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# **Urgent problems of constructing input- output tables and their use in forecast calculations**

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## **ABSTRACT**

*At present the information contained in national IOTs is mainly used in economic-mathematical models designed to estimate mid-term and long-term prospects of Russian economy. At the same time the traditional calculations and analyses of indirect and cumulated cost-shares and effects relegated to second place because they do not find demand among analysts and decision-makers.*

*This circumstance, as well as the long absence of detailed survey-based IOTs, forced IO researchers to use their own methods of operational working out of make-use tables of goods and services, their specification and regionalization. In this paper the authors present the experience of IOTs development based on the applied studies conducted during the last 8 years at the request of the Ministry of Economic Development of Russia. The studies were aimed at forecasting the indicators of Russia's socio-economic development in the cut of sectors and regions. The tables in use differ from the standard ones developed by Russian Statistical Agency. They are adapted to the features of a model complex in use that comprises a-spatial and spatial models (that is, models of national economy as whole and interregional models). The adaptation implies the use of basic prices and the maximum possible "naturalization" of the sectoral pattern of demand, that is, the approximation to real proportions of distribution of goods and services by kinds of economic activities.*

*The experience of construction of such modified tables and their use as informational base for IO models demonstrates that they have some advantages in comparison with standard tables, namely, the capability of faster development of the table for the base year of a forecast as well as the more adequate representation of sectoral and regional proportions between production and consumption.*

# **Urgent problems of constructing input-output tables and their use in forecast calculations**

## **Introduction**

The paper is based on the experience of developing multiregional input-output models (OMIOM) for purposes of long-term forecasting. The active studies in the field of long-term forecasting with the use of the OMIOM apparatus were recommenced in the Institute of Economics and Industrial Engineering SB RAS (Novosibirsk) in 2002 and go on till now.

The need in the explicit and predictable governmental socio-economic policy as well as the demand for development targets for long-term prospects initiated the elaboration of several long-term forecasts of the socio-economic development of the RF. The last of them is the forecast for 2030 developed by the Ministry of Economic Development<sup>1</sup>. It is adjusted with the account of the consequences of the world financial crisis and represents rather moderate growth rates of GDP till 2030, especially in comparison with the pre-crisis Conception of the Long-Term Development of the RF till 2020 (Kontceptcia...(2008)) that implied the possibility of reaching 4% average annual growth rate of the GDP even in the minimal variant. Now the conservative scenario foresees 3,6% growth rate, and the scenario of “innovation development” predicts 4,4% growth rate.

In the frames of the scenarios described above and on the base of the OMIOM apparatus the authors developed the sectoral and spatial aspects of the macroeconomic forecast till 2030. The task was reduced to forecasting the sectoral and spatial patterns of the Russian economy under the premises formulated in the external macroeconomic forecast of the Ministry of Economic Development and Trade. Under such conditions the sectoral and spatial forecasts produced on the base of the OMIOM answer the following question: “What would be sectoral and spatial patterns of the economy, if the Russian economy as a whole reaches the rates forecasted with the Ministry?”

## **Design of IO models in use**

At present the OMIOM apparatus in use includes 2 basic models called as “point” (dynamic IO model) and “spatial”(multiregional IO model) models.(Ershov, 2007) Forecasts for Russia as a whole are performed on the base of the optimization IO dynamic model. This model is used as an instrument for long-term forecasting in the cut of types of economic activities without taking spatial aspects of economic development into account. Spatial forecasts are performed on the base of the optimization multiregional IO model. Both models are constructed on the same database of 2010 in the cut of 40 sectors corresponding to the new nomenclature of

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<sup>1</sup> [http://www.economy.gov.ru/minec/activity/sections/macro/prognoz/doc20120428\\_010](http://www.economy.gov.ru/minec/activity/sections/macro/prognoz/doc20120428_010)

economic activities developed with the Rosstat in 2004 and harmonized with Classification of Economic Activities in the European Community (NACE rev.1.1). (The list of sectors is attached in Supplement 1). Both models produce forecasts to 2030.

