Deconstructing Pyatt's Apportioning Procedure. A real case scenario: the official Andalusian SAM.(MCSAN-05)

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Abstract

The goal of this paper is to tackle the multipliers analysis with a Social Accounting Matrix (SAM) that has the official SNA format. To that end, we have tried to use the usually applied apportioning procedure proposed by Pyatt (1985) on the Andalusian Official SAM (MCSAN-05). We have concluded that the conversion of the SNA format to the common formats used by SAMs and the Apportioning procedure are not necessary, and also that the results reached this way are not economically sound. We propose the use of a simple extended formalization of the SAM multipliers method, well adapted to the official format.

Keywords: Social Accounting Matrix, Impact Analysis, Apportioning procedure, multipliers analysis, Regional Accounts.

1. Introduction

Social Accounting Matrices are the consequence of the birth and evolution of Social Accounting. SAMs can be defined as the presentation in matrix form of the accounting systems comprising all the transactions, real or imputed, that take place in a national or regional economy (Moniche, 2003). Thus, a Social Accounting Matrix is a basic statistics information tool. In the set of structural statistics, its elaboration represents one of the biggest challenges for the Official Statistics Institutes. Its elaboration requires a thorough job that exceeds the resources with which a researcher usually counts. It offers very detailed information on the relationship among the sectors and accounts, showing the circular income flow in an economy for a given period of time.

In a Social Accounting Matrix each account is represented by a row and a column. By convention, the sources are indicated by rows and the destinations are indicated by columns, if the monetary and not the real flow are considered. This arrangement allows the identification of each transaction with only one entry, and its nature can be deduced by its position in the matrix. At the same time, each entry corresponds to a matrix (or vector) that can be described in detail, disjoint by products, activity branches, institutional sectors, qualification and gender of the workers, etc…., as required in any circumstance.

SAMs have multiple known advantages, among which we would highlight mainly two of them (King 1985):

1. With a SAM, a great amount of socioeconomic information of the reality under study (national or regional) is available, and it is arranged in a coherent way with a high descriptive power. As King says (King 1981), it integrates and relates the macro magnitudes in flows and when they are disjoint give information on the characteristics of the economic agents involved in the process of the circular income flow. Its use is particularly interesting to carry out analysis that allow relating the distribution of income and the qualification of employment, or the consumption vectors and the main income source of the households, for example. It also allows evaluating the degree of economic dependency of a territory on the exterior.

2. On the other hand (King 1985) it provides a statistical foundation for the elaboration of models, among them the Applied General Equilibrium Models (AGEM), which allows examining the consequences of the adoption of certain economic and social policies at a regional or national level.

The socio-demographic characteristics that can be included in SAMs sometimes become the object of specific studies since, when integrated in the matrix, they can provide much more detailed information on the prominence of the diverse agents, both as the income drivers or recipients, in the economic reality under study.

SAMs can be considered as a very valuable tool for the responsible of economic policies since they allow a deep understanding of what happens and the way it happens in an economy, giving answers to questions such as how to influence in the income allocation of those agents or which role have some entries or agents in the circulation flow, facilitating the analysis of the effects of given public policies.

SAMs were first introduced in “A Programme Growth for Britain 1960” (R. Stone and A. Brown, 1962). Its elaboration was a big project for the time, being an irrefutable proof of the great capacity of Stone and his team as Social Accountants. Pyatt was a member of this team and he can be considered also a pioneer and leader in the development of SAMs. The main goal of the first SAM was to serve as a starting point for the development of a model of economic growth.

R. Stone and Brown use them in their extended format in their studies and incorporate them in the 1969 System of National Accounts (SNA, 1968) as an alternative way of presenting the National Accounts, with the understanding that this other format facilitates the intuitive explanation of the accounts. It serves a pedagogic purpose.

From these works some other studies follow in the seventies (Thorbecke,1985): Pyatt et al (1972), Pyatt et al (1977), Thorbecker and Sengupta (1972) Adelman and Robinson (1978), Ahluwalia and Lysy (1979) Taylor et al (1980). Many of them are carried out from the World Bank, where the use of SAMs has been promoted to a large extent. In these works the commonly used format was standardized, and it differs from the one utilized by Stone and his team. These differences are mainly that they present less breakdowns in the accounts of production and income than the one used by Stone. This reduced format was used for studies on developing countries with precarious statistical systems and it has prevailed in the scientific community, being the one used in the majority of the SAM models, both lineal ones and AGEMs. Its structure will be reflected later in the work carried out by Adelman and Robinson (1978).

We will call this format a reduced SAM and it presents the structure that can be seen in Table 1.

SAMs have been used to analyze the economy from a structural point of view (Ferri and Uriel 2000; Polo and Valle, 2007), or to analyze via multipliers: the distribution of income or poverty (Kahn 1999; Bottiroli and Tragetti (1988); Curbello 1988; Llop and Manresa 2004; Ferri and Uriel 2000; Polo, Roland-Host and Sancho 1990; Pyatt and Round 1984 and 2006; Tarp et al. 2002) the comparison of economic structures of different countries, regions or through time (Cohen 2002; Battellini, Coli and Tartamella 2000; Cohen 1989; Tarp et al. 2002) public debt (Santos 2004), Environment (Xie 2000; Reiner and Roland-Host 2001; F. Miguel-Vélez et al 2009), sectoral policies like agriculture (Adelman and Robinson 1998; Rolnd-Host 1990; Rocchi, Romano and Stefani 2005) or tourism (Wagner 1997; Polo and Valle 2009; Akkemik 2012; Oosterhaven and Fan 2006).

Opposite to the reduced SAMs, the official SAMs have to follow the recommendations of the SNA (System of National Accounts). In the SNA SAMs the production account generally used is split into two different ones: goods and services and production, and the sectors account is split into at least four income accounts. Other disaggregation levels are also contemplated for other accounts like the capital and rest of the World. The structure of the SNA SAMs can be observed in Table 2. From now on, we will refer to the SAMs either as reduced SAMs or as SNA SAMs, respectively.

SNA SAMs offer more information than reduced SAMs. In previous works sometimes an extended SAM has been reduced and then used in a model. We think it is interesting to question whether if reducing the size of the matrix the results of the models used continue being sound.

Our goal in this paper is to study Pyatt's apportioning method and analyze the differences with the use of the SNA SAM. Our study is specifically located in the framework of lineal models. We will work with the first official SAM in Spain, elaborated for a region, Andalusia, for the year 2005.

There are three alternatives to reduce the size of a SAM while retaining its property of complete articulation, following Pyatt (2001): aggregation, consolidation and apportionment. The first of them, aggregation, is adequate to combine sub-accounts, but not complete accounts, since that would imply the duplication of concepts. The second one corresponds to the summary of the information into some of the accounts done by accounting consolidation of some of the accounts and the third one is Pyatt's apportioning method. Pyatt considers his method an alternative to the consolidation of the matrices and says that it "formalizes the common accounting practice of apportioning elements of costs (i.e. expenditures) to other accounts" (Pyatt 1985, p. 161).

The accounting consolidation seems like an intractable task due to the fact that it means to re-make the accounts to consolidate.

In that sense, Pyatt's method is much more useful, but, at the same time, in our view, it is too automatic, and for that reason we have been wondering about the economical sense of the outcomes y and its consequences on the multipliers of lineal models.

The rest of the paper is structured in the following way: section 2 is dedicated to the introduction of the Andalusian SAM (MCSAN-05), section 3 presents the reduced MCSAN-05 with the apportioning procedure while section 4 pictures the effects on the multipliers and section 5 draws some conclusions and further research actions.

2 . Structure of the MCSAN-05

The SAM for Andalusia 2005 (MCSAN 2005) (Statistical and cartographical Institute for Andalusia, 2011) is the official Regional SAM for Andalusia. Being a regional matrix, the estimation of flows with the rest of the world is complex in aspects such as the property income, current transfers and the regionalization of the public administration and enterprises flows.

