BUILDING A SYSTEM OF SYMMETRIC INPUT-OUTPUT TABLES – APPLICATION TO PORTUGAL, 2005

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Abstract

This paper describes the methodology used to build a complete system of symmetric input-output tables for Portugal, 2005 (product-by-product, using P60 ESA95 nomenclature) starting from supply and use tables (product-by-industry). A simple method of symmetrization is proposed, inspired in the product technology assumption but with no negative values.

Keywords: input-output tables; symmetric input-output tables; product-by-product; product technology

1. INTRODUCTION

This paper presents the methodology used to build a system of symmetric input-output (I-O) tables for Portugal, for the year 2005.

This work was carried out in the Department of Foresight and Planning and International Affairs (DPP) in 2008, with the aim of updating technical coefficients of DPP's input-output based model, MODEM[1], used for policy evaluation.

In fact, calibration of MODEM requires the availability of symmetric input-output tables (*i.e.*: product by product or industry by industry) for the Portuguese economy, in terms of a Total Flows table (at purchasers' prices) and its decomposition into tables for flows of Domestic Output at basic prices, of Imports CIF, of Taxes (net of Subsidies) on Products, and of Trade and Transport Margins.

This task was undertaken with the agreement of the Portuguese statistical office (Instituto Nacional de Estatística - INE), which supplied detailed (unpublished) information concerning the supply and use tables for 2005. We are grateful to INE's National Accounts team for their collaboration, which enabled the accomplishment of this task, in particular to Idílio Freire, Maria João David and Mafalda Ferreira.

This work was done following the rules and nomenclatures from the 1995 European System of Accounts (ESA95, European Commission, 1996) and taking into account the recommendations from the EUROSTAT Manual on Input-output tables (EUROSTAT, 2008). The sectoral breakdown of the I-O system follows the ESA95 product nomenclature P60[2], which is presented in Appendix 1.

The work evolved in the following stages:

[1] Dias and Lopes (2009).

[2] The number of products/branches considered in the system is actually 59 because the 60th product in the P60 nomenclature (code 99) refers to the services provided by extraterritorial organizations, which are not considered in the I-O tables because they are built under the territory's perspective (Portugal).

1. Construction of a disaggregated non-symmetric system of input-output (I-O) tables (426 products by 59 industries and final uses), P426×A60[3];

2. Aggregation of the P426×A60 system to a P60×A60 system (59 products by 59 industries and final uses);

3. Symmetrization of each table from the P60×A60 system, which became P60×P60 tables (59 products by 59 homogeneous branches producing those products), with the exception of the Domestic Output table;

4. Calculation of the symmetric I-O table for Domestic Output at basic prices through the difference between the matrix of Total Flows at purchasers' prices and the sum of the matrices for imports CIF, taxes (net of subsidies) and trade and transport margins.

Section 2 describes the construction of non-symmetric systems of I-O tables (stages 1 and 2) while section 3 presents the methodology used in the symmetrization of the I-O tables (stages 3 and 4) and section 4 concludes.

2. CONSTRUCTION OF NON-SYMMETRIC SYSTEMS OF I-O TABLES

This work was done on the basis of the following input-output detailed data, supplied by INE at the level of 426 products by 129 industries[4] and final uses:

- Supply and Use tables;
- Tables for Trade margins (MC), Transport Margins (MT), Value Added Tax (VAT), Other taxes on Products (OTP) and for Subsidies on Products (-Z).

INE supplied also estimates for the decomposition of trade and transport margins by type of margin (wholesale trade, retail trade, railway, road, sea and fluvial transportation) for total intermediate uses (for each of the 426 products).

[3] A60 is an ESA95 industry nomenclature. P426 (426 products) represents a disaggregation of P60.

[4] A disaggregation of A60 nomenclature.

From this data we built an input-output table for Total Flows at “Almost” Basic Prices of 426 products by 59 branches and final uses (which we will call FTabp), obtained from the table of Total Flows at Purchasers’ Prices (FTpp, supplied by INE), after subtracting the corresponding tables for Trade Margins (MC), Transport Margins (MT), VAT, Other Taxes on Products (OTP) and Z (table for Subsidies on Products multiplied by -1):

(1) FTabp = FTpp – MC – MT – VAT – OTP - Z

It should be noted that, for the MC and MT matrices we included, in the rows for trade and transport services (supplying the output corresponding to the margins) negative values equal to the total value of the respective margins applied to the various products (the remaining rows of the corresponding margins’ matrix), so that the sum of the elements of each column in these matrices equals zero. For that purpose it was necessary to estimate the breakdown of each type of margin and product by user industries, which was done proportionately to the corresponding breakdown for total trade or transport margins.

