[1]The Nonlinear Important Coefficients Input-Holding-Output Model

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ABSTRACT *In this paper nine input-holding--output tables at current price and constant price with 18 sectors for China during 1973-1999 are designed and compiled. Field of influence method is used to choose 47 important coefficients from nine Chinese input-holding-output tables. Using per employee occupied original value of fixed assets as independent variable, the nonlinear expression of each main coefficient is obtained by nonlinear regression method. The numeric experiment shows the prediction error of the models is smaller than RAS method.*

KEYWORDS: I*nput-holding-output table, important coefficient, nonlinear regression*

1. Foreword

Traditional input-output model is based on the homogeneity assumption and proportionality assumption. This makes the input-output model compact and convenient in national economy analysis, planning and forecasting in short time. But it is difficult when it is used to make long-time forecasting and planning. The reason is that there is big difference between homogeneity assumption, proportionality assumption and real economic situation. Law of diminishing marginal utility and increasing returns to scale rule lead to the nonlinear relation in real production. The nonlinear input-holding-output model should be researched to make input-output analysis more reasonable.

The nonlinear input-holding-output model research needs serial input-output tables in long period with the same sector classification. Because to compile a piece of national input-output table cost greatly, China compiles a piece of input-output table every five years. The sector classification and accounting system of these tables are changed frequently. The situation is alike in other countries. This makes the research on nonlinear input-output model progress slowly. Many scholars' work on nonlinear input-output model in the world is summed up as followed. Finally the nonlinear important coefficients input-holding-output model is presented, the forecasting precision of the model is contrasted with RAS method and its merits are evaluated.

2. Kinds of Nonlinear Input-Output Models

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2.1. Practicality Nonlinear Input-Output Models

Professor Chen, X.k. (1984) presented a kind of practicality nonlinear input-output model in 1981 based on 61 kind of main product input-output table of China in 1973. The form of the model is:

$$\mathbf{AX} + \mathbf{W}(\mathbf{X}) + \mathbf{Y} = \mathbf{X} \tag{1}$$

where $\mathbf{A}$ is technical coefficient matrix, $\mathbf{X}$ is the column vector of total output, $\mathbf{Y}$ is the column vector of final product, $\mathbf{W}$ is the column vector of other production consumption. The model divides the intermediate input into two parts. One part is the intermediate input to 61 kind of main product, the other part is the intermediate input to the other product besides the 61 kind of main products. The intermediate input to the other product is the function of $\mathbf{X}$ which can be written as $\mathbf{W}_i = \mathbf{W}_i(\mathbf{X}_1, \mathbf{X}_2, \cdots \mathbf{X}_n)$. The function can be decided by past years statistical data. To the different product, the form of the function is usually different too. Commonly they are linear function, power function or exponent function. R square of the models is close to 0.99. It is satisfied to apply these models in economic forecasting. It is the first research on nonlinear input-output model research in China. Though the research is limited in real product input-output model, the method is valuable for reference.

2.2. $\mathbf{A}$ is the nonlinear function of $\mathbf{X}$

Fujimoto Tokao (1986) set up the nonlinear input-output model:

$$A(\mathbf{X})\mathbf{X} + \mathbf{Y} = \mathbf{X} \tag{2}$$

in the arbitrary dimension abstract space without continuity of the input function $\mathbf{A}(\mathbf{X})$, where $\mathbf{X}$ is the column vector of total output, $\mathbf{Y}$ is the column vector of final product, $\mathbf{A}(\mathbf{X})$ is the mapping from $\mathbf{X}$ to $\mathbf{X}$ with the property $\mathbf{A}(0) = 0$ which is called input function. Based on these assumptions Fujimoto Tokao proved the existence, astringency and uniqueness of the solution to (2). It shows in some condition there is input function $\mathbf{A}(\mathbf{X})$ that makes (2) come into existence and have unique solution. The model proves there is unique nonlinear function of $\mathbf{A}$ about $\mathbf{X}$ in certain conditions in theory. But the expression of $\mathbf{A}(\mathbf{X})$ is not presented and model (2) can’t be used in practice.

