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Application of a novel matrix balancing approach to the estimation of UK input-output tables

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Abstract

We describe the first stage of the implementation of an international multi-region input-output model (MRIO) for the UK. The aim of the ongoing work is to develop a data optimisation procedure that allows the construction of integrated national input-output and environmental databases that can be used for an environmental MRIO model in the future. Thus the work will set the basis for numerous analyses of environmental impacts associated with UK trade flows, including detailed accounts of emissions embedded in trade flows to and from the UK over a period of time.

The work has been commissioned by the UK Department for Environment, Food and Rural Affairs (Defra) as a follow-up to a previous project where the most appropriate approach to constructing a robust account of impacts of trade and thus overall consumption in a headline indicator for Sustainable Development was identified (Wiedmann et al. 2006). In order to derive reliable and robust estimates for embedded emissions, it is important to explicitly consider the production efficiency and emissions intensity of a number of trading countries and world regions in an international trade model, which is globally closed and sectorally deeply disaggregated (Wiedmann et al. 2007).

In order to achieve this aim, initial data estimates need to be made, data constraints need to be defined and specific optimisation algorithms need to be developed and implemented. In this paper we describe a data framework that employs optimisation techniques balancing data according to constraints which are defined by existing/available data.

Optimisation routines will be used to impute missing data (Lenzen et al. 2006). We will employ techniques that

- incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
- allow considering the reliability of initial estimates and external constraints;
- are able to handle negative values and to preserve the sign of matrix elements if required;
- are able to handle conflicting external data.

The main outcome of this project stage is a time series of annual balanced input-output tables for the UK for each year from 1992 to 2004. The paper will present the theoretical framework of the model and the practical challenges of its implementation.

Keywords

input-output tables, RAS matrix balancing, constrained optimisation, international trade, multi-region input-output model, UK

Contents

1. Introduction

1.1. Project Background

In 2003, the UK Department for the Environment, Food and Rural Affairs (Defra) published a 'Framework for Sustainable Consumption and Production (SCP)', accompanied by a consultation paper setting out a basket of supporting sustainable development indicators. Respondents to the consultation reported that many of the indicators were difficult to interpret without a better understanding of the effect of structural change within the British economy, and in particular the extent to which any reductions in the environmental impact of the UK economy were being offset by increases in the impacts associated with the production of imports to the UK.

At the same time the launch of the SCP framework has led to an increasing policy focus on the environmental impacts of the products consumed by households within the UK, wherever those impacts occur, and to a demand for a better understanding of the life cycle impacts of the whole range of goods and services consumed by British households. More recently there has been an increasing emphasis on the need for British companies to take some responsibility for the upstream impacts of the goods which they sell or use, on the environmental impacts of particular products such as clothing which are heavily dependent upon imports, and on the importance of 'sustainability dialogues' between the UK Government and key trading partners. Attention is therefore focusing not just on the overall impacts of trade to and from the UK, but on which sectors, products and countries the trade relates to.

In 2005, Defra commissioned the Stockholm Environment Institute to identify the most appropriate approach to constructing an indicator for emissions embedded in trade flows to and from the UK (Wiedmann et al. 2006a)[1]. One of the conclusions from that study was that, in order to derive reliable and robust estimates for embedded emissions, it is important to explicitly consider the production efficiency and emissions intensity of a number of trading countries and world regions in an international trade model, which is globally closed and sectorally deeply disaggregated (Wiedmann et al. 2007).

While one of Defra's goals was to be able to produce a robust account of impacts of trade and thus overall consumption in a headline indicator for Sustainable Development, it was recognised that the adoption of such a consumption-based perspective – in addition to the 'traditional' approach of territorial emissions accounting – opened up the possibility of extending the range of policy and research applications considerably to cover sectoral, country and product analysis. (Druckman et al. 2007) recently reported a rise of 7.7% in total UK CO_2 emissions between 1990 and 2004 when estimated according to the consumption perspective and stress the "severe policy implications" in conjunction with any emission reduction targets.

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[1] Defra project ref. EV02001, 'Resource Flows'. Stockholm Environment Institute, York and Policy Studies Institute, London. Published by Defra, August 2006. http://www2.defra.gov.uk/research/project_data/More.asp?I=EV02001&M=KWS&V=EV02001&SCOPE=0

As a follow-up to our previous work, the current work[2] is the first stage of the implementation of an international multi-region input-output model for the UK (UK-MRIO). As a crucial part of an operational MRIO framework we develop a code protocol that processes data of any kind in a highly efficient way. In essence, this is a sophisticated computer programme that can ‘digest’ data from different countries and years in different classifications and valuations with data gaps and inconsistencies.

The main advantage of the model described in this paper is its flexibility towards the consistent integration of additional data in a process of a step-wise extension of the model as well as its (flexible) adaptation towards alternative research questions. Furthermore, the eventual model will allow a flexible breakdown of economic sectors if this is required to answer specific questions – a capability which is important for the most widespread applications (and therefore the associated cost-return rate of the project) in different areas such as global supply chain analysis, life cycle assessments or conventional environmental input-output analysis. An efficient data handling protocol of this type helps reducing cost and time requirements while at the same time allowing a consistent update of the model.

The Stockholm Environment Institute[3] at the University of York is collaborating with The Centre for Integrated Sustainability Analysis (ISA) at the University of Sydney[4] in this project to develop the required data and model basis.

1.2. Aim and scope of this work

For this stage of the UK-MRIO project, the aim is to develop and implement an initial, relatively small, data and model framework that is easily expandable without major adaptations. A data optimisation procedure will allow the flexible adaptation of national input-output and environmental databases for use in a multi-region environmental input-output model in the future. Thus the work will set the basis for multi-regional analyses of environmental impacts associated with UK trade flows, including detailed accounts of emissions embedded in trade flows to and from the UK over a period of time.

In order to achieve this aim, initial data estimates have been made, data constraints have been defined and specific optimisation algorithms have been developed and implemented. As a tangible outcome of the current project we have constructed a time series of input-output tables for the UK from 1992 to 2004 by using a modified RAS procedure for balancing (referred to as 'Conflicting RAS' or 'CRAS'). These tables are similar to the “Analytical IO Tables 1995” published by ONS[5], including symmetrical input-output tables (SIOT) as well as total requirement matrices (‘Leontief Inverses’) for each year from 1992 to 2004.

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[2] Defra project ref. EV02033, ‘Development of an Embedded Carbon Emissions Indicator. Stockholm Environment Institute, York and Centre for Integrated Sustainability Analysis (ISA), University of Sydney. Commissioned by Defra, December 2006. http://www2.defra.gov.uk/research/project_data/More.asp?I=EV02033&M=KWS&V=EV02033&SUBMIT1=Search&SCOPE=0

[3] http://www.sei.se

[4] http://www.isa.org.usyd.edu.au/index.html

[5] The UK Office for National Statistics, London

This paper is a follow-up of the work presented by Lenzen and colleagues at the 2006 Intermediate International Input-Output Conference in Sendai, Japan (Lenzen et al. 2006) which described the theoretical framework of the CRAS method. We now present a first 'real world' application of preparing the UK input-output time series.

1.3. Review of recent literature on the estimation of emissions embedded in international trade

A literature review was undertaken in the course of the ‘Resource Flows’ project[1] on models and approaches that are capable of estimating emission embodiments in international trade. Since the conclusion of the project and the publication of its findings (Wiedmann et al. 2006a), (Wiedmann et al. 2007) new research has been published. In this section we provide an update of the literature review on embedded emissions calculations.

A follow-up of a previous OECD study (Ahmad 2003; Ahmad and Wyckoff 2003) was undertaken by (Yamano et al. 2006). Using the sector harmonised OECD input-output tables, STAN bilateral trade data and IEA CO_2 emissions database for years around 1995 and 2000, the authors developed an international linked world economic model which covers 17 sectors and 42 countries/regions. CO_2 embodiments in international trade are derived from direct and indirect energy consumptions.

(Tunç et al. 2007) estimate the CO_2 content of imports to the Turkish economy by industrial sector in a single-region IO model. They find that the total estimated “CO_2 responsibility” for the Turkish economy in 1996 was 342 Mt, of which 17% are due to imported intermediate goods to be used in domestic production and 5% are due to imported goods to satisfy private and public consumption. The authors conclude that consumer-related environmental policies for CO_2 reduction will not necessarily be more effective than policies aimed at producers since the major part of CO_2 responsibility – domestically and imported – arises as a result of the production process.

(Limmeechokchai and Suksuntornsiri 2007) calculate energy and greenhouse gas embodiments of final consumption in Thailand for a number of years, taking into account greenhouse gases embedded in imported energy, in particular imported electricity.

The impact of different assumptions concerning the emissions embodied in imports in the case of Finland was tested by (Mäenpää and Siikavirta 2007). Using domestic emission intensities and data from the OECD study by Ahmad and Wyckoff (Ahmad and Wyckoff 2003) in a 139-sector single-region input-output model, the authors found relatively small differences: in the analysis for 1999 the net export of CO_2 from fossil fuel combustion changed from 4.2 to 3.6 Mt. Results for 1990-2003 show that Finland has increasingly been a net exporter of GHG emissions.

There are several follow-up applications of the MRIO model described by (Peters and Hertwich 2004). In (Peters and Hertwich 2006a) the authors use their MRIO model for a structural path analysis (SPA) across borders, thus enabling the investigation of international supply chains (on an aggregation level of 49 sectors). Embodied impacts in household and government consumption and exports are quantified, identifying high ranking impacts from imports, for example the household purchase of clothing from developing countries in the case of CO_2. Furthermore, the authors use SPA in a consumption and a production perspective, offering complementary insights, both in terms of analysis and policy.

Another application focuses on household consumption and impacts of imports to Norway (Peters and Hertwich 2006b). The study finds that household environmental impacts occurring in foreign regions represent 61% of indirect CO_2 emissions, 87% for SO_2, and 34% for NO_x, whereas imports represent only 22% of household expenditure in Norway. Furthermore, a disproportionately large amount of pollution embodied in Norwegian household imports can be traced back to developing countries.

All studies by Peters and Hertwich confirm the importance of considering regional technology differences in a multi-region model when calculating pollution embodied in trade. The pollution intensity of the electricity sector in China, for example, is 231 times higher for CO_2 and 1078 times higher for SO_2 than in Norway (Peters and Hertwich 2006c; see also Peters et al. 2007).

(Hoekstra and Janssen 2006) use a dynamic input-output model of two trading countries to explore the effects of taxes in different scenarios for environmental responsibility. The study is specified in a hypothetical framework and does not use empirical data.

The hypothesis that there is a shift of high polluting industries from developed countries to those with lower environmental standards (“pollution haven hypothesis”) is examined by (Wilting et al. 2006) for the Netherlands. Developments in emissions of CO_2, CH_4, N_2O, NO_x, SO_2 and NH_3 in Dutch industries from 1990 to 2004 are related to changes in trade patterns in the same period by using a structural decomposition analysis based on a single-region input-output model of Denmark. The analyses show that the export effect compensates the import effect for all air emissions except of CO_2, implicating that there is no net shift of pollution to abroad. Only CO_2 shows a small decrease in emissions resulting from trade effects, but the effect is too small to draw robust conclusions.

(Norman et al. 2007) create a 76 sector bi-national Canada-US EIO-LCA model by linking the national input-output models through trade flows by industrial sector. They find that US manufacturing and resource industries are about 1.15 times as energy-intensive and 1.3 times as GHG-intensive as Canadian industries, with significant sector-specific discrepancies in energy and GHG intensity. Accounting for trade can significantly alter the results of purely national life-cycle assessment studies, particularly for many Canadian manufacturing sectors. (Norman et al. 2007) show that the production and consumption of goods in one country often exerts significant energy- and GHG-influences on the other.

International trade can reduce overall CO_2 emissions if imported products are consumed that were produced with a lower carbon intensity than in the domestic industry. This is the case for trade between Japan and the US, for example. By using a two-region input-output model, (Ackerman et al. in press) estimate that in 1995, Japan–US trade reduced US industrial emissions by 14.6 million tons of CO_2-equivalent, and increased emissions in Japan by 6.7 million tons, for a global savings of 7.9 million tons. These quantities are less than one percent of each country's total emissions but trade of Japan and the US with the rest of the world reduced emissions by larger amounts, roughly four percent of each country's emissions. The authors estimate that US industry could cut its carbon emissions by more than half if it matched the environmental performance of industry in Japan.