The “point model” has 2 forecasting periods: 2011-2020 and 2021-2030. The model contains of common IO balances: the balances of production and distribution of outputs, the balances of labour resources, the balances of gross investments and constraints on foreign trade balances. The objective function maximizes the sum of final demand components such as the final demand of house holdings, of governmental institutions, of non-profit organizations attending to house holdings, and the accumulation (change in stocks) of circulating tangible assets. The realization of the dynamic IO model provides for the forecasted tables of distribution of goods and services in the economy for the last years of the forecast period as well as for all interim time points. The presence of the “point” model facilitates the subsequent statement and realization of the “spatial” model.

The “spatial” model consists of 9 regional blocks united with the conditions of interregional transportation-economic links and with the territorial pattern of final demand. Each region is presented with the semi-dynamic optimization IO model that provides for the calculation of the state of economy for the last year of a forecast period. The set of constraints in regional blocks duplicates the structure of the “point” model. The optimal solution of the model represents a consistent set of forecasted regional IO tables for the last year of a forecast period. The tables are produced in the cut of 8 federal districts and Tyumen oblast and 40 types of economic activities. (Regions are listed in Supplement 2, the formal statement of the model is written in Supplement 3) As distinct from the “point” IO model the “spatial” one is realized in the semi-dynamic statement, with the use of a forward recurrence: first the problem is solved for 2011-2020, and then it is solved for 2021-2030 from the base of 2020.

#### **Basic requirements to input information**

The specific of models in use determines some requirements to input information. The basic requirement may be formulated as the maximal approximation of derived proportions of distribution of goods and services to “natural” proportions. “Natural” proportions in this case mean those proportions that would take place under equality of prices of each commodity for all consumers. So, the IO tables required for our purposes will differ of standard tables developed in Rosstat.

First of all, outputs and demands in the tables must be estimated *in basic prices* as purchasers’ prices for different consumers vary much more then basic prices. Meantime, symmetric IO tables of Rosstat contain supplies in basic prices and the use in purchasers’ prices.

The second distinction relates to accounting *transportation margins*. The standard approach implies attributing inputs for intermediate consumption to those who pay for the used commodities and services. Such approach would not bring significant defects if a single national table were developed. But under regionalization each nationwide constraint for sector  $i$

$$x_i - \sum_j a_{ij} x_j = b_i \text{ (in simplified formulation)}$$

is transformed into the set of  $r$  regional constraints

$$x_i^r - \sum_j a_{ij}^r x_j^r - \sum_s x_i^{rs} + \sum_s x_i^{sr} = b_i^r$$

Such specifics of the interregional IO models in use requires for attributing transportation costs to transported products. As a result, in some sectors (for example, “extraction of natural gas”) total costs may exceed outputs. This approach simplifies the problem of distributing transport margins by sectors-consumers: it is suffice to estimate what part of transportation work falls at transportation of specific products, such as coal, timber, oil, etc. The estimate is based on of available statistical reports that perform outputs of oil and gas transportation sectors in money terms and the work of railroad transportation in natural terms.

### **Problems of regionalization**

All this implies that even if regional IO tables in their standard form were available, they would be inevitably re-calculated according to formulated requirements to input information. But in Russia regional tables are not developed at all, though some regional accounts of production and final consumption are published annually, with 3-year lag. So IO researcher faces the problem of regionalization of national IO tables.

The necessary condition of regionalization is availability of Russian IO table for a base year. But every new cycle of forecast calculations is carried out in the absence of the national IO table for the year of interest. The last survey-based IO table of Russian economy was built for 1995, and the last reliable calculated IO tables were published for 2003, in the out-of-date OKONKH nomenclature. In these conditions the authors have to up-date the estimated IO tables regularly, adjusting them to newly published figures of national and regional accounts, to changing sectoral nomenclature and administrative innovations in federal system of the country. The base year of forecast regularly moves: from 2005 to 2007, and to 2010 at present. These efforts allow keeping continuity of the model database.

