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TECHNICAL COEFFICIENTS CHANGE BY BI-PROPORTIONAL ECONOMETRIC ADJUSTMENT FUNCTIONS

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Abstract: Technical coefficients change is an important issue in empirical work of input-output modeling, due to the lack of recent I-O tables in most countries.

In the literature we find different approaches for updating input-output coefficients, most of them adjust technical coefficients 'along the row' starting from 'hypothetical output' given by a base year matrix and actual output (e.g. Conway, 1990). On the other hand we find – mainly in CGE models – variable input coefficients 'along the column' stemming from factor input equations derived from production or cost functions.

In this paper we attempt to combine these two approaches into a bi-proportional method. For the adjustment 'along the column' we use econometric factor input functions for total intermediates and for energy separately to arrive at a sum of non-energy intermediate inputs. For the adjustment 'along the row' we start from the adjustment functions described in the literature and put special emphasis on econometric specification and 'cointegration accounting' in order to overcome the usual more or less ad-hoc treatment in empirical applications.

Introduction

The issue of updating[1] I-O coefficients has had a long tradition in economics. The reasons for this can be found in the very nature of the construction of input-output tables and their coefficients: First, I-O coefficients represent averages of the underlying industry levels. Changes in the sectoral mix within one sector thus invalidate the estimated coefficients rather quickly. Second and more importantly, the tables describe input-output relations between sectors at a certain point of time. Since survey-based I-O-tables rely on a vast amount of data and their complexity usually entails a long construction period, I-O-tables get published with a considerable lag of time. The high cost of setting up such a table furthermore prohibits more frequent construction such that a time interval between the publication of two successive national tables of at least 5 years exists. The 1995 I-O table for Austria (see Statistics Austria, 2001) for example was released in summer of 2001 and was hence already 6 years old at the date of its publication.

It therefore does not come as a surprise that mainly the static nature of input-output tables was often criticised, since changes in the structure of the underlying economy over time might invalidate the direct usage of comparatively old tables. As a corollary many researchers have dealt with non-survey and partial survey methods for constructing more up-to-date input-output tables and thus overcome at least in part this lack of timeliness. The seminal work in this field of partial survey techniques was done by Stone and Brown (1962) who for the first time applied the so-called RAS approach to economic data. The basic idea of RAS is to utilize (usually more frequently available) data on the row and column sums of a matrix in order to update the structure of the matrix itself via an iterative procedure. Assuming an $n \times n$ matrix, RAS hence tries to accomplish the updating of n^2 coefficients using $2n$ data points (i.e. the row and column sum, respectively) which is clearly an underdetermined system once $n > 2$. Therefore, the approach relies on the concept of closeness, that is, those coefficients are obtained, that are closest to the original ones while at the same time obeying the new row and column sums (see e.g. Lecomber, 1975).

The RAS approach, however, cannot directly be implemented within disaggregated econometric models mainly because information needed to make the approach operable (i.e. sectoral row and column sums) has to be derived from the models which in turn rely on input-output tables. In the literature only a few applications of updating procedures within econometric I-O models can be found. Among those is the work of Conway (1990), which is described and applied also in Israilevich et. al. (1996) and which is also the basis of the updating procedure applied in the models described here. Ciaschini (1983) and Nyhus (1983) provide a slightly modified methodology of updating coefficients which was adopted within the family of INFORUM models. Most of these methods use hypothetical or predicted output calculated with fixed technical coefficients and time series data of output and then adjust the technical coefficients to be consistent with actual output data and with certain *a priori* restrictions. This method to a certain extent introduces a bias towards an adjustment 'along the row' of the technical coefficient matrix (the *absorption effect* in terms of the RAS methodology), whereas the column data mostly only enter in the form of restrictions. Implicit changes in the matrix of technical coefficients can be found in CGE models with factor input functions, mostly at the level of total intermediate inputs. This method is biased towards an adjustment 'along the column' assuming an equiproportional adjustment for all inputs in a certain industry, according to the factor S in RAS (*fabrication effect*).

In this paper we propose a combination of both methods yielding a bi-proportional method without fully integrating RAS. The first step is a description of the widely used adjustment 'along the row' mechanism. In this method predicted or hypothetical output is adjusted to actual output in econometric equations. As an alternative we put forward our bi-proportional adjustment mechanism consisting of (i) factor demand functions for energy and other intermediates as adjustment 'along the column' and (ii) new econometric specifications of adjustment of predicted (hypothetical) intermediate demand (by commodities, i.e. along the row) to actual intermediate demand. Our empirical applications show the relative importance of the column and row adjustment by looking at predictive capacity for the rows, if only the adjustment 'along the column' is applied and for the columns, if in the second step the adjustment 'along the row' is applied. From this comparison of the relative importance we derive conclusions for future research.

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[1] The term ‘updating’ here means the inclusion of recent data to obtain a better approximation of any matrix at time *t* than the same matrix at some earlier point in time (*t*-*k*) can provide.