In the UK, (Druckman et al. 2007)[6] present the findings of a two region input-output model (UK – Rest of World) based on work by (Proops et al. 1993) and developed further by the University of Surrey. The model takes into account a different economic structure of the Rest of the World but CO_2

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[6] See also (Jackson, Tim et al. 2006)

intensities that are identical to the UK[7] and is used to estimate total CO_2 emissions with different accounting principles for two years 1990 and 2004. The results is a clear increase when accounting in the consumption perspective, i.e. CO_2 emissions embedded in imports have increased significantly over time (by 7.7% between 1990 and 2004). The authors also stress the "absence of robust, up-to-date monetary datasets on which to build Environmental Input-Output analysis" and discuss the resulting negative implications for the robustness of this technique

Systematic environmental accounting alongside national economic accounting has long been recognised as a very useful source of information for ecological-economic modelling and (political) decision-making (see Lange 2007 for an introduction to a special issue of Ecological Economics on Environmental Accounting, Vol. 61, 2007). A new FP7 European Integrated Project, EXIOPOL, will contribute to the extension, consolidation and application of environmental-economic accounts in Europe. EXIOPOL stands for an 'Environmental Accounting Framework Using Externality Data and Input-Output Tools for Policy Analysis'[8]. EXIOPOL aims to develop estimates of external costs of a broad set of economic activities for Europe and to set up a detailed environmentally extended input-output framework including these estimates, in order to apply the results of this analysis to address policy questions in fields such as Integrated Product Policy or Sustainable Consumption and Production. One work area of the new project which was kick-started in April 2007 is the creation of a detailed input-output framework for the EU 25 which is extended with environmental information and will enable the creation of MRIO models in the future. The database will enable estimating environmental impacts and external costs of different economic sector activities, final consumption activities and resource consumption for countries in the EU (Tukker 2006; Tukker 2007).

A number of multi-region input-output models with world coverage using the GTAP database and results for environmental impacts embedded in trade are presented at the 16th IIOA Conference 2007, Istanbul (www.io2007.itu.edu.tr).

2. Methodology

2.1. Introduction

The implementation and application of a full multi-regional input-output framework poses three basic challenges: data availability, data reconciliation and computability. These issues and possible practical solutions are discussed in detail in (Wiedmann et al. 2006). In the following we focus on the preparing of input-output data for the model framework. The data system should be able to

- include data in different classifications,

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[7] (Druckman et al. 2007) say that the latter condition "is not particularly satisfactory as it assumes that the economic structure and energy mix in the Rest of the World can be approximated by those in the UK" but also point out that this "… is likely to underestimate the carbon associated with imports from the developing countries where less energy efficient and more energy intensive processes may dominate the economic structure."

[8] http://www.feem.it/Feem/Pub/Programmes/Sustainability+Indicators+and+Environmental+Valuation/Activities/200703-EXIOPOL.htm, see also http://www.seri.at/EXIOPOL

- handle conflicting data consistently,
- cope with suppressed data,
- estimate missing data,
- accommodate different years for the analysis of time series.

In essence, our data framework employs optimisation techniques that balance data according to constraints which are defined by existing/available data.

It is not a necessary condition to have analytical (symmetrical) input-output tables for an environmental input-output model. Supply and use matrices can be used instead as described in (Lenzen et al. 2004) and (Wiedmann et al. 2006b). This is a big advantage as supply and use tables are often available annually while analytical or symmetrical IO tables are usually not. It also allows using more up-to-date information as the time delay for publishing supply and use tables is shorter than for analytical tables (in the region of two to three years as opposed to more than five years for analytical tables). This is important because changes in the structure of domestic and foreign economies can be picked up more accurately if up-to-date input-output information is used.

2.2. Model layout

The basic layout of the model framework is depicted in Table 1. For the purpose of this project, which is to implement the model in principle with only a small number of trading partners at this stage, we choose to consider UK trade with three world regions, OECD Europe (Region e), OECD non-Europe (Region o) and non-OECD countries (Region w).[9]

The UK is represented with its full input-output data in supply and use format whereas the three world regions are represented by their (combined) coefficient matrices A (see Section XX). Imports to the UK are distinguished by region and by destination to intermediate ($\mathbf{U}^{ru}$) and final demand ($\mathbf{y}^{ru}$). At this stage, we only consider trade between the UK and the regions but not between the regions themselves and we only include CO_2 as environmental load ($\mathbf{E}^{r}$).

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[9] This decision was driven by data availability (see Section 3.2, page 24) and practical considerations.

Table 1: Multiregion input-output (MRIO) system employed in this work

		Intermediate demand				**Final demand**				**Total output**
		UK(u)	*Region e*	*Region o*	*Region w*	*UK(u)*	*Region e*	*Region o*	*Region w*	
UK(u)	$\mathbf{V}^{uu}$									$\mathbf{g}^{u}$
		$\mathbf{U}^{uu}$	¦	¦	¦	$\mathbf{y}^{uu}$	- - -	- - $\mathbf{y}^{ur}$ - -	- - -	$\mathbf{q}^{u}$
Region e		$\mathbf{U}^{eu}$	$\mathbf{A}^{e}$	¦	¦	$\mathbf{y}^{eu}$				$\mathbf{g}^{e}$
Region o		$\mathbf{U}^{ou}$	¦	$\mathbf{A}^{o}$	¦	$\mathbf{y}^{ou}$				$\mathbf{g}^{o}$
Region w		$\mathbf{U}^{wu}$	¦	¦	$\mathbf{A}^{w}$	$\mathbf{y}^{wu}$				$\mathbf{g}^{w}$
Primary inputs		$\mathbf{w}^{u}$	¦	¦	¦					
Total inputs	$\mathbf{q}^{u'}$	$\mathbf{g}^{u'}$	$\mathbf{g}^{e'}$	$\mathbf{g}^{o'}$	$\mathbf{g}^{w'}$					
Factor inputs (environmental loads)		$\mathbf{E}^{u}$	$\mathbf{E}^{e}$	$\mathbf{E}^{o}$	$\mathbf{E}^{w}$					

Legend to Table 1:

UK	United Kingdom (superscript u)
Region e	OECD Europe countries (superscript e)[10]
Region o	OECD non-Europe countries (superscript o)[11]
Region w	non-OECD countries = rest of the world (superscript w)
$\mathbf{U}^{uu}$	Domestic use matrix of the UK with elements u_{ij}^{uu} indicating the input of commodity *i* into industry *j*
$\mathbf{U}^{ru}$	Matrix of imports from region *r* into UK industries with u_{ij}^{ru} indicating the input of commodity *i* from region *r* into UK industry *j*
$\mathbf{V}^{uu}$	Domestic supply matrix of the UK with element v_{ij}^{uu} indicating the output of commodity *j* by industry *i*
$\mathbf{g}^{r}$	Vector of total output of industries in country/region *r* (the prime symbol ' denotes transposition)
$\mathbf{q}^{u}$	Vector of total output of UK commodities (the prime symbol ' denotes transposition)
$\mathbf{y}^{uu}$	Column vectors of total final domestic demand on UK production[12]
$\mathbf{y}^{ur}$	Column vectors of final export demand on UK production (exports of goods and services)
$\mathbf{y}^{ru}$	Column vectors of total final demand in the UK on production imported from region *r*
$\mathbf{A}^{r}$	Matrix of total interindustry requirements in region *r*, comprising interindustry requirements on domestic production ($\mathbf{A}_{\mathrm{d}}$) plus interindustry requirements of imports ($\mathbf{A}_{\mathrm{im}}$)
$\mathbf{w}^{u}$	Row vectors of primary inputs (income, surplus, taxes) into UK industries (note that $\mathbf{w}^{u}$ contains only value added items and no imports, because the latter are contained in the $\mathbf{U}^{ru}$ matrices.
$\mathbf{E}^{r}$	Row vector of (CO_2) emissions by industry in country/region *r*
- - -	Hyphens mean that data for this cell is implicitly included in data from other cells, i.e. import coefficients are included in matrices $\mathbf{A}^{r}$ and total UK export is aggregated in $\mathbf{y}^{ur}$

The next step is to derive (relative) coefficient matrices from the (absolute) transaction matrices. Defining input coefficient matrices $\mathbf{A}^{ru}$ with $a_{ij}^{ru} = u_{ij}^{ru} \big/ g_j^{u}$ and output coefficient matrix $\mathbf{B}^{uu}$ with

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[10] Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Luxembourg, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, Switzerland, Turkey, United Kingdom

[11] Canada, Mexico, United States, Australia, Japan, Korea, New Zealand

[12] Including demand of households, government, capital investment, valuables and changes in inventories.

$b_{ij}^{uu} = v_{ij}^{uu} / q_j^u$ the grey-shaded parts of Table 1 can be transformed into a compound direct requirements matrix:

Eq. 1
$$\mathbf{A}^* = \begin{pmatrix} 0 & \mathbf{B}^{uu} & 0 & 0 & 0 \\ \mathbf{A}^{uu} & 0 & 0 & 0 & 0 \\ \mathbf{A}^{eu} & 0 & \mathbf{A}^{e} & 0 & 0 \\ \mathbf{A}^{ou} & 0 & 0 & \mathbf{A}^{o} & 0 \\ \mathbf{A}^{wu} & 0 & 0 & 0 & \mathbf{A}^{w} \end{pmatrix}$$

Eq. 2 Setting
$$\mathbf{y}^* = \begin{pmatrix} 0 \\ \mathbf{y}^{u} \\ \mathbf{y}^{eu} \\ \mathbf{y}^{ou} \\ \mathbf{y}^{wu} \end{pmatrix}, \quad \mathbf{g}^* = \begin{pmatrix} \mathbf{g}^{u} \\ \mathbf{q}^{u} \\ \mathbf{g}^{e} \\ \mathbf{g}^{o} \\ \mathbf{g}^{w} \end{pmatrix},$$

with $\mathbf{y}^u = \mathbf{y}^{uu} + \mathbf{y}^{ur}$ = total final demand for the UK allows to calculate **A*** which satisfies the basic input-output relationship

Eq. 3
$$\mathbf{A}^* \mathbf{g}^* + \mathbf{y}^* = \mathbf{g}^* \iff \mathbf{g}^* = (\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{y}^* ,$$

where **I** is a suitable unity matrix. The compound Leontief inverse $(\mathbf{I} - \mathbf{A}^*)^{-1}$ contains compound total multipliers of intermediate demand and trade.

2.3. Balancing data by using the CRAS method

Optimisation requirements

A common problem in compiling and updating Social Accounting Matrices (SAM) or input-output tables is that of incomplete or inconsistent data. Missing or conflicting matrix elements may be due to a variety of reasons such as costly and therefore incomplete industry surveys, the suppression of confidential information and inconsistencies when sectors have to be disaggregated. The latter two are specific problems in the compilation of the UK-MRIO model (see Section 3, see also (Druckman et al. 2007).

External data points can be used to formulate a system of equations that constrain the unknown matrix elements. Constraints in this context are ‘fixed’ data values, i.e. any data points in the system that are known with sufficient accuracy. Any available and reliable data can serve as constraints. In order to constrain the preliminary estimate of IO tables or SAMs, it is important to incorporate as many sources of superior data as possible. However, unknowns usually outnumber external constraints, resulting in the system being underdetermined, that is exhibiting too many degrees of freedom to be solved analytically. The two most prominent numerical approaches for reconciling such an underdetermined system are probably the RAS method, and constrained optimisation.

During the past 40 years, both approaches have successfully tackled a number of challenges, leading to a number of useful features[13]: Ideally, the technique should

a) incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
b) allow considering the reliability of the initial estimate;
c) allow considering the reliability of external constraints;
d) be able to handle negative values and to preserve the sign of matrix elements if required;
e) be able to handle conflicting external data.

While all criteria have been addressed by constrained optimisation methods, there is currently no RAS-type technique that satisfies criterion e). In particular the inability of RAS to deal with conflicting external data represents a considerable drawback for practice, because for most statistical agencies such data are often rather the norm than the exception.

The most simple case of conflicting data is probably a situation in which two data sources are located that prescribe two different values for the same matrix entry, resulting in inconsistent constraints. When faced with such constraints, existing RAS variants adjust the respective matrix element in turn to both directly conflicting values, and thus enter into oscillations without ever converging to a satisfactory solution.

More generally, sets of external data can be conflicting indirectly amongst each other. (Cole 1992) mentions convergence problems, and gives a simple example as a matrix $\begin{pmatrix} a & b \\ c & 1 \end{pmatrix}$ with a, b, c ≥ 0, and with inconsistent row and column totals $\{1, 3\}^t$ and {1, 3}. In practice, indirect conflict might present itself for example when on one hand, data on final demand and gross output of wheat suggest a certain intermediate demand of wheat, however on the other hand this intermediate demand is too large to be absorbed by the flour milling sector. Further examples involving conflicting external information are GDP measures[14], and multi-national and regional input-output systems. In practice, such inconsistencies are often traced and adjusted manually by statisticians.[15]

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[13] (Lahr and de Mesnard 2004) provide a recent overview of extensions to the classic RAS technique.

[14] (Barker et al. 1984) (p. 475) write: "… we observed that the income, expenditure, production and financial estimates of data are typically inconsistent. The presence of such accounting inconsistencies emphasises the unreliable nature of economic data." See also (Smith et al. 1998).

[15] (Barker et al. 1984) (p. 475) remark that "…trading off the relative degrees of uncertainty of the various data items in the system in order to adjust the prior data to fit the accounting identities […] is essentially what national income accountants do during the last stages of compiling the accounts when faced mith major discrepancies between data from different sources". (Dalgaard and Gysting 2004) (p. 170) from Statistik Denmark report that many analysts responsible for compiling input-output tables favour manual adjustment, because "based on the experience that many errors in primary statistics are spotted in the course of a balancing process that is predominantly manual, compilers are typically convinced that a

In this work we use a new RAS variant that is able to handle conflicting external data and inconsistent constraints. We achieve this capability by introducing standard error estimates for external data. We build on previous RAS variants that satisfy the remaining criteria, and thus arrive at a RAS-type method that matches the capabilities of constrained optimisation. We will refer to this method as CRAS (Conflicting RAS).

