<u>Hybrid and monetarized input-output tables</u>

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<u>Abstract</u>

Input-Output tables are widely used in several types of analyses. Indeed, although born in an economic context, IOTs are more and more used for environmental impact assessment of product systems, e.g. Life Cycle Assessments, and for several other scenarios, e.g. the accounting of greenhouse gases. But, in order to perform as a multidisciplinary modeling tool, IOTs have to be robust and coherent with some defined requirements in many different levels, e.g. economic, mass, energy and so on.

The paper states that IOTs should reduce as much as possible the loss of information in order to achieve a more reliable analysis of physical variables and to behave as a proper multidisciplinary tool. Yet the use of monetary exchanges may even increase the loss of information, which already occurs when a common aggregation process is carried out to construct IOTs. These limits are the consequence of the fixed prices assumption adopted implicitly when a monetary framework is chosen for the Leontief demand-driven model.

In order to demonstrate such statement the paper focuses on the variability of behaviors within homogenous activity groups and the changeability of average prices. The latter are indeed dependent variables of final demand vectors hence a simple scenario analysis may jeopardize the proportionality between the predicted monetary and physical levels of production, which is strongly required in order to perform multidisciplinary analyses. Thus the monetary framework, including also a mixed-units case, may turn out to be inconsistent with fundamental physical laws, e.g. the Mass Conservation Law.

The paper concludes with some remarks and recommendations to avoid such drawback and to fulfill any desired balance. A numerical example is shown.

key words: Hybrid Input-Output tables, Monetary framework, Mass flow balance, Environmental impact assessment.

<u>1. Introduction</u>

IOTs (Leontief, 1941) have been widely used in an economic context and so were mostly constructed in monetary units, i.e the Monetary IOT (MIOT). In the last decades, when the effects on the environment have become more and more alarming, IOTs have been increasingly applied for environmental impact analyses. But analysts got that the monetary framework may be a limit for specific environmental impact assessments hence IOTs accounted in mass units were built, i.e. the Physical IOTs (PIOTs) (Stahmer et al, 1990; Nebbia, 2000; Mäenpää and Muukkonen, 2001)[1].

Statisticians and analysts have also combined the principles of MIOTs and PIOTs in a unique framework developing what has been defined as a hybrid framework (SEEA 2003). The latter has a really generic meaning. It can refer to a framework either where the MIOT has additional information about the exchanges between technosphere and biosphere, for example the natural resources and the emissions, which are not accountable in monetary units, or where part of the transactions in the economic system are accounted in mass terms. Hybrid IOTs (HIOTs) are used to enrich the analysis and, mostly in the second case, to overcome the limits encountered by both MIOT and PIOT (Hendrickson et al, 1998; Hoekstra, 2003; Weisz and Duchin, 2006). So HIOTs are believed to be the tool capable to address to both economic and environmental issues.

In addition to MIOTs and PIOTs there are few cases of IOTs published in other units, essentially to satisfy particular needs in specific fields (Stahmer, 2000). Hence there are IOTs in energy units (EIOTs), i.e. Joules or Watts, or furthermore in hours (TIOTs). According to all these experiences we can easily state that IOTs could be constructed in any unit of measurement. As consequence we could even have HIOTs accounted in monetary, mass and energy units plus hours, simultaneously.

In this paper the aim is to point out the importance of hybrid frameworks, as many authors have done since now (Hoekstra, 2003; Weitz and Duchin, 2006), but considering the loss of information and the consequent drawbacks as key factors in the choice of the units of measurement to adopt. To this aim section 2 and 3 introduce some topics known to most of IOTs practitioners but that will give the possibility to highlight some concepts useful in the following part. Section 4 deals with variability in homogenous groups of IOTs then section 5 introduce a drawback of the use of MIOTs. It follows a generalized approach to avoid the limit of a monetary framework and a numerical example in section 6 and 7, respectively. Finally the conclusions in section 8.

Before going on presenting the arguments of the paper, for clarity few words are necessary about the mathematical notations used hereafter. Capital letters indicate matrices and small letters vectors. With *diag(x)* or with the hat *(^)* I indicate a matrix with the values of vector *x* down the diagonal. A bar above a letter means that I refer to average values. The row by column matrix product is indicated with • while the Hadamard product with *.