So, the procedure of preparing input information for the model in use begins from the stage of developing *the national IO table for the base year*. The foundation of this stage is in that, against the criterion of ratio errors, the national table always exceeds any regional table by quality because sectoral balances of production and consumption are known for country at whole

with acceptable accuracy while analogous information about regions is not full. In part, the data about interregional deliveries of commodities and services are almost absent.

The next stage of developing the database is *regionalization* of the derived national IO table. There exist an extensive body of literature devoted to problem of updating and regionalization of SUTs and SIOTs. (Lahr, (2004); Temurshoev (2011)) These works develop and compare different techniques of updating, such as well known RAS methods, methods of squared differences and others. In our opinion, the formal methods must be applied in the last turn, after the use of all available direct and indirect data that might allow calculating amounts of inter-sectoral or interregional flows. The procedure of regionalization concerns to outputs, final consumption, capital accumulation, exports and imports.

#### Estimation of regional outputs

The main principle of the estimation of regional outputs is the trust to Rosstat data about outputs, value added and intermediate consumption as to the most complete and accurate source of information because they account for non-observed activities. So the necessary (but not sufficient) condition of controlling totals of estimated regional IO tables may be formulated as “*the sum of regional indicators must be equal to the same indicator estimated for the economy at whole*” In other words, in the sake of good quality of estimation *it is necessary to develop all set of regional IO tables simultaneously*. From this follows that it is impossible to estimate the quality of any regional IO table if it is developed separately. Nevertheless, some works of this kind were performed (Sayapova (2011); Serebryakov (2001)) for specific regions of Russia, such as Bashkiria, or Ivanovo.

The condition of developing the full set of regional IO tables may be fulfilled only with the use of basic prices that do not include indirect taxes and components of market prices that are included in national incomes, but not distributed among sectors-kinds of economic activities. In 2009 this non-distributed part of value added comprised 13% of GDP in market prices. Then, some part of GDP in basic prices cannot be distributed among regions. In 2009 this part was equal to 5% of GDP in basic prices. In such case methods of proportional increasing regional indicators remains to be the last solution. The single available reference point may be obtained from regional accounts that provide an approximate 15-sectoral structure of gross regional product for each region of Russia. At the same time analogous data about outputs and intermediate consumption are not developed.

Under such circumstances the task of estimating regional outputs in the cut of sectors gets independent value. It is fulfilled with the use of different hypotheses to different sectors. In

some sectors spatial structure of outputs may be assumed to be equal to the spatial structure of value added, or to the spatial structure of outputs of firms that are presented in statistical performance. In sectors of mono-products (such as fishing, forestry, extraction of coal, oil, gas, ferrous ores, and coke and oil products) the spatial structure of outputs may be calculated on the data of outputs in natural terms, that is, in average national prices. The calculations may be specified with the use of data about employment.

One of the most effective methods of regionalization implies the use of maximally detailed nomenclature for the national IO table with subsequent sectoral aggregation for regions. It is well known that regional differentials in technological coefficients stem from differentiation in intra-sectoral structure of output. So, if we assume the hypothesis about regional uniformity of inputs for intermediate consumption, build 1<sup>st</sup> quadrants of regional IO tables in detailed nomenclature and then aggregate them to lesser number of sectors, then the resulting approximation of regional tables will be evidently better in comparison to the variant in which we would keep the same number of sectors. Such method looks very promising on account of expected detailed IO tables of Russia for 2011.

One more trick for improvement of regional IO tables is the proper choice of sectoral nomenclature. The choice must provide for maximal number of reliable row balances of regional production and consumption, or, “edging totals” We may, for example, consider non-transportable kinds of economic activities. So, if we break sectoral aggregate E to 4 sectors - “Generating and distributing electric energy”, “Generating and distributing heat energy”, “Production and distribution of gas fuel” and “Gathering and distributing water”, it makes evident that interregional deliveries take place only for the first sub-sector while in the last 3 sectors production is conditioned only by intra-regional demand (row totals equal to zero).

