Comparative analysis of methods for assessing the value transfer in the formation of the final product

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1. Abstract

The aim of the research is a comparative analysis of different methods for assessing the transfer of the value added in the formation of the final product. This issue in terms of content is reduced to estimating degree of participation of domestic and foreign manufacturers in products' value creation for consumption, capital formation and exports. An overview of methods to solve this problem on the basis of input-output model is made in the paper. Russian researchers' approaches are also considered among the methods for assessing the transfer of value added. Review of methods is accompanied with an identification of degree of difference between the approaches based on mathematical analysis of Leontief inverse properties for NIOTs and WIOTs. Empirical calculations are also performed to assess the value added structure of the final domestic products through national and world input-output tables in the example of Russia. world input-output model is based on the WIOD. These calculations require appropriate modification of Russian input-output tables. Symmetric input-output tables are published as product-by-product tables in Russia, while WIOTs are published as industry-by-industry tables in WIOD. Therefore, the Russian symmetric tables are converted to the industry-by-industry form. Analysis of the results of calculations performed by various methods, allows us to decompose the cost of the final product on the value added produced by groups of domestic and foreign manufacturers.

The calculations show that the dependence on import of Russian final consumption and accumulation is significantly higher than assumed in assessing it on a gross basis . But research methods based on the global Input-Output models, show that the dependence on import of Russian economy is not as high as Russian researchers claim.

Due to the growing complexity of market relations within and between countries, the issue of estimation and calculation of individual components in the value of products becomes a matter of increasing importance in order to have an actual picture of economic relations. Previously, the proportion of goods produced from parts supplied from abroad, was extremely low. Nowadays the situation is diametrically opposite. There are very few industries that do not use foreign parts or components in production.

There are many variations of detail and assess of value of products, especially value added (VA), generated inside and outside of a country. They differ in the source data used, the degree of detail of the data and used input-output (IO) model. We consider various methods based on world and national IO models using as an information base world input-output tables for 41 countries and 35 industries (WIOT) and Russian national tables for 35 sectors (NIOT) in the form of industry - by - industry published in the World Input-Output Database (WIOD). Two approaches to assessing the value-added and its import part are analyzed in this paper:

1) Method based on national IO tables. Russian researchers [2].

2) Method based on world IO tables. European researchers [3].

2. **Kinds of IO models**

1) World IO models

Consider the mathematical description of the world input-output. model, as well as the model of national economy, based on IO tables. Table 1 shows the framework of the world IO table in the following notation:

x_{ij}^{rs} — the i-th industry production, imported from the country r to the country s production for j-th industry;

$\bar{y}_i^{rs}$— the i- th industry production, imported from the country r to the country s for final demand;

x_i^r — gross output of the i-th industry in the country r.

The sum of each row equals the gross output of the i-th industry in the r-th country: x_i^r in Table 1

This ratio can be written as an equation:

$$x_i^r = \sum_{s=1}^{m}\sum_{j=1}^{n} x_{ij}^{rs} + \sum_{s=1}^{n} \bar{y}_i^{rs}\ (i = 1, \dots, n; r = 1, \dots, m), \qquad (1)$$

where n denotes the number of industries , m - number of countries . Equation (1) includes three groups of variables, the amount of which exceeds the number of equations and therefore it is difficult to apply predictive and analytical calculations. The transition to a mathematical model requires certain assumptions, as in the case of IO model for the national economy.

Depending on the assumptions, we obtain different Input-Output models. In this article we consider two kinds of world IO models: Inter-Regional Input-Output model (IRIO) and Multi-Regional Input Output model (MRIO) – a model with trading coefficients.

A) World models.. IRIO

Main assumption is - a linear dependence of input and output $\frac{x_{ij}^{rs}}{x_j^s} = a_{ij}^{rs}$.. Each industry of each country is considered as an independent industry, i.e for m countries and n industries this is an analogue of national balance for *m*x*n* industries.

After the adoption of the above assumptions equation (1) can be rewritten as follows:

$$x_i^r = \sum_{s=1}^{m}\sum_{j=1}^{n} a_{ij}^{rs} x_j^s + \sum_{s=1}^{m} \bar{y}_i^{rs}, \qquad (i = 1, \dots, n; r = 1, \dots, m) \qquad (2)$$

This can be written in matrix form:

$$\text{X} = \text{AX} + \text{f} \qquad (3)$$

Where x denotes a *nm* vector of gross output (with m being the number of countries and *n* the number of
industries considered). A denotes the mn x mn matrix with each element
denoting the input used in a particular industry in one country per unit of gross output. f is the nxm vector of final demand. Being solved this system of equations looks as follows:

$$\text{X} = \text{Lf} \qquad (4)$$

Where I denotes the identity matrix. $L = (\text{I} - \text{A})^{-1}$ denotes the matrix of full costs, and the vector f with elements $f_i^r = \sum_{s=1}^{m} \bar{y}_i^{rs}$ is the vector of final demand.

Б) World models. MRIO.

The first assumption - is a linear dependence of output and input, i.e. $\frac{x_{ij}^s}{x_j^s} = a_{ij}^s$, where x_{ij}^s is the cost of i-th industry product (domestic and imported) per j-th industry output in the country s, a_{ij}^s are the coefficients of direct costs in the country s.