2. Input-output tables (IOTs)

IOTs are essentially models able to mimic the behavior of a system where it is assumed that the production functions are linear (Miller and Blair, 1985). Figure 1

[1] It is noteworthy to mention that already in the 1970, in a pioneering work, Leontief (1970) used

shows a generic symmetric IOT framework[2]. For simplicity we assume a close economy. It has to be noted that in practice each IOT, which is accounted in just one single-unit, does not include all the matrices shown in Figure 1 since determined flows can be accounted only in specific units of measurement.

So a MIOT is essentially made of the transaction matrix Z, the primary factors matrix L and the final demand Y since only these flows are monetizable. Instead a PIOT, which has all the matrices expressed in tons, includes the transaction matrix Z, the natural resources matrix R, the use and the supply of waste W_U and W_V and the emissions E. The same reasoning is valid for the energy IOT (EIOT), accounted in Joule, for the time IOT (TIOT) accounted in hours and so on.

In addition it is important to remember that some traded products are intangible or do not have an energy content, e.g. a service has not a mass. Hence some flows in the mass-accounting matrix Z could be accounted as zero even though a transaction occurs. So a HIOT, because of the possibility to combine all the above mentioned single-unit IOTs, is the only framework that potentially can include all the occurring flows within the economy and all the exchanges with the environment as depicted in Figure 1.

Figure 1 – A generic IOT

	Production units	Final demand	Total
Production units	Z	Y	g
Primary factors	L		
Natural resources	R		
Use of waste	W_U		
Supply of waste	$-W_V$		
Emissions	-E		
Total	q’		

A fundamental rule of symmetric IOTs, no matter which unit of measurement is adopted, is that a proper balance per each production unit has to be fulfilled, i.e. total inputs are equal to total outputs. Equation 1 shows such balances for the

2 A symmetric framework means essentially that the production units in the columns are exactly the same of those in the rows (Eurostat, 1996) .

monetary, mass and energetic frameworks[3]. The vectors *i* are proper summation vectors.

(1)

$$q'_M = Z'_M \cdot i_Z + L'_M \cdot i_L = Z_M \cdot i_Z + Y_M \cdot i_Y = g_M \text{ [MIOT]}$$
$$q'_P = Z'_P \cdot i_Z + R'_P \cdot i_R + W'_{U_P} \cdot i_W - W'_{V_P} \cdot i_W - E'_P \cdot i_E = Z_P \cdot i_Z + Y_P \cdot i_Y = g_P \text{ [PIOT]}$$
$$q'_E = Z'_E \cdot i_Z + R'_E \cdot i_R + W'_{U_E} \cdot i_W - W'_{V_E} \cdot i_W - E'_P \cdot i_E = Z_E \cdot i_Z + Y_E \cdot i_Y = g_E \quad \text{[EIOT]}$$

Of course if every production unit is coherently balanced and the interactions between them are properly taken into account, a natural consequence is that the whole system is fully balanced as stated in Equation 2. The system balance in Equation 2 is simplified since flows accounted both as input and as output are excluded from the calculation for obvious mathematical simplifications.

(2)

$$(L'_M \cdot i_L)' \cdot i = (Y_M \cdot i_Y)' \cdot i \qquad \textit{monetary system balance}$$
$$(R'_P \cdot i_R - E'_P \cdot i_E)' \cdot i = (Y_P \cdot i_Y)' \cdot i \qquad \textit{mass system balance}$$
$$(R'_E \cdot i_R - E'_E \cdot i_E)' \cdot i = (Y_E \cdot i_Y)' \cdot i \qquad \textit{energy system balance}$$

The first conditions states that remunerations to primary factors are equal to final consumption. The second indicate the Mass conservation Law, so as the energy balance. The first conditions in Equations 1 and 2 will be defined as 'monetary balances' while the latter two as 'physical balances'.

3. The demand driven model

The Leontief demand-driven model (Miller and Blair, 1995) is the most famous application of IOTs and it is used to assess the direct and indirect impact on the full system as consequence of a specific final demand vector. The procedure can be summarized in two steps as follows: first there is the accounting of a productive system, which is structured in fully balanced IOTs; afterwards this system works as model to asses the direct and indirect effects to satisfy any final demand vectors. We refer to the values in first step - *the accounting step* - as initial state values and are indicated with a superscripted *0* while the values obtained from the second step - *the simulation step* - as simulated values and are indicated with a superscripted *1*[4]. The Leontief demand-driven model is traced as follows:

(3)

$$g^1 = (I - A^0)^{-1} \cdot y^1 \qquad \textit{where} \quad A^0 = Z^0 / q^0$$

with *A* defined as the matrix of coefficients, which is the core of the Leontief model. A generic value of A, i.e a_{ij}, shows how much of input *i* is necessary in order to produce one single output of *j*. Equation 3 shows the total production g^1 necessary to satisfy the demand y^1, where the productive system is structured in A^0.