#### Estimation of regional amounts of final consumption

The next stage of regionalization is estimating regional amounts of final consumption. Initially, on the base of regional accounts it is possible to account for 75% of total final demand that concerns the published data about final consumption of house holdings in regions. The rest 25% falls to final consumption of state institutions (collective (public administration and military protection) and individual services (education, health and social services, culture and arts)). It is possible to estimate this part of final consumption on the base of hypothesis about non-transportability of products of these sectors. Then an amount of final consumption of these products in a region may be calculated as a function of already estimated outputs in the region. Such approach implies the uniformity of proportions between inputs and outputs by regions,

which follows from the general observation that inputs of other sectors in outputs of sectors in question comprise 2-3% of total outputs.

One of basic principles in solving the problem of regionalization of final consumption is in that sectoral pattern of final consumption, especially for house holdings, cannot differ by regions too much. Traditional differentiation is determined by differences in incomes and patterns of consumption between urban and rural population. Our method reveals maximal interregional differentiation in final consumption of sector “Public administration and military protection”. The share of this sector in the total final consumption of Far Eastern Federal District is equal to 31% while for Russia as a whole the same indicator equals to 13%.

#### Estimation of regional capital accumulation

The estimation of capital accumulation in regions is carried out proportionally to the published data about spatial structure of capital investments as well as to the data about technological structure of capital investments. This allows for distributing regional amounts of capital accumulation over 3 sectors: “Machinery”, “Construction” and “Realty, lease and other services”. The specific problem is that capital investments are presented in market prices, which requires for eliminating trade margins.

#### Estimation of regional exports and imports

Amounts of exports and imports of products may be regionalized by 3 methods: according to official statistics of foreign trade; attributing to real places of production and consumption; and distributing by border regions. The use of the first approach result in that Central Federal District will concentrate more than half of foreign trade turnover. The second approach implies that the share of product in regional exports corresponds to its share in regional production, that is, any producer takes part in exports and consumes imported products. To our opinion, the minimum of ratio errors in estimated spatial structure of foreign trade is reached on the use of the third method. As transportations in our model are represented in network statement, exports are distributed over regions from which products depart the country while imports are distributed over regions in which products initially arrive.

#### Estimation of regional amounts of intermediate consumption

The final stage of regionalization is estimating regional amounts of intermediate consumption. Having estimated amounts of regional outputs and final demand, it becomes possible to calculate the first approximation of symmetric IO tables on the base of hypothesis about regional uniformity of input coefficients. The resulting misbalances are treated in different ways. If misbalances are small, they are adjusted by proportional techniques. If misbalances are



significant, it requires for the use of additional informational resources: for example, tables of supply and use of electric power. If, for instance, to apply national average input ratios for electric power in Siberian federal District, this will always result in the excess of electric power in this region (positive regional trade balance), because the specifics of the main consumer – non-ferrous metallurgy – will not be accounted.

The availability of such additional information enables to reduce the use of formal methods of proportional adjusting rows and columns of regional IO tables. The simplified procedures may provide formally balanced tables, but sometimes they result in high, unexplainable interregional differentiation in input coefficients. So they may be applied only for eliminating small misbalances.

### **Results of long-term forecasts**

The estimated regional IO tables for 2010 are used as a database of OMIOM. The latest cycle of calculations resulted in forecast of spatial development of Russian economy for 2020 and 2030 in the cut of federal districts and Tyumen oblast. The external premises were formulated in the Ministry of Economic development and concerned to average growth rates of GDP, industrial outputs, capital investments, exports and imports, and final demand according to “conservative” (based on energy and raw materials) and “innovative” scenarios. The results of developed spatial forecast for the second scenario are presented in tables 1 and 2.

It is evident that the scenario turns out to be innovative not only for the national economy, but also for each region: shares of manufacturing increase everywhere, though in different degree. Nevertheless regions keep their traditional industrial specialization that may be changed only at the expense of creating new enterprises. But forecasting of spatial allocation of new industrial capacities remains to be unsolved problem.