1. An adjustment method 'along the row'

The starting point of the traditional adjustment 'along the row' used for example in regional input-output models as described in Conway (1990) and Israilevich et. al. (1996) is the commodity balance of the I-O model[2]. The total goods supply vector **Q** is made up of the imports vector **M** and the vector of domestic output **QA** equal to total demand, where **QH** is the intermediate demand vector and **F** is the final demand vector:

(1) $$\mathbf{Q} = \mathbf{QA} + \mathbf{M} = \mathbf{QH} + \mathbf{F}.$$

Introducing the technical coefficients matrix **A** (the sum of domestic and imported elements), **QH** can be substituted by the product of **A** and **QA**, where **F(M)** now stands for the final demand vector **F** minus the imports vector **M** :

(2) $$\mathbf{QA} = \mathbf{A} * \mathbf{QA} + \mathbf{F(M)},$$

with intermediate demand matrix **V** transformed to technical coefficients **A** = $[a_{ij}]$ as:

$a_{ij} = V_{ij}/QA_j$,

or in matrix notation:

(3) $$\mathbf{A} = \mathbf{V} * \mathrm{diag}(\mathbf{QA})^{-1}.$$

The vector **F(M)** can be thought of having a *k* dimension of final demand components, including private and public consumption, gross capital formation, exports, stock changes as well as imports. Out of this new vector **F(M)** a coefficient matrix **AF** = $[af_{ik}]$ is formed, whose elements are computed as $af_{ik} = fd_{ik}/ f_k'$, where $[f_k']$ are the elements of the transposed summation vector of final demand. Again, writing this relationship in matrix notation yields:

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[2] Both Conway (1990) and Israilevich et. al. (1996) actually apply the quadratic industry by industry matrix of technical coefficients within their methodology. The approach outlined here starts from the intermediate demand matrix (the USE matrix within the Make-Use system), therefore the approach of the aforementioned authors is also outlined along these lines.

(4) $\mathbf{AF} = \mathbf{F(M)} * \text{diag}(\mathbf{f})^{-1}$

Given matrices **A** and **AF** the following basic input-output relationship can be stated:

(5) $\mathbf{A} * \mathbf{QA} + \mathbf{AF} * \mathbf{F(M)} = \mathbf{QA}$.

In order to account for changes in both matrices **A** and **AF** over time, their coefficients serve as equilibrating forces in the quantity adjustment process of the models. The basic idea is to form a deterministic predictor of output **QA** over the entire historical period out of equation (5). Introducing time to the notation in (5) yields:

(6) $\mathbf{A_{t(0)}} * \mathbf{QA_t} + \mathbf{AF_{t(0)}} * \mathbf{F(M)_t} = \mathbf{QA^H_t}$.

In (6) the output vector $\mathbf{QA_t}$ is replaced by a corresponding deterministic predictor $\mathbf{QA^H_t}$, which is termed *predicted* or *hypothetical* output. It becomes immediately obvious from the notation that the time series of predicted output is obtained by inserting the actual values of output and final demand while the coefficient matrices **A** and **AF** remain constant at their base year levels (t(0)). The values of $\mathbf{QA_t}$ and $\mathbf{QA^H_t}$ will coincide only in this base year, but are likely to diverge from each other in every other year. The reason for this is due to the fact, that the variations in domestic output and final demand will not be sufficient to explain the entire variation in (6) due to unobserved changes in the coefficient matrices. Hence, the difference between output and its predicted counterpart can be attributed to the alterations taking place in the coefficients of **A** and **AF**. Now, the relationship of $\mathbf{QA^H_t}$ and $\mathbf{QA_t}$ over time can be stated as:

(7) $\mathbf{R_t} * \mathbf{QA^H_t} = \mathbf{QA_t}$,

where $\mathbf{R_t}$ is a diagonal matrix. The aim is to alter (and hence update) **A** and **AF** such that the entire variation in (6) is explained. Premultiplying both sides of (7) with $\mathbf{R_t}^{-1}$ and inserting the result for $\mathbf{QA^H_t}$ into (6) yields:

$$(8) \qquad \mathbf{A_{t(0)} * QA_t + AF_{t(0)} * F(M)_t = R_t^{-1} * QA_t,}$$

and

$$(9) \qquad \mathbf{R_t * A_{t(0)} * QA_t + R_t * AF_{t(0)} * F(M)_t = QA_t.}$$

That is, the coefficients of both matrices are updated at time *t* with a fixed factor along the rows derived from matrix $\mathbf{R_t}$. Note, that this ‘correction matrix’ $\mathbf{R_t}$ can be seen as the analogue to the first component of the RAS-approach of updating I-O-coefficients. Following the economic interpretation given by Stone and Brown (1962), it can be said that because $\mathbf{R_t}$ pre-multiplies **A** and **AF**, the unexplained variation from (6) is attributed to the technology of producing the output (row-wise multiplication with a constant, see also Snower (1990)).

In order to implement the approach within an overall model, the relationship between actual and hypothetical output is estimated econometrically within a separate block of equations, one for each industry under consideration. That is, the elements of the adjustment matrix $\mathbf{R_t}$ are derived via:

$$(10) \qquad QA_t = F(QA^H_t).$$

Both Conway (1990) and Israilevich et. al. (1996) apply log-linear models to assess the relationship between hypothetical and actual output.

The outlined method is widely used in large scale regional I-O models and can be seen as a pure 'along the row' adjustment mechanism. It is well known (see e.g. Snower (1990)), that the adjustment factors R and S in RAS have been given an economic interpretation as 'absorption' and 'fabrication' factors. The pure adjustment along the row puts the emphasis only on the absorption factor, thereby assuming that technical change can be fully accounted for by considering that certain inputs all change in a similar way in the production of different sectors.