It was elaborated for the year 2005 by the authors of this paper, in collaboration with the Statistical Institute for Andalusia. The matrix is a complete system of accounts, following the methodology of the current European System of Accounts (ESA95) and SNA93.

The structure of the table can be seen in the schematic matrix in table 3. Each entry has been labeled with an identifier that begins with letter "T" followed by a pair of numbers that indicate the situation in the matrix: the first digit indicates the row and the second one the column. The accounting balances are in capital and bold letters. As it can be seen in table 3, it only comprises 9 accounts, in contrast with the reduced format proposed by the ESA. This is due to the fact that the accounts of capital and rest of the world have been consolidated. A screen account for property income has been added. This account gathers the resources and destinations of the property income by institutional sectors and applies those incomes to the account of allocation of primary

income. It is, therefore, an account that includes flows that actually belong to the account of allocation of primary income, and that are contemplated apart so that they do not need to present the same breakdowns as that account. Since the SAM Is regional, the estimation of flows is very complex and we have had to resort to this reduction of information procedure.

For the homogeneous and non-homogeneous activities we have included 37 branches for the first two accounts.

The operating statement has breakdowns according to the educational level and gender of the wage earners and to the gender of the receivers of mixed revenues. The generated income has been sub-divided in thirteen categories, as it can be seen in Table 3.

There are 6 institutional sectors, with three categories for households, and also a differentiation of the companies in financial and non-financial companies. The approach used to segment the household sector in the MCSAN-05 is the main source of income, as recommended by the National Accounts system (SNA93). They can be seen in Table 4.

3. The Reduced MCSAN-05 with Pyatt's apportioning method.

To reduce the size of a SAM Pyatt's apportioning method has been applied quite generally (Pyatt 2001). Apportioning is a technique that originated with Leontief (1967) with the name of double inversion and used it to reduce the size of an I.-O table. It was later developed by Pyatt (1985) and applied to SAMs. When it was introduced, this method formalized the common accounting practice of apportioning elements of costs to other accounts.

The SAM is a matrix with m+n rows and columns, and can be partitioned into a set of m accounts which are to be retained, and n accounts to be eliminated by apportionment. Without loss of generality, the accounts are ordered so that those to be retained are leading. At this point, matrices A_{jk} corresponding to the matrices T_{jk} are defined by the relationship:

$$A_{jk} = T_{jk} \cdot \hat{y}_k^{-1}$$

Where $\hat{y}_k$ is a diagonal matrix formed from the vector y_k. This implies that elements of A_{jk} are proportions: an element of A_{jk} is given by the corresponding element of T_{jk} expressed as a proportion of the aggregate of all elements in the same column of the SAM.

With this notation, the following result describes the reduced SAM which can be obtained by apportionment:

If $B = A_{12}(I - A_{22})^{-1}$ exits[1], then there exists a reduced form SAM, T_{11}* such that $T_{11}^{*} = T_{11} + B * T_{21}$, where:

i) The row (and column) totals of T_{11}* are given by y_1 (and y_1') , and

ii) The column sums of B are all unity.

In Table 6 the MCSAN-05 is presented with the acronyms that will be used. It has 9 accounts, as already mentioned and presented in table 2. We will not use the conventional numeration and goods and services will be named C.1 instead of C.0., in order to facilitate the identification of the location of the sub-matrices.

The meaning of the acronyms for the MCSAN-05 can be seen in Annex I. The notation is as close as possible to the one used by Adelman and Robinson[2].

To proceed with the apportioning method we first rearrange the rows and columns in order to place the accounts that have to be eliminated ant the end. We will retain 5 accounts: Goods and Services C.1 (homogeneous activity branches), the operating statement C.3, income distribution C. 7. Capital C.8. and Rest of the World C.9.

The goal is to summarize the information provided in the 9 accounts in these 5 accounts. To this end, there must be a re-assignation. The accounts that have to be eliminated are C.2[3] (Production), C.4 property income, C.5. Allocation of primary income and C.6. Secondary (re)distribution of income (in kind).

The new matrix, once the rows and columns have been rearranged can be seen in Table 7.

With this new order of rows and columns, a partition takes place and the notation is simplified following Pyatt (1985). The matrix T_{11} comprises the accounts to be retained after the reduction process of the size of the SAM. See Table 8.

Two more sub-indexes will be used to name the sub-matrices situated into these four principal sub-matrices. For example, T_{1113} is the matrix that, belonging to principal sub-matrix T_{11} (the first two sub-indexes of that matrix) is situated in row 1 and column 3. Matrix T_{2145} is the matrix that, belonging to principal sub-matrix T_{21} is situated in row 4 and column 5. They will be called sub-matrices T_{ijkl}, where sub-indexes ij belong to the corresponding principal sub-matrix, sub-index k to the row and sub-index l to the column inside principal sub-matrix T_{ij}. See Table 9.

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[1] Where the inverse of a partitioned matrix is computed in the following way: Let A be a partitioned matrix in the following way:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

The inverse of A is then:

$$A^{-1} = \begin{bmatrix} F_1 & -F_1 A_{12} A_{22}^{-1} \\ -A_{22}^{-1} A_{21} F_1 & A_{22}^{-1}\left(I + A_{21} F_1 A_{12} A_{22}^{-1}\right) \end{bmatrix}$$

Where:

$$F_1 = \left(A_{11} - A_{12} A_{22}^{-1} A_{21}\right)^{-1}$$

[2] Note that we follow the conventional notation where matrices are in capital letters, vectors in small italics and scalars in small size.

[3] Account C.1 is of products since the symmetric Andalusian matrix contemplates homogeneous branches (goods and services). Although in the literature the commonly used matrices contemplate activity branches, in Spain they are made with products (homogeneous branches).

In the totals with this new numeration we obtain the y_{jl} where j is the sub-index corresponding to the column of the T_{ij} and l is the column corresponding to the position that the sub-matrix has in its principal sub-matrix.

The correspondences of the T_{ijkl} with the matrices previously defined can be consulted in Annex II.

The corresponding coefficients would be computed:

$$A_{ijkl} = T_{ijkl} \cdot \hat{y}_{jl}^{-1}$$

The coefficients matrix can also be expressed as a partitioned matrix with its principal sub-matrices as it can be seen in Table 10.

And the coefficients sub-matrices can also be denoted with the aforementioned notation, as can be seen in Table 11.

The correspondences of the A_{ijkl} with the matrices previously defined can also be consulted in Annex II.

We will subsequently analyze the results obtained on the reduced matrix and the multipliers.

With the procedure explained in section 4 we have obtained matrix $T_{11}^{*} = T_{11} + B * T_{21}$.

To observe in detail the operations and matrices T_{ijkl} y A_{ijkl} that are involved in each result (each submatrix T_{ijkl}*), see Table 12. It will facilitate the analysis of the results from a conceptual point of view.

In table 12 it can be observed that accounts T_{1113}*, T_{1114}*,T_{1115}*, T_{1125}*, T_{1133}*, T_{1143}*, T_{1145}* and T_{1154}* have not been affected by the apportioning process, so their values coincide with the economic concepts that they represent given their position in the matrix.

On the other hand, there are matrices that, in spite of having suffered a transformation, present the right numerical value given the concept and position that they have in the matrix. Those are: T_{1111}*, T_{1121}* and T_{1141}*.

In account T_{1111}*, that represents the intermediate consumption, $\boldsymbol{T_{1111}^{*} = T_{1111} + (A_{1211} * T_{2111})}$, an apportioning has taken place that consists in adding to the transportation and commercial margins the intermediate consumption distributed using goods and services as a criterion and using to that end the production matrix (the origin matrix on the I-O frame).This distribution has economic meaning since the results indicate that the intermediate consumption is distributed among the activity branches in the same proportion that those branches produce goods and services.