Table 1

Innovative scenario. Growth rates of macroeconomic indicators for 2011-2020

	RF	CFD	NWFD	SoFD	NCFD	VFD	UFD-T	Tyu	SiFD	FEFD
Outputs	104.4	104.6	104.5	104.6	105.2	104.4	103.6	102.4	104.5	104.7
Gross regional product	104.5	104.5	104.6	104.7	105.3	104.3	103.5	102.2	104.6	104.8
Industrial output	104.2	105.0	104.4	104.3	104.2	104.4	102.6	100.7	104.2	104.1
extraction	100.9	102.6	101.3	100.9	102.1	99.9	100.1	100.0	103.0	102.7
manufacturing	105.4	105.6	105.2	104.8	105.4	105.4	105.9	105.1	105.0	105.8
Final demand	104.7	104.5	104.8	105.1	105.3	104.7	104.7	104.5	104.9	105.1
Investments	107.0	106.0	107.3	106.9	107.9	107.5	106.4	106.3	107.8	107.5

Innovative scenario. Growth rates of macroeconomic indicators for 2021-2030

	RF	CFD	NWFD	SoFD	NCFD	VFD	UFD-T	Tyu	SiFD	FEFD
Outputs	104.3	104.4	104.4	104.6	104.8	104.3	103.7	102.5	104.3	104.9
Gross regional product	104.3	104.3	104.5	104.7	104.9	104.2	103.5	102.2	104.4	105.0
Industrial output	104.1	104.7	104.1	104.2	104.0	104.4	102.9	100.6	103.9	104.0
extraction	100.7	102.0	101.5	100.8	101.9	99.0	99.9	99.8	102.3	102.8
manufacturing	105.0	105.1	104.7	104.6	104.8	105.1	105.2	104.0	104.7	105.2
Final demand	104.1	104.0	104.2	104.3	104.5	104.0	104.1	103.7	104.3	104.6
Investments	105.7	105.1	105.9	105.5	106.1	105.9	104.9	104.4	105.2	106.3

The use of regionalized indicators in the OMIOM gave explainable regional differentiation in projected growth rates and levels of development. North-Caucasian region is leading by growth rates in both variants, but it remains to be outsider by per capita incomes during all forecast period. Meanwhile, Tyumen region (and. correspondingly, Urals Federal District) providing for lion's share of oil and gas extraction and transportations remains to be outsider by growth rates.

### Conclusion

This paper generalises 10-year experience of up-dating national IO tables and developing regional IO tables of Russian economy under circumstances of the absence of survey-based IO national table from 1995. The next survey-based table in detailed nomenclature for 2011 is under preparation now, which promises a chance to develop regional IO tables of better quality in the future, that is, to lessen the degree of proper "regionalization" of national IO table and, correspondingly, to increase the reliability of long-term spatial forecasts.

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## List of sectors

№	Types of economic activities
1	Agriculture
2	Hunting and forestry
3	Fishing, fish-farming
4	Extraction of solid fuel
5	Oil extraction
6	Gas extraction
7	Ferrous ore extraction
8	Non-ferrous ore extraction
9	Other minerals extraction
10	Food industry
11	Light industry
12	Wood industry
13	Pulp and paper industry
14	Publishing and printing
15	Coke
16	Oil products
17	Chemical products
18	Other non-metallic mineral products
19	Ferrous metals
20	Non-ferrous metals
21	Fabricated metal products
22	Machinery
23	Other industries
24	Generating and distributing electric energy
25	Generating and distributing heat energy
26	Production and distribution of gas fuel
27	Gathering and distributing water
28	Construction
29	Trade, repair of vehicles and others
30	Hotels and catering
31	Railway transport
32	Pipeline transportation
33	Other transportation and auxiliary activity
34	Communication
35	Financial activities
36	Realty, lease and other services
37	Public administration and military protection, mandatory social insurance
38	Education
39	Health and social services
40	Other public, social and personal utilities