Constraints on arbitrarily sized and shaped subsets of matrix elements

The RAS method – in its basic form – bi-proportionally scales a matrix $\mathbf{A}_0$ of unbalanced preliminary estimates of an unknown real matrix **A**, using **A**'s known row and column sums. The balancing process is usually aborted when the discrepancy between the row and column sums of $\mathbf{A}_0$ and **A** is less than a previously fixed threshold. (Bacharach 1970) has analysed the bi-proportional constrained matrix problem in great detail, in particular in regard to the economic meaning of bi-proportional change[16], the existence and uniqueness of the iterative RAS solution, its properties of minimisation of distance metric[17], and uncertainty associated with errors in row and column sum data and with the assumption of bi-proportionality. The origins of the method go back several decades ((Deming and Stephan 1940)). (Stone and Brown 1962), (Bacharach 1970), and (Polenske 1997) provide a historical background.

A special situation arises when some of the matrix elements of **A** are known in addition to its row and column sums, for example from an industry survey. The 'modified RAS' (MRAS) approach ((Paelinck and Walbroeck 1963); (Allen 1974; Lecomber 1975)) deals with this partial information as follows: the preliminary estimate $\mathbf{A}_0$ has to be "netted", that is the known elements are subtracted, and $\mathbf{A}_0$ contains 0 at the corresponding positions. The net $\mathbf{A}_0$ is then subjected to the standard RAS procedure, and the known elements are added back on after balancing.

In practice, situations can arise where, in addition to certain elements of **A**, some aggregates of elements of **A** are known. For example, a published table $\mathbf{A}^G$ of national aggregates may constitute partial information when constructing a multi-regional input-output system, or a more disaggregated national table. Accordingly, (Oosterhaven et al. 1986) add a "national cell constraint" to the standard row and column sum constraints. Similarly, (Jackson, R. W. and Comer 1993) use partition coefficients for groups of cells of a disaggregated base year matrix to disaggregate cells in an updated but aggregated matrix. (Batten and Martellato 1985) (p. 52-55)

(mainly) manual balancing process yields results of higher quality than those emanating from a purely automatic balancing of the accounts. From that point of view, the resources involved in manual balancing are justified as a very efficient consistency check on the accounts."

[16] When applied to the forecasting of monetary input-output matrices, bi-proportional changes have been interpreted as productivity, substitution or fabrication effects ((Leontief 1941); (Stone and Brown 1962)) affecting industries over time. (Miernyk 1976) view however is that the RAS method "substitutes computational tractability for economic logic", and that the production interpretation loses its meaning when the entire input-output table is balanced, and not only inter-industry transactions (see also (Giarratani 1975)).

[17] The RAS, Linear Programming and minimum information gain algorithms yield a balanced matrix estimate that is – in terms of some measure of multidimensional 'distance' – closest to the unbalanced preliminary estimate. When applied to temporal forecasting, this property is explained as a conservative hypothesis of attributing inertia to inter-industrial relations (Bacharach 1970), p. 26). While the classic RAS method is aimed at maintaining the value structure of the balanced matrix, the closely related Cross-Entropy methods (Robinson et al. 2001) are aimed at maintaining the coefficient structure.

discuss further constraints structures, involving intermediate and final demand data. (Gilchrist and St Louis 1999; Gilchrist and St Louis 2004) propose a three-stage "TRAS" for the case when aggregation rules exist under which the partial aggregated information $\mathbf{A}^{G}$ can be constructed from its disaggregated form **A**. Subjecting an input-output matrix to random censoring, these authors demonstrate that the inclusion of partial aggregated information into the TRAS procedure leads to superior outcomes than applying the standard RAS method. (Cole 1992) describes the general TRAS type that accepts constrained subsets of any size or shape. However, no TRAS variant deals with uncertainties, or handles negative matrix elements and conflicting external information.

Reliability of the initial estimate and external information

Another variant of the MRAS method takes into account the uncertainty of the preliminary estimates, and contains the occurrence of perfectly known elements as a special case ((Lecomber 1975; Lecomber 1975), with case studies in (Allen 1974) and (Allen and Lecomber 1975). This is accomplished by introducing a matrix **E** containing "reliability information" about the elements in $\mathbf{A}_0$. **E** instead of $\mathbf{A}_0$ is then balanced in order to take up the difference between the preliminary and true totals:

Eq. 4 $$\mathbf{A^*} = (\mathbf{A}_0 - \mathbf{E}) + \hat{\mathbf{r}}\mathbf{E}\hat{\mathbf{s}}$$

A* is the balanced estimate, and $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ are diagonal scaling matrices, as in the conventional RAS algorithm. Where $E_{ij} = 0$, A_{ij} remains unchanged during balancing. (Lecomber 1975; Lecomber 1975) also investigates the influence of errors in the "true" totals.

A shortcoming of Lecomber's approach is that the elements of **E** cannot be interpreted as standard deviations. If we follow Lecomber in maintaining $0 \le E_{ij} \le A_{0ij}$, and consider that RAS preserves the positive signs in **E**, then $A^*_{ij} \ge A_{oij} - E_{ij} \forall i, j$. In other words if E_{ij} were the standard deviations of A_{0ij}, then the balanced estimate **A*** could never go more than one standard deviation below the initial estimate $\mathbf{A}_0$. An upper limit for **A*** does not exist however. Thus, as Lecomber points out, the elements of **E** must be sufficiently large to ensure the controlling vectors are non-negative – but there is no method to ensure this, whilst still interpreting the elements of **E** as standard errors. Thus considering that conflicting external information may well diverge by more than one standard deviation, it follows that MRAS will not reach a solution under sufficiently inconsistent constraints, unless more (unspecified) information on errors is obtained.

(Lahr 2001) takes into account the uncertainties of external constraints in treating the tolerances of the RAS termination criteria as functions of the varying reliabilities of row and column sums. (Dalgaard and Gysting 2004) incorporate information about the reliability of external constraints (again row and column totals) into the balancing process as "confidence factors" $\boldsymbol{\lambda}$, and successively adjust the target totals $\mathbf{u}_n$ of the *n*th iteration as a weighted sum $u_{n,j} = \lambda_j^{n-1} u_{0,j} + (1 - \lambda_j^{n-1}) u_{n-1,j}$ of the initial unbalanced totals $u_{0,j}$ and the totals $u_{n-1,j}$ of the previous iteration. With subsequent iterations, the confidence factors $0 \le \lambda_j^n \le 1$ become smaller and smaller, thus gradually converging away from the unbalanced initial totals $\mathbf{u}_0$, towards the balanced totals $\mathbf{u}_\infty$. The innovation is that totals with high confidence ($\lambda_j \leq 1$) get adjusted away from the initial totals much slower than those totals with low confidence ($\lambda_j \geq 0$).

While both approaches consider the varying reliability of totals, they cannot deal with inconsistent totals. In applying conventional RAS scaling factors, Lahr's algorithm would always end up balancing matrix elements to satisfy only one of a number of conflicting external constraints. Similarly, for large enough n, Dalgaard and Gysting's algorithm would oscillate around those inconsistent totals $u_{n-1,j}$ with non-zero confidence.[18]

Negative elements

(Junius and Oosterhaven 2003) derive a generalised RAS ("GRAS") algorithm that can balance negative elements, by splitting the matrix **A** into positive and negative parts **P** and **N**, and balancing **A** = **P** – **N** according to

Eq. 5
$$\begin{aligned}\left(\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}\right)\mathbf{i} &= \mathbf{u}^* \\ \mathbf{i}\left(\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}\right) &= \mathbf{v}^*\end{aligned},$$

where **i** is the summation vector. Note that in order to minimise information gain, the balanced matrix $\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}$ conform to totals $\mathbf{u}^* = e\,\mathbf{u}$, $\mathbf{v}^* = e\,\mathbf{v}$, and $\mathbf{i}\,\mathbf{u}^* = \mathbf{i}\,\mathbf{v}^*$, where $e = 2.718\ldots$ is the base of the exponential function, and **u** and **v** are the prescribed row and column sum vectors, respectively, of **A** (Oosterhaven 2005)). The results $\{A_{ij}\}$ of GRAS have to be scaled down by e in order to satisfy the initially prescribed totals **u** and **v**.

In its basic formulation by (Junius and Oosterhaven 2003), GRAS neither incorporates constraints on subsets, nor does it deal with uncertainty and data conflict.

Constrained optimisation

Already (Bacharach 1970) has shown that the conventional RAS technique is equivalent to the constrained minimisation of an information gain function $f = \Sigma_{ij} A_{ij} \ln(A_{ij}/A_{0ij})$. Naturally, this circumstance leads to the parallel developments of both RAS and constrained optimisation techniques for the purpose of balancing input-output tables or SAMs. It is interesting to see that researchers working on either technique have faced almost the same challenges.

The basic structure of a constrained optimisation problem applied to SAMs is

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[18] (Dalgaard and Gysting 2004) do describe balancing matrices with "unequal net row and column sum" and "macro differences between supply and use". However, rather than inconsistencies in external information, this means correct differences in the sum over supply by *industry* and use by *product*, which naturally occur in asymmetric commodity-by-industry supply and use tables.

Eq. 6 Minimise $f(\mathbf{A}, \mathbf{A}_0)$, subject to $\Sigma_i A_{ij} = x_j$ and $\Sigma_j A_{ij} = x_i$,

where f is the objective function, and x_i and x_j are row and column totals. (Morrison and Thumann 1980) minimise a weighted sum of squares of deviations $f = \Sigma_{ij} (A_{ij} - A_{0ij})^2 / w_{ij}$, where the w_{ij} are the weights. They also explicitly describe the incorporation of external information referring to general subsets of matrix elements, into a Lagrange multiplier approach. Using a vectorised representation of $\mathbf{A} = \{a_i\}_{i=1,N\times N}$, a system of N_C constraints of any shape and size on $N\times N$ variables (including row and column totals) can be conveniently described in matrix notation:

Eq. 7 $\mathbf{G}\,\mathbf{a} = \mathbf{c}$,

Where the "aggregator matrix" $\mathbf{G}$ ($N_C \times N$) holds the coefficients linking the N variables a_i with the external data c_i on the N_C constraints.

(Byron 1978) incorporates variances $\mathbf{\Sigma}$ for the initial estimate $\mathbf{a}_0$ into a quadratic Lagrange function $f = (\mathbf{a} - \mathbf{a}_0)' \mathbf{\Sigma}^{-1} (\mathbf{a} - \mathbf{a}_0) + \boldsymbol{\lambda}'(\mathbf{Ga} - \mathbf{c})$, and uses the first-order conditions to solve for the Lagrange multipliers and the balanced SAM:

Eq. 8 $\boldsymbol{\lambda} = (\mathbf{G\Sigma G}')^{-1}(\mathbf{Ga}_0 - \mathbf{c})$,

Eq. 9 $\mathbf{a} = \mathbf{a}_0 - \mathbf{\Sigma G}\boldsymbol{\lambda}$.

(van der Ploeg 1982; van der Ploeg 1984; van der Ploeg 1988) elegantly extends Byron's formulation by a) adding disturbances $\boldsymbol{\varepsilon}$ to the external constraint information $\mathbf{c}$, so that $\mathbf{G}\,\mathbf{a} = \mathbf{c} + \boldsymbol{\varepsilon}$, and b) extending the unknown vector $\mathbf{a}$ with the unknown disturbances $\boldsymbol{\varepsilon}$, to a compound vector $\mathbf{p}$, distributed as

Eq. 10 $$\mathbf{p} = \begin{pmatrix} \mathbf{a} \\ \boldsymbol{\varepsilon} \end{pmatrix} \sim \mathrm{D}\left[\begin{pmatrix} \mathbf{a}_0 \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbf{\Sigma}_\mathbf{a} \\ \mathbf{\Sigma}_\mathbf{c} \end{pmatrix}\right] = \mathrm{D}[\mathbf{p}_0, \mathbf{\Sigma}]$$

with means $\mathbf{a}_0$ and 0, and variances $\mathbf{\Sigma}_\mathrm{a}$ and $\mathbf{\Sigma}_\mathrm{c}$. Exactly known constraints are a special case with the corresponding element of $\mathbf{\Sigma}_\mathrm{c}$ being zero. Extending $\mathbf{C} = (\mathbf{G}, -\mathbf{I})$, where $\mathbf{I}$ is the unity matrix, the generalised problem becomes

Eq. 11 Minimise $f = (\mathbf{p} - \mathbf{p}_0)' \mathbf{\Sigma}^{-1} (\mathbf{p} - \mathbf{p}_0)$, subject to $\mathbf{C}\,\mathbf{p} = \mathbf{c}$,

with solutions analog to Eq. 8 and Eq. 9. Since the solution for the Lagrange multipliers involves the inversion of $\mathbf{C\Sigma C}'$, computing times are strongly influenced by the sizes N and N_C of the SAM and constraint system. Both Byron and van der Ploeg go to great lengths in exploiting the sparse structure of the coefficients matrix, and in devising efficient algorithms in order to be able to solve large SAMs. In effect, it is the introduction of $\boldsymbol{\varepsilon}$ and $\mathbf{\Sigma}_\mathrm{c}$ that enables handling conflicting external data (van der Ploeg calls it "constraint violation"), because the disturbances $\boldsymbol{\varepsilon}$ in Eq. 10 and Eq. 11 allows the adjusted constraint value **G a** to deviate from its prescribed value **c**.