The next step was to estimate a first version of an Imports matrix, M(1) through the breakdown of the column vector (426×1) for total Imports of goods and services (CIF) by the various uses (59 intermediate uses, by 59 industries, and the various final uses). This breakdown was done for each row, proportionately to the corresponding row of the FTabp matrix after excluding Exports (for Imports of Goods) and after excluding Exports, Change in Inventories and Final Consumption Expenditure by Public Administrations and by Non-Profit Institutions serving Households (NPISH) (for Imports of Services).

Subsequently, a first version of an Import Duties matrix, ID(1) was calculated through the breakdown of each value of the import duties vector (426×1) proportionately to each row of M(1), *i.e*, assuming the same import duties’ rate across all product uses, for each imported product.

A first version of the Domestic Output I-O table, PN(1), was then calculated through the subtraction of M(1) and ID(1) from FTabp:

(2) PN(1) = FTabp – M(1) – ID(1)

PN(1) was then checked to verify if it filled the non-negativity condition for all intermediate and final consumption uses. A few cases of negative values were found, corresponding to products for which exports at basic prices exceeded domestic output, thus violating the initial assumption (when building M(1)) that there were no exports in the imports matrix. These situations were solved on a case by case basis after obtaining more detailed information from INE, including estimates about the percentage of total exports of each product that corresponded to re-exporting imported products without any kind of domestic transformation. This process led to the revision of the Imports, Import Duties and Domestic Output matrices, respecting the following identity:

(3) FTpp = PN + M + ID + VAT + OTP + Z + MC + MT

After obtaining this complete system of matrices for 426 products by 59 industries and final uses, we aggregated it to a system for P60 (59 products) × A60 (59 industries) and final uses.

3. SYMMETRIZATION OF THE SYSTEM OF I-O TABLES

3.1 Symmetrization methods – general features

Having obtained a system of I-O tables where the 1st and 3rd quadrants represent, respectively, intermediate flows of 59 products (P60) and k primary inputs (Gross Value Added components) to 59 industries (A60) (Appendix 2 – Figure 1), the symmetrization of these two quadrants was undertaken in order to convert heterogeneous branches (with secondary outputs) into "homogeneous" branches, producing only the corresponding product, i.e., converting a P60×A60 system (product by industry) into a P60× P60 system (product by product) (Appendix 2 – Figure 2).

The symmetric I-O tables must respect the following identities:

a) Total intermediate uses of each product and total uses of each primary input should have the same value for both Use Tables (non-symmetric) and Symmetric I-O Tables;

b) For symmetric I-O tables concerning Total Flows, the sum of the elements of each column (total intermediate and primary inputs used in the production of each product) should be equal to the output of the corresponding product.

To undertake symmetrization two assumptions (or a combination of them) can be considered regarding production technologies:

1 **Industry technology assumption**: the production technology for each product depends on the industry where the product is actually produced, corresponding to the technology of that industry, assuming that each industry has its own technology irrespective of its product mix;

2 **Product technology assumption**: each product has the same production technology irrespective of the industry where it is produced.

In reality, the production technology of each product should be a combination of these two assumptions, being closer to the product technology assumption for subsidiary production and to the industry technology assumption in the cases of by-products and of joint production (Eurostat, 2008, p. 314).

In order to facilitate the explanation of the various attempted symmetrization methods, we present first the terminology used and some definition equations which are common to the various methods.

Let MNS be a non-symmetric (product by industry) I-O table (1^{st} and 3^{rd} quadrants) and MS be the corresponding symmetric table (product by product), both with *m* rows and *n* columns (matrices of m×n dimension) where m=n+k, *n* being the total number of products/industries and *k* the number of primary inputs considered. These matrices' generic elements, of order (i,j), mns_{ij} e ms_{ij} , represent the quantity of product/primary input *i* used to produce the output of industry *j* (which, for the symmetric table[5], coincides with product *j*' output).