### List of regions and abbreviations

1. Central Federal District (CFD)
2. North-Western Federal District (NWFD)
3. Northern-Caucasian Federal District (NCFD)
4. Southern Federal District (SoFD)
5. Volga Federal District (VFD)
6. Urals Federal District excluding Tyumen oblast (UFD-T)
7. Tyumen oblast (Tyu)
8. Siberian Federal District (SiFD)
9. Far Eastern Federal District (FEFD)

### Formal statement of optimization multiregional input-output model of Russia's economy

#### Regional blocks of the model

Balances of production and distribution of products

$$x_i^{r0} + x_i^{r1} - \sum_{j=1}^n a_{ij}^{r0} x_j^0 - \sum_{j=1}^n a_{ij}^{r1} x_j^1 - \alpha_i^r z^r - \sum_{s \neq r} x_i^{rs} + \sum_{s \neq r} x_i^{sr} - \sum_{h=1}^3 v_i^{rh} + \sum_{h=1}^3 w_i^{rh} \geq b_i^r; i = 1, \dots, n; \quad (1)$$

corresponding restrictions for capital-forming sectors:

$$x_g^{r0} + x_g^{r1} - \sum_{j=1}^n a_{gj}^{r0} x_j^0 - \sum_{j=1}^n a_{gj}^{r1} x_j^1 - u_g^{r1} - \alpha_g^r z^r - \sum_{s \neq r} x_g^{rs} + \sum_{s \neq r} x_g^{sr} - \sum_{h=1}^3 v_g^{rh} + \sum_{h=1}^3 w_g^{rh} \geq b_g^r; g \in G; \quad (2)$$

corresponding restrictions for the sector of transport ( $i = \tau$ ):

$$x_\tau^{r0} + x_\tau^{r1} - \sum_{j=1}^n a_{\tau j}^{r0} x_j^0 - \sum_{j=1}^n a_{\tau j}^{r1} x_j^1 - \alpha_\tau^r z^r - \sum_{k,s:k \neq s} a_{\tau j}^{ks} x_j^{ks} - \sum_{h=1}^3 \sum_{j=1}^n c_{\tau j}^{rwh} v_j^{rh} - \sum_{h=1}^3 \sum_{j=1}^n c_{\tau j}^{rwh} w_j^{rh} \geq b_\tau^r; \quad (3)$$

Balances of labor resources:

$$\sum_{j=1}^n t_j^{r0} x_j^0 + \sum_{j=1}^n t_j^{r1} x_j^1 \leq T^r; \quad (4)$$

Balances of investments:

$$\sum_{j=1}^n k_{gj}^{r0} x_j^0 + \sum_{j=1}^n k_{ji}^{r1} x_i^1 - f_1(u_g^{r0}, u_g^{r1}) \leq 0; g \in G; \quad (5)$$

Regional foreign trade balances:

$$\sum_{j=1}^n \beta_j^r v_j^r - \sum_{j=1}^n \gamma_j^r w_j^r \geq Q^r; \quad (6)$$

Restrictions on outputs and increases in outputs:

$$x_j^{r0} \leq d_j^{r0}; x_j^{r1} \leq d_j^{r1}; j = 1, \dots, n; \quad (7)$$

Restrictions on maximum and minimum exports and imports (quotas on imports and exports):

$$q_j \leq \sum_r v_j^r \leq \bar{q}_j; p_j \leq \sum_r w_j^r \leq \bar{p}_j; j = 1, \dots, n; \quad (8)$$

#### System-wide restrictions

Restrictions on territorial pattern of final demand

$$z^r - \alpha^r z \geq 0; r = 1, \dots, R \quad (9)$$

Restrictions on maximum and minimum exports and imports:

$$\sum_{r=1}^R \sum_{j=1}^n \beta_j^r v_j^r - \sum_{r=1}^R \sum_{j=1}^n \gamma_j^r w_j^r \geq Q; \quad (10)$$

**Objective function:**

$$z \rightarrow \max \quad (11)$$

**List of symbols:****Variables:**

$x_i^{r0}$  - output of sector  $i$  of region  $r$  produced in the last year of forecasting period on production facilities that worked at the beginning of the period;

$x_i^{r1}$  - increase in output of sector  $i$  in region  $r$  for the period;