2. Adjustment 'along the column' : factor input functions

Starting again from the economic interpretation of RAS, a method that only considers adjustment along the column puts the emphasis on the 'fabrication' effect, i.e. it is assumed that different industries witness different changes in the overall productivity of their inputs, but that single inputs are affected in a proportional way. Such technical changes are implicitly assumed in CGE models, where KLEM production or cost functions are applied. Only in some cases fully endogenized I-O models for disaggregated types of inputs are formulated as in Tokutsu (1994). In most models substitution only takes place at the aggregated input level of capital, energy, other materials (intermediate inputs) and labour and repercussions on the single input level are assumed to be proportional.[3]

In the Austrian disaggregated macroeconomic model MULTIMAC similar cost functions have been introduced. Starting point is the separate treatment of energy transactions, such that all matrices and vectors can be split into an energy (e) and a non-energy (ne) part. The commodity balance for non-energy therefore becomes:

$$(11) \quad \mathbf{Q_{ne}} = \mathbf{A_{ne}} * \mathbf{QA} + \mathbf{F_{ne}}.$$

The technical coefficients matrix $\mathbf{A_{ne}}$ comprises the non-energy input in non-energy sectors as well as the non-energy input in energy sectors; **QA** is the total output vector (energy and non-energy).

At a first stage changes along the column are introduced. The sum of intermediate demand of an industry *i* , V_i gives the column restriction, as data for V are given in time series from National Accounts. The total input coefficient V_i/QA_i can now be modelled with factor demand functions or input functions. In MULTIMAC Generalized Leontief – cost functions with an extension for technical progress are chosen where the variable factors are the inputs of intermediate demand of an industry, V, with price p_v and labour input L with wage rate w,

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[3] Actually most studies do not fully describe all the implicit assumptions of input substitution for the technical change in the input-output matrices.

and a deterministic trend t representing technological progress. Starting point is the (short term) cost function for variable costs G:

$$(12) \qquad G = QA\left[\sum_i \sum_j a_{ij}\left(p_i p_j\right)^{1/2} + \sum_i d_{it}\, p_i t^{1/2} + \sum_i g_{tt}\, p_i t\right],$$

with p_i, p_j as the input prices of the variable factors.

Applying Shephard's Lemma we can derive factor demands, since the partial derivatives of (1) with respect to factor prices (p_v, w) yield the input quantities (V, L) :

$$(13) \qquad \frac{V}{QA} = \alpha_{VV} + \alpha_{VL}\left(\frac{w}{p_V}\right)^{1/2} + \gamma_{Vt} t^{1/2} + \gamma_{tt} t \,,$$

$$(14) \qquad \frac{L}{QA} = \alpha_{LL} + \alpha_{VL}\left(\frac{p_V}{w}\right)^{1/2} + \gamma_{Lt} t^{1/2} + \gamma_{tt} t \,.$$

The total input coefficient V_i/QA_i therefore changes due to changes in factor prices as well as technological change (deterministic trend).

Additionally substitution between energy and other intermediate inputs is further considered by modelling the energy demand by industries (see Kratena and Schleicher (2000)). This determines technological change in the sum of energy inputs $\sum a_e$ in each industry.

Therefore the sum of non-energy inputs (along the column) is given by:

$$(15) \qquad \sum a_{ne} = V / QA - \sum a_e \,,$$

which enters as the column restriction of the model.

The single coefficients of the technical coefficients matrix **A** = [a_{ij}] are given by the assumption of a fixed structure along the column comprised in a matrix $\Gamma = [\gamma_{ij}]$, such that multiplying the column sum of non energy coefficient $a_{ne,j}$ with the single elements of Γ yields the technical coefficients:

$$(16) \qquad a_{ij} = a_{ne,j} * \gamma_{ij}.$$

A side aspect of this approach is that the fixed structure within the column given by Γ also determines the feedback of the technical change on the price of intermediate demand p_v (see Kratena and Zakarias (2001)).

This final result of this method is a new matrix of non energy technical coefficients, $\mathbf{A_{ne}}$, as used in the commodity balance (11).

3. Adjustment 'along the row' : hypothetical and actual intermediate demand

The new matrix of non energy technical coefficients, $\mathbf{A_{ne}}$, represents the starting point for the second step, i.e. the adjustment 'along the row'. Therefore we use information from national accounts on final demand structures and imports. This information is combined with slightly changing structures of the bridge matrices which are derived from interpolations between input-output base years. This allows us to depart from the usual approach as applied by Conway (1990) and Israilevich et. al. (1996) and to assume that we can calculate actual intermediate demand as:

(17) $$\mathbf{QH_{ne,t} = QA_{ne,t} + M_{ne,t} - F_{ne,t}}.$$

This actual intermediate demand can be compared with a series of hypothetical intermediate demand for non-energy sectors ($\mathbf{QH_{ne,t}^{H}}$) calculated by applying the matrix of non energy technical coefficients, $\mathbf{A_{ne}}$, within the adjustment 'along the column' as described above:

(18) $$\mathbf{QH_{ne,t}^{H} = A_{ne,t} * QA_{t}}.$$

As already mentioned above both Conway (1990) and Israilevich et. al. (1996) modelled actual output within log-linear specifications. While Conway applies both series in levels and accounts for first order autocorrelation, Israilevich et. al. estimate the fraction of the two variables in first differences.