(Lecomber 1975), (Morrison and Thumann 1980), and (Harrigan and Buchanan 1984) explicitly note that the conventional Langrange multiplier procedure in Eq. 6 does not guarantee non-negative solutions. This is undesirable because negative matrix entries can present problems in input-output analysis ((Ten Raa and Van der Ploeg 1989)).

With the requirement of non-negativity, the constrained optimisation problem essentially becomes a *bounded* constrained optimisation. In general, one asks that the unknown SAM elements are within lower and upper bounds $l_i \leq a_i \leq u_i$. The mixing of equality and inequality conditions requires quadratic programming methods, which renders the solution of the optimisation problem considerably more complicated, as the expositions of (Harrigan and Buchanan 1984), (Zenios et al. 1989), and (Nagurney and Robinson 1992) may testify.

(Tarancon and Del Rio 2005) present an interesting variant of the bounded optimisation problem, by deriving lower and upper bounds from criteria for the stable structural evolution of input-output coefficients, and introducing supplementary variables to take up the slack between the bounds and the matrix entries. If the model turns out to be inconsistent because some constraints cannot be met within those bounds, then the analyst manually chosen certain constraints to be relaxed, until no variable exceeds the bounds.

Table 2: Recent extensions to RAS and optimisation techniques for balancing SAMs and input-output tables.

Criterion	RAS-type technique	Constrained optimisation
a)	(Gilchrist and St Louis 1999)	(Morrison and Thumann 1980)
b)	(Lecomber 1975; Lecomber 1975)	(Stone et al. 1942); (Byron 1978)
c)	(Lecomber 1975; Lecomber 1975); (Lahr 2001); (Dalgaard and Gysting 2004)	(van der Ploeg 1982)
d)	(Junius and Oosterhaven 2003)	(Harrigan and Buchanan 1984)
e)	This work	(van der Ploeg 1982)

CRAS – Constrained RAS

(Tarancon and Del Rio 2005) explicitly state that (p. 2) "… the RAS process cannot be developed with interval estimates of the margins. Hence, point estimates are used, which may carry an implicit error." On the other hand, compared to constrained optimisation techniques, RAS has enjoyed higher popularity, which is probably due to ease of programming. Considering that the use of RAS in statistical agencies requires the manual and therefore often tedious removal of inconsistencies in the constraint system, it would be desirable to have a RAS technique that deals with such common occurrences in a systematic and automated way. The description of such a RAS variant is the topic of this Section. We will base our derivation strongly on the GRAS notation of (Junius and Oosterhaven 2003).

In the standard GRAS method, the preliminary estimate $\mathbf{A}_0 = \mathbf{P}_0 - \mathbf{N}_0$ is alternately row- and column-scaled using diagonal matrices $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$, so that after the n-th round of balancing, $\mathbf{A}_n = \hat{\mathbf{r}}_{n-1}\mathbf{P}_{n-1}\hat{\mathbf{s}}_{n-1} - \hat{\mathbf{r}}_{n-1}^{-1}\mathbf{N}_{n-1}\hat{\mathbf{s}}_{n-1}^{-1}$. $\mathbf{A}_n$ is then subjected to the next scaling operation. GRAS uses scalers

Eq. 12
$$r_{n,i} = \frac{u_i^* + \sqrt{u_i^{*2} + 4\sum_j P_{n,ij}\sum_j N_{n,ij}}}{2\sum_j P_{n,ij}}, \text{ with}$$
$$P_{n,ij} = P_{n-1,ij}s_{n-1,j}, N_{n,ij} = N_{n-1,ij}s_{n-1,j}^{-1}, \text{ and}$$
$$s_{n-1,j} = \frac{v_j^* + \sqrt{v_j^{*2} + 4\sum_i P_{n-1,ij}\sum_i N_{n-1,ij}}}{2\sum_i P_{n-1,ij}}.$$

The algorithm converges if

Eq. 13
$$\begin{array}{l} \left\|\left(\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}\right)\mathbf{i} - \mathbf{u}^*\right\| < \delta\left\|\mathbf{u}^*\right\| \\ \left\|\mathbf{i}\left(\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}\right) - \mathbf{v}^*\right\| < \delta\left\|\mathbf{v}^*\right\| \end{array},$$

for a sufficiently small δ.

Incorporating constraints on arbitrary subsets of matrix elements

Consider now a generalised formulation of constraints as in $\mathbf{G}\ \mathbf{a} = \mathbf{c}$ (Eq. 7). Such a formulation includes constrained row and column sums, constraint single elements, constrained subsets, and negative elements as special cases. Constraints can include any number of elements, which may be fully, partly or non-adjacent.[19] Constraints may also exclude some of the row and column totals (compare (Thissen and Löfgren 1998), p. 1994). Let $\mathbf{G} = \mathbf{G}^+ - \mathbf{G}^-$ be a decomposition of the constraint coefficients matrix, analog to the decomposition $\mathbf{A} = \mathbf{P} - \mathbf{N}$ of $\mathbf{A}$. Let there be N_C constraints, and let $\mathbf{c}^* = e\ \mathbf{c}$. Eq. 11 can then be generalised to

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[19] Single-element constraints need not be part of the scaling procedure, but could be “netted out” using the “modified RAS” method.

Eq. 14
$$r_n = \frac{c_i^* + \sqrt{c_i^{*2} + 4\sum_j G_{ij}^+ a_{n-1,j} \sum_j G_{ij}^- a_{n-1,j}}}{2\sum_j G_{ij}^+ a_{n-1,j}} \quad \text{and}$$
$$a_{n,j} = a_{n-1,j} r_n^{\operatorname{Sgn}(G_{ij})}, \qquad \text{with } i = n - \left\lfloor \frac{n}{N_C} \right\rfloor N_C .$$

In Eq. 14, the negative elements in Eq. 11 have been replaced with negative coefficients on positive elements, but otherwise the formulation is exactly the same. There is only one scaler r_i for each constraint i, and these scalers are applied consecutively for all $i = 1,\ldots,N_C$.[20] The r_i and a_j are calculated alternately. The GRAS feature of scaling negative elements by the inverse of the positive scaler is evident in the exponent $\operatorname{Sgn}(G_{ij})$ in Eq. 14. The algorithm converges if

Eq. 15
$$\left\|\mathbf{Ga} - \mathbf{c}^*\right\| < \delta \left\|\mathbf{c}^*\right\| ,$$

for a sufficiently small δ.

Incorporating reliability and conflict of external data

In cases of inconsistent constraints brought about by conflicting external data, the termination condition (11) may never be met, and GRAS has to be terminated if the distance function between the constraints **c** and their realisations **G a** does not improve anymore, that is if for two subsequent iterations $n - 1$ and n

Eq. 16
$$\left\|\mathbf{Ga} - \mathbf{c}^*\right\|_n - \left\|\mathbf{Ga} - \mathbf{c}^*\right\|_{n-1} < \delta ,$$

for a sufficiently small δ. Following this termination, we propose a GRAS-type algorithm that modifies the constraints **c*** as well:

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[20] The symbol $\lfloor \; \rfloor$ in equation 10 is the floor function and refers to the largest integer smaller than the number inside.

$$r_n = \frac{c^*_{n,i} + \sqrt{c^{*2}_{n,i} + 4\sum_j G^+_{ij} a_{n-1,j} \sum_j G^-_{ij} a_{n-1,j}}}{2\sum_j G^+_{ij} a_{n-1,j}},$$

Eq. 17 $$c^*_{n,i} = c^*_{n-1,i} - \mathrm{Sgn}\left(c^*_{n-1,i} - \sum_j G_{ij} a_{n-1,j}\right) \times \mathrm{Min}\left(\left|c^*_{n-1,i} - \sum_j G_{ij} a_{n-1,j}\right|, \alpha\sigma_i\right)$$

$$\text{with } c^*_{0,i} = c^*_i\text{, } a_{n,j} = a_{n-1,j} r_n^{\mathrm{Sgn}(G_{ij})}\text{, and } i = n\text{-}\left\lfloor \frac{n}{N_C} \right\rfloor N_C\text{,}$$

where $0 \le \alpha \le 1$ and σ_i is the standard error of c_i. We refer to this algorithm as CRAS ('Conflicting RAS'). The essence of this idea is that once GRAS terminates in oscillations without reaching convergence, the original external constraints c_i can clearly not all be satisfied simultaneously, and either some of them or all of them must be erroneous. In order to achieve convergence, the c_i must be modified "towards" their realisations $\{\mathbf{Ga}\}_i$. Since each constraint is known to a higher or lower degree of accuracy. Therefore, an amount $\alpha\sigma_i$ is added or subtracted from each $c^*_{n-1,i}$, depending on the sign Sgn($c^*_{n-1,i} - \sum_j G_{ij} a_{n-1,j}$). The constant α can be chosen freely: The higher its value, the more rapid the adjustment process, but also the more inaccurate the adjustment. Note that in order to prevent overshoot in situations where the realisation $\{\mathbf{Ga}\}_i$ is closer to the c_i than σ_i, the maximum adjustment allowed is $\left| c^*_{n-1,i} - \sum_j G_{ij} a_{n-1,j} \right|$. With constraint values modified as in Eq. 17, the termination criterion of CRAS is equal to that in Eq. 15.

3. Data sources and data preparation

3.1. UK input-output data

One important part of the work involves the provision of meaningful initial data. The closer these initial estimates are to the 'real' data, the more accurate the balanced results will be. The starting basis for our calculations in this project will be the currently available input-output data from ONS. Additional information such as the transition matrix from basic to purchaser's prices in the Analytical Tables 1995 form other crucial information about the structure of imports and other data.

In the UK, input-output data are collated and published regularly by the Office for National Statistics as part of the National Accounting framework (ONS 2006), (Mahajan 2006).[21] The data are presented in various formats of which those with the highest numbers of sectors and detailed inter-industry transactions including those with foreign countries are most relevant for this

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[21] See http://www.statistics.gov.uk/inputoutput.

project.[22] For the years from 1992 to 2004 the following tables are currently publicly available (numbers in brackets show the numbers of sectors or headings in the tables; excluding totals and sub-totals):

ONS Table 1: Domestic output at basic prices (123)

ONS Table 2: Supply of products in basic and purchasers' prices, including trading margins and taxes less subsidies on products (123)

ONS Table 3a: Demand for products - The 'Combined Use' matrix - Intermediate demand (123 x 124) (all intermediate consumption at purchasers' prices, except of Gross Value Added and Total Output which are at basic prices)

ONS Table 3b: Demand for products - The 'Combined Use' matrix - Final demand (123 x 11) (all at purchasers' prices)

ONS Table 8: Summary analysis of domestic output at basic prices (supply matrix, 30 x 30)

Additional IO analyses (not relevant for the current project):

ONS Table 4: Household final consumption expenditure by functional heading (123 x 43)

ONS Table 5: General government final consumption by type of service (123 x 8)

ONS Table 6: Gross fixed capital formation (123 x 39)

ONS Table 7: Production accounts by sector and for the whole economy (summary table)

Expanding supply tables

For the purpose of this project it is advantageous to have initial estimates of supply tables in 123 sector breakdown. Published data however show complete supply tables by 30 industries only (ONS Table 8) and much of the data even at this level of aggregation is considered disclosive. A request to ONS to provide supply tables at 123 industries by 123 products was not granted on the grounds that this would be contrary to current statistics legislation[23], even on the proviso that they are not published (Gazley 2007).

We have therefore reverted to Eurostat which also publishes these tables (Eurostat 2007). The IO data from ONS is consistent with the European System of Accounts (ESA 95) and is regularly submitted to Eurostat. However, the Eurostat publications show supply tables in a 59 sector resolution and thus in a more detailed format than the 30 sector supply tables published by ONS.

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[22] http://www.statistics.gov.uk/about/methodology_by_theme/inputoutput/latestdata.asp. Due to an ongoing major programme of modernisation of the UK National Accounts, the annual updating of the accounts in the *Blue Book 2007* through the existing supply and use tables is not taking place in 2007 and the latest annual benchmark data will not be incorporated until 2008. In 2007 ONS is not producing Input-Output Annual Supply and Use Tables or Input-Output Analyses for the year 2005 (Beadle 2007).

[23] This policy is outlined on page 301 of the UK Input-Output publication (ONS 2006).

These 59x59 supply tables are available for the year 1995 to 2004 and have been expanded to 123x123 tables by using the following procedure.

