MEASURING THE EFFICACY OF INVENTORY WITH A DYNAMIC INPUT-OUTPUT MODEL

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Abstract

This paper extends the recently developed Dynamic Inoperability Input-Output Model (DIIM) for assessing productivity degradations due to disasters. Inventory policies are formulated and incorporated within the DIIM to evaluate the impact of inventories on the resilience of disrupted interdependent systems. The inventory DIIM can provide practical insights to preparedness decision making through explicit tradeoff analysis of multiple objectives, including inventory costs and economic loss reductions. The model is demonstrated in several illustrative examples to depict various nuances of inventory policies. The paper then culminates in a case study that utilizes input-output and inventory accounts from the Bureau of Economic Analysis.

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ABSTRACT

This paper extends the recently developed Dynamic Inoperability Input-Output Model (DIIM) for assessing productivity degradations due to disasters. Inventory policies are formulated and incorporated within the DIIM to evaluate the impact of inventories on the resilience of disrupted interdependent systems. The inventory DIIM can provide practical insights to preparedness decision making through explicit tradeoff analysis of multiple objectives, including inventory costs and economic loss reductions. The model is demonstrated in several illustrative examples to depict various nuances of inventory policies. The paper then culminates in a case study that utilizes input-output and inventory accounts from the Bureau of Economic Analysis.

KEY WORDS

Inventory, Interdependent systems, Input-output analysis, Preparedness, Decision making

1. INTRODUCTION

The focus of infrastructure risk management and decisionmaking has recently shifted from prevention and protection of infrastructure systems from disruptive events to recovery and response. For example, the US Department of Homeland Security (DHS) through its National Infrastructure Protection Plan [DHS 2006] has highlighted that the US must prepare for the inevitable occasion when a disruptive event occurs, stressing risk management strategies that "strengthen national preparedness, timely response, and rapid recovery in the event" of an attack or disaster. Furthermore, DHS [2006] underscores the need for instituting preparedness and resilience plans for critical infrastructure and key resources (CI/KR) of the nation.

Discussions of preparedness and resilience appear in Haimes [2006] and Haimes et al. [2008], where the connection is made between preparedness activities prior to a disruptive event to the resilience achieved following the disruptive event. Resilience is defined as the "ability to cushion or mute potential losses" [Rose 2004] from a disruptive event. In general, economic resilience is defined as the ability or capability of a system to absorb or cushion against damage or loss [Holling 1973, Perrings 2001]. Increasing the resilience of a sector reduces its recovery time as well as the associated economic losses.

Of particular interest to the discussion of preparedness and resilience are interdependencies among critical infrastructure and economic systems. The operation of such critical systems, or essential services, without interruption is of incredible importance, and failing to prepare can result in "widespread uncertainty about restoration of services, lack of viable economic and social networks, serious loss of public confidence, and even social collapse" [La Porte 2006]. The interdependence of such essential services and the private infrastructure components of supply chains is well documented (e.g., Rinaldi et al. [2001], Little [2002], Kormos and Bowe [2006]). Due to the interdependencies among production activities in various sectors of the economy, a disruption in production can have far-reaching effects. One significant means of preparedness and resilience in a production environment comes from the availability of inventory. The above motivates this work to strengthen our ability to model the impact of inventory policies on interdependent infrastructure systems.

Several risk-based interdependency modeling schemes have been developed recently, including the Inoperability Input-Output Model (IIM) [Haimes and Jiang 2001; Jiang and Haimes 2004; Santos and Haimes 2004]. A derivative of the IIM which models the dynamic recovery of interdependent sectors and evaluates the effect of risk management strategies on that recovery is the Dynamic IIM (DIIM) [Lian and Haimes 2006]. While the IIM and its derivatives successfully measure the effects of certain risk management strategies, they are unable to account for strategies that add resilience through inventory. This paper integrates the DIIM with an inventory model to quantify the efficacy of inventory strategies employed in interdependent infrastructure sectors and other members of a supply chain.

Ultimately, the model provides a metric quantifying how different risk management strategies involving inventory will affect recovery following a disruption, as depicted in Figure 1. From left to right, the first component in Figure 1 depicts various preparedness strategies used to reduce the effects of a disruptive event and the DIIM parameters that vary with each strategy. They serve as inputs to the Inventory DIIM, which quantifies inoperability experienced by different sectors of the economy over time and quantifies the economic losses resulting from a disruptive event. Such inoperability trajectories and economic losses are calculated for each strategy, and the strategies are compared with a multiobjective framework where tradeoffs between costs and benefits are calculated.

2. METHODOLOGICAL BACKGROUND

Discussed in this section are several models of inventory, including previous input-output-based representations of inventory, and the risk-based interdependency model used in this paper, the Dynamic Inoperability Input-Output Model.

2.1. Inventory Philosophies

Presented in this section are discussions of several key inventory control approaches found in practice and in the operations management literature. The philosophies of many of these methods are perhaps contradictory to the concept of preparedness.

Early inventory models include the economic order quantity (EOQ), Wagner-Whitin, base stock, and (Q, r) models, and each calculate a reorder point that attempts to minimize the cost of manufacturing. While each has varying levels of assumptions and data requirements, a basic insight from these models is that there is a tradeoff between customer service and inventory [Hopp and Spearman 2000]. That is, inventory costs are reduced at the cost of meeting customer service levels under conditions of random demand. The previously mentioned inventory models were deemed more appropriate, though highly restrictive, for purchasing environments and not production environments.

A shift to a more supply chain-oriented approach was developed with the just-in-time (JIT) philosophy. The JIT approach is designed around the arrival of required materials to a production workstation precisely when needed [Hopp and Spearman 2000], thereby reducing the amount of on-hand inventory to nearly zero. Difficulties arising from the “zero inventory” tenet of JIT led companies to adapt the approach, though the desire to maintain small amounts of inventory still remains.

An operations management philosophy that emphasizes minimal inventory has significant implications for participants in an interdependent supply chain. Chittister and Haimes [2004] observe that an information technology-driven shift to JIT has "reduced the operational buffer zone in most infrastructures." Kleindorfer and Saad [2005] note two types of risks to supply chains: risks arising from problems in coordinating supply and demand, and risks arising from disruptions to normal activities. A disruption to a supply chain participant who has no ability to stave off inoperability, either by on-hand inventory or some other means, can have ripple effects throughout its interconnected supply chain. Chopra and Sodhi [2004] remark that while "bare-bones inventory levels decrease the impact of overforecasting demand, they simultaneously increase the impact of a supply chain disruption. Similarly, actions taken by any company in the supply-chain can increase risk for any other participating company." Kleindorfer and Saad [2005] observe that implementing a policy of reduced inventory "may result in increasing the level of vulnerability, at both the individual firm level and across the supply chain."

The effects of supply chain disruptions on profitability can be potentially very large [Heal et al. 2006]. Hendricks and Singhal [2005] found that companies involved in supply chain disruptions did not recover quickly from those disruptions in terms of operating performance (i.e., costs, sales, profits) and stock performance. Little [2005] suggests that New York City's recovery following the terrorist attacks of September 11, 2001, would have been hampered had more organizations taken a JIT philosophy. Chopra and Sodhi [2004] observe that many companies have plans in place to reduce the effects of recurrent, low-impact risks but are unprepared for low-probability, high-impact events, much like the disruptive events for which many interdependent organizations should be preparing.

LaPorte [2006] notes that critical infrastructure systems cannot operate as "normal organizations," where production or distribution breakdowns can occur without substantial penalties, as such essential services are expected to be available in real time, without fail, regardless of operating conditions. Emergencies are "times when essential services are most needed by vulnerable populations and those whose job it is to assist them" [LaPorte 2006].

Note that "inventory" is discussed here as a means to maintain operability following a disruptive event, not to affect the relief effort following such an event (e.g., Whybark [2007]).

2.2. Input-Output Representations of Inventory

While several different modeling approaches have been developed recently to describe interdependencies among infrastructure sectors (e.g., [Brown et al. 2004], [Bagheri and Ghorbani 2007], [Buzna et al. 2007]), the form of the input-output model [Leontief 1936, 1951a, 1951b, 1966] lends itself to integration with an inventory model. Potentially lacking in depicting the complexity of certain infrastructure interdependencies, the input-output model more than compensates with model interpretability and a wealth of interdependency data gathered by U.S. Census Bureau, the Bureau of Economic Analysis (BEA), and other U.S. federal agencies.