In MULTIMAC a different approach is pursued in this regard. In order to provide clear treatment of the underlying long run relationship between the two variables of interest, the econometric specification is carried out along the lines of the two step method proposed by Engle and Granger (1987) in estimating an error correction model. The first step in estimating such a system is to determine the order of integration of the respective time series. For the present application the Augmented Dickey-Fuller test was used to determine the number of unit roots within the series. The test results using various variants of the ADF test (including constant or trend or both as well as different numbers of lags) for both series are shown in tables A1 to A4 in the appendix. The results indicate that both series can be said to be I(1) in almost every industry. At this point it must, however, be mentioned, that the ADF tests applied are subject to a considerable small sample bias. To overcome these problem, some authors such as Banerjee et. al. (1986) and Kremers et. al. (1992) recommended to test the significance of the residual of (19) in the final error correction model (20) stated below by means of a standard t-test. The hypothesis of a cointegrating relationship is hence accepted if the estimated parameter of the error correction term in (20) is significantly (and negatively) different from zero, which is the case for each industry. Hence estimation of the error correction model was continued for every industry. The long run equation can be stated as follows:

$$(19) \qquad \log\left(QH_{ne,i,t}\right) = \alpha_i + \beta_i \log\left(QH^{H}_{ne,i,t}\right) + \varepsilon_{i,t}$$

The estimated value of β_i is crucial in determining the log run behaviour of the adjustment factor appearing in the *i*-th main diagonal of the adjustment matrix $\mathbf{R_t}$ from equation (9). An estimated value close or equal to one indicates that the underlying coefficients do not change over time. Consequently a value larger than one indicates rising overall demand of the respective good over time and vice versa. The magnitude of the estimated parameter of course also determines the amount by which the coefficients will be altered each time period. This can also be seen from figure 1 below, which shows the course of actual and hypothetical output for industry *Research and Development, Business Services* (which is industry 34 in the models terminology) over time.

Figure 1: Hypothetical and actual intermediate output for *Research and Development, Business Services*

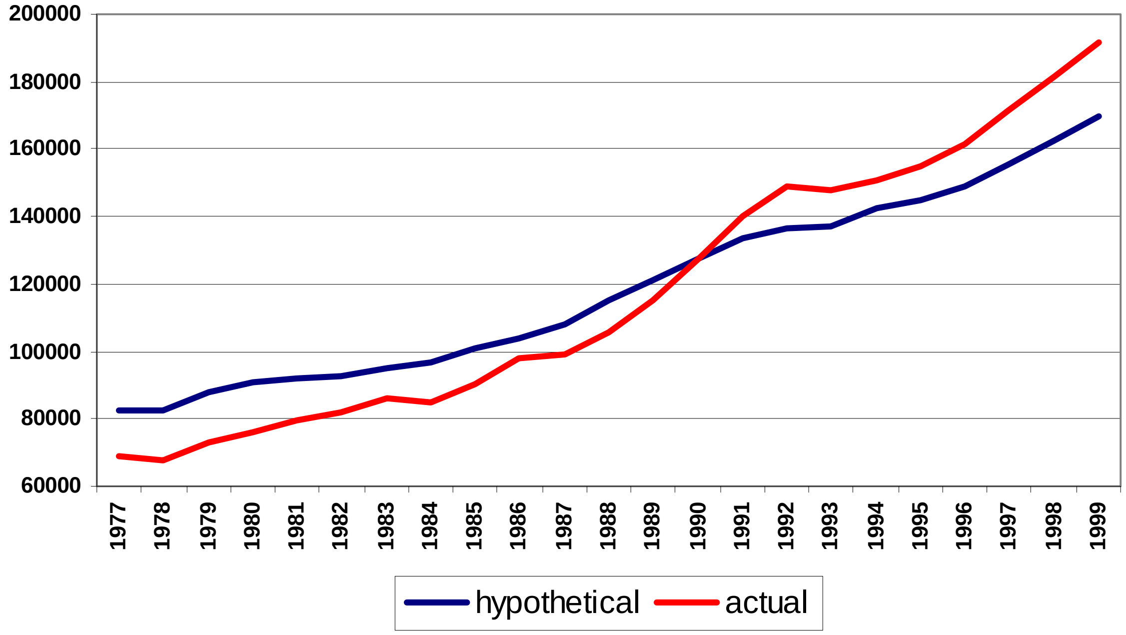


Figure 1 shows that actual output is generally steeper than hypothetical output for the series under investigation, while the two series take the same value in 1990 (as required by definition). The plot also indicates, that the respective good is growing in importance over time. This can easily be seen by examining e.g. the year 1977 more closely. The hypothetical output series shows the demand for *Research and Development, Business Services* in every year assuming the Input-Output relations from 1990. Obviously actual demand for the good under investigation was lower than hypothetical demand in 1977, indicating, that the good was actually less important than it was in 1990. In the years after 1990 the picture consequently changes and the series of actual output is above hypothetical output. The growing importance of *Research and Development, Business Services* is of course also reflected in the estimated value of β_{34}, which is equal to *1.64* (see table A5 in the appendix) and statistically significantly larger than one.

The estimate of the long run relationship in equation (19) is accepted as establishing an cointegrated relationship if the residuals do not exhibit serial correlation. Again an ADF test is used to test the residual series. The results, which generally show that the resulting residual series are indeed I(0), are shown in table A6 in the appendix. Except for industries 17 and 24 the results are clearly in favour of a stationary behaviour of the residual series. However, both in industry 17 and 24 the estimated value of the parameter for the error correction term turned out to be significantly different from zero, and hence the Engle-Granger two step methodology is also applied in these cases. The error correction model is expressed as follows:

$$(20) \quad \Delta \log\left(QH_{ne,i,t}\right) = \delta_i + \lambda_i \Delta \log\left(QH^{H}_{ne,i,t}\right) + \gamma_i\left(\varepsilon_{i,t-1}\right) + \mu_{i,t} \,.$$

The third term on the right hand side is the so called error correction term, whose estimated parameter must necessarily be negative. In case of deviations of the two series from their long run trend, this term – as its name implies – corrects for these short run fluctuations by forcing the relationship back towards the long run path.