Suppressed (confidential) data points were estimated and filled in manually in the original 59x59 Eurostat supply tables in such a way that industry and commodity totals would change less than 1% and that the highest value in any one row or column would always be at the crossing of industry and corresponding commodity (diagonal of primary products). These tables were then expanded to 123x123 sectors by using total output of industries and commodities as given in ONS Table 2. Vertical expansion from 59 to 123 sectors was done by applying the proportions of total domestic supply of 123 products to all rows of the supply matrix. Accordingly, horizontal expansion was performed by applying the proportions of total output of 123 industries to all columns of the supply matrix. Information on the principal product as a percentage of total industry output and of total commodity output (i.e. the proportion of diagonal vs non-diagonal elements, provided in ONS Table 2) was then used as a constraint for balancing the supply tables.

Creating domestic use tables in basic prices

Combined use tables for intermediate and final demand are provided by ONS in 123 sector format (ONS Tables 3a and 3b; (ONS 2006). Two modifications need to be made before these tables can be used in the MRIO model; they need be converted from purchasers' prices to basic prices and imports need to be subtracted in order to obtain the domestic use tables for intermediate and final demand ($\mathbf{U}^{uu}$ and $\mathbf{y}^{u}$ in Table 1). The 'transition matrix' published by ONS in the 'UK Input-Output Analytical Tables 1995' achieves both steps in one go by combining imports, trading margins and taxes less subsidies in one table (Ruiz and Mahajan 2002). We use this transition matrix from 1995 for two purposes, a) to create the initial estimates of the imports matrices $\mathbf{U}^{eu}$, $\mathbf{y}^{eu}$, $\mathbf{U}^{ou}$, $\mathbf{y}^{ou}$, $\mathbf{U}^{wu}$, $\mathbf{y}^{wu}$, and b) to derive a domestic use matrix in basic prices for each year 1992 to 2004. More specific information, such as transition and/or imports matrices for years other than 1995 – which would have made our initial estimates more accurate – was not available from ONS (Mahajan 2007), (see also Druckman et al. 2007).

Whilst the Use and Transition tables are provided in product by industry form, the published Imports table is in product by product form, which according to (Ruiz and Mahajan 2002) was calculated by applying RAS to known product column totals of a product by industry table. As a first step, hence, the imports table was necessarily re-engineered into a product by industry table by re-applying RAS to the published industry column totals. The resulting product by industry Imports table was then subtracted from the published product by industry Transition matrix to obtain a Transition matrix that referred only to Distributors' trading margins and Taxes less subsidies on products. Finally, the domestic Use table in basic prices was obtained by subtracting the Transition and Imports tables from the original Use table.

The lack of structural data on imports and margins for any year other than 1995 necessitated the assumption that there had been no change in the relative amount consumers pay/receive in imports, taxes/subsidies and distribution margins from 1995 to other years.[24] The total amount of

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[24] Note that this problem (the difficulty of converting Use tables from purchasers' to basic prices and from combined to domestic layout because of lack of published data due to confidentiality guidelines being followed by ONS) is also well documented by (Druckman et al. 2007).

imports, taxes/subsidies and margins of each product, is, however, known, and included as a constraint on the data.

The method to split up the total imports matrix into contributions from the three world regions is described in Section 3.3 below.

3.2. Non-UK input-output data

Absolute figures for IO analysis that correspond to the Rest of the World (ROW) are not available. Thus, in a MRIO model, this region can not be explicitly, but only structurally modelled. As such, only the technical coefficients estimated for the ROW will be included, and not total levels of transactions between foreign economic sectors.

There are only a few databases worldwide that hold input-output tables for the whole or large regions of the world economy. The most important are OECD, GTAP, IDE-JETRO and Eurostat.[25] In the following we examine the suitability of those databases for our ROW approximation.

The OECD Input-Output Database has recently been updated with the 2006 edition (Ahmad et al., 2006;Wixted et al., 2006;Yamano and Ahmad, 2006). The first edition of this collection of IO tables dates back to 1995 and covered 10 OECD countries spanning the period 1968 to 1990. The first update to this was the 2002 edition of the database, which increased the country coverage to 18 OECD and 2 large non-OECD countries, spanning the period 1992 to 1997. The 2006 edition has continued this expansion and includes 37 countries (28 OECD and 9 non-OECD) further strengthening the ability of the database to allow the analysis of global issues. These latest tables are based around the year 2000 for most countries, though for some, more recent years are provided (for example, 2003 for Mexico). Figure 1 shows the coverage of global GDP of the respective editions of OECD IO tables. For a broad overview of potential uses of 'harmonised' Input-Output tables see (Wixted et al. 2006).

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[25] Compare with the summary on databases of international input-output transactions from (Wixted et al., 2006):12-14).

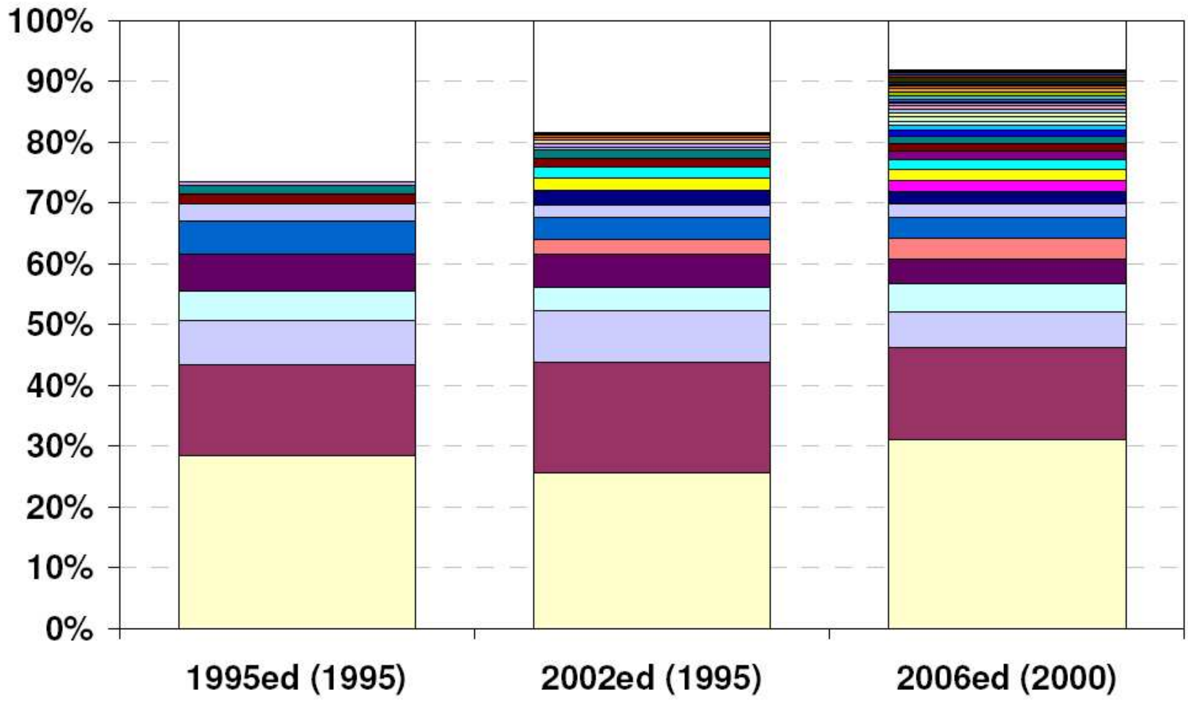

Figure 1: **OECD IO Database of global GDP by edition (from (Ahmad et al. 2006; Yamano and Ahmad 2006).**

The OECD database however does not offer aggregated IO tables for world regions or even the whole world economy. For this purpose, individual tables would have to be aggregated separately.

OECD input-output and trade data are also used by the econometric model GINFORS (Global INterindustry FORecasting System) which was applied in the European MOSUS project[26] (Lutz et al., 2005) (see also (Meyer et al., 2003a;Meyer et al., 2003b) as well as (Giljum et al., 2006b) (Giljum et al., 2006a).[27] However, although time series from 1980 to 2002 for trade data are provided, the model does not aggregate data from individual countries and thus there are no input-output data for larger world regions (see also (Wiedmann et al., 2007).

The Institute of Developing Economies (IDE-JETRO, http://www.ide.go.jp) offers a database of international input-output tables for Asia and the US for the years 1985, 1990, 1995 and 2000. The general layout is depicted in Table 3 (the most detailed industrial classification comprises 76 sectors.). The tables also include import and export matrices for Hong Kong, the EU and the "Rest of the World" but obviously this is not the same as technology matrices. Basically, IDE-JETRO define the ROW matrix as a residual of import matrices from the national IO tables after separating out all the import matrices from the member countries[28], and treat it as exogenous data

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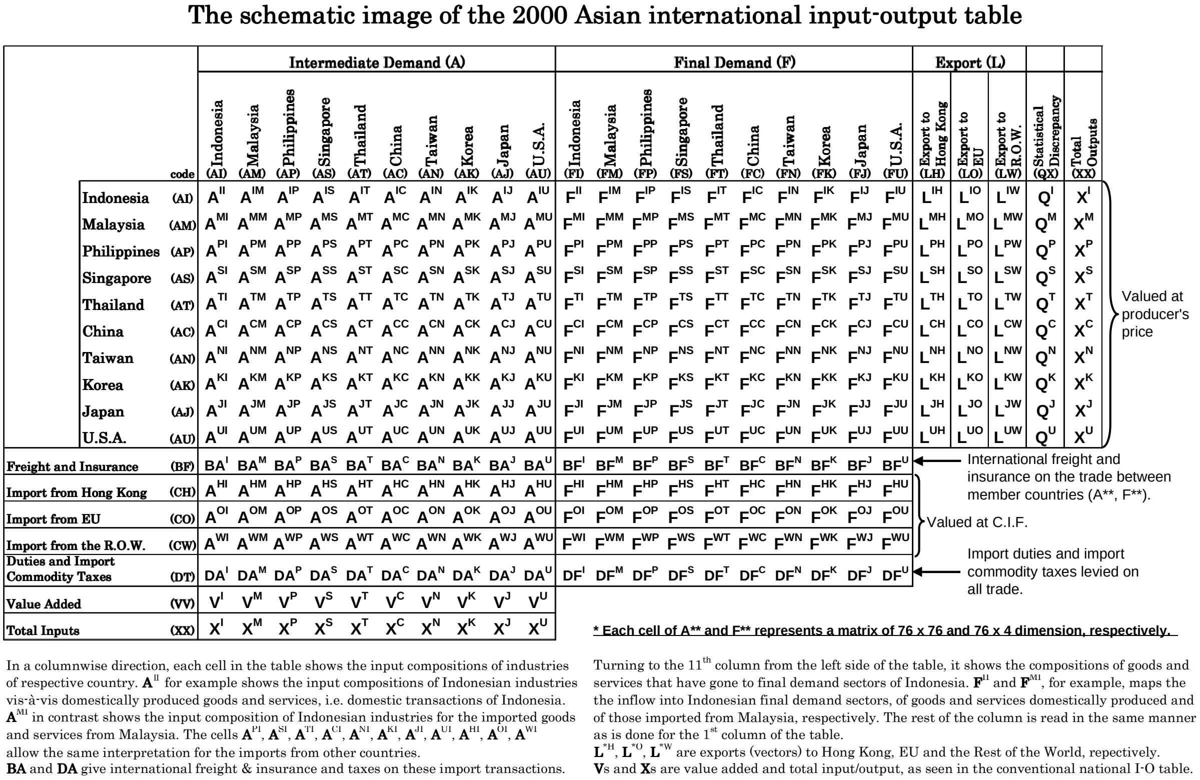
[26] "Is Europe sustainable? MOdelling opportunities and limits for restructuring Europe towards SUStainability", see http://www.mosus.net.

[27] As a follow-up of the MOSUS project it is intended to set up a global multi-country input-output model in order to quantify embodied natural resource requirements and to calculate comprehensive material flow indicators such as Total Material Consumption (TMC) (Giljum, 2005).

[28] Japan, USA, China (mainland), Korea, Taiwan, Indonesia, Malaysia, Thailand, the Philippines, Singapore, Hong Kong, and EU.

to the Leontief inverse system. Henceforth, an A matrix for the ROW is not estimated (Inomata, 2007).

Table 3: Schematic illustration of the 2000 Asian international input-output table from IDE-JETRO (Inomata, 2007).