The major work in combining inventory modeling and input-output modeling follows from the Sequential Interindustry Model (SIM) [Romanoff and Levine 1977], an input-output model for sectors whose production depends on anticipating demand or responding to demand. The SIM distinguishes between three events in the production process: demand stimulus occurs when goods are orders, yield or supply happens when goods are delivered, and production yield occurs when goods are produced [Okuyama 2004]. Further work and extensions to SIM appear in Romanoff and Levine [1981, 1993], Okuyama [2004], and Okuyama et al. [2004], among others.

The SIM allows for three different production modes: anticipatory, where production occurs in anticipation of future orders (e.g., agriculture, many manufacturing industries); responsive, where production takes place after receipt of customer orders (e.g., construction, service industries); and just-in-time, where production takes place and goods are delivered as the order is placed.

Inventory is often used in industries following the anticipatory production mode. That is, the SIM describes how a sector's production contributes to other sectors, to final demand, and to inventory, with the intent of the model being to account for lagging and leading effects in modeling industry output. The use of SIM differs from how the concept of inventory would be used in the context of preparedness risk management decisionmaking, where the goal is not to model the contributions to inventory but to model the dynamic effects of withdrawing from inventory to maintain operability.

Other work using input-output analysis in the supply chain management field involves work by Lovell [1992], Grubbstrom and Tang [2000], and Albino et al. [2002], among others

2.3. Inventory Represented in Bureau of Economic Analysis Data

Inventory is published by the BEA in the form of national income and product account (NIPA) tables [BEA 2009]. Such tables, updated quarterly (recently, on monthly bases), provide the inventories in manufacturing and trade sectors in dollar terms and as a ratio of inventory-to-sales. Examples of such data are available in the BEA website, and are also published within BEA's flagship journal, *Survey of Current Business* (see, for example, BEA [2007]). The inventory-to-sales ratio is particularly interesting, as it can be used to describe the likelihood that inventory will change due to some change in demand. Miller and Blair [1985] suggest that any inventory figures maintained in input-output accounts do not fit the traditional definition of inventory, those goods held by the industry that is producing the product. In input-output accounting, inventory accounts for changes in an industry's final product regardless of which industries hold those products. Miller and Blair [1985] provide an example of coal inventories held by electric power plants being classified as coal inventories. For certain sectors, e.g., manufacturing sectors, the NIPA tables may well describe the needs of the inventory model discussed subsequently despite the discrepancy noted by Miller and Blair [1985].

2.4. The Inoperability Input-Output Model (IIM) and its Derivatives

Taking advantage of the rich databases collected by the U.S. Census Bureau, BEA, and others, the input-output model was extended to describe how inoperability, or a proportion of "dysfunctionality," propagates through a set of interconnected infrastructure sectors with the Inoperability Input-Output Model (IIM) [Haimes and Jiang 2001, Santos and Haimes 2004, Santos 2006]. Inoperability is interpreted as the percentage to which a particular sector does not satisfy the as-planned level of production output. The IIM, a linear transformation of the Leontief input-output model, is presented in Eq. (1) for an economy of *n* sectors, resulting in matrices of size $n \times n$ and vectors of length *n*.

$$\mathbf{q} = \mathbf{A}^*\mathbf{q} + \mathbf{c}^* \Rightarrow \mathbf{q} = \left[\mathbf{I} - \mathbf{A}^*\right]^{-1}\mathbf{c}^* \tag{1}$$

The vector **q** represents the inoperability vector, the elements of which measure the proportion of "unrealized" production per as-planned production. Demand perturbation is represented with vector $\mathbf{c}^*$, whose elements quantify reduced final demand as a proportion of total as-planned output and which could follow a disruption for a number of reasons, including the result of a forced demand reduction from diminished supply and due to lingering consumer fear or doubt. The matrix $\mathbf{A}^*$ is the normalized interdependency matrix describing the extent of economic interdependence between sectors of the economy. The row elements of $\mathbf{A}^*$ indicate the proportions of additional inoperability that are contributed by a column sector to the row sector. Further details about the derivation of IIM parameters can be found in Santos [2006]. Applications of the IIM include a study of the 2003 Northeast blackout [Anderson et al. 2007], the effect of cyber attacks [Andrijcic and Horowitz 2006], and sequential decisionmaking among interdependent infrastructure systems [Santos et al. 2008], among others.

Though few attempts have been made to use dynamic input-output models to describe recovery from disruptive events [Okuyama 2007], one exception is the Dynamic Inoperability Input-Output Model (DIIM). The DIIM [Haimes et al. 2005a, 2005b, Lian and Haimes 2006], extends the IIM form in Eq. (1) to the widely-accepted dynamic model discussed in Miller and Blair [1985]. The DIIM is provided in discrete-time form in Eq. (2).

$$\mathbf{q}(t+1) = \mathbf{q}(t) + \mathbf{K}\left[\mathbf{A}^*\mathbf{q}(t) + \mathbf{c}^*(t) - \mathbf{q}(t)\right] \tag{2}$$

Definitions of $\mathbf{q}(t)$, $\mathbf{A}^*$, and $\mathbf{c}^*(t)$ are the same as those for their counterparts in the static IIM, except that $\mathbf{q}(t)$ and $\mathbf{c}^*(t)$ describe those values at a specific time t. $\mathbf{K}$ is a matrix with resilience coefficients $k_1,\ldots, k_n$ on the diagonals and zeroes elsewhere. The resilience coefficient k_i represents the ability of Sector i to recover following a disruptive event, where the greater k_i values correspond to a faster response by the sector. Derivations of the DIIM and its parameters may be found in Lian and Haimes [2006].

A metric of interest in a DIIM analysis is the total economic loss for the entire economy over some span of time, caused by the sector inoperabilities at each time period and the effect of those inoperabilities on total output. Total economic loss, Q, found in Eq. (3), sums the economic loss for each of τ time periods. Economic loss for Sector i is found by multiplying total output of the sector, x_i, with inoperability of the sector, q_i, and total economic loss is the sum of all sector losses. Eq. (3) assumes that total output, **x**, is time-invariant.

$$Q = (\mathbf{x})^T \sum_{j=1}^{\tau} \mathbf{q}(j) \tag{3}$$

It is assumed that the values of $q_i(0)$, T_i, and $c_i^*(t)$ are controllable through the risk management decisionmaking process (e.g., preparedness strategies addressing system hardening could alter the value of $q_i(0)$, pre-positioned recovery supplies could reduce T_i, storage activities could affect $c_i^*(t)$). Therefore, the effectiveness of strategies which alter these states of the systems can be measured and compared with the value of Q obtained from the DIIM. Nevertheless, minimizing Q is only one of a number of objectives in a multiobjective optimization framework for performing tradeoff analyses among preparedness strategies.

3. The Inventory DIIM

The ability of the DIIM to measure the efficacy of risk management strategies that lead to manipulations of model parameters $q_i(0)$, T_i, and $c_i^*(t)$ was discussed previously, and the ability of the model to measure such strategies is important. However, Eq. (2) lacks the ability to capture risk management strategies that maintain essential services, or delay the onset of inoperability By combining the DIIM with an inventory model, one can capture how a preparedness initiative such as improving inventory stock can minimize the effects of a disruptive event. Described in this section is the *Inventory DIIM,* a formulation that will allow decisionmakers to model the efficacy of risk management strategies which involve the implementation of inventory policies. The efficacy of each policy can be evaluated by quantifying how inventory delays inoperability, how operability in interdependent sectors is then sustained, and how economic losses are reduced.

Terms used in the model are defined as follows:

$q_i(t)$ *Inoperability*. This quantity is the standard DIIM definition of inoperability of Sector i at the end of time t, or the lack of ability to deliver goods to others sectors and consumers.

$p_i(t)$ *Production inoperability*. The production inoperability of Sector i at the end of time t, similar to $q_i(t)$, describes inoperability of the production process due only to a physical disruption in that process. Note that $q_i(0) \equiv p_i(0)$ if no inventory is in place.