4. Empirical Results

Since the primary goal of the proposed approach within MULTIMAC is to provide a better approximation of the column sums of the matrix to be updated (due to the additional feature of a one time adjustment of the row sum according to separately estimated values from Generalized Leontief cost functions), the empirical investigations will concentrate on assessing this issue.

Because of the usual adjustment along the rows of the matrix is taking place *after* the column adjustment in MULTIMAC, the column sums will again deviate from the sums estimated within the cost functions. That is, the column sums of the updated matrix will in general not to be equal to the share of total intermediate demand per industry as estimated within the cost functions. This is of course also true for the traditional approach applied in the literature, however, the new approach should provide an improvement in meeting this target.

To implement this experiment, two different versions of MULTIMAC were set up, one including the proposed approach and the other implementing the method of Conway (1990). In order to compare the results, the second model consists of the same equations and data as the first one, except of course for the updating itself.

Given the two models, the respective adjustment factors, that is the elements of $\mathbf{R_t}$, were derived via relationship (7). Given these estimates the elements of the coefficient matrix for intermediate demand, $\mathbf{A_{ne}}$, are easily derived, which was done for the year 1995 in the following experiment. Summing up the coefficients over each column yields the estimated value of the summation vector of $\mathbf{A_{ne}}$, which can be compared to the intermediate demand coefficient estimated in the cost function. This coefficient is equal to the true observed values as obtained from National Accounts, since the experiment is conducted for a year within the historical period of our model. To ensure the outcome of the cost function is equal to the true value, we included the residuals series within the equation. The following table provides the results of the experiment for both models.

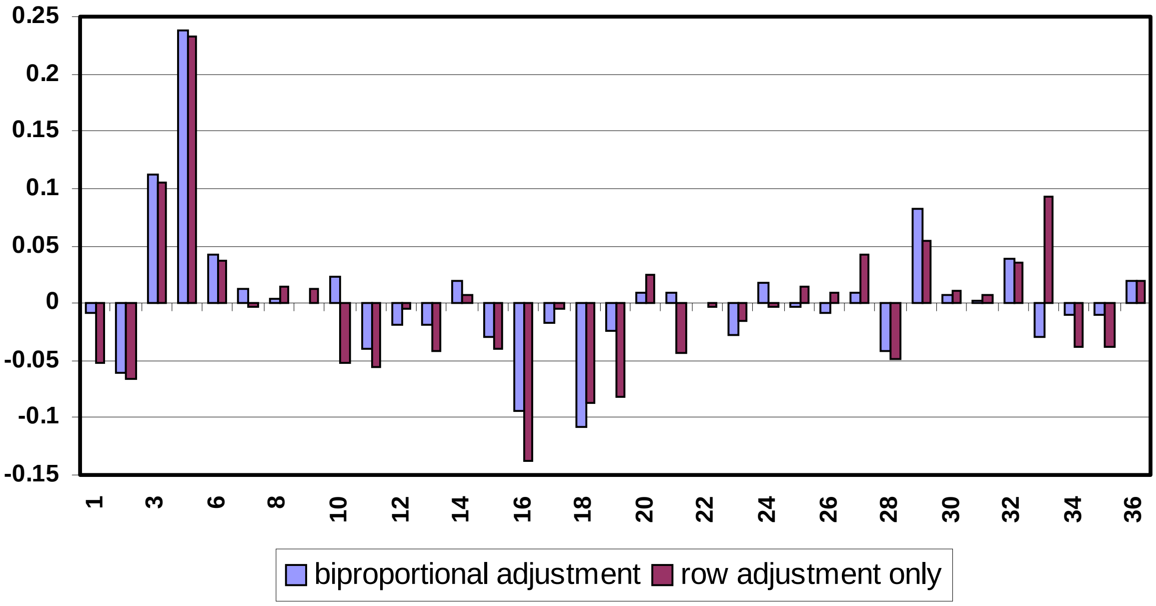
Table 1: Deviations of the column sum of the updated matrix from the estimated total input coefficient given by the cost function, 1995

	s.s.d.[1]	$\sum \lvert dev. \rvert$	Mean	Variance
Column and row adjustment	0,11	1,19	0,0025	0,0032
Row adjustment only	0,14	1,54	-0,0028	0,0030

[1] *s.s.d denotes the sum of squared deviations*

The sum of squared deviations is 0,11 in the first model and 0,14 in the second, which confirms the presumption that the new approach is performing better on the selected criterion of meeting the values from the column sums. The new method hence provides a better approximation to the true column sum than the usual adjustment 'along the row'. The difference is, however, not very large, at least in terms of the squared deviations. Since the ratio of the sum over the absolute value of the deviations between the models is almost the same as for the sum of squared deviations (around 0.77 in both cases), the result does obviously not depend on outliers among the industries. This outcome is also confirmed by a visual inspection of the deviations per industry as plotted in figure 2 below.

Figure 2: Deviations per industry, 1995



5. Conclusions and Future Research

This paper proposed an alternative way of approaching the issue of updating IO coefficients within multisectoral econometric models, by supplementing the usual row adjustment with a one time adjustment of the column sums of the matrix under investigation. The respective column sums are derived from sectoral Generalized Leontief cost functions, which yield the share of total intermediate demand in output for each industry. The empirical results indicate that the new approach provides an improvement in updating the matrix under investigation, since the deviations from the given column sum are smaller than in the traditional approach. There appear at least two issues for future research, which are both related to the fact that the proposed approach still lacks the implementation of a full bi-proportional adjustment. One possible way in resolving this would be to incorporate further feedback effects after the row sum was adjusted, since this causes the column sum to deviate from the summation coefficient estimated in the cost functions. This would, however, introduce a considerable increase in the interdependencies of the overall system and it is not yet clear whether the convergence could still be achieved. Another possibility would therefore be to further exploit the time series behaviour of the intermediate demand vector $\mathbf{QH_{ne}}$. The changes in the importance of certain single elements of this vector could provide information which could be used to adjust the structure *within* the column of certain industries - e.g. of those industries exhibiting comparatively large deviations from the row sum as obtained from the cost functions.