[3] For simplicity only the three most used balances are shown.

[4] In reality the accounting step should refer to the construction of Supply and Use tables from which the IOTs are derived (Stone, 1961; UN, 1968). Here for simplicity I avoid to introduce these tables.

Notice that for completeness it is always preferable to apply the model only when the matrix of coefficients Z includes all the transactions between production units in the productive system and this can be only pursued using either monetary matrix of coefficients A, in case of market economies, or a hybrid mixed-units matrix of coefficients A, as stated above[5]. Notice that in Equation 3 g^1 and y^1 have the same units of measurement of the matrix of coefficients.

Going further, once obtained the new production g^1 as shown in Equation 3 all the possible impact assessments can be derived from, e.g. the request of primary factors and natural resources, or the emissions discharged, as follows:

$$(4) \qquad b^1 = \hat{C}^0 \cdot g^1 \quad where \quad \hat{C}^0 = C^0 \cdot diag(q^{-1})$$

where C^0represents any of the matrices *L, R, T, E* in Figure 1 and b^1 any chosen impact category. Equation 4 shows any impact assessment determination in order to satisfy a specific consumption bundle y^1. Of course a simulated impact level should be calculated respecting the balances properties of IOTs, as stated in Equation 1 and 2, in order to have reliable results. In such a case, we can surely state that IOTs are robust transdisciplinary tools, able to address sustainability tasks.

The rest of the text will be so focused on what are the consequences of using a monetary framework in terms of impact assessment and system balances.

For the sake of clarity, with monetary framework I'll particularly refer to MIOTs although to some extend the result can be valid for hybrid frameworks whenever a transaction is accounted in a monetary unit rather than a physical one.

We will figure out for instance if the monetary framework breaks the Mass Conservation Law. Furthermore if that happens, we will try to investigate the cause of such drawback.

This will done trying to assess the underlying volume production obtained in the monetary framework and check if the fundamental physical balances are not violated.

4. Variability within homogenous categories and monetary framework assumptions

IOTs are powerful tools since include all the productive activities belonging to a system. The number of single activities and products may be very high hence an aggregation is somehow always necessary for computational reasons. The underlying aggregation principle consists of grouping together products according to some specific characteristics and, similarly, activities according to their principal production (UN, 1999; Eurostat, 2006). In this way both rows and columns of an IOT show the sum of all the similar products and activities that so are said *homogenous*. Let us indicate with $j=1,\dots,n$ the set of homogenous activities and with $i=1, \dots,$m the

[5] A service, for example, is not a tangible flow so it cannot be accounted for in mass terms. Service could be then included if expressed in monetary terms, in hours, days or units of service, such as reports.

set of homogenous products. In order to apply the Leontief model - as traced in Equations 3 and 4 - the transaction matrix Z has to be invertible hence $n=m$.

Yet, nevertheless the great effort in the aggregation process is to set together similar activities and products as much as possible by mean of refined procedures, there could still be some differences appearing in the homogenous groups since current productive systems are becoming more and more complex and diversified. But, once the system of classification is defined, the process of aggregation pursued and an IOT constructed, few possibilities still remain to preserve the underlying variability within the groups.

The only type of variability that can be preserved in the IOTs relates to the outputs valuation, in other words the variability that may occur when homogenous activities, although using the same techniques, produce similar outputs but with different qualities and so differently valued on the market. For example two consultancy companies may both produce a report but the market price charged may diverge since having different skills[6]. In this case the compensation to employees or the profits, both indicated with *w,* may differ as shown in Equation 5.

$$(5) \qquad w_{v,k_1} > w_{v,k_2} \rightarrow p_{i,j_A} > p_{i,j_B} \qquad with \;\; k_1 \; and \; k_2 \in \;\; j = i; \; j_A \neq j_B \neq j$$

where v indicates a generic category of labor or capital. Equation 5 shows how two activities included in the same homogenous group charge different price to purchasers. Notice that the variability is for aggregates outside the transaction matrix[7].