$x_i(t)$ *Total anticipated output*. This value describes the total output anticipated to be produced by Sector i between the end of time t–1 and the end of time t.

$X_i(t)$ *Inventory level*. This quantifies the amount of inventory in Sector i remaining at the end of time t.

k_i *Resilience coefficient*. The resilience coefficient of Sector i describes the ability of Sector i to recover following a disruptive event, which includes the extent to which Sector i is interconnected with other potentially inoperable sectors, a greater value of k_i results in faster reduction of sector inoperability.

l_i *Repair coefficient*. The repair coefficient of Sector i describes the ability of the sector to recover production capability from an initial disruptive event, with no interdependent effects included in its calculation, a greater value of l_i results in faster recovery of production capability.

a_{ij}^* *Normalized technical coefficient*. Same as the IIM and DIIM definition, this technical coefficient is *ij*th entry in the normalized interdependency matrix describing the dependence of the *i*th sector on the *j*th sector.

c_i^* *Demand perturbation*. Same as previous definitions, this quantifies the proportion of reduced final demand from the total nominal output of Sector *i*.

Regarding $X_i(t)$, inventory is typically divided into three main categories [Sipper and Bulfin, Jr. 1997]: *raw materials*, or components used for production; *works in process*, or components that have begun the transformation into finished goods; and *finished goods*, or goods ready for sale to consumers. Here, inventory is meant to describe finished goods, or some other method by which the sector's ability to provide goods to other sectors and to final consumers is maintained. In sectors where an inventory of finished goods is impractical, $x_i(t)$ can describe redundancies that allow the sector to maintain some ability to provide output. For example, finished goods in the oil and gas sector could include excess stored gasoline ready for use, and redundancies in the electric power sector, where energy cannot be stored, could include back up generators that can provide some amount of total output to be maintained.

3.1. Production Inoperability

In previous works (e.g., Lian and Haimes [2006], Santos [2006]), sector inoperability $q_i(t)$ is determined either by a reduction in demand or some initial inoperability $q_i(0)$ whose cause is not necessarily specified. In the discussion found here, sector inoperability is caused by some production inoperability experienced as a result of a disruption. For example, oil and gas refining may be halted as a result of production facility outage caused by a hurricane. Generally speaking, the inoperability experienced by Sector *i* in this discussion is derived from production inoperability p_i.

In the work of Lian and Haimes [2006], the resilience coefficient of Sector i, k_i, was introduced to measure the speed of recovery for that sector in the DIIM. The repair coefficient is used similarly here, describing the speed with which a sector's physical inoperability is improved. For example, if a terrorist event renders a production facility 20 percent inoperable, l_i would describe how quickly that production facility is repaired to regain full or nearly full operability, regardless of any inoperability experienced in other interconnected sectors. The calculation of l_i, similar to the derivation of k_i from Lian and Haimes [2006], is found in Eq. (4).

$$l_i = \frac{\ln\left[p_i(0)/p_i(T_i)\right]}{T_i} \tag{4}$$

The calculation of production inoperability p_i only describes Sector i's ability to recover from a physical inoperability. It differs from inoperability q_i in that it does not measure any interdependent inoperability effects. Production inoperability for Sector i at time t is calculated in Eq. (5), similar to the inoperability calculation in Lian and Haimes [2006], where $p_i(0)$ is the initial production inoperability experience by Sector i.

$$p_i(t) = e^{-l_i t} p_i(0) \tag{5}$$

For many sectors, recovery from a disruption occurs exponentially [Haimes et al. 2005a, 2005b, Cimellaro et al. 2005, Cimellaro et al. 2006], and Eq. (5) models production recovery in this manner. If a sector is known to recover in a different way, an equation for $p_i(t)$ can reflect that. Production recovery is meant to be interpreted as a discrete-time glimpse at a continuous-time model. That is, $p_i(t)$ is of interest for $t = 1, 2, 3,\ldots$, where t represents whole days. Because of this, if the Inventory DIIM is used in real-time or for a specific case study, known values of $p_i(t)$ can be used in the analysis.

Sector inoperability is initialized with Eq. (6)**Erro! Fonte de referência não encontrada.**. Naturally, if enough inventory is available to cover the output reduction caused by the initial production inoperability, defined as $p_i(0)x_i(0)$, then $q_i(0) = 0$. If some inventory is available, but not enough to cover the output reduction, sector inoperability is a function of the fraction of inventory used to meet the output reduction. Finally, if no inventory is available, the initial sector inoperability is brought about entirely by the physical production inoperability experienced by the sector.

$$q_i(0) = \begin{cases} 0 & \text{if } X_i(0) \geq p_i(0)x_i(0) \\ 1 - \dfrac{X_i(0)}{p_i(0)x_i(0)} & \text{if } 0 < X_i(0) < p_i(0)x_i(0) \\ p_i(0) & \text{if } X_i(0) = 0 \end{cases} \quad (6)$$

3.2. Sector Inoperability

A dynamic calculation of sector inoperability, found in matrix notation in Eq. (2), describes how sectors recover from an initial inoperability in one or more sectors. Including inventory in the calculation of sector inoperability requires a piecewise calculation which considers four different cases. The equation for sector inoperability of Sector i is shown in Eq. (7). Sector inoperability is solved for Sector i at time $t + 1$, that is $q_i(t + 1)$. It is assumed that the total number of sectors under study is n.

$$
q_i(t+1)=\begin{cases}
q_i(t)+k_i\left[c_i^*(t)-q_i(t)+\sum_{j=1}^{n}a_{ij}^*q_j(t)\right] & \text{if } X_i(t+1)\geq p_i(t+1)x_i(t+1)\\
\max\begin{cases} p_i(t+1)-\dfrac{X_i(t+1)}{x_i(t+1)}\\ q_i(t)+k_i\left[c_i^*(t)-q_i(t)+\sum_{j=1}^{n}a_{ij}^*q_j(t)\right]\end{cases} & \text{if } 0<X_i(t+1)<p_i(t+1)x_i(t+1)\\
\max\begin{cases} p_i(t+1)\\ q_i(t)+k_i\left[c_i^*(t)-q_i(t)+\sum_{j=1}^{n}a_{ij}^*q_j(t)\right]\end{cases} & \text{if } X_i(t+1)=0,\ X_i(t)>0\\
q_i(t)+k_i\left[c_i^*(t)-q_i(t)+\sum_{j=1}^{n}a_{ij}^*q_j(t)\right] & \text{if } X_i(t+1)=X_i(t)=0
\end{cases} \tag{7}
$$

Calculation of $q_i(t + 1)$ is divided into four cases, discussed in the order that they appear in the piecewise formulation in Eq. (7).

3.2.1. Case 1, where $X_i(t + 1) \geq p_i(t + 1)x_i(t + 1)$

During a disruption, the amount of reduced production is dictated, at least initially, by production inoperability $p_i(t)$. That is, if $x_i(t)$ dollars worth of production is produced at the end of time t, then $p_i(t)x_i(t)$ calculates the amount of production that cannot be used as total output. If the amount of inventory $X_i(t)$ in Sector i at time t is large enough to cover the amount of total output reduced as a result of the physical disruption to production, then Case 1 occurs. Since enough inventory is available to cover total output, the value of sector inoperability $q_i(t)$ of Sector i at time t is simply calculated from inoperability of both sectors in the previous time period. That is, $q_i(t)$, while not accounting for the physical disruption of Sector i due to inventory, can include interdependent disruption from one of the other sectors that became inoperable. The equation for calculating sector inoperability for Sector i in this first case is the basic discrete-time DIIM model for dynamic inoperability found in Eq. (8). This is a scalar representation of Eq. (2).

$$q_i(t+1) = q_i(t) + k_i\left[c_i^*(t) - q_i(t) + \sum_{j=1}^{n} a_{ij}^* q_j(t)\right] \qquad (8)$$

3.2.2. Case 2, where $0 < X_i(t+1) < p_i(t+1)x_i(t+1)$

In this situation, Sector *i* has inventory available at time *t* to alleviate some of the burden of the physical disruption to production, but it does not have enough to cover the total output requirement for the time period. That is, inventory is less than $p_i(t)x_i(t)$ at time *t*. Therefore, the inoperability of the sector will be dictated either by the production inoperability remaining after inventory is exhausted or by the interdependent inoperability resulting from the other sectors. The maximum of these two values represents the inoperability experienced by Sector *i*.