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Appendix

The appendix first shows the results of various ADF tests for unit roots in the time series of actual and hypothetical intermediate demand as well as the residuals from the long run equation (19) above. The notation *n* in the tables denotes the usual ADF test including neither a constant nor a trend. Both variants including only a constant (*c*) and including both a constant and a trend (t) are also shown. The test was conducted including no lag as well as one and two lags, where for the sake of brevity the results of only the first two variants (lag 0 and 1) are displayed. The stated critical values are the so called MacKinnon critical values for the rejection of the hypothesis of a unit root in the investigated series. The tests were conducted within Eviews 3.1. Table A6 finally shows the industry classification adopted within MULTIMAC along with the corresponding two-digit NACE codes.

Table A1: ADF statistic for hypothetical intermediate demand (QH_{ne}^{H})

	lag 0			lag 1		
Industry	n	c	t	n	c	t
1	2.60	-0.60	-1.69	2.66	-1.52	-1.85
7	6.76	0.77	-3.90	4.93	0.77	-3.22
8	4.16	1.48	-0.74	2.03	0.46	-1.55
9	5.31	0.25	-1.17	2.51	0.69	-2.06
10	5.30	0.85	-1.28	2.72	0.62	-2.14
11	5.54	1.65	-1.29	2.58	0.52	-1.61
12	4.95	1.02	-1.92	2.77	0.18	-2.02
13	9.64	3.30	-2.22	4.31	1.78	-0.71
14	6.97	2.26	-0.46	3.78	2.03	-0.67
15	5.57	2.19	-0.42	2.62	1.01	-0.99
16	3.60	-0.53	-1.58	2.97	-2.34	-2.81
17	1.47	-0.42	-2.51	1.14	-0.93	-4.00
18	4.65	1.15	-2.48	3.24	0.86	-2.50
19	5.00	0.20	-1.74	2.30	-0.37	-2.41
20	9.04	2.55	-1.77	3.49	1.21	-1.07
21	6.86	1.70	-1.27	2.77	0.64	-1.78
23	8.33	1.53	-1.67	3.56	1.05	-1.73
24	9.04	1.54	-1.33	4.72	1.73	-1.33
25	8.96	2.16	-1.30	3.21	0.92	-1.37
26	7.27	0.64	-2.07	2.91	0.38	-2.52
27	10.61	2.76	0.03	3.18	2.10	-0.68
28	4.49	-0.45	-2.32	2.48	-0.41	-2.73
29	11.17	3.50	0.58	4.75	3.34	0.25
30	10.38	4.39	0.25	3.35	2.12	0.89
31	10.29	3.25	-0.90	3.90	1.71	-0.30
32	10.65	2.47	-1.79	4.23	1.63	-1.16
33	11.03	3.80	-1.61	4.48	2.05	-0.36
34	9.73	2.41	-1.89	3.68	1.06	-1.14
35	6.93	1.53	-2.74	3.45	0.18	-2.11
36	11.79	2.85	0.00	3.91	2.44	-0.62
1% Critical Value	-2.67	-3.75	-4.42	-2.68	-3.77	-4.44
5% Critical Value	-1.96	-3.00	-3.62	-1.96	-3.00	-3.63
10% Critical Value	-1.62	-2.64	-3.25	-1.62	-2.64	-3.25

Table A2: ADF statistic for first difference of hypothetical intermediate demand (ΔQH_{ne}^{H})

	lag 0			lag 1		
Industry	n	c	t	n	c	t
1	-3.42	-4.87	-4.93	-1.68	-2.50	-2.40
7	-2.15	-6.15	-6.21	-1.13	-6.19	-6.42
8	-2.13	-3.00	-3.20	-2.14	-3.16	-3.83
9	-2.43	-3.56	-3.74	-1.28	-2.11	-2.22
10	-2.05	-3.50	-3.62	-1.16	-2.51	-2.59
11	-1.77	-3.20	-3.29	-1.77	-3.36	-3.96
12	-2.11	-3.77	-3.69	-1.74	-3.43	-3.55
13	-0.85	-3.83	-4.42	-0.46	-2.89	-4.31
14	-1.84	-3.63	-4.57	-0.84	-2.47	-3.43
15	-1.54	-2.85	-3.21	-1.39	-2.96	-3.58
16	-2.40	-4.55	-4.91	-1.53	-2.39	-2.25
17	-3.54	-3.77	-3.70	-3.38	-4.04	-3.86
18	-2.52	-4.26	-4.56	-1.45	-3.12	-3.41
19	-1.83	-3.32	-3.22	-1.39	-2.90	-2.82
20	-0.64	-3.22	-3.55	-0.22	-2.40	-2.92
21	-1.42	-3.11	-3.24	-1.14	-2.88	-3.15
23	-1.37	-3.78	-4.06	-0.47	-2.36	-2.60
24	-1.54	-4.61	-5.29	-0.69	-3.35	-4.16
25	-0.99	-3.21	-3.41	-0.69	-2.76	-3.16
26	-1.48	-3.87	-3.86	-0.86	-3.34	-3.29
27	-0.67	-2.34	-3.33	0.10	-1.23	-2.11
28	-2.38	-4.36	-4.25	-1.62	-3.94	-3.87
29	-1.13	-2.93	-4.86	0.35	-1.23	-2.42
30	0.14	-1.92	-2.83	0.53	-1.05	-2.22
31	-0.58	-3.17	-3.77	-0.14	-2.08	-3.00
32	-0.87	-3.90	-4.48	-0.31	-3.23	-4.01
33	-0.64	-3.63	-4.42	-0.21	-2.06	-3.41
34	-0.78	-3.65	-3.85	-0.39	-2.59	-3.05
35	-0.96	-4.08	-3.93	-0.51	-2.49	-2.58
36	-0.82	-2.77	-4.00	-0.13	-2.29	-3.49
1% Critical Value	-2.68	-3.77	-4.44	-2.68	-3.79	-4.47
5% Critical Value	-1.96	-3.00	-3.63	-1.96	-3.01	-3.65
10% Critical Value	-1.62	-2.64	-3.25	-1.62	-2.65	-3.26