2.3. Planting industry nonlinear input-output model

National planting industry nonlinear input-output model (He, J.M.1996) divides the planting industry into 13 sectors. Based on agriculture nonlinear input-output flow table in constant price it got the nonlinear function relation between intermediate flow and seeding area. Furthermore it got the expression of nonlinear function of final product about seeding area of 13 planting sectors. The model is:

$$\mathbf{A}(\mathbf{X})\mathbf{M}\overline{\mathbf{X}} + \mathbf{M}\overline{\mathbf{Y}} = \mathbf{M}\overline{\mathbf{X}} \tag{3}$$

where

$$\mathbf{A}(\mathbf{X}) = \begin{pmatrix} a_{11}(\mathbf{X}_1) & a_{12}(\mathbf{X}_2) & \cdots & a_{1n}(\mathbf{X}_n) \\ a_{21}(\mathbf{X}_1) & a_{22}(\mathbf{X}_2) & \cdots & a_{2n}(\mathbf{X}_n) \\ & & \vdots & \\ a_{n1}(\mathbf{X}_1) & a_{n2}(\mathbf{X}_2) & \cdots & a_{nn}(\mathbf{X}_n) \end{pmatrix}$$

M=diag ($m_1, m_2, \cdots, m_n$), m_i is the seeding area of the planting sector i, $\overline{\mathbf{X}} = \left(\overline{\mathbf{X}}_1, \overline{\mathbf{X}}_2, \cdots \overline{\mathbf{X}}_n\right)^T$ is the column vector of total output of unit seeding area, $\overline{\mathbf{Y}} = (\overline{\mathbf{Y}}_1, \overline{\mathbf{Y}}_2, \cdots \overline{\mathbf{Y}}_n)^T$ is the column vector of final product of unit seeding area, $\mathbf{X} = \mathbf{M}\overline{\mathbf{X}}$ is the column vector of the total output.

There is nonlinear function relation between intermediate flow and seeding area, while total output equals to total output of unit seeding area (constant) multiplies seeding area, so the relation between intermediate flow and total output is nonlinear. Model (3) is called as nonlinear input-output model. The merit of model (3) is that it relaxed the proportionality assumption of traditional input-output model. The model is easy to calculate and powerful to forecast. But the model is limited in planting industry. It made no research on other industry.

2.4. The nonlinear input-holding-output model included invariable input

The nonlinear input-holding-output model included invariable input (Wang, H.J. ,1998) divided the intermediate input into invariable input and variable input. Invariable input is the input factor which amount is not increased as the amount of output increased. Variable input is the input factor which amount is increased as the amount of output increased. With the assumption, there is equation (4) in horizontal direction and (5) in vertical direction.

$$\sum_{j=1}^{n} \mathbf{X}_{ij}(\mathbf{X}_j) + \overline{\mathbf{Q}}_i + \mathbf{Y}_i = \mathbf{X}_i, \qquad i = 1,2,\cdots n \tag{4}$$

$$\sum_{i=1}^{n} \mathbf{X}_{ij}(\mathbf{X}_j) + \sum_{i=1}^{n} \overline{\mathbf{Q}}_{ij} + \mathbf{N}_j = \mathbf{X}_j, \qquad j = 1,2,\cdots n \tag{5}$$

$\mathbf{X}_{ij}(\mathbf{X}_j)$ is the variable input that has nonlinear relation with total output, $\overline{\mathbf{Q}}_i$ is the invariable input of sector i, $\mathbf{Y}_i$ is the final demand of sector i , $\mathbf{X}_i$ is the total output of sector i, $\overline{\mathbf{Q}}_{ij}$ is the amount of invariable input of sector i used in sector j, $\mathbf{N}_j$ is the prime input of sector j. (4) and (5) is the nonlinear input-holding-output model included invariable input. The existence and uniqueness of the solution to model is proved, but the expression of nonlinear function between intermediate flow and total output is not given. It is difficult to apply the model in practice.

2.5. Calculable nonlinear dynamic input-output model

Zhang J.S. (2000) divided national economy into 6 sectors: agriculture, industry, construction, transportation and communication, commerce and non-material production sector. It unified the calculable CEG model with Leontief linear dynamic input-output model and got calculable nonlinear dynamic input-output model with 6 sectors of China. The form of it is:

$$\mathbf{Y}(t) = \mathbf{A}\mathbf{Y}(t) + \tilde{\mathbf{B}}\mathbf{Y}(t+1) - (\mathbf{I} - \boldsymbol{\delta})\tilde{\mathbf{B}}\mathbf{Y}(t) + \mathbf{c}(t) \tag{6}$$

where $\mathbf{A} = \begin{bmatrix} a_{11} & \cdots & a_{16} \\ \vdots & & \vdots \\ a_{61} & \cdots & a_{66} \end{bmatrix}$ is technical coefficient matrix, $\mathbf{Y}(t) = \begin{bmatrix} \mathbf{Y}_1(t) \\ \vdots \\ \mathbf{Y}_6(t) \end{bmatrix}$ is the column vector of total output, $\mathbf{c}(t) = \begin{bmatrix} c_1(t) \\ \vdots \\ c_6(t) \end{bmatrix}$ is the column vector of consumption demand, $\widetilde{\mathbf{B}}=\begin{bmatrix} 0 & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & b_{26} \\ b_{31} & b_{32} & \cdots & b_{36} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}$ is the capital demand coefficient matrix, where