The differences of valuation may be also due to market conditions as bulk purchases and this can be included in the above case when there is a reduction of profits or, in general, of a whatever value added component. Also auxiliary activities could be included in this case because the cost paid can be less than that charged on the market due to the internalization of the activity.

So generalizing, what we define as different outputs valuation refers to the well-known condition in the input-output Literature of prices differing per purchaser (Weisz and Duchin, 2006). This variability is intrinsic in the collected data and it is preserved after the aggregation process. It follows that the underlying relation between the monetary and the quantitative transaction matrices is always as follows:

$$(6) \qquad Z_M = Z_Q * P^0$$

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[6] It can be assumed that for one hour of consultancy, two workers with different skills may both use the laptop, turn on a desk lamp, drink a coffee and so on, although at the end the two services have a different value. In this case an immaterial good makes the difference, the skill, and this quality distinction may be embodied in the different salary paid to the workers.

[7] This statement implies that the physical input and output structure of homogenous products is exactly the same no matter who is the producer or purchaser. Such structure will be an average of all the productive recipes. This assumption could be somehow overcome but this would go beyond tha aims of the paper.

where P^0 is the matrix of prices in the accounting step and * indicates the Hadamard matrix product[8].

Yet the use of a monetary transaction matrix in the Leontief demand-driven model implies that there is a fixed vector of prices, or said in different words, each product is sold to a fixed price, which is practically the average price at the accounting step. A consequence of this is that the monetized variables are always proportional to the quantities and the vector of average prices works as conversion factor. This can be treated as a loss of information that may determine a drawback in physical balances as shown in the next section.

In the light of what just said about average prices in a monetary framework, Equation 6 can be rewritten as follows:

$$(7) \qquad Z_{Q|M} = diag(\bar{p}^0)^{-1} \cdot Z_M \quad \text{where} \quad \bar{p}_0 = [diag(q_M) \cdot diag(q_P)^{-1}] \cdot i$$

Equation 7 shows the assumed relation between monetary and underlying quantitative transactions when a monetary framework is used to implement the Leontief model. Said in different words, Equation 7 provides the procedure by which quantitative levels, or indirectly any impact assessment in physical units, are obtained once monetary model is carried out. I will try to investigate if such indirect quantitative levels obtained in the simulation step respect the balance laws.

5. <u>Monetary framework and physical balances</u>

In this session it is shown why the results of a simulated scenario, which are obtained by a monetary framework, may fail to respect physical balances laws.

Before starting, in order to make a neat distinction between the monetary values and quantities I use Latin letters for the former and the Greek letters for the latter. Furthermore when speaking about the coefficient matrix A I refer to an "outlay coefficients" in case of a monetary framework and to a "technical coefficient" when referring to the quantity framework.

Let us start with Equation 10 where a generic outlay coefficient is decomposed.

$$(8)\ a_{i,j} = \frac{z_{ij}}{q_j} = \frac{\chi_{ij} \cdot p_{ij}}{(\sum_{j'} \chi_{jj'} \cdot p_{jj'} + \square_j \cdot p_j^C)} = \frac{\chi_{ij} \cdot p_{ij}}{(\sum_{j'} \chi_{jj'} + \square_j)} \cdot \frac{p_{ij}}{\bar{p}_j} = \alpha_{ij} \cdot \frac{\bar{p}_i}{\bar{p}_j} \cdot \frac{p_{ij}}{\bar{p}_i} = \alpha_{ij} \cdot \bar{v}_{ij} \cdot s_{ij}$$

where $\chi_{i,j}$ is an element of the physical matrix Z_Q , γ is the volume of final demand, p_{ij} is the price paid by activity *j* to purchase *i* and p^C is the consumer price.

Equation 8 shows that the outlay coefficient is the product of three aggregates, the technical coefficient α, the ratio of average prices $\bar{v}$ and the price variability factor *s*. The second coefficient $\bar{v}$ is a scale factor and is the ratio between the prices used implicitly in the monetary framework, while the third one is a variable that keeps

[8] This relation is valid for any monetizable matrix but here for simplicity I limit to the transaction matrix case.

the information of different prices charged to purchaser. It follows from Equation 8 that the outlay coefficients and the technical coefficients are always proportional in any simulated scenario by a fixed scale factor $\bar{v}_{ij}$, if s_{ij} is invariant.