Production inoperability is reduced from its current state by the effect of the remaining inventory. That is, production inoperability is reduced by a fraction representing the proportion of inventory $X_i(t+1)$ used to meet $p_i(t+1)x_i(t+1)$. The other operand is from Eq. (8).

$$q_i(t+1) = p_i(t+1)\left[1 - \frac{X_i(t+1)}{p_i(t+1)x_i(t+1)}\right] = p_i(t+1) - \frac{X_i(t+1)}{x_i(t+1)} \qquad (9)$$

3.2.3. Case 3, where $X_i(t+1) = 0$, $X_i(t) > 0$

The third case describes the occasion where sector *i* had some amount of inventory at time *t*, but that inventory is completely depleted at time $t + 1$. Here, sector inoperability takes on one of two different values: the value of production inoperability, $p_i(t+1)$, or the value of interdependent inoperability, calculated using Eq. (8). Sector inoperability is considered to be the maximum of these two values.

The reasoning behind this calculation is that if $X_i(t+1) = 0$ and $X_i(t) > 0$, then $t + 1$ is the first time period where production inoperability can be reflected entirely in sector inoperability. That is, Sector *i* will be inoperable to the extent that either its production is inoperable or the other sectors upon which Sector *i* relies are inoperable.

3.2.4. *Case 4, where $X_i(t + 1) = X_i(t) = 0$*

The final case represents the situation where there has been no inventory available for at least two time periods. Here, the effect of a lack of inventory has already been accounted for in the calculation of sector inoperability. Production inoperability is implicit in the calculation, as it was used to calculate sector inoperability at least two periods prior. Henceforth, only interdependent effects are explicitly calculated. In this case, sector inoperability is simply calculated using Eq. (8).

3.3. Updating Inventory

Naturally, the inventory of on-hand finished goods is depleted as those goods are used to compensate for production inoperability. The value of on-hand finished goods inventory is updated according to Eq. (10), which represents the reduction in inventory brought about by the amount of output that cannot be produced, calculated as a function of $p_i(t)$ and $x_i(t)$.

$$X_i(t+1) = \max\begin{cases} X_i(t) - p_i(t)x_i(t) \\ 0 \end{cases} \tag{10}$$

4. Illustrative Examples

The examples in this section illustrate the use of inventory as a risk management strategy for delaying inoperability in a system of interdependent economic sectors. The example is based on a two-industry example adapted from Miller and Blair [1985].

Interdependency parameters of the two sectors are provided in Table 1. Assume that the values in the table are in 10^3 dollars. It is assumed that the values in the table are constant across all time periods of interest, here in days, that is $\mathbf{x}(t) = \mathbf{x}(t + 1) = \cdots = \mathbf{x} = (1000, 2000)^T$.

Based on Table 1, the normalized technical coefficient matrix, required for Inventory DIIM calculation, is found in Eq. (11).

$$\mathbf{A}^* = \begin{bmatrix} \frac{150}{1000} & \left(\frac{500}{2000}\right)\left(\frac{2000}{1000}\right) \\ \left(\frac{200}{1000}\right)\left(\frac{1000}{2000}\right) & \frac{100}{2000} \end{bmatrix} = \begin{bmatrix} 0.15 & 0.50 \\ 0.10 & 0.05 \end{bmatrix} \tag{11}$$

The resilience coefficient matrix **K** assumed in all examples is found in Eq. (12).

$$\mathbf{K} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix} \tag{12}$$

For all three examples, there is no deviation from nominal final demand. Recovery is plotted for 30 days.

4.1. Inventory DIIM Example 1

For the first example, consider a disruption that results in a 15 percent inoperability in Sector 2. In vector form, this corresponds to the following initial production inoperability $\mathbf{p}(0)$.

$$\mathbf{p}(0) = \begin{bmatrix} 0 \\ 0.15 \end{bmatrix} \tag{13}$$

Assuming that neither sector has instituted any sort of inventory policy to delay such an initial inoperability, recovery near-zero inoperability is shown in Figure 2.

Note from Figure 2 that Sector 2 experienced an immediate inoperability which led to an interdependent inoperability in Sector 1. The total effect that inventory has on inoperability can be calculated from total economic loss, Q, across all sectors during the entire recovery period. The economic loss associated with Figure 2 is $Q_{\text{no inventory}}$ = \$2,154,130. That is, the initial inoperability to Sector 2 resulted in a combined economic loss across both sectors of nearly \$2.2 million over the span of 30 days.

However, if Sector 2 has inventory on hand of \$400,000, an amount that can cover over one day's worth of the sector's total output lost due to inoperability, the economic loss is not quite as drastic. The initial vector of inventory on hand is provided in Eq. (14).

$$\mathbf{X}(0) = \begin{bmatrix} 0 \\ 400 \end{bmatrix} \tag{14}$$

Figure 3 depicts the initial amount of inventory in Sector 2 maintaining operability for some time following the disruption. The economic loss associated with Sector 2's inventory policy is $Q_{\text{inventory}}$ = \$1,638,580. From this example, one can conclude that holding \$400,000 worth of inventory in Sector 2 would result in a reduction of over \$500,000 in total economic loss experienced by both sectors. Table 2 provides a few values of $p_i(t)$ and $q_i(t)$ for both inventory policies for several time periods.

Note how $p_2(t)$ improves due to the repair of the production inoperability such that when inventory in Sector 2 runs out in period 2, the sector inoperability experienced in period 3 is not as great as the original $p_2(0) = 0.15$.

4.2. Inventory DIIM Example 2

In this example, a disruption results in 20 and 15 percent inoperability in Sectors 1 and 2, respectively. The initial production inoperability vector $\mathbf{p}(0)$ for the two sectors appears in Eq. (15).

$$\mathbf{p}(0) = \begin{bmatrix} 0.20 \\ 0.15 \end{bmatrix} \tag{15}$$

The recovery of the two sectors is shown in Figure 4. The economic loss associated with the initial inoperability in Eq. (15) is $Q_{\text{no inventory}}$ = \$3,654,390, or over \$3.6 million in economic losses combined for both sectors over the 30 day recovery period.

Assume, as in the first example, that Sector 2 has an initial on-hand inventory worth \$400,000 of output, shown in Eq. (14).

Figure 5 depicts the recovery of the two sectors. Note how inoperability of Sector 2 is delayed, and that the recovery trajectory of Sector 1 is steeper than the "no inventory" case in Figure 4. This delay is also witnessed in Table 3, which presents values that populate Figure 5 for the first five time periods. The total economic loss for the inventory policy is $Q_{\text{inventory}}$ = \$3,033,990. This means that an inventory policy which requires \$400,000 worth of on-hand inventory in Sector 2 provides a total economic impact of more than \$600,000.

4.3. Inventory DIIM Example 3

The third example illustrates the use of inventory policies in both sectors. The initial inoperabilities, same as the previous example, are 20 and 15 percent, respectively, for Sectors 1 and 2. The initial production inoperability vector **p**(0) is found in Eq. (15), the graphical depiction of sector recovery is given in Figure 4, and the associated economic loss is $Q_{\text{no inventory}}$ = \$3,654,000.

The *no inventory* strategy is compared in this example to the situation where an initial inventory worth \$400,000 is maintained in both sectors. The vector of initial inventory **X**(0) is found in Eq. (16).

$$\mathbf{X}(0) = \begin{bmatrix} 400 \\ 400 \end{bmatrix} \tag{16}$$

A depiction of recovery of the sectors is found in Figure 6. The associated total economic loss when the sectors adopt the inventory policy found in Eq. (16) is $Q_{\text{inventory}}$ = \$2,453,000. One can conclude that a total initial inventory worth \$800,000 across both sectors can reduce the total economic loss of the specified disruption by \$1,200,000. Table 4 displays production inoperability and sector inoperability for the two inventory policies for the first five time periods.