Let X be the Production matrix (a n×n matrix where the generic element x_{ij} represents the output of product i produced by industry j) and XRD and XPD be diagonal matrices

[5] It should be noted that the expression "symmetric table" used in this paper does not correspond to the concept of symmetric matrix from the matrix algebra.

(n×n) where the generic element of the principal diagonal represents, respectively, total output for each industry j (before symmetrization), xr_j and for each product j, xp_j .

Let ANS and AS be the matrices of vertical technical coefficients calculated respectively from MNS and MS, representing their generic elements, ans_{ij} and as_{ij}, respectively, the quantity of product/primary input i used in the production of one unit of industry j' output (before symmetrization) or of product j (after symmetrization). These coefficients are calculated in the following way:

(4) $ans_{ij} = mns_{ij} / xr_j$ and (5) $as_{ij} = ms_{ij} / xp_j$

or, in matrix notation:

(6) $ANS = MNS*XRD^{-1}$ and (7) $AS = MS*XPD^{-1}$

Re-arranging (6) and (7) in order to MNS and MS we have:

(8) $MNS = ANS*XRD$ and (9) $MS = AS*XPD$

Let ANS_j and AS_i be the columns of order j and i from ANS and AS, representing, respectively, industry j (before symmetrization) and product i (after symmetrization) productive technologies.

3.2. Method 1 for symmetrization (pure industry technology assumption)

Assumption 1 (industry technology), applied in a strict way, implies that the productive technology for each product in the symmetric table is a weighted average of industry technologies (observed in the non-symmetric table) where the weights represent the share of each industry in that product's output:

(10) $AS_i = \sum_j (x_{ij}/xp_i) *ANS_j$

From (10) we obtain the following matrix equation for AS:

(11) $AS = ANS*X' *XPD^{-1}$

Combining equations (11), (9) and (6), we obtain the formula for symmetrization under the industry technology assumption[6]:

(12) $MS = ANS*X' = MNS*XRD^{-1}*X'$

3.3. Method 2 for symmetrization (pure product technology assumption)

Assumption 2 (product technology), applied in a strict way, implies that the productive technology for each industry (observed in the non-symmetric table) is a weighted average of product technologies where the weights represent the share of each product in that industry's output:

(13) $ANS_j = \sum_i (x_{ij}/xr_j) *AS_i$

From (13) we obtain the following matrix equation for ANS, as a function of AS:

(14) $ANS = AS*X*XRD^{-1}$

Re-arranging (14) in order to AS and combining with (8) we have:

(15) $AS = ANS*XRD*X^{-1} = MNS* X^{-1}$

Combining equations (15) and (9) we obtain the formula for symmetrization under the product technology assumption[7]:

(16) $MS = MNS*X^{-1} *XPD$

3.4. Appreciation of methods 1 and 2 and presentation of the adopted method (method 3)

Both methods 1 and 2 have the advantage of leading to results that ensure both identities a) and b) (presented in section 3.1) in just one iteration.

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[6] This formula is equivalent to the one presented in Eurostat (2008), p.349 (Model B: Product-by product input-output table based on industry technology assumption), although using a different terminology.

[7] This formula is equivalent to the one presented in Eurostat (2008), p.349 (Model A: Product-by product input-output table based on product technology assumption), although using a different terminology.

These two methods were applied to the non-symmetric table of Total Flows at Purchasers' prices for 2005 (product by industry).

The application of method 2 (product technology assumption) led to a table with a large number of negative values for intermediate consumptions, which was unacceptable.

On the other hand, method 1 (industry technology assumption), although having the merit of generating only positive values for intermediate consumptions, led to production technologies that, in some cases, did not look reasonable, *e.g.* the existence of agricultural inputs for the production of office machinery and computers.

A third method was then attempted that had the advantages of generating no negative values for any intermediate uses and of leading to plausible production technologies. Nevertheless, this method has to inconvenient of not ensuring identity a) (presented in section 3.1) in the first iteration, thus implying the need to perform, at least, one more iteration in order to ensure both identities a) and b).

