Matrix Homothety and GLS-based Extension of RAS Method

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RAS method is based on parametrical multiplicative model of updating matrix element to provide a similarity (or closeness etc.) between target and initial matrices. In general it implicates that initial matrix homothety with its center in null matrix and some scalar ratio is considered according to RAS logic as an etalon for target matrix. Unfortunately, homothetic transformation does not allow to obtain target matrix with both (row and column) marginal totals given, but there is unique possibility to use matrix homothety concept for constructing the weakened version of target matrix with given sum of all elements. In accepting RAS multiplicative pattern of similarity one may suppose the initial matrix multiplied by proper homothety ratio estimate to be the best approximation of required target matrix.

The basic idea of this paper is to introduce a matrix of unknown (and independent) factors and to consider its Hadamard product with weakened version of target matrix in the framework of generalized multiplicative model. Rather natural way to satisfy row and column total constraints is to disturb a factor matrix in some minimalistic manner, e.g. in accordance with GLS principles. The result is a mathematical programming problem with quadratic goal function and two sets of linear constraints. In contrast to seemingly similar Harthoorn and van Dalen method (1987), the proposed goal function summarizes the squares of relative factor deviations from a priori unknown constant level.

The developed optimization model is fully represented in matrix notation, and its analytical solution is obtained in form suitable for sensitivity analysis implementation in the cases when the constraints are assumed non-binding. Therefore the paper contains a variety of mathematical details including algorithmic scheme for updating partitioned matrices with very large dimensions. In practice the proposed GLS-based method generates much more compact factor distributions in comparison with RAS method.