In account T_{1121}*, that represents the net value added, $\boldsymbol{T_{1121}^{*} = A_{1221} * T_{2111}}$ and account T_{1141}*, that represents the consumption of fixed capital $\boldsymbol{T_{1141}^{*} = A_{1241} * T_{2111}}$ an apportioning has taken place that consists in the distribution of the value added and the consumption of fixed capital, respectively, from activity branches to goods and services using to that end the production matrix (the origin matrix on the I-O frame).As in the previous case, that distribution has economic meaning for the same reason given there.

There are 6 accounts with results that do not agree with the expected values given their position in the resulting matrix.

The clearest examples are accounts T_{1135}* and T_{1155}*. In these two sub-matrices sub-matrix T_{2145}, (*tffr, (vector of current transfers from the RoW)* is distributed. This concept should go entirely in the account T_{1135}*, since it represents a source of income for the institutional sectors from the Rest of the World. This causes that the cross of the account Rest of the world with Rest

of the World (T_{1155}*), which is a flow that is not in the Spanish National Accounting and does not have economical sense, has a value of 2634 thousands of Euros. This proofs that the entries that have not been incorporated in other accounts are included in this position to balance the matrix. This amount has been distributed with the accounts current transfers among resident institutional sectors and current transfers paid to the Rest of the World, as it can be seen in the formula:

T_{1155}*= A_{1254}*(I-A_{2244})-1*T_{2145}

If the amounts in T_{1135}* and T_{1155}* are added, the result is 14.988, that coincides with the value of current transfers to the RoW in the MCSAN-05.

For the cases that follow we observe again the same situation than in the previous cases: a flow that should be entirely imputed to account 7 (use of disposable income) is applied also to the Rest of the World, in a proportion difficult to explain since there are some accounts that appear many times without a economical sense for the computations (property income is used to distribute taxes).

In the account T_{1151}* only should appear imports of goods and services, with a total of 63.036, while it can be observed that the amount is bigger, 65.336, since part of *taxes less subsidies on products* has been included (more precisely, 2.300). If this amount is added to 11.371, the value of T_{1131}*, the result is 13.671, which is the value that appears in T(5,1) in the MCSAN-05 and that should be entirely in the entry T_{1131}*.

In the account T_{1152}* only should appear compensation of employees to the RoW, with a total of 1.255, while it can be observed that the amount is bigger, 10.268, since part of *net generated income* has been included (more precisely, 9.013). If this amount is added to 86.803, the value of T_{1132}*, the result is 95.816, which is the value that appears in T(5,3) in the MCSAN-05 and that should be entirely in the entry T_{1132}*.

Note that if we observe the matrices used for that distribution:

$$(A_{1252}*-(I-A_{2223}*A_{2232})^{-1}*(-A_{2223})+A_{1253}*(I-A_{2232}*(I-A_{2223}*A_{2232})^{-1}*(-A_{2223}))+A_{1254}*(-(I-A_{2244})^{-1})*(-A_{2243})*(I-A_{2232}*(I-A_{2223}*A_{2232})^{-1}*(-A_{2223})))$$

The matrix A_{2223} = *Apip (the coefficients vector of property income payment)* appears 6 times and the matrix A_{2232} = *Apir (the coefficients vector of property income receiver)* appears 4 times. So, part of *taxes less subsidies on products* and of *net generated income* are assigned with this information, in a quite obscure way, participating the matrices so many times and in so many different ways. It is impossible to understand the underlying economic sense of this distribution.

We have to add that with this apportioning procedure, in comparison to the consolidation, there is a loss of information on concepts as: T_{1252} = pitr *(property income to the RoW (scalar)); T_{1253} = tptr (vector of taxes less subsidies on production to the RoW); T_{1254} = tftr (vector of current transfers to the RoW) y T_{2244} = TF (matrix of current transfers),* which are accounts that do not appear in the reduced matrix (table 13). This fact can be observed comparing tables 1 and 12.

Being an automatic procedure in which the only condition imposed is that the row and column totals must be equal, there are some rigidities that prevent from having a more economically sound solution in many cases.

In view of the results, it can be understood the reason why until this moment there haven't been objections to the method proposed by Pyatt to reduce the size of a SAM: when only accounts 1 and 2 are reassigned, the procedure has full economic sense because the distribution is carried

out with the account of production, the origin table, which makes sense for the allocation of the intermediate consumption and the rest of balances. This application has been carried out by some authors. It is also true that other applications included the reduction of the size of more complex SAMs and did not realize the drawbacks of the procedure (Isla et al., 2002).

Until now not many official SAMs where available, but now that the standard is introduced it is natural to assume that the ones to be produced from now on will respect this format. This is the case of the SAMs for Canada (Siddiqi, Y. and Salem, M. 2012), Portugal 1997, 1998 and 1999 (Santos, S. 2007), Sudafrica (Statistics South Africa 2010), UK (Stuttard, N. and Frogner, M. 2003), Netherland (Timmerman, J.G. and Van de Ven, P. J. M. 1994; P. van de Ven et al. 1999) and Italy (Battellini et al 2003). Many of these SAMs have been elaborated under the Leadership Group on Social Accounting Matrices (LEG - Leadership Group on SAM, 2003).

It is plausible that the multipliers are also affected by the reduction of the size of the SAM. To study this issue, we will introduce in the next section the comparison of the multipliers with a SAM and a reduced SAM with the MCSAN-05 as a case study.

4. Effects on the multipliers

4.1. Multipliers models for SAMs and reduced SAM

Following Stone (1985) the SAM multipliers have their origin in the Leontief inverse multipliers and in Goodwin (1949) and other developments of the type II multipliers (See Morillas 1983). The first application of the multipliers, even if only partially, in a SAM was Copeland and Henry (1975) although the most popular and well known application can be seen in Pyatt et al (1977), and its procedure is the one that is now the reference for this kind of works.

SAM multipliers models are useful tools to know the direct, indirect and induced effects caused by a shock in an exogenous variable[4]. In these models, the reaction to changes in the exogenous demand is reflected in modifications in offer and demand to balance the endogenous accounts.

It is necessary to determine case to case which accounts will be considered exogenous. In practice, these accounts are Public sector, Capital and Rest of the world.

These multipliers have been used to analyze the impact of the changes in an investment, in the public sector expenditures or in the development strategies over the income distribution.

The multipliers model is essentially a lineal model that uses a SAM to study the effects on the income distribution of different shocks. It can be represented like in Aldelman, I and Robinson, S. (1986), with a partitioned SAM, with some columns specified as exogenous and some rows excluded:

[4] The model assumes infinitely Price-elastic supply and completely price-inelastic demand. (Fan and Oosterhaven, 2005)

$$A^* = \begin{bmatrix} A & 0 & C \\ V & 0 & 0 \\ 0 & Y & T \end{bmatrix}$$

A*= matrix of SAM coefficients (n+m+k,n+m+k)

A = matrix of input-output coefficients (n,n)

V = matrix of value added coefficients (m,n)

Y = matrix of income distribution coefficients (k, n)

C = matrix of expenditure coefficients (n,k)

T = matrix of inter-institutional transfer coefficients (k,k)

n= number of sectors

m= number of value added categories

k= number of endogenous institutions

Given the choice of exogenous accounts, the balance equations can be written:

$$\begin{bmatrix} x \\ v \\ y \end{bmatrix} = A^* \begin{bmatrix} x \\ v \\ y \end{bmatrix} + \begin{bmatrix} e^x \\ e^v \\ e^y \end{bmatrix};$$

x= vector of sectoral supply (n,1)

v= vector of value added by categories (m,1)

y= vector of institutional incomes (k,1)

e^x= vector of exogenous sectoral demand (n,1)

e^v= vector of exogenous value added (m,1)

e^y= vector of exogenous institutional incomes (k,1)

Inverting A*, we can write the multiplier matrix equation relating changes in sectoral supply, value added, and institutional income to changes in the exogenous variables:

$$\begin{bmatrix} x \\ v \\ y \end{bmatrix} = M \begin{bmatrix} e^x \\ e^v \\ e^y \end{bmatrix};$$

where $M = (I - A^*)^{-1}$

The outputs show the results of an exogenous shock on production *x*, factors income, *v*, and institutional sectors income, *y*.