4.4. Inventory DIIM in Decisionmaking

Described previously, the ultimate purpose of the Inventory DIIM is to provide a metric with which to determine the efficacy of preparedness risk management options and with which to compare such options. The Inventory DIIM can model a broad variety of preparedness option possibilities, e.g., system hardening, prepositioned supplies, on-hand inventory, through parameters $q_i(0)$, T_i, $c_i^*(t)$, and $X_i(0)$.

As illustrated by the three examples, there is an opportunity to compare several inventory policies with a multiobjective decisionmaking framework. Consider, for example, the risk management strategies provided in Table 5, which includes the options and their associated model parameters illustrated in the three examples. Assume the same disruptive event from Examples 2 and 3, shown in Eq. (15), where Sector 1 is initially 20 percent inoperable and Sector 2 is 15 percent inoperable. Assume that seven risk management strategies are devised, each with different levels of on-hand inventory in place to dampen the effects of a disruptive event in the two sectors. Note from the table that the resilience coefficients are held constant for all strategies, signifying that values of $q_i(0)$, T_i, and $c_i^*(t)$ are held constant across all strategies, i.e., no other preparedness options than on-hand inventory are considered here.

Of interest in Table 5 from a decisionmaking perspective are the values of total initial on-hand inventory, measured in dollars, and the total economic loss Q, also measured in dollars. A decisionmaker's objectives would be to minimize both of those values. Note that while both objectives are measured in dollars, they are likely noncommensurate. That is, total on-hand inventory is money spent by a single or multiple sectors in the economy individually, and possibly driven by government policy, while total economic loss would be experienced by all sectors. Additionally, when dealing with critical infrastructure systems in particular, economic loss is a surrogate for the ability to provide essential services for the well-being of citizens, for the recovery effort following a disruption, among others.

A plot of the Pareto-optimal strategies appears in Figure 7. Strategies D and E are dominated.

Other objectives for consideration could include, depending on the disruption, minimizing the number of lives lost, minimizing the number of homes without a particular essential service, among others.

5. Case Study

In this case study, BEA's national input-output accounts and inventory-to-sales data were utilized to parameterize the inventory DIIM. Here, inventory-to-sales ratio will be interpreted as a measure of the capability of a sector to react to and absorb the impact of temporary loss of production capacity. For example, the July 2007 estimate of the inventory-to-sales ratio for the manufacturing sector was estimated at 1.34 [BEA 2007]. This number can be interpreted as a ratio of the value of inventory (i.e., finished goods on hand) relative to sales. Hence, we could assume that anything in excess of 1 for this ratio could be allocated for subsequent delivery cycles/periods. The proposed inventory DIIM is demonstrated using the 15-sector classification scheme shown in Table 6.

Suppose that a disaster causes a 20 percent temporary loss to the production capacity of industry sectors at the national level. This percentage corresponds to the initial production inoperability, which was denoted by $p_i(0)$ in Eq.(6). Data from historical events could be used in quantifying and calibrating the values of production inoperability. Examples of events where some notion of production inoperability were derived include a regional blackout [Anderson et al. 2007] and a major earthquake [Gordon et al. 1998], among others.

Since the aim of this case study is to demonstrate the inventory DIIM using actual input-output data and inventory-to-sales ratios, we assumed for simplicity that the 20 percent initial production inoperability is uniformly applied to all the 15 sectors. How this initial production inoperability could evolve over time is dictated by the dynamic formulations of sector inoperability found in Sections 3.2 and 3.3. Due to sector interdependencies, this production loss will further create ripple effects. Without inventory in place, economic losses will be intuitively higher. In previous DIIM implementations, only contrived examples of risk management options were considered. Hence, the purpose of this case study is to compare model results with and without inventory, ultimately providing intuitions on: (i) how previous DIIM implementations could potentially overestimate economic losses without consideration of inventory, and (ii) how the proposed inventory DIIM could be used to explicitly model inventory policies in the context of managing interdependent sector productivity degradations.

The DIIM results are depicted in Figure 8. The dark lines correspond to the DIIM results when no inventory is assumed, while the light lines correspond to the results of the proposed inventory DIIM formulation. Note that due to the data limitations in inventory-to-sales ratios, only three sectors were assumed to maintain inventories, namely *Manufacturing* (S5), *Wholesale trade* (S6), and *Retail trade* (S7). It is also assumed that sectors would recover within a simulated 30-day horizon. Results indicate the delayed effects of production capacity loss in sectors that maintain inventories (S5, S6, and S7). Due to sector interdependencies, results also show the cascade of benefits to other sectors that are assumed to have no inventories in place, such as *Agriculture, forestry, fishing, and hunting* (S1), *Mining* (S2), *Utilities* (S3), *Transportation and* *warehousing* (S8), *Information* (S9), *Finance, insurance, real estate, rental, and leasing* (S10), *Professional and business services* (S11), and *Other services, except government* (S14). The remaining sectors did not receive distinguishable benefits (i.e., the lighter line coincides with the darker line), such as *Construction* (S4), *Educational services, health care, and social assistance* (S12), *Arts, entertainment, recreation, accommodation, and food services* (S13), and *Government* (S15). Considering the integrated benefit to the economy comprising of the 15 sectors, results show that maintaining inventories could curtail the propagation of inoperability. Likewise, inventories could also drive down economic losses as will be explained in subsequent analysis.

To demonstrate the multiobjective aspect of this case study in the same manner as the hypothetical example in Figure 7, consider the following inventory strategies as risk management options: No inventory in any sector, Inventory for Sector S5 only, Inventory for Sector S6 only, Inventory for Sector S7 only, and Inventory for all sectors (S5, S6, and S7).

It is assumed that the costs associated with the above options are directly proportional to the value of the inventories themselves, which can be deduced from the GDP accounts. For each option, inventory cost is plotted versus the economic loss determined from multiple runs of the DIIM.

Figure 9 shows the Pareto optimal frontier. In general, economic losses are reduced as more sectors are allowed to maintain inventories. When S6 and S7 are separately analyzed, it was found that maintaining inventory for S7 alone is inferior, since its cost is higher than S6. The Pareto optimal frontier comprises of solutions ranging from “without inventory”, at a cost of zero, to maintaining inventories for combined Sectors S5, S6, and S7, at a cost of $35 billion. It should be noted that the efficacy of inventory becomes more significant as production inoperabilities (or disruptions to production capacities) increase. In a relatively stable economy where supply and demand are fairly predictable and resilient to perturbations, the DIIM charts would depict “flat lines” almost coinciding with zero inoperability. Since inventories require cost, this stable economy assumption would favor the JIT philosophy where inventories are desired to be at minimum. Nevertheless, such is not always the case. In the US, for example, the adverse effects of natural disasters (e.g., hurricanes) to workforce and commodity flows are highly unpredictable. These workforce and commodity flow disruptions ultimately degrade the production capacities of most sectors, hence justifying investments in inventories.

6. CONCLUSIONS

Due to the interdependencies among production activities in various economic and infrastructure sectors, an adverse impact to production brought about by any number of disruptive events can have far-reaching effects. Recent emphasis has been placed on improving resilience in these sectors, namely in their ability to respond and recover to such disruptive events. Previous risk-based interdependency models, including the IIM and DIIM, were developed to measure the efficacy of several different preparedness options designed to improve sector resilience.

This paper describes an important extension to this modeling paradigm, the Inventory DIIM, which models the efficacy of inventory policies in making economic and infrastructure sectors more resilient to disruptive events. Thus, the Inventory DIIM provides a metric with which one can compare a number of preparedness strategies in a multiobjective framework. Its usefulness is illustrated with three pedagogical examples and also a case study using actual inventory ratio data provided by the Bureau of Economic Analysis.

Although a proof of concept in the case study demonstrates the execution of the Inventory DIIM using actual data, further refinements have been identified. Many economic and infrastructure sectors, particularly those providing "essential services" for which preparedness planning is of major interest, are not of the nature where finished goods inventory is appropriate or perhaps possible, e.g., electric power, and future work includes adapting the model to such sectors. As BEA inventory data are available only for national-level economic sectors, an opportunity exists to collect regional inventory surveys from local private and public organizations to understand how national-level data can be interpolated to local inventory decision making. Further, the relationship between firm-level inventory and disruptions and regional- and national-level inventory and disruptions should be modeled.