Table A3: ADF statistic for actual intermediate demand (QH_{ne})

	lag 0			lag 1		
Industry	n	c	t	n	c	t
1	2.62	0.07	-2.20	3.21	0.32	-1.45
7	0.76	-1.10	-3.26	1.06	-1.17	-2.98
8	1.04	-0.61	-2.22	1.43	0.18	-1.29
9	1.07	-0.68	-1.90	0.81	-0.80	-2.22
10	0.83	-1.88	-1.70	0.89	-1.54	-1.12
11	1.39	-0.06	-2.04	1.74	0.31	-1.66
12	0.13	-2.10	-3.43	1.47	-0.32	-1.38
13	0.32	-0.58	-3.64	0.32	-0.60	-4.00
14	1.75	-0.89	-3.67	2.26	-0.33	-2.89
15	-0.10	-3.02	-3.14	0.08	-2.88	-3.01
16	2.34	-1.38	-2.11	1.90	-1.09	-2.01
17	-2.83	-1.57	-2.24	-2.53	-1.68	-2.47
18	1.72	0.15	-2.51	1.56	0.14	-3.01
19	3.27	0.81	-1.60	2.99	0.96	-1.37
20	2.13	-2.27	-1.22	1.42	-1.82	-1.13
21	2.39	0.71	-0.71	1.54	0.39	-1.16
23	-0.84	-2.33	-2.09	-0.64	-1.19	-0.32
24	0.10	-2.68	-3.10	0.41	-1.32	-1.80
25	1.42	-0.98	-3.87	1.91	-0.75	-3.16
26	-2.57	0.44	-2.34	-1.93	0.37	-3.08
27	-0.52	-0.62	-0.65	-0.25	-1.62	-1.49
28	4.81	2.55	0.20	2.91	1.93	-0.25
29	-0.60	-1.59	-0.86	-0.67	-1.82	-1.12
30	5.89	3.69	2.34	2.88	3.00	2.25
31	3.90	-0.36	-2.89	2.29	-0.16	-3.96
32	6.02	0.30	-2.77	3.18	0.17	-3.16
33	4.70	1.89	-1.47	1.27	0.54	-1.74
34	6.67	1.78	-1.18	2.43	1.07	-2.42
35	2.13	-0.10	-2.49	1.29	-0.69	-3.18
36	0.94	-0.86	-1.22	0.54	-0.91	-1.73
1% Critical Value	-2.67	-3.75	-4.42	-2.68	-3.77	-4.44
5% Critical Value	-1.96	-3.00	-3.62	-1.96	-3.00	-3.63
10% Critical Value	-1.62	-2.64	-3.25	-1.62	-2.64	-3.25

Table A4: ADF statistic for first difference in actual intermediate demand (ΔQH_{ne})

	lag 0			lag 1		
Industry	n	c	t	n	c	t
1	-4.52	-6.34	-6.30	-2.17	-3.51	-3.51
7	-5.30	-5.53	-5.41	-4.12	-4.69	-4.60
8	-5.99	-6.28	-6.47	-3.31	-3.64	-3.94
9	-3.91	-4.00	-3.90	-3.41	-3.77	-3.61
10	-5.35	-5.48	-5.65	-3.35	-3.55	-3.86
11	-4.85	-5.38	-5.58	-4.04	-4.93	-6.17
12	-8.77	-9.20	-9.06	-3.11	-3.31	-3.37
13	-5.58	-5.89	-5.81	-5.58	-6.31	-6.29
14	-5.06	-6.34	-6.19	-3.04	-4.43	-4.32
15	-5.29	-5.21	-5.11	-4.88	-4.82	-4.81
16	-3.74	-4.60	-4.50	-2.67	-3.54	-3.43
17	-3.83	-4.57	-4.60	-4.24	-5.89	-6.05
18	-4.04	-4.50	-4.65	-3.70	-4.50	-4.97
19	-3.54	-4.91	-5.23	-2.03	-3.18	-3.61
20	-3.16	-3.84	-4.11	-2.03	-2.52	-2.71
21	-3.17	-3.58	-3.91	-2.55	-3.13	-3.53
23	-7.33	-7.20	-7.64	-3.29	-3.24	-3.68
24	-7.68	-7.57	-7.60	-4.71	-4.69	-4.97
25	-5.71	-6.58	-6.42	-4.40	-7.05	-7.03
26	-3.34	-4.14	-4.34	-2.25	-3.09	-3.45
27	-3.26	-3.17	-3.78	-2.81	-2.74	-3.76
28	-2.42	-3.24	-4.11	-0.86	-1.76	-2.44
29	-4.11	-4.01	-4.32	-2.82	-2.74	-3.26
30	0.25	-0.79	-1.96	0.48	-0.62	-2.35
31	-2.69	-3.93	-3.80	-2.46	-6.17	-6.04
32	-1.79	-4.41	-4.33	-0.71	-3.58	-3.48
33	-1.31	-2.37	-2.71	-0.83	-1.67	-1.97
34	-1.19	-2.52	-2.96	-0.86	-2.80	-3.07
35	-3.10	-4.71	-4.58	-2.28	-3.78	-3.68
36	-3.60	-3.67	-3.59	-2.47	-2.57	-2.52
1% Critical Value	-2.68	-3.77	-4.44	-2.68	-3.79	-4.47
5% Critical Value	-1.96	-3.00	-3.63	-1.96	-3.01	-3.65
10% Critical Value	-1.62	-2.64	-3.25	-1.62	-2.65	-3.26