The schematic image of the 2000 Asian international input-output table

		Intermediate Demand (A)										Final Demand (F)										Export (L)				
	code	Indonesia (AI)	Malaysia (AM)	Philippines (AP)	Singapore (AS)	Thailand (AT)	China (AC)	Taiwan (AN)	Korea (AK)	Japan (AJ)	U.S.A. (AU)	Indonesia (FI)	Malaysia (FM)	Philippines (FP)	Singapore (FS)	Thailand (FT)	China (FC)	Taiwan (FN)	Korea (FK)	Japan (FJ)	U.S.A. (FU)	Export to Hong Kong (LH)	Export to EU (LO)	Export to R.O.W. (LW)	Statistical Discrepancy (QX)	Total Outputs (XX)
Indonesia	(AI)	A^{II}	A^{IM}	A^{IP}	A^{IS}	A^{IT}	A^{IC}	A^{IN}	A^{IK}	A^{IJ}	A^{IU}	F^{II}	F^{IM}	F^{IP}	F^{IS}	F^{IT}	F^{IC}	F^{IN}	F^{IK}	F^{IJ}	F^{IU}	L^{IH}	L^{IO}	L^{IW}	Q^{I}	X^{I}
Malaysia	(AM)	A^{MI}	A^{MM}	A^{MP}	A^{MS}	A^{MT}	A^{MC}	A^{MN}	A^{MK}	A^{MJ}	A^{MU}	F^{MI}	F^{MM}	F^{MP}	F^{MS}	F^{MT}	F^{MC}	F^{MN}	F^{MK}	F^{MJ}	F^{MU}	L^{MH}	L^{MO}	L^{MW}	Q^{M}	X^{M}
Philippines	(AP)	A^{PI}	A^{PM}	A^{PP}	A^{PS}	A^{PT}	A^{PC}	A^{PN}	A^{PK}	A^{PJ}	A^{PU}	F^{PI}	F^{PM}	F^{PP}	F^{PS}	F^{PT}	F^{PC}	F^{PN}	F^{PK}	F^{PJ}	F^{PU}	L^{PH}	L^{PO}	L^{PW}	Q^{P}	X^{P}
Singapore	(AS)	A^{SI}	A^{SM}	A^{SP}	A^{SS}	A^{ST}	A^{SC}	A^{SN}	A^{SK}	A^{SJ}	A^{SU}	F^{SI}	F^{SM}	F^{SP}	F^{SS}	F^{ST}	F^{SC}	F^{SN}	F^{SK}	F^{SJ}	F^{SU}	L^{SH}	L^{SO}	L^{SW}	Q^{S}	X^{S}
Thailand	(AT)	A^{TI}	A^{TM}	A^{TP}	A^{TS}	A^{TT}	A^{TC}	A^{TN}	A^{TK}	A^{TJ}	A^{TU}	F^{TI}	F^{TM}	F^{TP}	F^{TS}	F^{TT}	F^{TC}	F^{TN}	F^{TK}	F^{TJ}	F^{TU}	L^{TH}	L^{TO}	L^{TW}	Q^{T}	X^{T}
China	(AC)	A^{CI}	A^{CM}	A^{CP}	A^{CS}	A^{CT}	A^{CC}	A^{CN}	A^{CK}	A^{CJ}	A^{CU}	F^{CI}	F^{CM}	F^{CP}	F^{CS}	F^{CT}	F^{CC}	F^{CN}	F^{CK}	F^{CJ}	F^{CU}	L^{CH}	L^{CO}	L^{CW}	Q^{C}	X^{C}
Taiwan	(AN)	A^{NI}	A^{NM}	A^{NP}	A^{NS}	A^{NT}	A^{NC}	A^{NN}	A^{NK}	A^{NJ}	A^{NU}	F^{NI}	F^{NM}	F^{NP}	F^{NS}	F^{NT}	F^{NC}	F^{NN}	F^{NK}	F^{NJ}	F^{NU}	L^{NH}	L^{NO}	L^{NW}	Q^{N}	X^{N}
Korea	(AK)	A^{KI}	A^{KM}	A^{KP}	A^{KS}	A^{KT}	A^{KC}	A^{KN}	A^{KK}	A^{KJ}	A^{KU}	F^{KI}	F^{KM}	F^{KP}	F^{KS}	F^{KT}	F^{KC}	F^{KN}	F^{KK}	F^{KJ}	F^{KU}	L^{KH}	L^{KO}	L^{KW}	Q^{K}	X^{K}
Japan	(AJ)	A^{JI}	A^{JM}	A^{JP}	A^{JS}	A^{JT}	A^{JC}	A^{JN}	A^{JK}	A^{JJ}	A^{JU}	F^{JI}	F^{JM}	F^{JP}	F^{JS}	F^{JT}	F^{JC}	F^{JN}	F^{JK}	F^{JJ}	F^{JU}	L^{JH}	L^{JO}	L^{JW}	Q^{J}	X^{J}
U.S.A.	(AU)	A^{UI}	A^{UM}	A^{UP}	A^{US}	A^{UT}	A^{UC}	A^{UN}	A^{UK}	A^{UJ}	A^{UU}	F^{UI}	F^{UM}	F^{UP}	F^{US}	F^{UT}	F^{UC}	F^{UN}	F^{UK}	F^{UJ}	F^{UU}	L^{UH}	L^{UO}	L^{UW}	Q^{U}	X^{U}
Freight and Insurance	(BF)	BA^{I}	BA^{M}	BA^{P}	BA^{S}	BA^{T}	BA^{C}	BA^{N}	BA^{K}	BA^{J}	BA^{U}	BF^{I}	BF^{M}	BF^{P}	BF^{S}	BF^{T}	BF^{C}	BF^{N}	BF^{K}	BF^{J}	BF^{U}					
Import from Hong Kong	(CH)	A^{HI}	A^{HM}	A^{HP}	A^{HS}	A^{HT}	A^{HC}	A^{HN}	A^{HK}	A^{HJ}	A^{HU}	F^{HI}	F^{HM}	F^{HP}	F^{HS}	F^{HT}	F^{HC}	F^{HN}	F^{HK}	F^{HJ}	F^{HU}					
Import from EU	(CO)	A^{OI}	A^{OM}	A^{OP}	A^{OS}	A^{OT}	A^{OC}	A^{ON}	A^{OK}	A^{OJ}	A^{OU}	F^{OI}	F^{OM}	F^{OP}	F^{OS}	F^{OT}	F^{OC}	F^{ON}	F^{OK}	F^{OJ}	F^{OU}					
Import from the R.O.W.	(CW)	A^{WI}	A^{WM}	A^{WP}	A^{WS}	A^{WT}	A^{WC}	A^{WN}	A^{WK}	A^{WJ}	A^{WU}	F^{WI}	F^{WM}	F^{WP}	F^{WS}	F^{WT}	F^{WC}	F^{WN}	F^{WK}	F^{WJ}	F^{WU}					
Duties and Import Commodity Taxes	(DT)	DA^{I}	DA^{M}	DA^{P}	DA^{S}	DA^{T}	DA^{C}	DA^{N}	DA^{K}	DA^{J}	DA^{U}	DF^{I}	DF^{M}	DF^{P}	DF^{S}	DF^{T}	DF^{C}	DF^{N}	DF^{K}	DF^{J}	DF^{U}					
Value Added	(VV)	V^{I}	V^{M}	V^{P}	V^{S}	V^{T}	V^{C}	V^{N}	V^{K}	V^{J}	V^{U}															
Total Inputs	(XX)	X^{I}	X^{M}	X^{P}	X^{S}	X^{T}	X^{C}	X^{N}	X^{K}	X^{J}	X^{U}															

Valued at producer's price

International freight and insurance on the trade between member countries (A**, F**).

Valued at C.I.F.

Import duties and import commodity taxes levied on all trade.

*** Each cell of A** and F** represents a matrix of 76 x 76 and 76 x 4 dimension, respectively.**

In a columnwise direction, each cell in the table shows the input compositions of industries of respective country. $\mathbf{A}^{II}$ for example shows the input compositions of Indonesian industries vis-à-vis domestically produced goods and services, i.e. domestic transactions of Indonesia. $\mathbf{A}^{MI}$ in contrast shows the input composition of Indonesian industries for the imported goods and services from Malaysia. The cells $\mathbf{A}^{PI}$, $\mathbf{A}^{SI}$, $\mathbf{A}^{TI}$, $\mathbf{A}^{CI}$, $\mathbf{A}^{NI}$, $\mathbf{A}^{KI}$, $\mathbf{A}^{JI}$, $\mathbf{A}^{UI}$, $\mathbf{A}^{HI}$, $\mathbf{A}^{OI}$, $\mathbf{A}^{WI}$ allow the same interpretation for the imports from other countries.
BA and **DA** give international freight & insurance and taxes on these import transactions.

Turning to the 11th column from the left side of the table, it shows the compositions of goods and services that have gone to final demand sectors of Indonesia. $\mathbf{F}^{II}$ and $\mathbf{F}^{MI}$, for example, maps the the inflow into Indonesian final demand sectors, of goods and services domestically produced and of those imported from Malaysia, respectively. The rest of the column is read in the same manner as is done for the 1st column of the table.
$\mathbf{L}^{*H}$, $\mathbf{L}^{*O}$, $\mathbf{L}^{*W}$ are exports (vectors) to Hong Kong, EU and the Rest of the World, repectively.
Vs and **X**s are value added and total input/output, as seen in the conventional national I-O table.

The European System of Accounts ESA 95 has established a compulsory transmission of tables of the input-output framework by the European Member States. In detail this concerns annual supply- and use-tables, five-yearly symmetrical input-output tables, symmetrical input-output tables of domestic production and symmetrical input-output tables of imports. All these tables cover the period from 1995 onwards and are harmonised by Eurostat's standardised questionnaire, which distinguishes 60 products (classification CPA P60) and 60 industries (NACE Rev.1 A60). Currently, IO data are available for 24 European Member States and Norway.[29] However, there are no aggregated IO tables for parts or the whole of Europe.[30] See also (Huppes et al., 2006) for a critique of the IO data situation in Europe.

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[29] http://epp.eurostat.ec.europa.eu/pls/portal/url/page/PGP_DS_ESA_IOT/PGE_DS_ESA_01

[30] The Regional Economics Department at the University of Groningen offers some EU inter-country input-output tables for download (www.regroningen.nl/index_en.html), the most recent one from 1985 featuring six interlinked EU countries.

A linked IO model with world coverage is described by (Shimpo and Okamura, 2006). According to this source, Keio University is compiling an inventory of IO tables from more than 60 countries by sending questionnaires to national statistical offices in the world and conducting surveys on website. However, no more information could be retrieved from the website.

GTAP (Global Trade Analysis Project) is a global network of researchers and policy makers conducting quantitative analysis of international policy issues and is coordinated by the Center for Global Trade Analysis, Purdue University, USA (http://www.gtap.agecon.purdue.edu). Products from GTAP include data, models, and utilities for multi-region, applied general equilibrium analysis of global economic issues. The GTAP 6 data base (Dimaranan 2006) describes bilateral trade patterns, production, consumption and intermediate use of commodities and services of the global economy in 2001. The data is disaggregated to 57 sectors and 87 countries/regions and thus the data base is able to capture some detail of interactions between domestic sectors as well as international trading partners. Aggregated data (e.g. one large IO table of the world economy) is not available for GTAP 6, although two aggregations of the GTAP 5 data base can be purchased (10 sectors x 66 regions and 57 sectors x 10 regions).

GTAP data are used in several studies with MRIO models for the calculation of impacts embodied in trade (see (Wiedmann et al., 2007). Whereas (Chung, 2005) aggregates the data into nine regions of the world, Nijdam and colleagues (Nijdam et al. 2005) construct technological matrices for three world regions from the GTAP input-output tables, representing OECD Europe, OECD non-Europe and non-OECD countries.

The Netherlands Environmental Assessment Agency (MNP) (Wilting, 2007) courteously provided us with the technical coefficient matrices for 1997 used in the study by (Nijdam et al. 2005) which are based on the GTAP 5 database as well as with a similar dataset for the year 2001 (based on the GTAP 6 database). These six technological matrices were derived from GTAP coefficient 'cost structure of firms' and distinguish 30 economic sectors. The coefficients include both domestic as imported inputs. By using these coefficients it is assumed that the imports of a certain region are produced with the technology of that region (see (Nijdam et al. 2005): 151).

We use the six tables to apply constraints to the MRIO for the three non-UK regions. Due to the lack of data for any years but 1997 and 2001, static technical coefficients are assumed for three time periods: 1997 and earlier (using 1997 technical coefficients); 1998-2000 (using average coefficients); and 2001 and later (using 2001 technical coefficient).

Table 4: Country coverage of three world regions in the UK-MRIO model as adopted from (Nijdam et al. 2005)

Region e **OECD Europe**	**Region o** **OECD non-Europe**	**Region w** **non_OECD**
Austria	Canada	All other countries
Belgium	Mexico	
Czech Republic	United States	
Denmark	Australia	
Finland	Japan	
France	Korea	
Germany	New Zealand	
Greece		
Hungary		
Iceland		
Ireland		
Italy		
Luxembourg		
Norway		
Poland		
Portugal		
Slovak Republic		
Spain		
Sweden		
Switzerland		
Turkey		
United Kingdom*)		

*) Due to the original model purpose of (Nijdam et al. 2005) the A matrix from Region e includes technical coefficients from the UK and excludes those from the Netherlands. However, CO_2 emissions were compiled differently in order to be more in line with the purpose of the UK-MRIO model and thus include CO_2 emissions for the Netherlands and exclude those for the UK (see Section 3.4 below).

3.3. Trade data

Imports matrices

A very important component of the MRIO system is separate matrices for imports to (UK) intermediate and final demand for each of the three world regions. These are not part of the annual ONS publications and a total imports matrix has only been published once as part of the 1995 Analytical Tables ("Imports Use matrix at basic prices, Product by Product", (Ruiz and Mahajan 2002). We have described in Section XX above how we made use of this information to derive imports matrices for all years of the time series. In the following Section we describe the method to split up the total imports matrix into contributions from the three world regions, i.e. to create $\mathbf{U}^{eu}$, $\mathbf{y}^{eu}$, $\mathbf{U}^{ou}$, $\mathbf{y}^{ou}$, $\mathbf{U}^{wu}$, and $\mathbf{y}^{wu}$ in Table 1.