We can now go further, checking the conditions that allow the indirect respect of physical balances in a monetary framework. In the following equation a simplified monetary balance (see Equation 1) is decomposed applying the result of Equation 8. η indicates the total volume produced.

$$
\begin{gathered}
\sum_j \left(a_{ij} \cdot g_j\right) + y_i = \bar{p}_i \cdot g_i \\
\sum_j \left(\left(\alpha_{ij} \cdot \frac{\bar{p}_i}{\bar{p}_j} \cdot s_{ij}\right) \cdot \left(\bar{p}_j \cdot g_j\right)\right) + \bar{p}_i \cdot s_{i;y} \cdot \gamma_i = \bar{p}_i \cdot g_i \\
\sum_j \left(\chi_{ij} \cdot s_{ij}\right) + s_{i;y} \cdot \gamma_i = g_i \\
\bar{s}_i = 1
\end{gathered}
\tag{9}
$$

Equation 9 shows that, in order to have indirectly the respect of physical balances, the parameter of prices variability s has always to be equal to 1 on average.

We can now check what happen with the average of s in the accounting state and in the simulated state when the monetary framework is applied for the Leontief model.

In the accounting state, where the demand is y^0, we have that:

$$
\bar{s}_i^0 = \frac{\sum_j (s_{ij}^0 \cdot \chi_{ij}^0) + (s_{i;C}^0 \cdot \gamma_i^0)}{\sum_j (\chi_{ij}^0 + \gamma_i^0)} = \frac{\sum_j (\frac{p_{ij}}{\bar{p}_i} \cdot \chi_{ij}^0) + \frac{p_i^C}{\bar{p}_i} \cdot \gamma_i^0)}{\sum_j (\chi_{ij}^0 + \gamma_i^0)} = \frac{\sum_j (p_{ij} \cdot \chi_{ij}^0) + (p_i^C \cdot \gamma_i^0)}{\sum_j (\chi_{ij}^0 + \gamma_i^0) \cdot \bar{p}_i} = 1
\tag{10}
$$

From Equation 10 we can state that the physical balance of the product i is respected in the accounting step. This is an obvious result since the average price is based on the accounting data. We can now move to the next equation to see what happens in the simulated state, where the demand is indicated with $y^1 (\neq y^0)$ and the input of material with $\chi_{i,j}^1 = \alpha_{i,j} \cdot \eta_j^1$.

$$
\begin{aligned}
\bar{s}_i^1 &= \frac{\sum_j (s_{ij}^0 \cdot \chi_{ij}^1) + (s_{i;C}^0 \cdot \gamma_i^1)}{\sum_j (\chi_{ij}^1 + \gamma_i^1)} = \frac{\sum_j (\frac{p_{ij}}{\bar{p}_i} \cdot \chi_{ij}^1) + \frac{p_i^C}{\bar{p}_i} \cdot \gamma_i^1)}{\sum_j (\chi_{ij}^0 + \gamma_i^0)} = \frac{\sum_j (p_{ij} \cdot \chi_{ij}^1) + (p_i^C \cdot \gamma_i^1)}{\sum_j (\chi_{ij}^1 + \gamma_i^1) \cdot \bar{p}_i} \\
&= \frac{\sum_j (p_{ij} \cdot \chi_{ij}^1) + (p_i^C \cdot \gamma_i^1)}{\sum_j (\chi_{ij}^1 + \gamma_i^1) \cdot \frac{\sum_j (p_{ij} \cdot \chi_{ij}^0) + (p_i^C \cdot \gamma_i^0)}{\sum_j (\chi_{ij}^0 + \gamma_i^0)}} = \frac{\sum_j (\chi_{ij}^0 + \gamma_i^0) \cdot \sum_j (p_{ij} \cdot \chi_{ij}^1) + (p_i^C \cdot \gamma_i^1)}{\sum_j (\chi_{ij}^1 + \gamma_i^1) \cdot \sum_j (p_{ij} \cdot \chi_{ij}^0) + (p_i^C \cdot \gamma_i^0)} = \frac{\bar{p}^1}{\bar{p}^0} \nrightarrow 1
\end{aligned}
\tag{11}
$$

Equation 11 shows that the condition on $\bar{s}_i$ in Equation 9 is violated hence the physical balance of the product i too. This because $\bar{s}_i^1$ has a distribution where the average is function of the demand and may change case by case.