Table A5: ADF statistic for the residuals of the long run equation ($\varepsilon_{i,t}$)

	lag 0			lag 1		
Industry	n	c	t	n	c	t
1	-4.31	-4.40	-5.03	-2.45	-2.52	-3.26
7	-2.47	-2.91	-2.80	-2.34	-2.65	-2.83
8	-2.48	-2.86	-3.83	-1.19	-1.51	-2.43
9	-2.91	-3.19	-3.07	-2.68	-2.98	-2.97
10	-3.82	-3.76	-3.73	-3.01	-2.96	-2.94
11	-2.94	-2.91	-2.89	-2.06	-2.03	-2.03
12	-3.99	-4.45	-5.21	-2.08	-2.43	-3.16
13	-5.67	-5.65	-6.53	-3.72	-4.18	-4.33
14	-2.17	-2.51	-3.21	-1.05	-1.20	-2.50
15	-5.12	-5.07	-5.10	-4.26	-4.25	-4.40
16	-4.51	-4.44	-4.37	-4.11	-4.04	-3.99
17	-0.43	-1.69	-1.63	-0.65	-1.75	-2.39
18	-3.99	-4.19	-4.25	-3.36	-3.60	-3.75
19	-5.79	-5.73	-5.72	-5.10	-5.04	-5.18
20	-6.23	-6.49	-6.38	-3.90	-4.15	-4.20
21	-5.76	-6.01	-5.83	-3.77	-4.05	-3.97
23	-5.98	-6.31	-6.58	-3.12	-3.42	-3.66
24	-1.15	-1.23	-1.51	-1.47	-1.46	-2.05
25	-3.60	-3.83	-4.59	-2.46	-2.73	-3.43
26	-2.99	-2.95	-2.91	-3.90	-3.86	-3.86
27	-3.73	-3.78	-3.73	-3.54	-3.50	-3.67
28	-2.74	-2.70	-2.65	-2.07	-2.05	-2.05
29	-3.81	-3.85	-4.24	-3.57	-3.63	-4.32
30	-3.54	-3.51	-3.48	-2.72	-2.68	-2.63
31	-4.58	-4.53	-4.47	-3.98	-3.91	-3.82
32	-2.73	-2.64	-4.97	-1.61	-1.64	-4.08
33	-4.72	-4.77	-4.06	-2.01	-2.10	-1.87
34	-2.99	-3.06	-3.18	-3.74	-3.96	-3.99
35	-2.39	-3.18	-3.00	-1.16	-1.80	-1.77
36	-2.68	-2.74	-2.63	-2.51	-2.62	-2.55
1% Critical Value	-2.63	-3.64	-4.26	-2.64	-3.65	-4.27
5% Critical Value	-1.95	-2.95	-3.55	-1.95	-2.96	-3.56
10% Critical Value	-1.62	-2.61	-3.21	-1.62	-2.62	-3.21

Table A6: The industry classification in MULTIMAC

	Model Industry	2-digit NACE codes
1	Agriculture, Forestry and Fishing	1,2,5
2	Mining of Coal and Lignite	10
3	Extraction of Crude Petroleum and Natural Gas	11
4	Gas Supply	
5	Manufacture of Refined Petroleum Products	23
6	Electricity, Steam and Hot Water Supply	40
7	Collection, Purification and Distribution of Water	41
8	Ferrous & Non Ferrous Metals	27
9	Non-metallic Mineral Products	13, 14, 26
10	Chemicals	24
11	Metal Products	28
12	Agricultural and Industrial Machines	29
13	Office Machines	30
14	Electrical Goods	31, 32
15	Transport Equipment	34,35
16	Food and Tobacco	15, 16
17	Textiles, Clothing & Footwear	17, 18, 19
18	Timber & Wood	20
19	Paper	21
20	Printing Products	22
21	Rubber & Plastic Products	25
22	Recycling	37
23	Other Manufactures	33, 36
24	Construction	45
25	Distribution	50, 51, 52
26	Hotels and Restaurants	55
27	Inland Transport	60
28	Water and Air Transport	61, 62
29	Supporting and Auxiliary Transport	63
30	Communications	64
31	Bank, Finance & Insurance	65, 66, 67
32	Real Estate	70, 71
33	Software & Data Processing	72
34	R&D, Business Services	73, 74
35	Other Market Services	92, 93, 95
36	Non-market Services	75, 80, 85, 90, 91
37	Statistical Differences	