4.2. Multipliers model for a SNA SAM.

If the accounts of the SNA are used, Adelman and Robinson's model used for reduced SAMs need to be extended. The extension consists in a model that contains more flows and accounting equalities, just like the SAM multipliers model is an expansion of Leontief´s model. The extension does not mean a loss of clarity but a gain in richness and detail. In this model, we can define the matrix T* as follows:

The SAM would be represented by matrix T. The meaning of the acronyms can be seen in Annex I.

If, for example, the Rest of the world (last row and column) is considered exogenous, the resulting model would be:

The meaning of the acronyms can also be consulted in Annex I.

So that:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix} = A^* \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix} + \begin{bmatrix} e^{y_1} \\ e^{y_2} \\ e^{y_3} \\ e^{y_4} \\ e^{y_5} \\ e^{y_6} \\ e^{y_7} \\ e^{y_8} \end{bmatrix}$$

If matrix A* is inverted, the multipliers matrix is obtained. They determine the equations that explain, with the structure given in the initial matrix, the relative changes in the:

- Y_1: Production of Good and services (by products) value at purchaser's prices.
- Y_2: Domestic production by industries value at basic prices.
- Y_3: Generation of income by primary input categories.
- Y_4: Property of income
- Y_5: Allocation of primary income by institutional sectors.
- Y_6: Secondary (re)distribution of income (in kind) by institutional sectors.
- Y_7: Disposable income (institutional sectors)
- Y_8: Capital

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix} = M \begin{bmatrix} e^{y_1} \\ e^{y_2} \\ e^{y_3} \\ e^{y_4} \\ e^{y_5} \\ e^{y_6} \\ e^{y_7} \\ e^{y_8} \end{bmatrix}$$

where $M = (I - A^*)^{-1}$

Here we present a theoretical comparison of the multipliers obtained with the reduced SAM and the SNA SAM:

Vector y_1 or y_2 (depending on the choice made when considering the production disjoint in activity branches or in goods) would be the equivalent to x= vector of sectoral supply (n,1).

Vector y3 would be the equivalent to v= vector of value added by categories (m,1)

And finally, y7 would be the equivalent to y= vector of institutional incomes (k,1)

Now we present the results of both multipliers for the MCSAN-05, in Table 14.

As in Leontief multipliers, the economical meaning of element m_{ij} of matrix M is that an exogenous unitarian shock in branch j (which can be interpreted either as the corresponding factor or the corresponding income account), produces an increase of m_{ij} in branch i. For example the element m24,5=0.5902 means that an increase in the hostel services demand (24) of an Euro implies an increase in the production of food, beverages and tobacco of 0,5902 Euros, (valued at acquisition price).

We will now compare the multipliers resulting with this procedure (SNA SAM multipliers) and the multipliers obtained with the reduced SNA (with Pyatt's method), for the MCSAN-05, considering in both cases the external sector (rest of the world) as exogenous.

Due to space limitations we cannot present all the results. We have considered the multipliers resulting for an exogenous shock in demand exclusively for the four principal sectors in the Andalusian Economy regarding their position in the ranking of sectors attending to the total production in Andalusia: (20) Construction, (5) Food, beverages and tobacco industry, (28) Real State and renting services (24) Hotel and restaurant services. We will highlight the effects of this shock on these sectors and on the factors income, institutional sectors income and capital. We can point out some effects looking at Table 13.

If we apply an impulse in demand of 10.000 € in sector (20) Construction, which is the one with a higher production in Andalusia, we will see:

- An increase in the total Andalusian production (acquisition prices) of the Food, beverage and tobacco products of 2.702 Euros (m20,5=0.2702), when using a SNA SAM. If we use the apportioning method instead, this shock will cause an increase in the Andalusian production of the Food, beverage and tobacco products of 1.355 Euros (mp20,5=0.1355). With this last approximation, we would have underestimated this production in 1.347 Euros (50%).

- In general, the total Andalusian production (acquisition prices) would increase in 52.651 Euros, being $\sum_{j=1}^{37} m_{j,20} = 5.2654$, when using the SNA SAM. On the other hand, using the apportioning method, the multiplier with a reduced SAM would show an increase in production of 42.807 Euros. With this last approximation, we would have underestimated this production approximately by 19%.

- This shock would also affect the factors income. It would increase the salaries of men with primary education, illiterate or without studies in 1.121 Euros, when using the SNA SAM. Using the apportioning method, the increase would be of 1.147 Euros (2% higher).

- In general, the factors income would increase in 15.346 Euros when using the SNA SAM. Using the reduced SAM this increase would be of 15.400 Euros (1% lower). The difference is not very high in this case, although we can observe much higher divergences in the case of other sectors, like food, beverage and Tobacco, with a 23% of discrepancy.

- The household's disposable income is also modified by this initial shock in Construction. For example, Household 3 increases its disposable income in 2.675 Euros with the SNA SAM. Using the reduced SAM this increase is of 2.565 Euros, 4% lower. In the global flow of disposable income perceived by the institutional sectors there is an increase of 15.463 Euros as a consequence of the shock in the SNA SAM. This increase is of 15.200 for the reduced SAM, 2% lower. The difference is not very high in this case, although we can observe much higher divergences in the case of other sectors, like food, beverage and Tobacco, with a 18% of higher increase.

We can observe then how these divergences appear in every multiplier. Some of them are remarkable and others are only small differences. We want to underline the fact that the sectors chosen are not necessarily those with greater disagreement but those more representative of the Andalusian economy.

There are also sectors that are involved in the valuation differences like (22) Wholesale trade and commission trade, except of motor vehicles and motorcycles and (23) Retail trade, except of motor vehicles and motorcycles, repair of personal and household goods, which are relative to commercial margins and are the ones that show a greater divergence. For example, a shock in the hotel and restaurant services would have a multiplier in (23) a 3017% higher in the reduced SAM.

The differences are due to the fact that the data in the reduced SAM have lost the economic sense they should have in the accounting equalities in a SAM, as already mentioned in section 3.

In table 13, other effects can also be seen. In summary, the multipliers are different using the SNA SAM and the reduced SAM. This is against Pyatt's claim in (Pyatt 2001, p. 151): "Moreover, this can be shown to be a general result: the elimination of any set of accounts by apportionment does not change the accounting multipliers that are implied by the consequent SAM for those accounts that remain."

This is, in our view, an important aspect to be taken into account since the impact studies or the evaluation models of public policies using reduced SAMs will throw different results and conclusions that those carried out with a SNA SAM.

5. Conclusions

SAMs allow a deep understanding of the structure of an economy and of the influences of public policies in the income allocation of the agents. They constitute very valuable tools for the responsible of economic policies, and this is even more so for the regional SAMs, given the fact that a regional matrix includes a more complex estimation of flows with the rest of the world. The lineal model provides an easy answer to the effects of any stimulus under consideration. We have proposed a lineal model with straightforward and intuitive results, once the initial matrix is known. In this paper we have proposed the application and interpretation of the multipliers for the MCSAN-05.

SAMs have evolved with time, according to the evolution of the National Accounting. It is also worth noting that in the development of SAMs it has had a great impact the improvement of the statistical tools and agencies in providing more detailed information of the transactions in an Economy, resulting in extended SAMs, which enrich the analysis of the reality. This fact has to be taken into account when considering the effects of the reduction of the size of a SAM with Pyatt's apportioning method.