There is a range of international trade statistics that specify trade volumes in both f.o.b. and c.i.f. valuation. However, these statistics only detail the amounts of commodities traded between countries but not their usage by industries (elements U_{ij}^{ru} flow matrices). In other words, it is in general not possible to find information on the spatial origin of every intermediate and final import, disaggregated according to the consuming sector in the country of destination (see also (Boomsma et al. 1991, pp.7-8). This is mainly because of the considerable cost, time and resources that are associated with conducting international industry surveys (Round 1978; Round 1978).

One solution to the generation of an initial (pre-balancing) estimate of off-diagonal trade flow matrices is to use *trade coefficients* (a non-survey approach)

Eq. 18
$$c_i^{rs} = \frac{u_i^{rs}}{\sum_r u_i^{rs}} \quad \text{with} \quad \sum_r c_i^{rs} = 1 ,$$

describing the percentage of imports of commodity i into country s (here the UK) that come from country r. These trade coefficients can then be applied to an entire row of the national imports matrices $\{M_{ij}^{s}\}$ and imported final demand vectors $\{f_i^{s}\}$ in order to yield breakdown according to country of origin:

Eq. 19
$$U_{ij}^{rs} = c_i^{rs} M_{ij}^{s} \quad \text{and} \quad y_i^{rs} = c_i^{rs} f_i^{s} .$$

This procedure assumes that the trade coefficients are identical for all entries along a row of the imports matrix, that is for all using domestic industries. Additionally, for years without separate import matrices (which is the case for the UK), an initial estimation of import coefficients can be made by assuming the relative importance of the usage of commodity by industry is constant over time.

Trade data

International trade data are available from a variety of sources including ONS, Eurostat, OECD and UN Statistics. In addition, and especially for the years 1999 to 2004, UK specific trade data are also available from HM Revenue & Customs' 'Statistics and Analysis of Trade Unit' (http://www.uktradeinfo.com) which formed the main data source for trade data used in this project.

However, when compiling the trade data we encountered major problems, e.g.

- trade in services is not included in the standard databases
- concordance matrices had to be constructed in order to convert the data to a 123 sector format
- data for the years 1996 to 1998 from HMRC was in a different classification than the data available from www.uktradeinfo.com (and had to be purchased).
- data for the years 1992 to 1995 were not available at all.

Trade in goods

We obtained data of *trade in goods* in 5 digits SITC (Standard International Trade Code) format from HM Revenue & Customs for the years of 1999-2004 (www.uktradeinfo.com). The dataset for each year is available for 240 countries on approximately 2,500 different products. According to the requirements for this project, we needed to compile the trade dataset in the format of three world regions by 123 economic sectors. A country concordance matrix was used to compile the 240 countries into the three regions – European OECD countries, Non-European OECD countries and the rest of the world. A commodity concordance matrix was used to convert the 2,500 products in SITC format into 123 input-output categories in terms of the "Classification of 123 Input-Output industry/product groups by Standard Industrial Classification (SIC) 2003 and NACE Revision 1.1" as provided in "UK input-output analysis – edition 2006"[31], and the concordance matrix between SITC and SIC 2003 as provided by Eurostat[32].

For the years of 1996-1998, data of *trade in goods* in 4 digits SITC format had to be purchased from HM Revenue & Customs. The dataset only provides the data (imports or exports) between the UK and the other countries which have trade transactions in particular commodities, but no data entry is made if there is no transaction (not even 'zero'). This results in an inconsistency of the country list in every product. For example, under the category of "growing of cereals and other crops", the dataset provides the trade data for both France and Austria since the two countries have transactions with the UK for cereals products in a particular year. However, under the category of "farming of cattle, sheep, goats, horses, asses, mules and hinnies; dairy farming", the dataset only shows the data for France but not Austria since Austria did not have trade transactions with the UK for this particular product. Therefore, the number of European countries which had the trade transaction with the UK for the category of "growing of cereals and other crops" is 9, but the number changes to 5 for the category of "farming of cattle, sheep, goats, horses, asses, mules and hinnies; dairy farming". In order to compile the data to the standard format of three regions with 123 input-output sectors, we had to take five steps to achieve this. Firstly we sorted the dataset by country alphabetically and separated the trade data between EU OECD countries, Non-EU OECD countries and the rest of world in three different files. Secondly we sorted the data in each file by SITC category; and then run the subtotal for each SITC category; extracted and saved the subtotals to a new file for each region. Each subtotal contains the information of summation of each SITC product in all countries in each region. Thirdly we compared the SITC lists between the three regions (three different files); there are 1034 products categories in the list of EU OECD countries, 1030 categories in Non-EU OECD countries and 1028 categories in the rest of world. In order to make the lists consistent, we manually assigned "zeros" to the missing categories in Non-EU OECD countries and the rest of world lists to make a consistent list of SITC categories of 1034 for all regions. Fourthly we created a concordance matrix between the 4-digits SITC format (1034 sectors) with 123 IO sectors. At last, we distinguished between EU OECD countries, Non-EU OECD countries and the rest of world.

For the early years (1992-1995) there was no trade data available at all. Enquiries with professional data providers suggested by HMRC resulted in no response. Due to this unavailability

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[31] ONS website http://www.statistics.gov.uk/downloads/theme_economy/Input_Output_Analyses_2006_edition.pdf

[32] Eurostat website http://ec.europa.eu/eurostat/ramon/relations/index.cfm?TargetUrl=LST_REL&StrLanguageCode=EN&IntCurrentPage=2

of trade data on country level for these years we used a linear trendline from the year 1996 to 2004 and projected backwards the figures for each input-output sector between 1992 and 1995.

As a last step, we compared our dataset of *trade in goods* with the totals for imports in goods from the Supply and Use tables provided by ONS. The summation of the three regional trade data in each IO sector matches with the ONS totals for imports in the region of ±20% or better. In order to be consistent with IO data, we derived the percentage breakdown of each IO sector for the three regions for each year by using the compiled dataset of *trade in goods*; then multiplied the percentage breakdown with the sectoral ONS total for imports in goods to generate the compiled *trade in goods* dataset in the format of three world regions by 123 sectors.

Trade in Services

Data on the UK *trade in services* is available from the 'Pink Book' published annually by ONS (ONS 2006). By courtesy of ONS we obtained Excel tables of trade in services data for the years of 1997 to 2004 (Lowes 2007). The trade in services data has 11 categories with distinction between 31 regions and countries. Similarly to the process of compiling the data of *trade in goods*, we firstly aggregated the data into three regions. For the category of EU OECD countries, the data is available for 1997-2003, which is represented as “EU 15” in the original dataset. The “EU 15” is replaced by “EU 25” in 2004 dataset, we assumed that the new 10 EU countries have same trade pattern to “Philippines”. Therefore to generate EU OECD in 2004, we used the EU 25 figures minus ten times the Philippine’s services imports to the UK. For the category of Non OECD countries, most of individual countries data are available except Norway, Czech Republic and Poland. We assumed that the three countries have the same trade pattern as South Korea. Therefore, we add all available Non EU OECD countries data plus three times the figures of South Korea. To generate the figures of the rest of world, we deducted the EU OECD and Non EU OECD from the world totals.

Finally, we assigned the 11 services categories to the 57 IO services sectors by generating the percentage breakdowns for the 11 services categories between the three world regions, and then multiplying with the ONS totals for imports in services. This results in trade in services data for the three world regions by 57 IO services sectors which are consistent with the total imports figures provided in the annual Supply and Use Tables. Again, for the early years 1992 to 1996, where no trade in services data is available, we used the trend for the year 1997 to 2004 for each sector and projected backwards.

3.4. Carbon dioxide emissions and intensities

Sectoral carbon dioxide emissions estimates for the UK economy can be found in the Environmental Accounts, which are published bi-annually by the Office for National Statistics (ONS 2007). The data distinguishes emissions from 91 production and two household activities (travel and non-travel) and is available for the full time period from 1992 to 2004 covered by the multi-regional model. For the years 2000, 2001, 2003 and 2004 CO_2 emissions were allocated from sector “Mining of metal ores” (SIC92: 13) to sector “Other mining and quarrying” (SIC92: 14) (minor in size), because no economic activities were recorded for these years (industry output

= £0). To retain as much detail as possible in the MRIO model, the carbon dioxide emissions data were further disaggregated to the 123 sector level of the supply and use tables. In the absence of better information, CO_2 emissions were broken down proportionally to total industry output. For example, emissions of sector e_j can be broken down into two sub-sectors e_{j1} and e_{j2} given available information on total industry output g_{j1} and g_{j2} by

Eq. 20
$$e_j = e_{j1} + e_{j2} = \frac{g_{j1}}{g_j} e_j + \frac{g_{j2}}{g_j} e_j$$
$$with\ g_j = g_{j1} + g_{j2}$$

As a direct consequence, CO_2 intensities d_{j1} and d_{j2} in these sub-sectors will be equal to the CO_2 intensity in the aggregate sector d_j, that is

Eq. 21
$$d_{j1} = \frac{e_{j1}}{g_{j1}} = \frac{e_j}{g_j} = d_j = \frac{e_{j2}}{g_{j2}} = d_{j2}$$

As the 91 production sectors of the Environmental Accounts could not be directly mapped onto the 123 sectors of the SUT publication without further aggregation, only 76 different UK-specific CO_2 intensities are distinguished in the multi-regional model across the 123 production sectors (see also Wiedmann et al. 2006).

CO_2 emission data for the rest of the world were taken from the database provided by the International Energy Agency (IEA 2006). The data is restricted to CO_2 emissions from fuel combustion. Even though the IEA database covers all years from 1992 and 2004, emission data was only compiled for the years 1997 and 2001 in the absence of other input-output data for non-UK regions. The data is consistent with the IPCC's sectoral approach (see IEA, 2006: chapter 5). However, in order to gain a more complete picture of CO_2 emissions embodied in products imported to the UK, emissions from international marine bunkers and international aviation were included as well. CO_2 emissions in 140 countries as distinguished in the IEA database were aggregated into the three world regions (OECD-Europe, OECD non-Europe, non-OECD) of the MRIO model. Equally, 31 sectors of the IEA data were mapped into the 30 sectors distinguished in the MRIO model for non-UK regions. In this context it was assumed that all CO_2 emissions from energy production arise in the energy sector even if it was auto-generated by another sector.

CO_2 intensities for non-UK regions were derived by dividing sectoral CO_2 emissions of a particular region by total sector industry outputs. However, while monetary data for the UK is provided in British pounds (£), non-UK regions are recorded in US dollar ($). In general, to deal with differences in currencies in multi-regional models two approaches are available: adopt a mixed units approach, such that the national production and demand data is kept in the national currency, and trade matrices are recorded in mixed units, where units are constant across any one row of the MRIO table, but not across any column. The second option is to convert the output data of all regions to a single currency. Due to the uni-directional nature of the multi-regional model developed here, total industry output vectors for the non-UK regions were converted from US dollars into British pounds (£) using purchasing power parities[33] provided by the Organisation for Economic Co-operation and Development (OECD, 2007). Due to differences in classification between the input-output and the IEA data, 18 different CO_2 intensities could finally be derived for the 30 sectors distinguished for the non-UK regions in the model.

4. Matrix balancing using CRAS

4.1. First experiences

- Estimates on errors were obtained from combining standard sampling errors from the ONS published Annual Business Inquiry Quality Measures database with respective figures from the main Annual Business Inquiry.[34] Relative standard errors were calculated and regressed against the totals.
- We had to implement non-unitary coefficients for the principal product constraints, this additional feature slowed down the CRAS routine considerably.

4.2. Production of Symmetrical IO Tables - Technology assumptions in a supply-use representation

The deliverable of this project is a time series of balanced (monetary) input-output tables for the UK (“Analytical IO Tables”, “Leontief Inverses”, “Symmetrical Input-Output Tables”) for the years 1992 to 2004, based on the initial estimates and constraints compiled for each. Such a time series is very useful when carrying out a number of analyses, including long-term Structural Decomposition Analysis (SDA) (see e.g. De Bruyn 2000; Jacobsen 2000; Kagawa and Inamura 2001; Hoekstra and van den Bergh 2002; Kagawa et al. 2002; Alcántara and Duarte 2004; Lenzen 2006; (Llop, 2007); (Dietzenbacher and Stage, 2006)) and trend analyses. Such analyses allow the identification of driving factors that contribute most strongly to growing environmental pressure and unsustainability.

This section addresses the question of technology assumptions in a supply-use representation as used in the UK-MRIO model.

The United Nations Handbook on input-output table compilation (United Nations 1999) distinguishes two basic technology assumptions: In the industry technology assumption[35], the production recipe is unique to an industry, while products’ input recipes are a weighted sum over industries’ production recipes. In the commodity technology assumption, the input recipe is unique

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[33] Purchasing Power Parities (PPPs) are currency conversion rates that both convert to a common currency and equalise the purchasing power of different currencies. In other words, they eliminate the differences in price levels between countries in the process of conversion.