$$\mathbf{b}_{2j}=\boldsymbol{\beta}_j\left(\frac{\mathbf{f}_j/\mathbf{a}_j}{\mathbf{q}_j}\right)^{1/(\boldsymbol{\delta}_j-1)}\times(1-\mathbf{a}_{1j}-\cdots-\mathbf{a}_{6j}) \tag{7}$$

$$\mathbf{b}_{3j}=(1-\boldsymbol{\beta}_j)\left(\frac{\mathbf{f}_j/\mathbf{a}_j}{\mathbf{q}_j}\right)^{1/(\boldsymbol{\delta}_j-1)}\times(1-\mathbf{a}_{1j}-\cdots-\mathbf{a}_{6j}) \tag{8}$$

In (7) and (8), $\mathbf{f}_j$ is the fixed assets rent of sector j. $\mathbf{q}_j$ is the value added price index of sector j, $\boldsymbol{\delta}_j$ is the substitute elasticity coefficient of capital and labor of sector j, $\mathbf{a}_j$ and $\boldsymbol{\beta}_j$ are parameters. According to (7) and (8), the elements of $\widetilde{\mathbf{B}}$ are expressed as function of price, which reflect the condition that capital and labor substitute each other. The elements value of $\widetilde{\mathbf{B}}$ can be estimated by actual data.

$\mathbf{I}-\boldsymbol{\delta}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\delta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-\delta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, $\boldsymbol{\delta}_2$ is the fixed capital depreciation rate of industrial product, $\boldsymbol{\delta}_3$ is the fixed capital depreciation rate of construction industry product. The economic meaning of expression (6) is:

Total output or supply = Demand of intermediate input + Investment demand + Consumption demand (9)

The model eliminated homogeneity assumption, proportionality assumption and unsubstituted assumption of input-output model, unified the calculable CEG model with Leontief linear dynamic input-output model. But the model is based on input-output table of 6 sectors. The sector classification is too cursory to research real economy and social problem. When the sector classification is more than 6 sectors, the parameters of the model is difficult to decide. Anyway this research has made great progress on nonlinear input-output model research. Synthetically analyzed all the nonlinear input-output models presented by other scholars, the problem needed to be solved urgently is to find a kind of nonlinear input-output model which is calculable and applicable simply. It can be applied in economy analysis and forecast directly. To the aim, the paper provides a kind of nonlinear input-holding-output model of important coefficients.

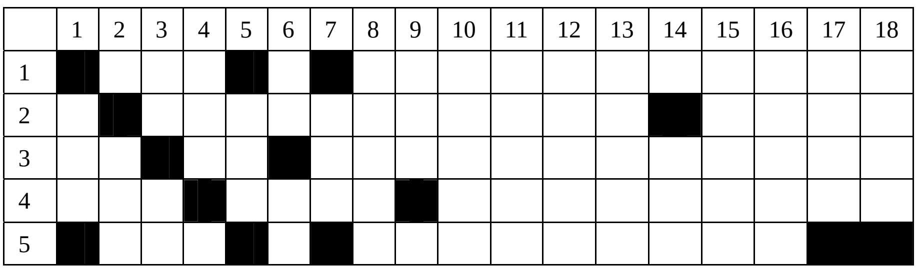
3. Nonlinear input-holding-output model of important coefficients

At present the main difficulty encountered in the nonlinear input-output model research is that the number of technical coefficient is overabundance. It's difficult to make each of them nonlinear. For example, in Chinese 1997 input-output table there are 124 sectors. The technical coefficient matrix have 15376 elements, where more than 50 percent of them are smaller than $(0.1)^{-6}$. Demonstration shows there are one to four important coefficients in one row. These important coefficients play vital role in input-output model. The concept of important coefficient has appeared in literature（Xu & Gould, 1991；Xu & Madden, 1991；Sonis & Hewings, 1992）. They provide a basic economic structure; in the sense that these entries of the technical coefficients matrix are the most likely to spread growth impulses along the economic structure (Aroche-Reyes, 2002) .We called these coefficients as important coefficients. The others are named as unimportant coefficients. But they are not used in nonlinear input-output model research up to now. To resolve the main difficult with low cost, the paper provides a kind of nonlinear input-holding-output model of important coefficient. The method reduces more than 97 percent work of before. Field of influence method and tolerable limits methods are the common methods to decide important coefficients (Xu, J. 2003). There are many factors that make technical coefficients change frequently. Many research (Shi, Y.Y. , 1999; Na, R.S. & Tang, H.W. , 1998) shows price and technology are main factors that make technical to change. So this research uses China 1973, 1981, 1983, 1987, 1990, 1992, 1995, 1997 and 1999 input-holding-output tables with 18 sectors (Table 1) at 1990 constant price as data base. These tables were designed by CAS in the nonlinear and dynamic input-holding-output technology and application item. They have eliminated the influence of price to technical coefficients. Applied Cobb-Douglas product function theory, the paper uses per employee occupied original value of fixed assets at constant price as index to scale technology level in production.