The apportioning procedure has economic meaning when the account de eliminated is the one of Goods and Services. If there are more accounts to be eliminated, some results have economic meaning like for example the intermediate consumption, where an apportioning has taken place that consists in adding to the transportation and commercial margins the intermediate consumption distributed using goods and services as a criterion and using to that end the production matrix (the origin matrix on the I-O frame).

On the other hand, we observe two other different effects of the apportioning procedure: first, for some accounts the economic flows are misplaced in the resulting matrix. For example, a

flow that should be entirely imputed to account 7 (use of disposable income) is applied also to the Rest of the World, in a proportion difficult to explain since there are some accounts that appear many times without a economical sense for the computations (property income is used to distribute taxes). Also, the cross of the account Rest of the world with Rest of the World in the reduced SAM (T_{1155}*), which is a flow that is not in the Spanish National Accounting and does not have economical sense, has a value of 2634 thousands of Euros. Second: with the apportioning procedure, in comparison to the consolidation, there is a loss of information on concepts as: T1252 = pitr (property income to the RoW (scalar)); T1253 = tptr (vector of taxes less subsidies on production to the RoW); T1254 = tftr (vector of current transfers to the RoW) y T2244 = TF (matrix of current transfers), which are accounts that are displayed as blank cells in the reduced matrix.

When comparing SNA SAM multipliers with the multipliers obtained with the reduced SNA (with Pyatt's method), for the MCSAN-05, considering in both cases the external sector (rest of the world) as exogenous we observe big differences, for example, a shock in the hotel and restaurant services would have a multiplier in (23) a 3017% higher in the reduced SAM, which is against Pyatt's claim that the elimination of any set of accounts by apportionment does not change the accounting multipliers that are implied by the consequent SAM for those accounts that remain. The differences are due to the fact that the data in the reduced SAM have lost the economic sense they should have in the accounting equalities in a SAM.

We conclude that the impact studies or the evaluation models of public policies using reduced SAMs will throw different results and conclusions that those carried out with a SNA SAM.

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Table 1: Reduced SAM

ACCOUNT	PRODUCTION	PRIMARY INPUT CATEGORIES	INSTITUTIONAL SECTORS	CAPITAL	REST OF THE WORLD	TOTAL
PRODUCTION	Intermediate Consumption		Consumption	Invesment	Exports	
PRIMARY INPUT CATEGORIES	Value Added				Compensation of employees from the RoW	
INSTITUTIONAL SECTORS	Tax less subsidies on products	Generated Income	Current transfers		Current transfers from the ROW	
CAPITAL	Consumption of fixed capital		Saving		Net Lending of the ROW	
REST OF THE WORLD	Imports	Compensation of employees to the RoW	Current transfers to the ROW			
TOTAL						

Source: Own elaboration

Table 2. SNA SAM

Table 20.2. Consolidated SNA matrix with sub-accounts

Account	Codes	Goods and services 1	Production 2	Primary distribution of income 3 & 4	Secondary distribution of income 5	Use of income 6	Capital 7	Rest of the world, current and capital 10 &11	*Total*
0. Goods and services	1		Intermediate consumption			Final consumption expenditure	Gross capital formation	Exports of goods and services	
			1 883			1 399	414	540	4 236
I. Production	2	Output and taxes on products less subsidies							
		3 737							3 737
II.1. Primary distribution of income	3 4		*Net domestic product*	Property income				Compensation of employees and property income from ROW 1/	
			1 632	353				69	2 054
II.2. Secondary distribution of income	5			*Net national income*	Current tax on income, wealth etc. and current transfers			Current taxes on income, wealth etc. and current transfers from ROW	
				1 661	1 096			10	2 767
II.4. Use of income	6				*Net disposable income*	Adjustment for the change in net equity of households on pension funds		Adjustment for the change in net equity of households on pension funds from ROW	
					1 632	11		0	1 643
III.1. Capital	7		Consumption of fixed capital			*Net saving*	Capital transfers and acquisitions less disposals of non-produced assets	Capital transfers receivable (+) /payable (–) and acquisitions less disposals of non-produced assets by ROW	
			222			233	61	– 3	513
V.I, V.II and V.III.1. Rest of the world, current and capital	10 11	Imports of goods and services		Compensation of employees and property income to ROW 1/	Current tax on income, wealth etc. and current transfers to ROW	Adjustment for the change in net equity of households on pension funds to ROW	*Net lending (+)/ Net borrowing (–)* of the total economy		
		499		40	39	0	38		616
Total		4 236	3 737	2 054	2 767	1 643	513	616	

1/ Including taxes on production and imports from (cell 3&4, 10&11) and to (cell 10&11 and 3&4) ROW.

Source: SNA1993, page 580

Table 3: SCHEMATIC PRESENTATION OF MCSAN-05 (Millions of euros)

ACCOUNT			1. Good and services (products) 37 1….37	TOTAL ECONOMY							RoW	TOTAL
				2. Production (industries) 37 38…74	3. Generation of income (primary input categories) 12 75…87	4. Property income 1 88	5. Allocation of primary income (institutional sectors) 6 89…94	6. Secondary (re)distribution of income (in kind) (institutional sectors) 6 95…100	7. Use of disposable income (institutional sectors) 6 101…106	8. Capital 1 107	REST OF THE WORLD 1 108	
			C.1	C.2	C.3	C.4	C.5	C.6	C.7	C.8	C.9	
1. Good and services (products) 37 1…37		C.1	Trade and transport margins **T(1,1)** **0**	Intermediate consumption **T(1,2)** **130.976**					Final consumption **T(1,7)** **106.140**	Gross fixed capital formation* **T(1,8)** **41.570**	Exports of goods and services **T(1,9)** **42.423**	
TOTAL	2. Production (industries) 37 38…74	C.2	Output **T(2,1)** **244.403**									
TOTAL	3. Generation of income (primary input categories) 13 75…87	C.3		**NET VALUE ADDED** (basic prices) **T(3,2)** **94.779**							Compensation of employees from the RoW **T(3,9)** **2.293**	
TOTAL	4. Property income 1 88	C.4					Property income payed **T(4,5)** **24.795**					
TOTAL	5. Allocation of primary income (institutional sectors) 6 89…94	C.5	Tax less subsidies on products **T(5,1)** **13.671**		**GENERATED INCOME, NET** (basic prices) **T(5,3)** **95.816**	Property income received **T(5,4)** **19.834**						

Table 3(continues): SCHEMATIC PRESENTATION OF MCSAN-05 (Millions of euros)

	ACCOUNT		1. Good and services (products) 37 1….37	TOTAL ECONOMY							RoW	TOTAL
				2. Production (industries) 37 38…74	3. Generation of income (primary input categories) 13 75…87	4. Property income 1 88	5. Allocation of primary income (institutional sectors) 6 89…94	6. Secondary (re)distribution of income (in kind) (institutional sectors) 6 95…100	7. Use of disposable income (institutional sectors) 6 101…106	8. Capital 1 107	REST OF THE WORLD 1 108	
			C.1	C.2	C.3	C.4	C.5	C.6	C.7	C.8	C.9	
ECONOMY	6. Secondary (re)distribution of income (in kind) (institutional sectors) 6 95…100	C.6					**RENGIONAL INCOME, NET** T(6,5) **105.684**	Current transfers T(6,6) **74.819**			Current transfers from the RoW T(6,9) **14.988**	
	7. Use of disposable income (institutional sectors) 6 101…106	C.7						**(ADJUSTED) DISPOSABLE INCOME, NET** T(7,6) **110.528**	Adjustment for the change in net equity of households on pension funds reserves T(7,7) **32**			
	8. Capital 1 107	C.8		Consumption of fixed capital T(8,2) **18.648**					**SAVING, NET** T(8,7) **4.388**		**NET LENDING OF THE ROW** T(8,9) **16.434**	
RoW	REST OF THE WORLD 1 108	C.9	Imports of goods and services T(9,1) **63.036**		Compensation of employees to the RoW T(9,3) **1.255**	Property income to the RoW T(9,4) **4.961**	Taxes less subsidies on production to the RoW T(9,5) **-1.157**	Current transfers to the RoW T(9,6) **10.143**		Capital transfers to the RoW T(9,8) **-2.100**		
TOTAL												

*Including Changes in inventories

* For clarity in the exposition we will not use the conventional numeration and goods and services will be named C.1 instead of C.0.