What happens is that for any given monetary request of the *i*-product by the *j*-activity:

$$\text{(12)} \quad \begin{array}{l} if \quad s_{i,j} < 1 \quad \rightarrow \; physical \quad underproduction \quad of \quad i-product \\ if \quad s_{i,j} < 1 \quad \rightarrow \; physical \quad overproduction \quad of \quad i-product \end{array}$$

Then in the base state the underproduction and the overproduction are counterweighed, instead when the demand changes this does not happen anymore. This is explained by the fact that after the simulation the average prices change as consequence of variation of market transactions magnitude, which is determined by the new final demand. But in the monetary framework, because the vector of prices, which is the set of average prices in the base state, is kept the same also in the simulated state, the indirectly assumed proportion between the monetary and the physical production is wrong causing the break the physical balances. In other words, the monetary framework loses the information about the variability of prices fundamental to model the physical relations, generating imprecise estimations of physical flows and of relative impact assessment[9]. Then it is important to notice that the more the averages of prices change, the more imprecise the assessment is. Hence it follows that great changes in the demand vector – or in the simulated scenario - respect to the base state may generate totally wrong estimations.

It is noteworthy to say that a problem may rise even in the economic level since also the ratio of the remuneration of primary factors changes according to the market exchanges, hence attention must be put when tracing the value added chain (Duchin, 2004) in a monetary framework.

Finally is noteworthy to say that the exposed problem encountered by MIOT is also valid for hybrid frameworks whenever a flow is accounted in monetary terms and there is variability in the quality of the homogenous products. For example this can be the case of some services. It is important to keep in mind that balance discrepancies will occur only for those units of measurement that can take into account the flow where prices differ. For example if the price of electricity differs and we use a monetary framework, the unbalance will be for the energy but not for the mass. But, however, also the magnitude of the impact on the other levels may be affected due to the consequent under or overproduction in the energy sector. For example the request of extracted oil may be wrongly estimated.

[9] Notice that here prices change as consequence of the switch within the purchasers' share and not for changes in produced volumes. This means that the exposed drawback may be valid also for General Equilibrium Models.

6. A physical framework

In order to have an IOA respecting all the possible balance flows simultaneously or to avoid wrong estimations of specific impact assessments, an obvious solution consists of using a hybrid framework that excludes all the monetary flows from the transaction matrix relying only on the physical exchanges. For example services can be expressed in hours or units of service, e.g. number of reports. Equation 3 can be so written as follows:

(13) $$\eta^1 = (I - A^0) \cdot \gamma^1 \qquad where \quad A^0 = \chi^0 \cdot diag\left(\eta^{0^{-1}}\right)$$

In this approach firstly there is the determination of the total production in hybrid terms, i.e. η^1, and only in a second step the impact assessments in the several units of measurement where all the relative balances are fulfilled, using specific conversion coefficients. But this time the latter are tailored on the hybrid production of the base state , i.e. η^0.

The determination of the monetary accounts and of the economic impact assessment can be fulfilled similarly to the physical accounts but considering a matrix P of prices and a matrix of salaries W, if they cause the price variability[10]. Hence we can determine the average prices and salaries only after the simulation and, based on them trace the value added chain.

(17) $$\overline{p}^1_i = \frac{\sum_j \left[(\alpha_{i,j} \cdot \eta^1_j) \cdot p_{i,j}\right] + \gamma_i \cdot p_{i,\gamma}}{\eta^1_i}; \quad \overline{w}^1_i = \frac{\sum_j \left[(w_{i,j} \cdot \eta^1_j)\right]}{\eta^1_i} \quad .$$

In this way the IOA keeps the full information on the products quality without breaking the physical and economic balances or failing in the assessment of specific flows. However it has to be said that currently an analyst does not have the whole information to accomplish this method hence by this paper, showing a drawback of the monetary framework, I wish to provide a further motivation for boosting the statistical offices to deliver the economic data divided in quantities and prices.

7. Numerical example

Here a very simple economy with three sectors/products, i.e. Agriculture, Manufacture and Services, is depicted. The aim of the exercise is to show the differences that may exist using a pure monetary or, alternatively, a fully physical IOT, in a situation where prices differ per purchaser. For simplicity only the economic and mass level will be analyzed. Figure 2 shows the same economy but accounted first in physical terms and then in monetary ones. Furthermore the matrices of prices and salaries are showed in order to move back and forth from one

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[10] If the salaries are fixed what changes are the profits.

level to the other. I assume the Manufacture and Services have different prices per purchaser. The different price charged by the Manufacture is due to market condition and hits the profits. Instead for the Services the difference is due to a better-paid labor service, which denotes a higher quality of service due to different skills. This can be seen in the third row of the matrix of salaries.