This method uses the **product technology assumption** as a starting point but, instead of applying the pure formula presented in section 3.3 (which has the inconvenient of generating negative values), it uses the production technologies observed for each industry in the non-symmetric I-O table (Use Table) as a first iteration for the corresponding product technologies. Therefore, it is assumed, in a first step, that that the production technology for each product j, $AS_j(1)$, is identical to the industry technology where this product is the main output, ANS_j:

(17) $AS_j(1) = ANS_j$

therefore:

(18) $AS(1) = ANS$

Combining (18) with equations (6) and (9) we arrive to:

(19) $MS(1) = MNS*XRD^{-1}*XPD$

This first iteration does not ensure identity a) referred in section 3.1, i.e., the equality between total intermediate uses of each product (and total uses of each primary input) in MS(1) and MNS matrices.

Therefore a second iteration (and last) was performed, calculating MS through the multiplication of each row of MS(1) by the ratio of the respective row sums for matrices MNS and MS(1), with the exception (for the I-O table of Total flows) of the last row (Net Operating Surplus). In this way identity a) was ensured and identity b) was subsequently guaranteed through the calculation of the Net Operating Surplus row residually (difference between a row for total output of each product and the sum of the remaining rows of MS(2)):

(20) $MS_i = MS(1)_i *[MNS_i *in]/[MS(1)_i *in]$ for all $i \neq NOS$

(21) $MS_{NOS} = XP - \sum_{i \neq NOS} MS_i$

where MS_i, $MS(1)_i$ and MNS_i are the ith rows of MS, MS(1) and MNS, **in** is a (n×1) unit vector, NOS is Net Operating Surplus and XP is a row vector for product outputs.

The option for performing only two iterations in this symmetrization procedure and for adjusting column sum discrepancies through one unique row (Net Operating Surplus, NOS) instead of performing a multiple iteration RAS procedure to distribute discrepancies across all matrix elements was because NOS is, by definition, already a residual variable, which may take either positive or negative values, and also to ensure a greater coherence in the method of symmetrization of the various matrices within the system, to which the same method was applied. It should be noted that the column sum restriction (equal to the product's output) applies only to the Total Flows matrix.

This method of symmetrization was applied to all I-O tables of the system with the exception of the table for Domestic output at basic prices (PN), which was calculated from the remaining symmetrized matrices of the system (matrix of Total flows at purchasers' prices less matrices for margins, taxes, subsidies and imports):

(22) PN = FTpp – MC – MT - VAT – ID - OTP – Z - M

Finally we checked PN to verify if it filled the non-negativity condition for all intermediate uses. Very few cases of negative values (of a very small amount) were still found, which were resolved by setting those values to zero and compensating them through the element of each row presenting a larger value (to ensure the maintenance if identity a)). The imports matrix was corrected afterwards through the addition of the symmetric of the corresponding correction made in the Domestic Output matrix, for each matrix element, in order to maintain constant the sum of PN and M matrices, corresponding to the matrix of total flows at basic prices (FTbp):

(23) FTbp = PN + M = FTpp – MC – MT - VAT – ID - OTP – Z

The final symmetric system of I-O tables for Portugal, 2005 is published in Dias (2008). The symmetric I-O tables for Total Flows at basic prices, Domestic Output at basic prices and Imports, as well as the non-symmetric I-O table for Total Flows at purchasers' prices (Use table), are also available in excel format in the site of European Unions' statistical office, Eurostat, at: http://epp.eurostat.ec.europa.eu/portal/page/portal/esa95_supply_use_input_tables/data/workbooks. Those tables are downloadable from the workbooks for Portugal (Portugal_Suiot_090211u.xls, within the zip file for Portugal, sheets 'use05' (use table) 'siot05','dom05' and 'imp05').

4. CONCLUDING REMARKS

This paper described the methodology used to build a system of symmetric input-output tables for Portugal, 2005 (product-by-product, using P60 ESA95 nomenclature) starting from supply and use tables (product-by-industry).

Two methods of symmetrization were attempted, based on the pure "industry technology" and "product technology" assumptions, but the results were not satisfactory due to the presence of a large number of negative values, in the case of the "product technology" assumption and of some unrealistic production technologies, when the "industry technology" assumption was used.

Trying to avoid the abovementioned shortcomings, a third method was implemented (and adopted), inspired in the product technology assumption but using, as a first iteration for each product's technology, the observed corresponding industry technology.