[34] See http://www.statistics.gov.uk/abi/quality_measures.asp. Standard sampling errors and 'Coefficients of Variation' are given in http://www.statistics.gov.uk/abi/downloads/ABI_Quality_Measures.xls.

[35] This assumption could also be called *“assumption of fixed product sales structures”* according to (Thage, 2005) and (Yamano and Ahmad, 2006). Both publications argue in favour of the compilation of industry-by-industry tables based on this assumption and present a number of advantages.

to a product, while industries' production recipes are a weighted sum over their primary and joint products.

In the UN Handbook, technology assumptions are dealt with for symmetrical input-output coefficients tables (SIOT). However, both industry and commodity technology assumption can be represented using supply-use formulations without the need for producing a SIOT. In the following we will use the standard United Nations notation (United Nations 1999), except for the supply matrix, which we will call **V** instead of M signifying the older term "make matrix". Let a single-region supply-use transaction block **T** be represented by

Eq. 22 $$\mathbf{T} = \begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix},$$

with **U** being a product-by-industry use matrix, showing the input U_{ij} of commodity i into industry j, and **V** being a industry-by-product supply matrix, with V_{ij} showing the output by industry i of commodity j. This block formulation is well known in the input-output literature ((Gigantes 1970; Schinnar 1978)).

Let **T** satisfy the national accounting identity

Eq. 23 $$\begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \end{bmatrix} + \begin{bmatrix} \mathbf{y}_c \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{q} \\ \mathbf{g} \end{bmatrix},$$

where $[\ \mathbf{1}\ \mathbf{1}\ ]^t$ is the row summation vector, $\mathbf{y}_c$ is a vector of final demand of products, and **q** and **g** are vectors of gross output of products and industries, respectively. Let

Eq. 24 $$\begin{bmatrix} 0 & \mathbf{B} \\ \mathbf{D} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{q}}^{-1} & 0 \\ 0 & \hat{\mathbf{g}}^{-1} \end{bmatrix}$$

be a supply-use coefficients block, where the hat symbol ("^") denotes a diagonalised vector. **B** is called the (product-by-industry) use coefficients matrix, and **D** is called the (industry-by-product) market share matrix.

Industry technology assumption

Industry technology assumes an input recipe that is characteristic for an industry; this is in essence the use matrix **U**. Therefore, the supply-use blocks in Eq. 12 basically assume industry technology. The UN input-output handbook(United Nations 1999, Eqs. 4.4 and 4.12), provides instructions for combining use coefficients and market shares into either a symmetrical industry-by-industry input-output coefficients matrix

Eq. 25 $$\mathbf{A}_{\mathrm{I,ii}} = \mathbf{DB} \ ,$$

or a symmetrical product-by-product input-output coefficients matrix

Eq. 26 $$\mathbf{A}_{\mathrm{I,cc}} = \mathbf{BD} \ .$$

These matrices are used in either the industry-by-industry input-output model (United Nations 1999, Eq. 4.10)

Eq. 27 $$(\mathbf{I} - \mathbf{DB})\mathbf{g} = \mathbf{D}\mathbf{y}_{\mathrm{c}} \ ,$$

or in the product-by-product input-output model, (United Nations 1999, Eq. 4.9)

Eq. 28 $$(\mathbf{I} - \mathbf{BD})\mathbf{q} = \mathbf{y}_{\mathrm{c}} \ .$$

Using the compound supply-use-block formulation as in Eq. 24, a compound Leontief Inverse can be written as

Eq. 29 $$\mathbf{L}_{\mathrm{I}}^{*} = \begin{bmatrix} \mathbf{I} & -\mathbf{B} \\ -\mathbf{D} & \mathbf{I} \end{bmatrix}^{-1} \ .$$

Using the partitioned inverse of (Miyazawa 1968), Eq. 29 can be written as

Eq. 30 $$\mathbf{L}_{\mathrm{I}}^{*} = \begin{bmatrix} \mathbf{I} + \mathbf{BL}_{\mathrm{I,ii}}\mathbf{D} & \mathbf{BL}_{\mathrm{I,ii}} \\ \mathbf{L}_{\mathrm{I,ii}}\mathbf{D} & \mathbf{L}_{\mathrm{I,ii}} \end{bmatrix} ,$$

where $\mathbf{L}_{\mathrm{I,ii}} = (\mathbf{I} - \mathbf{DB})^{-1}$ is the Leontief Inverse of the industry-by-industry input-output model. Considering the series expansion $\mathbf{BL}_{\mathrm{I,ii}}\mathbf{D} = \mathbf{B}\left(\mathbf{I} + \mathbf{DB} + (\mathbf{DB})^2 + \mathbf{...}\right)\mathbf{D}$, Eq. 30 can be simplified to

Eq. 31 $$\mathbf{L}_{\mathrm{I}}^{*} = \begin{bmatrix} \mathbf{L}_{\mathrm{I,cc}} & \mathbf{BL}_{\mathrm{I,ii}} \\ \mathbf{L}_{\mathrm{I,ii}}\mathbf{D} & \mathbf{L}_{\mathrm{I,ii}} \end{bmatrix} ,$$

with $\mathbf{L}_{\mathrm{I,cc}} = (\mathbf{I} - \mathbf{BD})^{-1}$ being the Leontief Inverse of the product-by-product input-output model (see Eq. 28).

Hence, when supply and use matrices are handled in integrated blocks, the compound Leontief inverse elegantly reproduces both product-by-product and industry-by-industry models in one formulation.

Commodity technology assumption

Commodity technology assumes an input recipe that is characteristic for a product. Once again, the UN (United Nations 1999), Eq. 4.17) provides instructions for combining use coefficients and the supply matrix into a symmetrical product-by-product input-output coefficients matrix:

Eq. 32 $$\mathbf{A}_{\mathrm{C,cc}} = \mathbf{UV}^{-1} \ .$$

In essence, $\mathbf{A}_{\mathrm{C,cc}}$ holds the input recipe for products produced by industries. The corresponding product-by-product input-output model is

Eq. 33 $$(\mathbf{I} - \mathbf{A}_{\mathrm{C,cc}})\mathbf{q} = \mathbf{y}_{\mathrm{c}} \ .$$

In the context of the commodity technology assumption, the supply-use block assumes a different shape; the coefficients matrix is now

Eq. 34 $$\begin{bmatrix} \mathbf{A}_{\mathrm{C,cc}} & 0 \\ \mathbf{D} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{q}}^{-1} & 0 \\ \mathbf{V}^{-1} & 0 \end{bmatrix} .$$

The compound Leontief Inverse can then be written as

Eq. 35 $$\mathbf{L}^{*}_{\mathrm{C}} = \begin{bmatrix} \mathbf{I} - \mathbf{A}_{\mathrm{C,cc}} & 0 \\ -\mathbf{D} & \mathbf{I} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{L}_{\mathrm{C,cc}} & 0 \\ \mathbf{DL}_{\mathrm{C,cc}} & \mathbf{I} \end{bmatrix} ,$$

where $\mathbf{L}_{\mathrm{C,cc}} = (\mathbf{I} - \mathbf{A}_{\mathrm{C,cc}})^{-1}$ is the Leontief Inverse of the product-by-product input-output model (see Eq. 33).

Once again, when supply and use matrices are handled in integrated blocks, the compound Leontief Inverse elegantly reproduces both product-by-product and industry-by-industry models in one formulation.

Further information on the treatment of industry technology and commodity technology in a make-use framework can be found in this literature: Viet 1994; Kop Jansen & ten Raa 1990; ten Raa & van der Ploeg 1989; ten Raa 1994a,b; UN 1993 and 1999; Wachsmann 2005; Thage 2005; Yamano and Ahmad 2006.

5. Discussion of strengths and limitations of the current UK-MRIO model

Work on this project is still in progress and results for CO_2 emissions embedded in trade cannot be given at this stage. We will therefore focus on a discussion of strengths and weaknesses of the model and on first experiences made during the compilation of the initial data estimates and the balancing procedure. The following list provides an overview of the main assumptions and limitations of the current model and a discussion of possible improvements. Apart from the usual limitations of environmental input-output models (Wood et al. 2007), the peculiarities of this model are as follows.

- Great care was taken to obtain an accurate picture of imports to the UK from the three world regions. We have used specific UK trade data, detailing imports of goods and services from all countries in the world (subsequently aggregated to three world regions) by 5-digit SITC code (subsequently aggregated to 123 input-output sectors). Total imports were brought in line with totals in the official SUTs provided by ONS. However, no information on the structure of imports to intermediate and final demand was available, other than one imports matrix for from the Analytical Tables 1995. Hence we had to assume that the relative proportions of imports to domestic production would not change over time, a potentially far-reaching and undesirable assumption. It is hoped that the modernisation of UK National Accounts (Beadle 2007) in 2008 will provide more up-to-date information on the structure of imports to the UK.
- We do not consider all possible trade flows between the four trading partners in the model (UK plus three world regions). This is due to the fact that imports (exports) matrices between the three world regions are not available and would take a great deal of resources to compile. Therefore our model only considers trade to and from the UK. The effect of not considering extra-UK trade on the estimation of emissions embedded in UK trade is thought to be small (Lenzen et al. 2004)report feedback loop effects of 1.5%).
- A future version of the model should include explicit (and more) countries as trading partners (instead of world regions) for which it is easier to obtain imports/export matrices. Logically, such a model would include the main individual trading partners of the UK. In this context, it would make sense to put international trade data on a more consistent basis, e.g. by exploiting the UN Comtrade data base.
- There are several advantages when using (more) individual countries in a future model. Supply and Use tables can be used instead of A matrices which improves data coverage for time series. This will allow increasing the numbers of economic sectors to well over 30 as most SUTs are provided in greater detail by national statistical offices. Not least, environmental data from individual countries can be used providing much improved sector specifity of CO_2 emissions and other environmental load factors. For this purpose, country-specific NAMEAs can be utilised. Finally, a mixed units approach can be adopted when including explicit trading partners (see Lenzen et al., 2004).
- Due to the original setup of the (Nijdam et al. 2005) model, the A matrix from Region e (OECD Europe countries) includes technical coefficients from the UK and excludes those from the Netherlands. Thus, the economic structure of this region is not exactly in line with the actual trading partners of the UK, but the associated error should be relatively small given the

fact that both the UK and the Netherlands are developed western economies. The errors associated with the sector aggregation (30 sectors for the three regions vs 123 sectors in the UK) as well as the unavailability of coefficient matrices for all years are thought to constitute a further reaching limitation of the model.

- For CO_2 emissions, however, we have included CO_2 emissions for the Netherlands in Region e and exclude those for the UK (see Section XX), thus partially correcting the discrepancy mentioned above.
- A limitation is posed by detail and classification differences between the economic and environmental accounts published in the UK: full correspondence can only be established at the 76 sector level. For more policy relevant analysis in many important sectors such as food, transport or energy more detail is required. Apart from the need to urge the Office for National Statistics to reconcile this classification issue and provide more detailed data, the next version of the model will use more sophisticated estimation methods using detailed emission estimates from other databases such as CEDA (Suh, 2005) or the Japanese environmental and economic accounts to break-down (CO_2) emissions. This will allow the distinction of 123 instead of only 76 emission intensities across the input-output sectors and help to further improve the relevance of direct and embodied emission estimates associated with goods and services produced in the UK.
- Equal limitations are imposed by the availability of emission intensities for non-UK regions for all years between 1992 and 2004. While it does not seem realistic to compile a complete country-specific input-output database for the whole time period, the next version of the multi-regional model will use information from economic accounts as published, for example, by the United Nations or Eurostat to estimate region-specific emission intensities for all years included (in addition to specific country data). This will further improve the reliability of embodied emission estimates associated with imports to the UK.

6. Conclusions

The completion of the first stage of a UK specific multi-region input-output model has achieved its goal, providing the basis for a specific and robust estimation of CO_2 emissions embedded in UK trade. It's main features and strenghts are:

- UK-MRIO explicitly models the trade of the UK with three world regions and the associated flow of CO_2 emissions
- UK-MRIO distinguishes 123 sectors of domestic production and trade
- UK-MRIO looks at a complete time series from 1992 to 2004
- thus UK-MRIO is the most comprehensive and most robust estimation of CO_2 emissions embedded in UK trade to date.
- for these reasons UK-MRIO is most relevant and indispensable for UK national and international environmental policy (see also Druckman et al. 2007)

In the course of the UK-MRIO project we also constructed symmetric input-output tables for each year from 1992 to 2004. This fills a current gap in UK input-output data as 'Analytical Tables' are only produced every five years with the last one being from 1995. Analytical Tables for the year 2000 will not be produced at all due to a major National Accounts modernisation program at ONS (Beadle 2007).

The UK-MRIO model is the first 'real world' application of the new matrix balancing procedure CRAS (Constrained RAS), developed at the University of Sydney, proving its ability the provide useful results in an empirical context.

The current model is major step towards a fully fledged multi-region input-output model featuring multidirectional trade of a substantial number of UK trading partners.

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