Figure 2- The economic system at the accounting step

HIOT		Agriculture	Manufacture	Services	Demand	Total
Agriculture	kg	0	1	2	4	7
Manufacture	kg	2	0	3	3	8
Service	hours	2	2	0	1	5
Labor	hours	3	2	2		
Resources	kg	8	9	0		
Emissions	kg	3	2	5		
Production	kg	7	8	0		

Prices		Agriculture	Manufacture	Service	Consumer
Agriculture	€/kg	1	1	1	1
Manufacture	€/kg	0.8	0.8	0.8	1
Service	€/hour	1.5	2	1.5	2

Salaries		Agriculture	Manufacture	Service	Demand
Agriculture	€/hour	0.5	0.5	0.5	0.5
Manufacture	€/hour	0.5	0.5	0.5	0.5
Service	€/hour	0.5	1	1	1

MIOT		Agriculture	Manufacture	Service	Demand	Total
Agriculture	€	0	1	2	4	7
Manufacture	€	1.6	0	2.4	3	7
Service	€	3	4	0	2	9
Labor	€	1.5	1	1.6		
Profits	€	0.9	1	3		
Production	€	7	7	9		
Resources	kg	8	9	0		
Emissions	kg	3	2	5		

Now let us assume that there is a change in the demand vector and we want to assess the consequent environmental impact and distribution of the generated income, i.e. the value added chain. To this aim the monetary and the fully physical hybrid model, as exposed at the end of the previous section, are used and then a comparison of the results is carried out. Figure 3 shows the new vector of the demand and the consequent results.

It can be seen that results are dissimilar denoting a divergence of the two approaches when dealing with differences of prices and, consequentially, of salaries. The hybrid framework performs better because takes into account the variability occurring in the system. Differences can be seen in the request of resources and in the discharged emissions but even in the monetary production and in the distribution of income.

Figure 3 – Results of the hybrid and monetary approaches

New demand				
	Physical		Monetary	
Agriculture	kg	8.0	€	8.0
Manufacture	kg	3.0	€	3.0
Service	hours	1.5	€	3.0
			total	14.0

New production							
	Hybrid framework				Monetary framework		
	Hybrid		Monetary		Monetary		Difference
Agriculture	kg	12.6	€	12.6	€	12.4	-1.45%
Manufacture	kg	11.4	€	9.7	€	9.5	-1.92%
Service	hours	7.9	€	14.1	€	13.8	-2.33%

Mass balance					Difference
	Hybrid fram.		Monetary framework		
Resources	kg	27.2	kg	26.4	-2.84%
Emission	kg	16.2	kg	15.7	-3.11%
Demand	kg	11.0			
Final balance	kg	0.000			

Value added chain					Difference
	Hybrid fram.		Monetary framework		
Labor	€	6.6	€	6.5	-1.72%
Profits	€	7.4	€	7.5	1.52%
Total	€	14.0	€	14.0	0.00%

Notice that the mass balance cannot be done for the monetary framework and even using the average prices to transform the monetary demand in mass terms, the final balance would be not equal to zero as instead it is for the hybrid framework.

Finally it is noteworthy to say that also an intermediate approach, where just the Services are accounted in economic terms, differences in the results occur although of a smaller extent. These results are not presented here.

8. Conclusions

In this paper it is shown that the use an input-output model is preferable in a complete physically accounted framework whenever there are divergences of price in the homogenous products of a category as many authors have already stated although with different emphasis. Differences of price, which are often neglected by the final user of MIOTs, may be caused by the aggregation process itself or by the market conditions and they can be quite common in the real world. In this backdrop, the use of monetary transactions may mislead the assessment of the exact magnitude of some flows, physical and economic, other than imply a loss of information, reducing the potentialities of an IOA. In this paper I try to deepen the consequences of ignoring the limits of a monetary framework in practical analysis using concept not strictly linked to a proper economic environment.

Furthermore I am also confident that the approach here presented may ameliorate the economic analysis because the quality of primary factors, a relevant variable for economists, can be taken into account without any effort.

However I am aware that nowadays all the information necessary to avoid monetary transaction in favor of hybrid framework are not yet available for researchers and analysts. Hence by this paper I would also to encourage the offices of statistics to collect and deliver as much as possible data in quantity units since they, in combination with prices, assure more reliable analyses, making feasible to address to transdisciplinary issues.

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