Table 1. Sector serial number and sector name in recompiled input-output tables

Sector No.	Sector Name
1	Agriculture
2	Coal mining
3	Petroleum and natural gas extraction
4	Other minerals mining and dressing
5	Food, textile, papermaking and furniture
6	Petroleum refining and coking
7	Chemical industry
8	Construction materials
9	Smelting and pressing of metals and metal products
10	Mechanic industry
11	Transportation equipment and manufacture
12	Electrical machinery
13	Instruments, meters, cultural and Clerical machinery and other manufactures
14	Electric, gas and water
15	Construction
16	Transportation and communication
17	Commerce
18	Other services

The popular method field of influence (Sonis & Hewings, 1992) is adopted to choose important coefficients. To each piece of 1973-1999 China input-holding-output tables with 18 sectors at constant price, applied field of influence method, and got their important coefficients set. According to the frequency that each important coefficient appeared in the set, 47 important coefficients are chosen. The distribution of them can be seen in figure 1. (The number in the figure is sector serial number.)



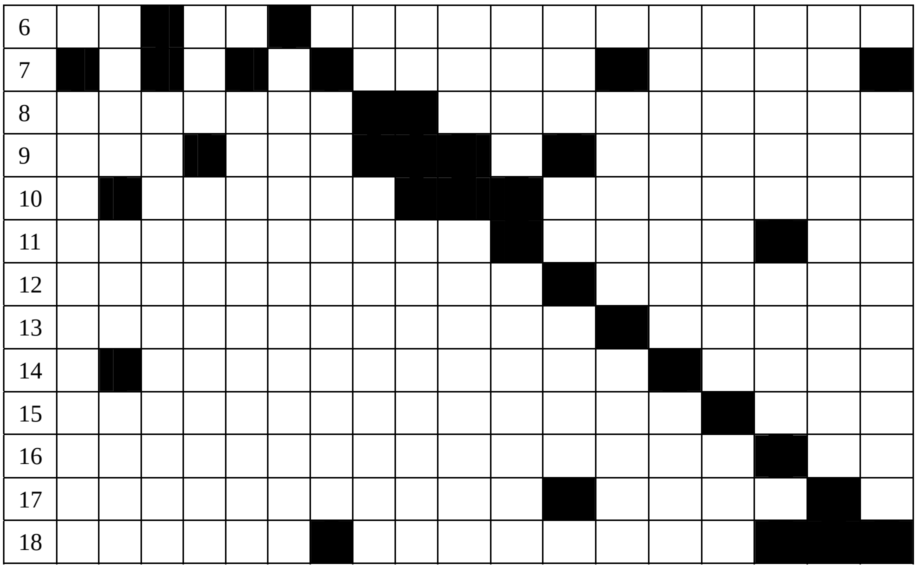


Figure1. The distribution of important coefficients in 18 sectors technical coefficient matrix

To each important coefficient a_{ij}, using per employee of sector j occupied original value of fixed capital at constant price $\mathbf{R}_j$ as independent variable, adapting nonlinear regression method, we get the nonlinear model of a_{ij} and $\mathbf{R}_j$. For example:

$$a_{51} = \underset{(11.352)}{0.1370} + \underset{(8.067)}{0.0544} \ln(\mathbf{R}_1), \quad R^2 = 0.9029, \quad F = 65.07 \tag{9}$$

$$a_{9\ 12} = \underset{(-3.024)}{-0.0136} + \underset{(6.499)}{0.0909} \ln(\mathbf{R}_{12}), \quad R^2 = 0.8579, \quad F = 42.24 \tag{10}$$

The nonlinear functions of other important coefficients are not listed here in detail. The values of R^2, F and t reveal model (9) and (10) can be accepted.