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FIGURES

Note that Figure 1 was attached as a TIFF file. The remaining eight figures, created in Excel, are found below. The original Excel file is attached.

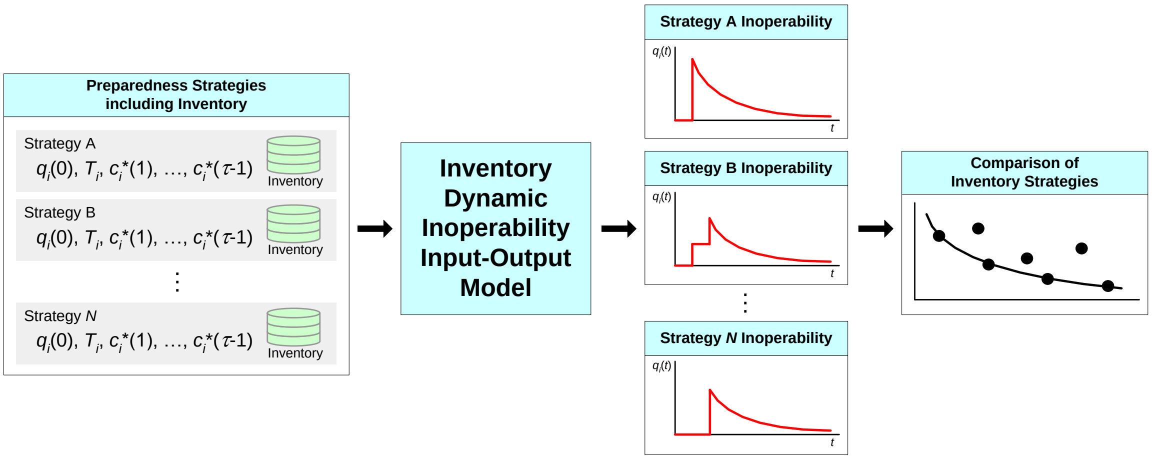

Figure 1. Depiction of modeling preparedness strategies which involve inventory policies.

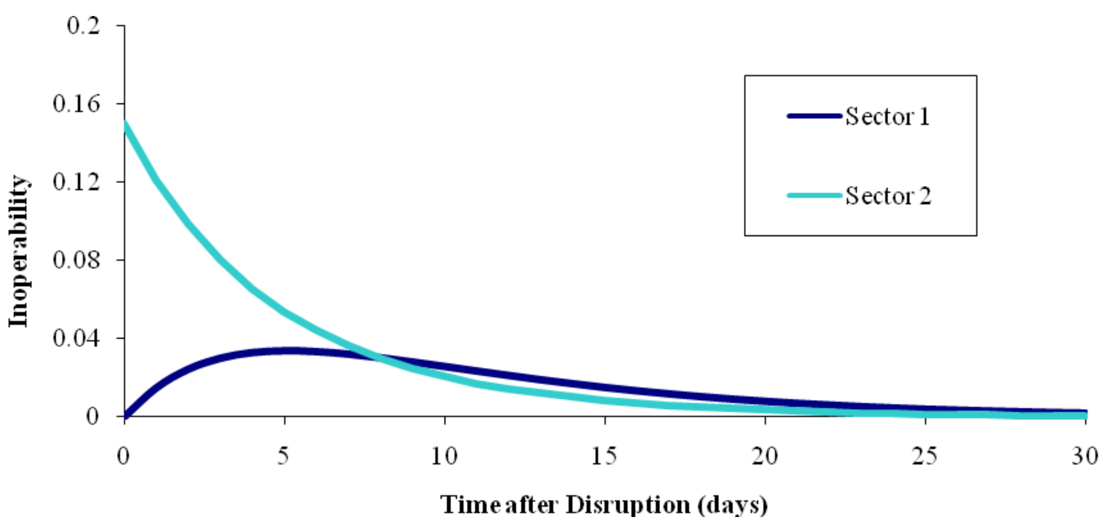

Figure 2. Depiction of inoperabilities of both industries in Example 1 without the inclusion of inventory.

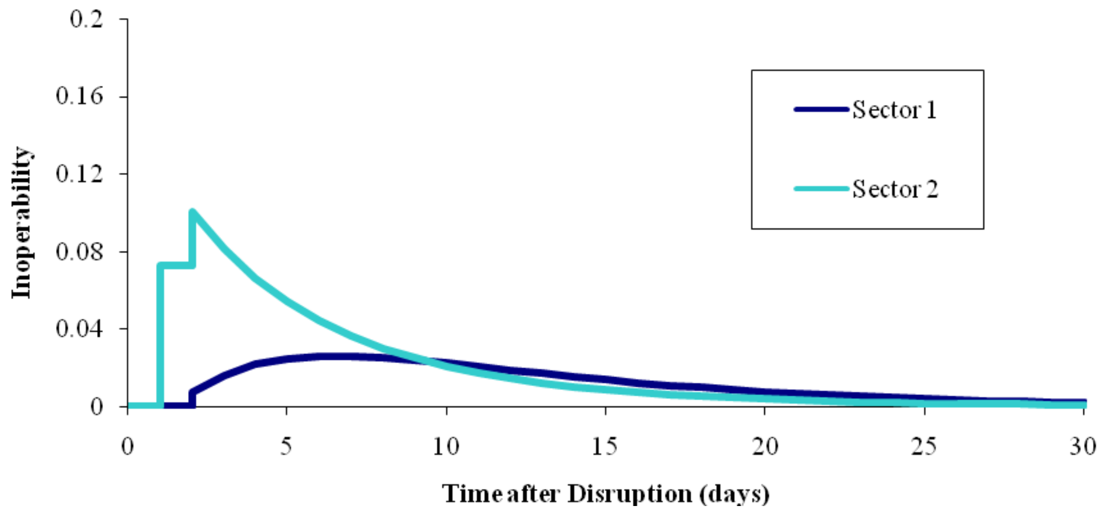

Figure 3. Depiction of inoperabilities of both industries in Example 1 including Sector 2 inventory from (14).

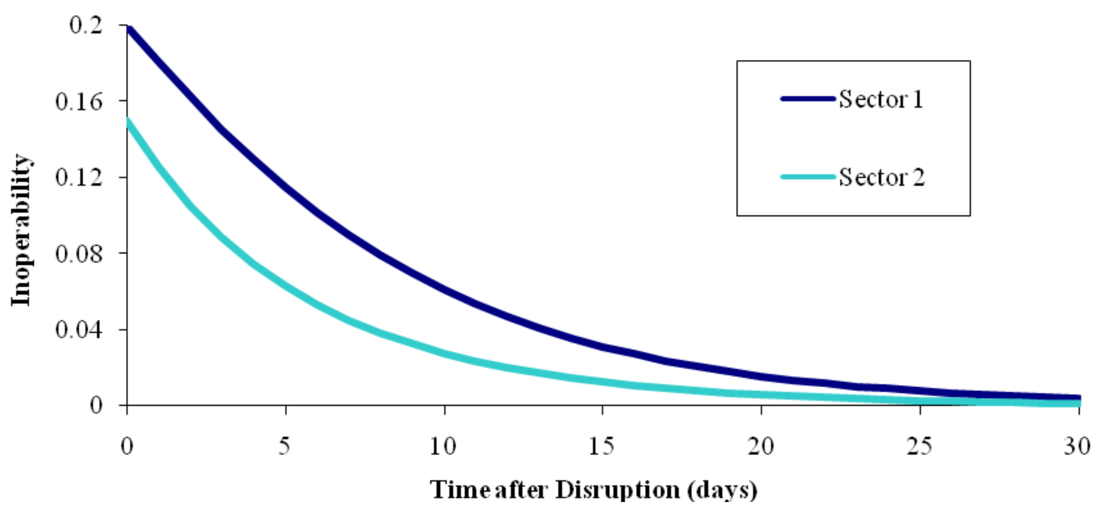

Figure 4. Depiction of inoperabilities of both industries in Example 2 without the inclusion of inventory.