$x_i^{rs}$  - transportation of products of sector  $i$  from region  $r$  to region  $s$  in the last year of forecasting period;

$x_i^{sr}$  - transportation of products of sector  $i$  from region  $s$  to region  $r$  in the last year of forecasting period;

$Z^r$  - value of final demand of region  $r$  in the last year of forecasting period;

$v_i^{rh}$  - export of products of sector  $i$  of region  $r$  in the last year of forecasting period in direction  $h$ ;

$w_i^{rh}$  - import of products of sector  $i$  of region  $r$  in the last year of forecasting period in direction  $h$ ;

$u_g^{r1}$  - gross investment of region  $r$  in the last year of the period (in the part of capital-forming sector  $g$ ) that are calculated as a sum of investments in the base year  $u_g^{r0}$  and increases in investments

$$\sum_{k=1}^T \Delta u_g^{r0}(k) \quad (T - \text{length of the period});$$

$z$  - a value of maximized part of final demand in the last year of the period;

$\alpha^r$  - a share of region  $r$  in maximized part of final demand in the last year of the period.

**Parameters:**

$a_{ij}^{r0}$  - input-output coefficients providing a value of output of sector  $j$  of region  $r$  in the last year of the period not exceeding a base value;

$a_{ij}^{r1}$  - input-output coefficients providing an increase in output of sector  $j$  of region  $r$  over the period;

$\alpha_i^r$  - a share of products (services) of sector  $i$  of region  $r$  in maximized part of final demand in the last year of the period;

$a_{vj}^{ks}$  - transport costs of region  $r$  for transportation of a product unit of sector  $j$  from region  $k$  to region  $s$  in the last year of the period;

$b_i^r$  - fixed part of final demand of sector  $i$  of region  $r$  in the last year of the period;

$c_{vj}^{r,h}$  - transport costs of exporting a product unit of sector  $j$  of region  $r$  in the last year of the period by direction  $h$ ;

$c_{vj}^{r,wh}$  - transport costs of importing a product unit of sector  $j$  of region  $r$  in the last year of the period by direction  $h$ ;

$t_i^{r0}$  - labour coefficients providing a value of output of sector  $i$  of region  $r$  in the last year of the period not exceeding a base value;

$t_i^{r1}$  - labour coefficients in the last year of the period providing an increase in output of sector  $i$  of region  $r$  over the period;

$k_{gi}^{r0}$  - capital coefficients maintaining output of sector  $i$  of region  $r$  over the period on the level attained in the base year (in the part of costs of capital-forming sector  $g$ );

$k_{gi}^{r1}$  - capital coefficients providing an increase in output of sector  $i$  of region  $r$  for the period (in the part of costs of capital-forming sector  $g$ );

$u_g^{r0}$  - base value of investment in the part of costs of capital-forming sector  $g$  of region  $r$ ;

$f(u_g^{r0}, u_g^{r1})$  - dependence function of total investment of region  $r$  for the period on its base value and its value attained in the last year of the period (for a given law of growth);

$\beta_i^r$  - coefficients converting domestic basic prices (in rubles) into foreign market prices (in dollars) for products of sector  $i$  exported from region  $r$  in the last year of the period;  
 $\gamma_i^r$  - coefficients converting domestic basic prices (in rubles) into foreign market prices (in dollars) for products of sector  $i$  imported by region  $r$  in the last year of the period;  
 $T^r$  - restrictions on number of labour resources of region  $r$  in the last year of the period;  
 $Q^r$  - restrictions on trade balance of region  $r$  in the last year of the period;  
 $d_i^{r0}, d_i^{r1}$  - restrictions on values of output variables and on increases in outputs of region  $r$  in the last year of the period;  
 $q_i, \bar{q}_i$  - maximum and minimum exports of products of sector  $i$  in the last year of the period;  
 $p_i, \bar{p}_i$  - maximum and minimum imports of products of sector  $i$  in the last year of the period;  
 $Q$  - restriction on value of national trade balance in the last year of the period.