Source: Instituto de Estadística y Cartografía de Andalucía (2011) http://www.juntadeandalucia.es/institutodeestadisticaycartografia/mcsan/index.htm

Table 4. Primary input categories in the MCSAN-05

3. Generation of income (primary input categories)	Wages and Salaries	Men primary education, illiterate and without studies
		Men secondary education (first stage) and corresponding training and labor orientation
		Men secondary education (second stage) and training and labor orientation that needs the possession of a diploma of the first or second stage of secondary education.
		Men higher education and Ph.D.
		Women primary education, illiterate and without studies
		Women secondary education (first stage) and corresponding training and labor orientation
		Women secondary education (second stage) and training and labor orientation that needs the possession of a diploma of the first or second stage of secondary education.
		Women higher education and Ph.D.
	Social Contributions	
	Other net taxes on production	
	Entrepreneurial net operating surplus	
	Mixed revenues	Mixed revenues Men
		Mixed revenues Women

Source: Instituto de Estadística y Cartografía de Andalucía (2011)
http://www.juntadeandalucia.es/institutodeestadisticaycartografia/mcsan/index.htm

Table 5. Institutional sectors in the MCSAN-05

1	Household	Household 1 Households with wages as the main source of income (wage earners).
		Household 2 Households where the main source of income is income for self-employment, capital and property income and other regular incomes.
3		Household 3 Households where the main source of income comes from pensioners (retired, disabled, widowhood, etc), subsidies and unemployment benefits, other regular social benefits.
4	Non-Financial Companies	
5	Financial Companies	
6	Public Administration and Non-profit institutions serving households (NPISHs)	

Source: Instituto de Estadística y Cartografía de Andalucía (2011)
http://www.juntadeandalucia.es/institutodeestadisticaycartografia/mcsan/index.htm

Table 6. MCSAN-05

	C.1	C.2	C.3	C.4	C.5	C.6	C.7	C.8	C.9	Total
C.1	TTM	X	0	0	0	0	C	*fc*	*ex*	y_1
C.2	P	0	0	0	0	0	0	0	0	y_2
C.3	0	VA	0	0	0	0	0	0	*cefr*	y_3
C.4	0	0	0	0	*pip*	0	0	0	0	y_4
C.5	TP	0	GI	*pir*	0	0	0	0	0	y_5
C.6	0	0	0	0	NI	TF	0	0	*tffr*	y_6
C.7	0	0	0	0	0	DI	AD	0	0	y_7
C.8	0	*cfc*	0	0	0	0	*s*	0	nlr	y_8
C.9	*im*	0	*cetr*	pitr	*tptr*	*tftr*	0	cttr	0	y_9
Total	Y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	

Source: Own elaboration

Table 7. Rearranged MCSAN-05

	C.1	C.3	C.7	C.8	C.9	C.2	C.4	C.5	C.6
C.1	TTM	0	C	*fc*	*ex*	X	0	0	0
C.3	0	0	0	0	*cefr*	VA	0	0	0
C.7	0	0	AD	0	0	0	0	0	DI
C.8	0	0	*s*	0	nlr	*cfc*	0	0	0
C.9	*im*	*cetr*	0	*cttr*	0	0	pitr	*tptr*	*tftr*
C.2	P	0	0	0	0	0	0	0	0
C.4	0	0	0	0	0	0	0	*pip*	0
C.5	TP	GI	0	0	0	0	*pir*	0	0
C.6	0	0	0	0	*tffr*	0	0	NI	TF

Source: Own elaboration

Table 8. Partitioned matrix

	C.1	C.3	C.7	C.8	C.9	C.2	C.4	C.5	C.6
C.1	T11					T12			
C.3									
C.7									
C.8									
C.9									
C.2	T21					T22			
C.4									
C.5									
C.6									

Source: Own elaboration

Table 9: Sub-matrices in the partitioned matrix.

	C.1	C.3	C.7	C.8	C.9	C.2	C.4	C.5	C.6
C.1	T_{1111}	0	T_{1113}	T_{1114}	T_{1115}	T_{1211}	0	0	0
C.3	0	0	0	0	T_{1125}	T_{1221}	0	0	0
C.7	0	0	T_{1133}	0	0	0	0	0	T_{1234}
C.8	0	0	T_{1143}	0	T_{1145}	T_{1241}	0	0	0
C.9	T_{1151}	T_{1152}	0	T_{1154}	0	0	T_{1252}	T_{1253}	T_{1254}
C.2	T_{2111}	0	0	0	0	0	0	0	0
C.4	0	0	0	0	0	0	0	T_{2223}	0
C.5	T_{2131}	T_{2132}	0	0	0	0	T_{2232}	0	0
C.6	0	0	0	0	T_{2145}	0	0	T_{2243}	T_{2244}
Total	y_{11}	Y_{12}	y_{13}	y_{14}	y_{15}	y_{21}	y_{22}	y_{23}	y_{24}

Source: Own elaboration

Table 10: Coefficients partitioned matrix

	C.1	C.3	C.7	C.8	C.9	C.2	C.4	C.5	C.6
C.1	A11					A12			
C.3									
C.7									
C.8									
C.9									
C.2	A21					A22			
C.4									
C.5									
C.6									

Source: Own elaboration

Table 11: Coefficients sub-matrices in the coefficients partitioned matrix

	C.1	C.3	C.7	C.8	C.9	C.2	C.4	C.5	C.6
C.1	A_{1111}	0	A_{1113}	A_{1114}	A_{1115}	A_{1211}	0	0	0
C.3	0	0	0	0	A_{1125}	A_{1221}	0	0	0
C.7	0	0	A_{1133}	0	0	0	0	0	A_{1234}
C.8	0	0	A_{1143}	0	A_{1145}	A_{1241}	0	0	0
C.9	A_{1151}	A_{1152}	0	A_{1154}	0	0	A_{1252}	A_{1253}	A_{1254}
C.2	A_{2111}	0	0	0	0	0	0	0	0
C.4	0	0	0	0	0	0	0	A_{2223}	0
C.5	A_{2131}	A_{2132}	0	0	0	0	A_{2232}	0	0
C.6	0	0	0	0	A_{2145}	0	0	A_{2243}	A_{2244}

Source: Own elaboration

Table 12: Reduced SAM with Pyatt's (1985) apportioning method.