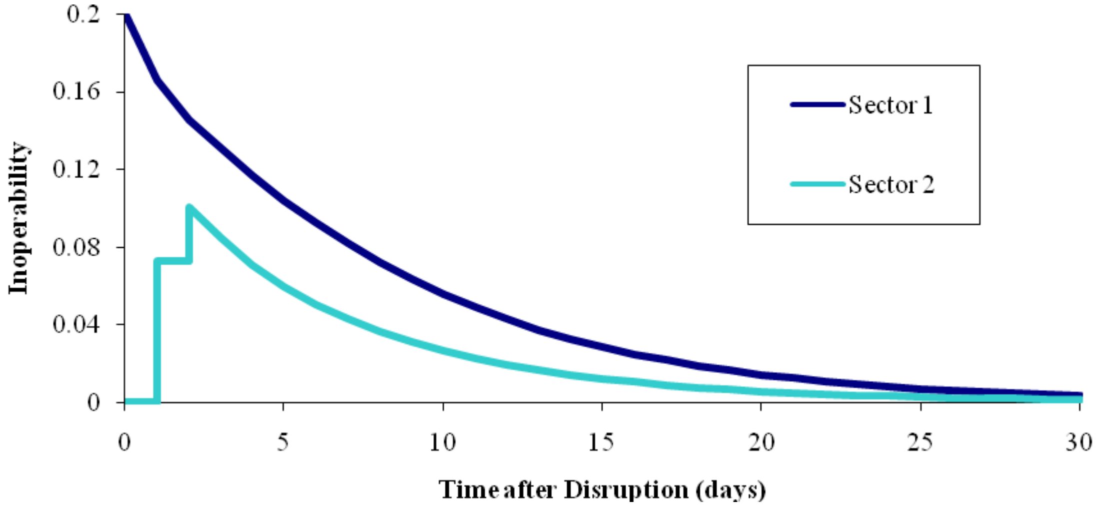

Figure 5. Depiction of inoperabilities of both industries in Example 2 including Sector 2 inventory from Erro! Fonte de referência não encontrada.**.**

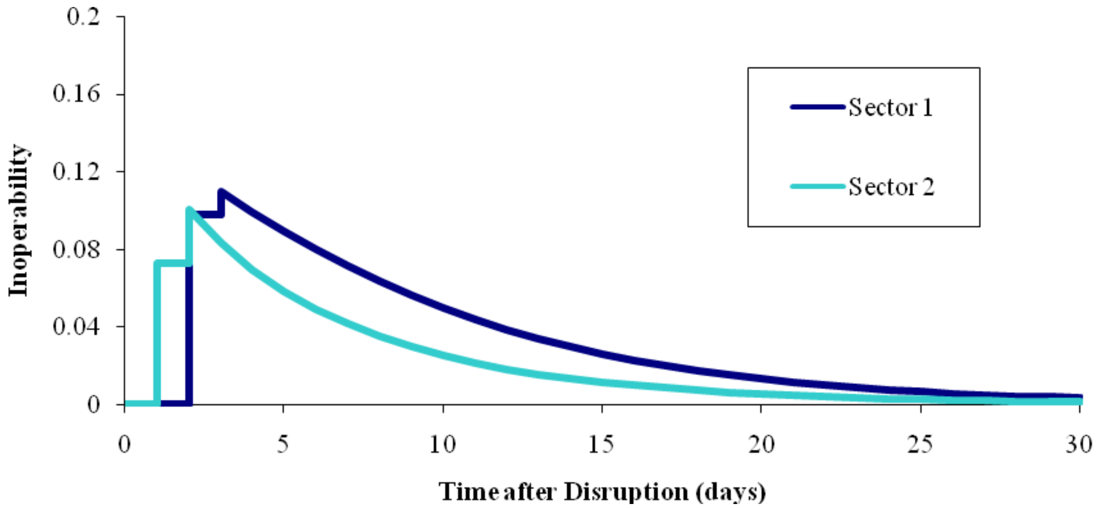

Figure 6. Depiction of inoperabilities of both industries in Example 3 including Sector 2 inventory from (17).

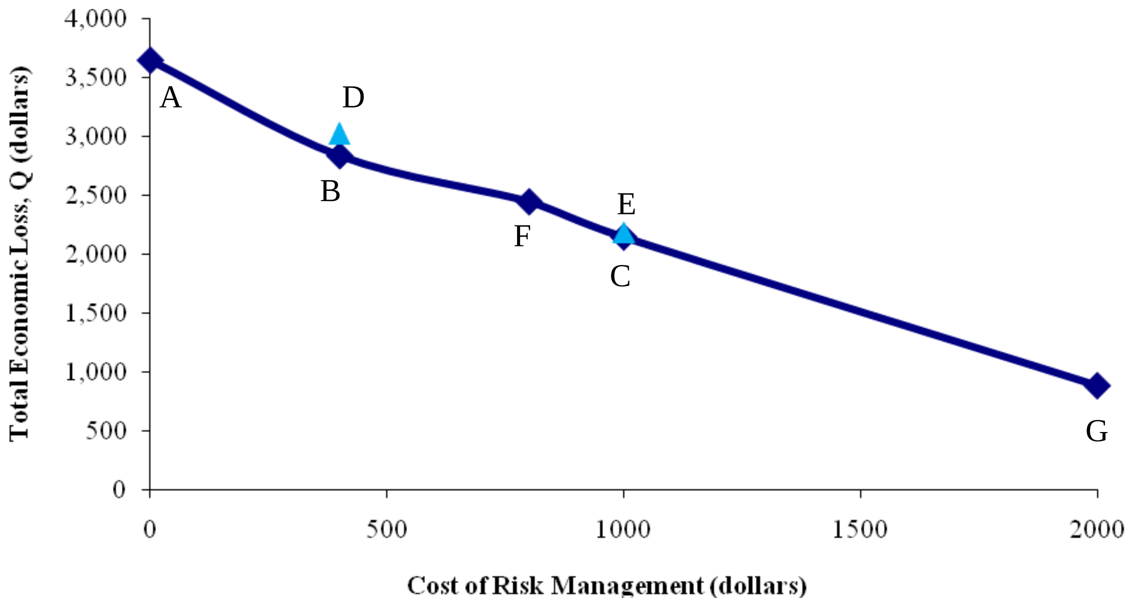

Figure 7. Depiction of the Pareto-optimal strategies from Table 5.

