_{1j},\ \pi_j$	...	$z_{1n},\ \pi_n$	$f_1,\ \pi^f$	$x_1,\ \pi_1$
...	...		...		...	...	...
Agent i	$z_{i1},\ \pi_1$	...	$z_{ij},\ \pi_j$	...	$z_{in},\ \pi_n$	$f_i,\ \pi^f$	$x_i,\ \pi_i$
...	...		...		...	...	...
Agent n	$z_{n1},\ \pi_1$		$z_{nj},\ \pi_j$	...	$z_{nn},\ \pi_n$	$f_n,\ \pi^f$	$x_n,\ \pi_n$
Exogenous final supply	$\bar{v}_1,\ \pi_1$	...	$\bar{v}_j,\ \pi_j$	...	$\bar{v}_n,\ \pi_n$		
Total input	$x_1,\ \pi_1$	...	$x_j,\ \pi_j$	...	$x_n,\ \pi_n$		

Table 4. The value I-O table with input prices
(homogeneity by columns)

		Input Prices	Output Prices	
		Supply-driven model	Supply-driven Model	Demand-Driven Model
Physical Model	Primal (by rows)	$\mathbf{p}$	identity	$\bar{\mathbf{x}}$
	Dual (by columns)	$\bar{\mathbf{x}}$	$\mathbf{x}$	$\mathbf{p}$
Value Model	Primal (by rows)	$\boldsymbol{\pi}$	identity	$\mathbf{x}$
	Dual (by columns)	$\mathbf{x}$	$\tilde{\mathbf{x}}$ (time-lag values)	$\boldsymbol{\pi}$
Richness of results		Good	Poor	Good
Credibility		No	Yes	Yes

Table 5. Solutions of the models