Account	C.1 Goods and services (37)	C.3 Generation of income (13)	C.7 Used of disposable income (6)	C.8 Capital (1)	C.9 Rest of the world (1)	Total
C.1 Goods and services (37)	***Intermediate consumption 130976*** $T_{1111}^{*} = T_{1111}+(A_{1211}*T_{2111})$	***0***	***Final consumption 106140*** $T_{1113}^{*} = T_{1113}$	***Gross fixed capital formation 41571*** $T_{1114}^{*} = T_{1114}$	***Exports of goods and services 42423*** $T_{1115}^{*} = T_{1115}$	***321110***
C.3 Generation of income (13)	***Net Value Added 94779*** $T_{1121}^{*} = A_{1221}*T_{2111}$	***0***	***0***	***0***	***Compensation of employees from the RoW 2292*** $T_{1125}^{*} = T_{1125}$	***97071***
C. 7 Used of disposable income (6)	***11371*** $T_{1131}^{*} = (A_{1234}*(-(I-A_{2244})^{-1}*-A_{2243}*(I-A_{2232}*(I-A_{2223}*A_{2232})^{-1}*-A_{2223})))*T_{2131}$	***86803*** $T_{1132}^{*} = (A_{1234}*(-(I-A_{2244})^{-1}*-A_{2243}*(I-A_{2232}*(I-A_{2223}*A_{2232})^{-1}*-A_{2223})))^{*} T_{2132}$	***Adjustment for the change in net equity of households on pension funds reserves 32*** $T_{1133}^{*} = T_{1133}$	***0***	***12354*** $T_{1135}^{*} = A_{1234}*(I-A_{2244})^{-1}*T_{2145}$	***110560***
C.8 Capital (1)	***Consumption of fixed capital 18648*** $T_{1141}^{*} = A_{1241}*T_{2111}$	***0***	***SAVING, NET 4388*** $T_{1143}^{*} = T_{1143}$	***0***	***NET LENDING OF THE ROW 16435*** $T_{1145}^{*} = T_{1145}$	***39471***
C. 9 Rest of the world (1)	***65336*** $T_{1151}^{*} = T_{1151}+(A_{1252}*-(I-A_{2223}*A_{2232})^{-1}*(-A_{2223})+A_{1253}*(I-A_{2232}*(I-A_{2223}*A_{2232})^{-1}*(-A_{2223}))+A_{1254}*(-(I-A_{2244})^{-1})*(-A_{2243})*(I-A_{2232}*(I-A_{2223}*A_{2232})^{-1}*(-A_{2223})))*T_{2131}$	***10268*** $T_{1152}^{*} = T_{1152}+(A_{1252}*-(I-A_{2223}*A_{2232})^{-1}*(-A_{2223})+A_{1253}*(I-A_{2232}*(I-A_{2223}*A_{2232})^{-1}*(-A_{2223}))+A_{1254}*(-(I-A_{2244})^{-1})*(-A_{2243})*(I-A_{2232}*(I-A_{2223}*A_{2232})^{-1}*(-A_{2223})))^{*} T_{2132}$	***0***	***Capital transfers to the RoW -2100*** $T_{1154}^{*} = T_{1154}$	***2634*** $T_{1155}^{*}= A_{1254}*(I-A_{2244})-1*T_{2145}$ $A_{1254}*(I-A_{2244})^{-1}*T_{2145}$	***76138***
Total	***321110***	***97071***	***110560***	***39471***	***76138***	

Source: Own elaboration

TABLE 13 Comparison of SAM multipliers for MCSAN-05

MCSAN-05 Andalusia 2005 Thousands of €			SNA SAM MCSAN-05				REDUCED SAM MCSAN-05				DIFFERENCES			
			5 Food, beverages and tobacco	20 Construction	24 Hotels and restaurant services	28 Real State and renting services	5 Food, beverages and tobacco	20 Construction	24 Hotels and restaurant services	28 Real State and renting services	5 Food, beverages and tobacco	20 Construction	24 Hotels and restaurant services	28 Real State and renting services
C.0 GOODS AND SERVICES (PRODUCTS)		Branches 1-4	…	…	…	…	…	…	…	…	…	…	…	…
	5	Food, beverages and tobacco	1,4061	0,2702	0,5902	0,2830	1,3223	0,1355	0,3183	0,1457	-6%	-50%	-46%	-49%
		Branches 6-19	…	…	…	…	…	…	…	…	…	…	…	…
	20	Construction	0,3868	1,8596	0,5093	0,9158	0,4734	1,8508	0,5242	0,9279	22%	0%	3%	1%
	21	Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of automotive fuel	0,0311	0,0415	0,0390	0,0406	0,0575	0,0642	0,0633	0,0641	85%	54%	62%	58%
	22	Wholesale trade and commission trade, except of motor vehicles and motorcycles	0,0177	0,0082	0,0113	0,0081	0,1483	0,0885	0,1281	0,0891	739%	983%	1033%	1007%
	23	Retail trade, except of motor vehicles and motorcycles, repair of personal and household goods	0,0076	0,0036	0,0051	0,0037	0,1670	0,1039	0,1584	0,1106	2104%	2796%	3017%	2882%
	24	Hotels and restaurant services	0,1470	0,1844	1,2032	0,1928	0,1572	0,1647	1,1911	0,1771	7%	-11%	-1%	-8%
		Branches 25-27	…	…	…	…	…	…	…	…	…	…	…	…
	28	Real State and renting services	0,1990	0,2372	0,2883	1,2734	0,2086	0,2137	0,2745	1,2550	5%	-10%	-5%	-1%
		Rest of branches to 37	…	…	…	…	…	…	…	…	…	…	…	…
		Subtotals	4,3192	5,2651	5,1423	5,3321	4,3619	4,2807	4,3606	4,4965	1%	-19%	-15%	-16%

Source: Own elaboration

TABLE 13 (continues): Comparison of SAM multipliers for MCSAN-05

MCSAN-05 Andalusia 2005 Thousands of €			SNA SAM MCSAN-05				REDUCED SAM MCSAN-05				DIFFERENCES			
			5 Food, beverages and tobacco	20 Constructio n	24 Hotels and restaurant services	28 Real State and renting services	5 Food, beverages and tobacco	20 Constructio n	24 Hotels and restaurant services	28 Real State and renting services	5 Food, beverages and tobacco	20 Constructio n	24 Hotels and restaurant services	28 Real State and renting services
C.2 GENERATION OF INCOME (primary input categories)	1	Men primary education, illiterate and without studies	0,0616	0,1121	0,0776	0,0782	0,0825	0,1147	0,0825	0,0792	34%	2%	6%	1%
	2	Men secondary education (first stage) and corresponding training and labor orientation	0,1095	0,1748	0,1509	0,1352	0,1362	0,1780	0,1609	0,1371	24%	2%	7%	1%
	3	Men secondary education (second stage) and training and labor orientation that needs the possession of a diploma of the first or second stage of secondary education.	0,0853	0,1103	0,1222	0,1086	0,1037	0,1108	0,1297	0,1100	22%	0%	6%	1%
	4	Men higher education and Ph.D.	0,1361	0,1726	0,1728	0,1854	0,1643	0,1722	0,1786	0,1880	21%	0%	3%	1%
	5	Women primary education, illiterate and without studies	0,0193	0,0168	0,0315	0,0182	0,0251	0,0166	0,0338	0,0183	30%	-1%	7%	1%
	6	Women secondary education (first stage) and corresponding training and labor orientation	0,0404	0,0339	0,0684	0,0401	0,0473	0,0335	0,0736	0,0406	17%	-1%	7%	1%
	7	Women secondary education (second stage) and training and labor orientation that needs the possession of a diploma of the first or second stage of secondary education.	0,0441	0,0435	0,0705	0,0538	0,0492	0,0428	0,0748	0,0546	12%	-1%	6%	2%
	8	Women higher education and Ph.D.	0,0908	0,1049	0,1284	0,1246	0,1060	0,1035	0,1318	0,1267	17%	-1%	3%	2%
	9	Social Contributions	0,1615	0,2165	0,2259	0,2051	0,1969	0,2176	0,2382	0,2077	22%	1%	5%	1%
	10	Other net taxes on production	0,0037	0,0143	0,0104	0,0526	0,0009	0,0144	0,0109	0,0574	-77%	1%	4%	9%
	11	Entrepreneurial net operating surplus	0,1695	0,2365	0,2108	0,4206	0,2118	0,2389	0,2232	0,4483	25%	1%	6%	7%
	12	Mixed revenues Men	0,2175	0,2470	0,3086	0,2205	0,2837	0,2499	0,3323	0,2246	30%	1%	8%	2%
	13	Mixed revenues Women	0,0657	0,0515	0,1171	0,0626	0,0791	0,0511	0,1270	0,0640	20%	-1%	8%	2%
		Subtotals	1,2052	1,5346	1,6951	1,7055	1,4866	1,5440	1,7973	1,7568	23%	1%	6%	3%
C.6 USE OF DISPOSABLE INCOME (INSTITUTIONAL SECTORS)	1	Household 1	0,6748	0,8660	0,9361	0,8942	0,8083	0,8611	0,9778	0,9112	20%	-1%	4%	2%
	2	Household 2	0,2257	0,2586	0,3244	0,2675	0,2791	0,2576	0,3438	0,2735	24%	0%	6%	2%
	3	Household 3	0,2167	0,2675	0,2908	0,3139	0,2414	0,2565	0,2931	0,3210	11%	-4%	1%	2%
	4	Non-Financial Companies	-0,0109	-0,0147	-0,0139	-0,0234	-0,0132	-0,0147	-0,0145	-0,0247	21%	0%	4%	6%
	5	Financial Companies	0,0152	0,0197	0,0200	0,0267	0,0179	0,0194	0,0206	0,0278	18%	-2%	3%	4%
	6	Public Administration and Non-profit institutions serving households (NPISHs)	0,1208	0,1492	0,1588	0,1692	0,1281	0,1401	0,1559	0,1715	6%	-6%	-2%	1%
		Subtotals	1,2424	1,5463	1,7162	1,6481	1,4616	1,5200	1,7767	1,6804	18%	-2%	4%	2%