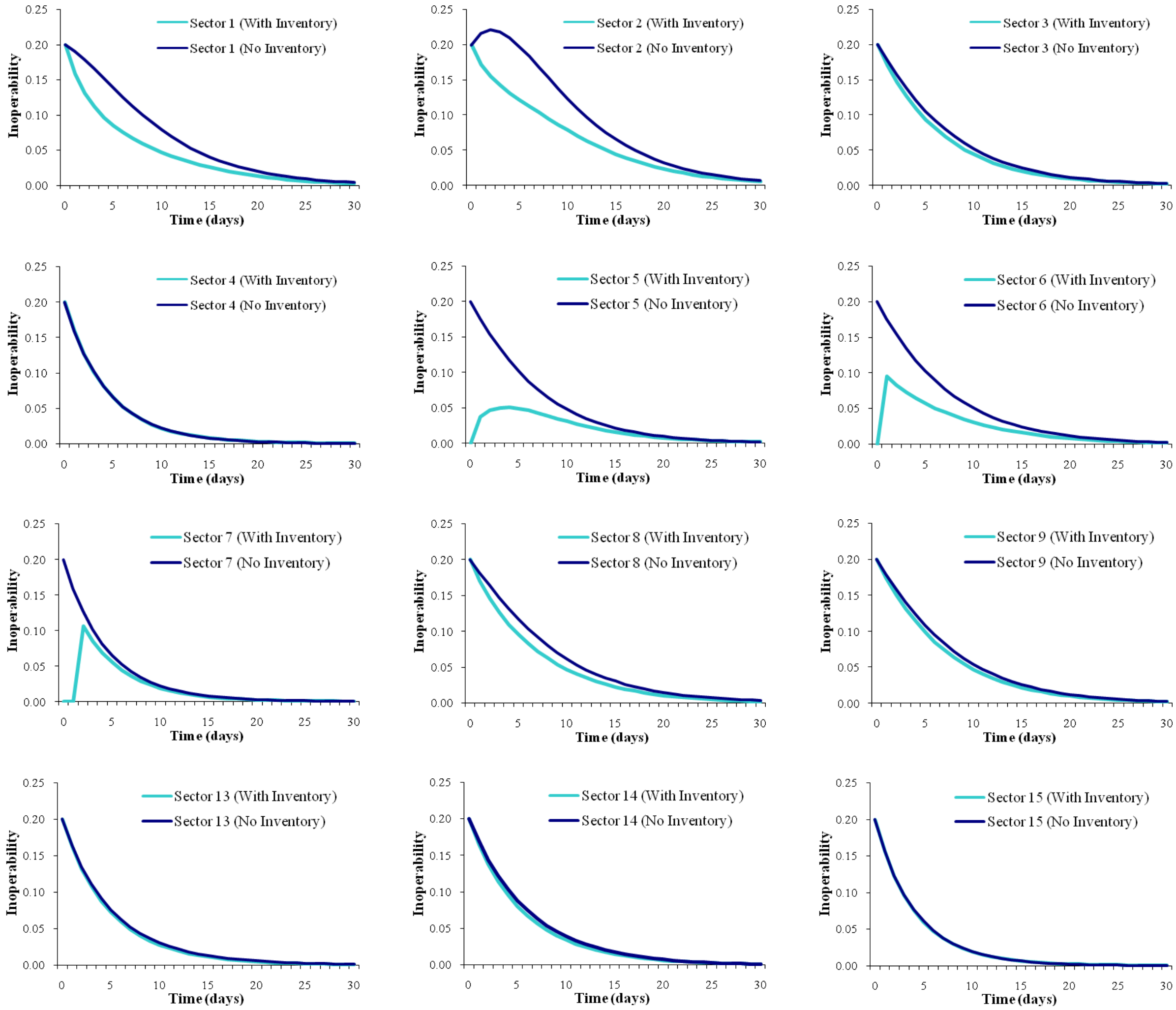

Figure 8. DIIM results with inventory (light line) and without inventory (dark line), see Table 6 for sector descriptions.

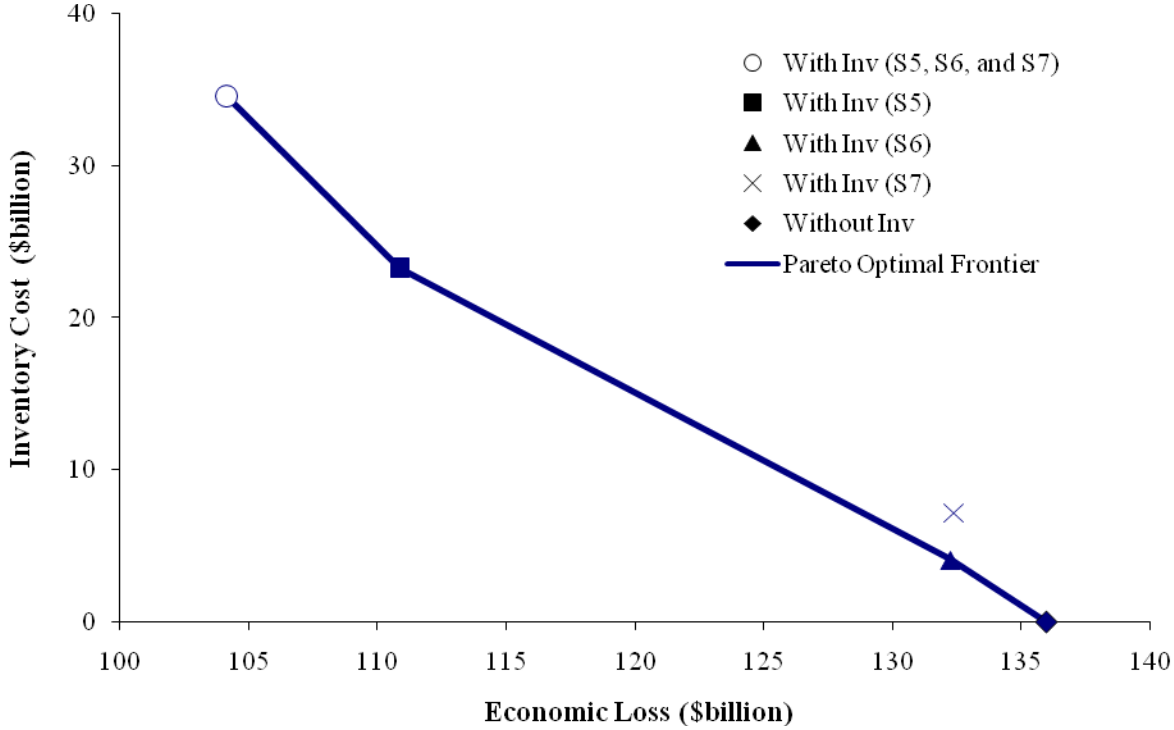

Figure 9. Economic loss versus inventory cost for several inventory policies.

TABLES

Table 1. Daily as-planned commodity flows for the two-sector example (in 10^3 dollars), from Miller and Blair [1985].

Industries	1	2	Final Demand	Total Output
1	150	500	350	1000
2	200	100	1700	2000
Value Added	650	1400		
Total Input	1000	2000		

Table 2. Production and sector inoperability recovery for Example 1.

			Without Initial Inventory		With Initial Inventory	
t	$p_1(t)$	$p_2(t)$	$q_1(t)$	$q_2(t)$	$q_1(t)$	$q_2(t)$
0	0.000	0.150	0.000	0.150	0.000	0.000
1	0.000	0.123	0.015	0.122	0.000	0.073
2	0.000	0.101	0.025	0.099	0.007	0.101
3	0.000	0.082	0.030	0.081	0.016	0.082
4	0.000	0.067	0.033	0.066	0.022	0.066
5	0.000	0.055	0.034	0.054	0.025	0.054

Table 3. Production and sector inoperability recovery for Example 2.

			Without Initial Inventory		With Initial Inventory	
t	$p_1(t)$	$p_2(t)$	$q_1(t)$	$q_2(t)$	$q_1(t)$	$q_2(t)$
0	0.200	0.150	0.200	0.150	0.200	0.000
1	0.164	0.123	0.181	0.126	0.166	0.073
2	0.134	0.101	0.163	0.105	0.145	0.101
3	0.110	0.082	0.146	0.089	0.131	0.084
4	0.090	0.067	0.130	0.075	0.117	0.071
5	0.074	0.055	0.115	0.063	0.104	0.060

Table 4. Production and sector inoperability recovery for Example 3.

			Without Initial Inventory		With Initial Inventory	
t	$p_1(t)$	$p_2(t)$	$q_1(t)$	$q_2(t)$	$q_1(t)$	$q_2(t)$
0	0.200	0.150	0.200	0.150	0.000	0.000
1	0.164	0.123	0.181	0.126	0.000	0.073
2	0.134	0.101	0.163	0.105	0.098	0.101
3	0.110	0.082	0.146	0.089	0.110	0.083
4	0.090	0.067	0.130	0.075	0.099	0.070
5	0.074	0.055	0.115	0.063	0.090	0.059

Table 5. Details of seven risk management strategies for dealing with a disruption to both sectors of a two-sector example, in 10^3 dollars.

Strategy	Initial On-Hand Inventory $X_1(0)$	$X_2(0)$	Total	Q	Δ from baseline
A	0	0	0	3,654	0
B	400	0	400	2,841	814
C	1,000	0	1,000	2,154	1,500
D	0	400	400	3,034	620
E	0	1,000	1,000	2,193	1,461
F	400	400	800	2,453	1,201
G	1,000	1,000	2,000	886	2,769

Table 6. BEA sectors used in the case study.

Code	Name
S1	Agriculture, forestry, fishing, and hunting
S2	Mining
S3	Utilities
S4	Construction
S5	Manufacturing
S6	Wholesale trade
S7	Retail trade
S8	Transportation and warehousing
S9	Information
S10	Finance, insurance, real estate, rental, and leasing
S11	Professional and business services
S12	Educational services, health care, and social assistance
S13	Arts, entertainment, recreation, accommodation, and food services
S14	Other services, except government
S15	Government