4. Numerical Experiment

To test the prediction precision of the nonlinear models of the important coefficients, RAS method and the nonlinear models of the important coefficients are used to predict technical coefficients on main diagonal of 1997 Chinese input-output table with 18 sectors at constant price. The prediction error of the two methods is compared in table 2.

Table 2. Comparing the prediction error between RAS method and the nonlinear models of the important coefficients

Coefficients	RAS (%)	the ICs (%)	Coefficients	RAS (%)	the ICs (%)
a_{11}	15.7	8.7	$a_{10\ 10}$	-11.4	6.6
a_{22}	-14.4	6.8	$a_{11\ 11}$	-8.5	4.7
a_{33}	-14.3	-7.1	$a_{12\ 12}$	12.7	8.7
a_{44}	-12.3	8.6	$a_{13\ 13}$	9.6	-6.4
a_{55}	9.4	5.1	$a_{14\ 14}$	-15.6	8.4
a_{66}	11.5	7.3	$a_{15\ 15}$	11.7	9.2
a_{77}	6.3	6.8	$a_{16\ 16}$	-12.1	-3.8
a_{88}	12.3	-6.5	$a_{17\ 17}$	-12.7	5.6
a_{99}	10.8	6.1	$a_{18\ 18}$	-15.4	7.2

In table 2 almost all the absolute prediction error of the important coefficient method is smaller than that of RAS method.

5. Conclusion

From the view of labor and capital, the paper gives us a kind of nonlinear model of important coefficients based on nine Chinese input-holding-output tables at constant price with 18 sectors. These nonlinear input-holding-output models reflect the actual economy situation more truly and can be applied in national economy forecast. They are easy to calculate and the data they need is easy to get.

References

Aroche-Reyes(2002) Structural Transformations and Important Coefficients in the North American Economies, *Economic Systems Research*, 14, pp.257-273.

Bullard,C.W. & Sebald,A.V.(1977) Effects of parametric uncertainty and technological change on input-output models, *Review of Economics and Statistics*, 59, pp.75-81.

Chen, X. (1990) Input-occupancy-output analysis and its application in China, in: M. Chatterji & R.E. Kuenne (eds) Dynamics and Conflict in Regional Structural Change (London, Macmillan), pp. 267–278.

Chen, X. (1998) Input-occupancy-output analysis and its application in Chinese economy, International Journal of Development Planning Literature, 13, pp. 105–134.

Chen, X., Cao, J., Xue X. & Lu Z. (1992) Input-Occupancy-Output Analysis of the Urban and Rural Economies of China (Beijing, Science Press) (in Chinese).

Chen, X.K. & Xue, X.W.(1984) A non-linear input-output model in physical units and its application in China. *Proceedings of the Seventh International Conference on Input-Output Techniques*(United Nations), pp.201—209.

Fujimoto Tokao(1986) Nonlinear Leontief model in abstract spaces, *Journal of mathematical Economics,* 15, pp.151-156.

He, J.M.(1996) Research on national planting nonlinear input-output model, *Journal of Systems Science and Systems Engineering,* 6,pp.64-72.

Na, R.S. & Tang, H.W.(1998) Research on the relation between interest rate, wage and product price change and technical matrix change, *Journal of Systems Science and Systems Engineering,* 4, pp.29-33.

Shi, Y.Y.(1999) Input-output analysis on China economic structure change, *The theory and practice of system engineering,* 10, pp.74-76

Sonis,M. & Hewings,G.J.D.(1992) Coefficient change in input-output models: theory and applications, *Economic Systems Research,* 4, pp.143-157.

Wang, H.J.(1998) Research on several problems of national economy system nonlinear input-holding-output theory and application, doctor's degree paper(Beijing, graduate school of Chinese Academy of Sciences), pp.1-8.

Xu, J.(2003), Research on some theoretic and practical issues in water conservancy input-holding-output analysis, doctor's degree paper of graduate school of Chinese academy of sciences(Beijing, graduate school of Chinese Academy of Sciences), pp.23-34

Xu, S. & Gould, P.(1991)The grad field of input-output models and the nature of coefficients, *Economic Systems Research,* 3, pp. 367-378.

Xu, S. & Madden, M.(1991)The concept of important coefficients in input-output models, Flexibility and Extensions in Regional Input-Output Modelling(edited by J. J.LI. Dewhurst, G. J. D. Hewings& R.C. Jensen, Aldershot, Avebury), pp. 66-97.

Zhang, J.S.(2000) The calculable non-linear dynamic input-output model(Beijing, Tinghua publication), pp.258-263.