Source: Own elaboration

ANNEX I

ACRONYMS FOR THE MCSAN-05	**ACRONYMS FOR THE COEFFICIENTS MATRIX FOR THE MCSAN-05**
TTM = matrix of trade and transport margins	*ATTM = the coefficients matrix of trade and transport margins*
X= matrix of intermediate consumption	*AX= the coefficients matrix of intermediate consumption*
C= matrix of actual final consumption	*AC= the coefficients matrix of actual final consumption*
fc= vector of fixed capital formation and changes in inventories	*afc= the coefficients vector of fixed capital formation and changes in inventories*
ex= vector of exports of goods and services	*AP= the coefficients matrix of Output*
P= matrix of Output	*AVA= the coefficients matrix of net valued added*
VA= matrix of net valued added	*apip = the coefficients vector of property income payment.*
cefr= vector of compensation of employees from the RoW.	*ATP= the coefficients matrix of taxes less subsidies on products.*
pip = vector of property income payment.	*AGI= the coefficients matrix of generated income, net.*
TP= matrix of taxes less subsidies on products.	*apir= the coefficients vector of property income receiver.*
GI= matrix of generated income, net.	*ANI= the coefficients matrix of national income, net.*
pir= vector of property income receiver.	*ATF= the coefficients matrix of current transfers.*
NI= matrix of national income, net.	*ADI= the coefficients matrix of disposable income, net*
TF= matrix of current transfers.	*AAD=the coefficients matrix of adjustment for the change in net equity of households on pension funds reserves*
tffr= vector of current transfers from the RoW.	*acfc= the coefficients vector of consumption of fixed capital*
DI= matrix of disposable income, net	*as= the coefficients vector of saving, net*
AD= matrix of adjustment for the change in net equity of households on pension funds reserves	
cfc= vector of consumption of fixed capital	
s= vector of saving, net	
nlr= *net lending of the RoW (scalar).*	
im= vector of imports of goods and services	
cetr= vector of compensation of employees to the RoW.	
pitr= *property income to the RoW (scalar).*	
tptr= vector of taxes less subsidies on production to the RoW.	
tftr= vector of current transfers to the RoW.	
cttr= *capital transfers to the RoW (scalar).*	
y_j=*total for row j*	

Source: Own elaboration

ANNEX II

ELEMENT OF THE PARTITIONET SAM MATRIX	ELEMENT OF THE PARTITIONET SAM COEFFICIENTS MATRIX
T_{1111} = *TTM (matrix of trade and transport margins)*	A_{1111} = *ATTM (the coeficients matrix of trade and transport margins)*
T_{1113} = *C (matrix of actual final consumption)*	A_{1113} = *AC (the coeficients matrix of actual final consumption)*
T_{1114} = *fc (vector of fixed capital formation and changes in inventories)*	A_{1114} = *afc (the coeficients vector of fixed capital formation and changes in inventories)*
T_{1115} = *ex (vector of exports of goods and services)*	A_{1115} = *aex (the coeficients vector of exports of goods and services)*
T_{1125} = *cefr (vector of compensation of employees from the RoW).*	A_{1125} = *acefr (the coeficients vector of compensation of employees from the RoW).*
T_{1133} = *AD (matrix of adjustment for the change in net equity of households on pension funds reserves)*	A_{1133} = *AAD (the coeficients matrix of adjustment for the change in net equity of households on pension funds reserves)*
T_{1143} = *s (vector of saving, net)*	A_{1143} = *s (the coeficients vector of saving, net)*
T_{1145} = nlr *(net lending of the RoW (scalar)).*	A_{1145} = nlr *(the coeficient of net lending of the RoW (scalar)).*
T_{1151} = *im (vector of imports of goods and services)*	A_{1151} = *im (the coeficients vector of imports of goods and services)*
T_{1152} = *cetr (vector of compensation of employees to the RoW).*	A_{1152} = *cetr (the coeficients vector of compensation of employees to the RoW).*
T_{1154} = cttr *(capital transfers to the RoW (scalar)).*	A_{1154} = cttr *(the coeficient of capital transfers to the RoW (scalar)).*
T_{1211} = *X (matrix of intermediate consumption)*	A_{1211} = *X (the coeficients matrix of intermediate consumption)*
T_{1221}= *VA (matrix of net valued added)*	A_{1221}= *VA (the coeficients matrix of net valued added)*
T_{1234} = *DI (matrix of disposable income, net)*	A_{1234} = *DI (the coeficients matrix of disposable income, net)*
T_{1241} = *cfc (vector of consumption of fixed capital)*	A_{1241} = *cfc (the coeficients vector of consumption of fixed capital)*
T_{1252} = pitr *(property income to the RoW (scalar)).*	A_{1252} = pitr *(the coeficient of property income to the RoW (scalar)).*
T_{1253} = *tptr (vector of taxes less subsidies on production to the RoW).*	A_{1253} = *tptr (the coeficients vector of taxes less subsidies on production to the RoW).*
T_{1254} = *tftr (vector of current transfers to the RoW).*	A_{1254} = *tftr (the coeficients vector of current transfers to the RoW).*
T_{2111}= *P (matrix of Output)*	A_{2111}= *P (the coeficients matrix of Output)*
T_{2131}= *TP (matrix of taxes less subsidies on products).*	A_{2131}= *TP (the coeficients matrix of taxes less subsidies on products).*
T_{2132}= *GI (matrix of generated income, net).*	A_{2132}= *GI (the coeficients matrix of generated income, net).*
T_{2145} = *tffr (vector of current transfers from the RoW).*	A_{2145} = *tffr (the coeficients vector of current transfers from the RoW).*
T_{2223} = *pip (vector of property income payment).*	A_{2223} = *pip (the coeficients vector of property income payment).*
T_{2232} = *pir (vector of property income receiver).*	A_{2232} = *pir (the coeficients vector of property income receiver).*
T_{2243} = *NI (matrix of national income, net).*	A_{2243} = *NI (the coeficients matrix of national income, net).*
T_{2244} = *TF (matrix of current transfers).*	A_{2244} = *TF (the coeficients matrix of current transfers).*

Source: Own elaboration