Symmetrization procedures lead, almost inevitably, to some data distortion, but they are necessary to enable input-output analysis and modeling. Whenever detailed information is available concerning the technology of off-diagonal productions (from the production matrix, in the supply table) a method based on a hybrid technology assumption (such as the method presented in Eurostat, 2008, p.349, Model E) might be more appropriate than the method proposed in this paper, which is based on the absence of such detailed information.

The proposed method should be used cautiously and it is more appropriate when the weight of secondary production is relatively low across all industries.

Although simpler than some other existing symmetrization methods such as the Almon procedure (Almon, 2000), it appears to the author that the proposed method is not necessarily inferior in terms of the quality of its results.

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APPENDIX 1 – Products/homogeneous branches considered in the system of symmetric input-output tables for Portugal, 2005 (P60)

Code	Description
01	Products of agriculture, hunting and related services
02	Products of forestry. logging and related services
05	Fish and other fishing products; services incidental of fishing
10	Coal and lignite; peat
11	Crude petroleum &nat.gas; serv.to oil&gas extract. excluding surveying
12	Uranium and thorium ores
13	Metal ores
14	Other mining and quarrying products
15	Food products and beverages
16	Tobacco products
17	Textiles
18	Wearing apparel; furs
19	Leather and leather products
20	Wood& wood prod. and cork (except furnit.); articles Straw& plaiting mat.
21	Pulp. paper and paper products
22	Printed matter and recorded media
23	Coke. refined petroleum products and nuclear fuels
24	Chemicals. chemical products and man-made fibres
25	Rubber and plastic products
26	Other non-metallic mineral products
27	Basic metals
28	Fabricated metal products except machinery and equipment
29	Machinery and equipment n.e.c.
30	Office machinery and computers
31	Electrical machinery and apparatus n.e.c.
32	Radio. television and communication equipment and apparatus
33	Medical. precision and optical instruments. watches and clocks
34	Motor vehicles. trailers and semi-trailers
35	Other transport equipment
36	Furniture; other manufactured goods n.e.c.
37	Secondary raw materials
40	Electrical energy. gas. steam and hot water
41	Collected and purified water. distribution services of water
45	Construction work
50	Trade, maint., repair of motor vehic. and motorcyc.; retail sale of auto. fuel
51	Wholesale trade and trade services. except of motor vehicles and motorcycles
52	Retail trade except motor vehic.&motorcyc.; repair.Pers.& household goods
55	Hotel and restaurant services
60	Land transport; transport via pipeline services
61	Water transport services
62	Air transport services
63	Suppouting and auxiliary transport services; travel agency services
64	Post and telecommunication services
65	Financial interm.. except insurance and pension funding services
66	Insurance & pension funding. except compulsory social .sec.
67	Services auxiliary to financial intermediation
70	Real estate services
71	Renting machin.&equip. without operator and of personal and househ. goods
72	Computer and related services
73	Research and development services
74	Other business services
75	Public administration and defence services; compulsory social security serv.
80	Education services
85	Health and social work services
90	Sewage and refuse disposal services. sanitation and similar services
91	Membership organization services n.e.c.
92	Recreational. cultural and sporting services
93	Other services
95	Private households with employed persons

APPENDIX 2

Figure 1 - Non-symmetric table (Use table)

	59 Industries (A60)	Total uses of interm. &primary inputs	C	G	NPISH	GFCF	VE	ACOV	Exports	Total uses
59 Products (P60)	Intermediate inputs (1st quadrant)		Final Uses (2nd quadrant)							
Compensation of employees	Primary inputs (GVA components) (3rd quadrant)									
Other taxes on production										
Other subsidies on production (-)										
Consumption of fixed capital										
Net Operating Surplus										
Industry Output										

Figure 2 - Symmetric input-output table

	59 homogeneous branches (P60)	Total uses of interm. &primary inputs	C	G	NPISH	GFCF	VE	ACOV	Exports	Total uses
59 Products (P60)	Intermediate inputs (1st quadrant)		Final Uses (2nd quadrant)							
Compensation of employees	Primary inputs (GVA components) (3rd quadrant)									
Other taxes on production										
Other subsidies on production (-)										
Consumption of fixed capital										
Net Operating Surplus										
Product Output										