The second assumption is the constancy of the so-called trade coefficients. $\text{g}_\text{i}^\text{rs}$ denote the trade coefficients, reflecting the proportion of r- th country in the total consumption (manufacturing and non-manufacturing) of i-th industry products in the country s. According to the definition of the trade coefficients, the following equalities must take place:

$$\sum_{\text{r}=1}^{\text{m}} \text{g}_\text{i}^\text{rs} = 1 \quad (\text{i} = 1, \dots, \text{n})$$

Trade coefficients are calculated from formulas

$$\text{g}_\text{i}^\text{rs} = \frac{\text{z}_\text{i}^\text{rs}}{\text{z}_\text{i}^\text{s}}$$

where

$$z_i^{rs} = \sum_{j=1}^{n} x_{ij}^{rs} + \bar{y}_i^{rs}$$

z_i^{rs} denotes product volume of i-th industry of the country r imported to the country s for intermediate consumption and final demand;

$$z_i^{s} = \sum_{r=1}^{m}\sum_{j=1}^{n} x_{ij}^{rs} + \sum_{r=1}^{m} \bar{y}_i^{rs}$$

z_i^{s} denotes total output of i-th industry consumed in the country s (as an intermediate consumption and final demand).

Z_j^s - GVA produced by the j-th industry of s-th country: $Z_j^s = x_j^s - z_j^s$ (gross value added).

Thus:

$$g_i^{rs} = \frac{\sum_{j=1}^{n} x_{ij}^{rs} + \bar{y}_i^{rs}}{\sum_{r=1}^{m}\sum_{j=1}^{n} x_{ij}^{rs} + \sum_{r=1}^{m} \bar{y}_i^{rs}}$$

Assumption of a linear dependence of input and output, and the constancy of the trade coefficients are realistic enough, so the resulting model is useful in practice. After the adoption of the above assumptions equation (1) can be rewritten as:

$$x_i^r = \sum_{s=1}^{m} g_i^{rs} \left(\sum_{j=1}^{n} a_{ij}^s x_j^s + \bar{y}_i^s\right) = \sum_{s=1}^{m}\sum_{j=1}^{n} g_i^{rs} a_{ij}^s x_j^s + \sum_{s=1}^{m} g_i^{rs} \bar{y}_i^s \; (i = 1, \dots, n; r = 1, \dots, m) \quad (5)$$

In matrix form the MRIO model can be written as:

$$X = GA'X + G\bar{Y} \qquad (6)$$

where $G = (G^{rs})$ is a block-diagonal matrix (nm × nm), with each block G^{rs} being a diagonal matrix of coefficients g_i^{rs}. A ' is also a block-diagonal matrix X, $\bar{Y}$ – are the compositions of the country vectors $X^r, \bar{Y}^r$.

Solution of the matrix equation has the following form:

$$X = (G^{-1} - A')^{-1}\bar{Y} \qquad (7)$$

In its economic content B = $(G^{-1} - A')^{-1}$ – is the matrix of full costs per unit of final demand.

Vectors of gross output X used in IRIO model and in MRIO model – are absolutely identical. But the vectors of final demand f and $\bar{Y}$ are different. In the first model we use production of goods within the country used at the final demand of all countries $f_i^s = \sum_{r=1}^{m} \bar{y}_i^{sr}$.

In the second model we deal with the final demand of products from all countries by the considered country $\bar{y}_i^s = \sum_{r=1}^{m} \bar{y}_i^{rs}$..

As it can be seen, matrices of the full costs in IRIO and MRIO models - have significant differences, though derived from the same input-output table. Vectors of final demand f and $\bar{Y}$ are not the same, too.

Both input-output models presented contain *n* x *m* equations and 2*n* x *m* variables: *m*x*n* variables $\mathbf{x}_i^r$ and *m*x*n* variables $\bar{\mathbf{y}}_i^r$. However, while the MRIO model is determined by the n * n * m (matrix A') + m * m * n (matrix G) parameters, the IRIO model uses n * n * m * m parameters (matrix A). So IRIO model has much more governing parametres than MRIO model. It should be noted, that final demand $\bar{\mathbf{y}}_i^r$ differs from the final demand of i-th industry in the national input-Output model for r-th country. The first indicator reflects the final consumption and accumulation of the i-th product in the r-th country and the difference between them equals balance of exports and imports.

2) *National Input-Output with divided domesticand improted consumption.*

National Input-Output model consists of a National Input-Output Table (NIOT) divided into domestic and imported consumption, and the system of equations defined by this table. We will consider the case of industry-by-industry NIOT. Table 2 shows the framework of the national balance sheet:

the following notation:

Xn_{ij} – is the volume of domestic i-th industry product, supplied for j-th industry. For the r-th country it equals: $Xn_{ij} = x_{ij}^{rr}$.
IMn_{ij} – is the volume of imported i-th industry product, supplied for j-th industry. For the r-th country it equals: $IMn_{ij} = \sum_{s=1}^{m} x_{ij}^{sr}$.
$IMki_i$ – – is the volume of imported i-th industry product, supplied for final demand. For the r-th country it equals:$IMki_i = \sum_{s=1, s\neq r}^{m} \bar{y}_i^{sr}$
Z_j – is Gross value added of the i-th industry. For the r-th country it equals $Z_j = Z_j^r$
$\bar{y}_i$ – i-th industry domestic product for final demand. For the r-th country it equals: ($\bar{y}_i = \bar{y}_i^{rr}$)
E_i – is the export of the i-th industry product. For the r-th country it equals: $\sum_{s=1,s\neq r}^{m}(\sum_{j=1}^{n} x_{ij}^{rs} + \bar{y}_i^{rs})$
KIo_i – is the volume of i-th industry domestic product, supplied for final uses. For the r-th country it equals:$KIo_i = \sum_{s=1}^{m} \bar{y}_i^{rs}$
y_i – is final uses of i-th industry product. $y_i = \bar{y}_i + E_i - \sum_{j=1}^{n} IMn_{ij}$.
x_i – is i-th industry gross output. For the r-th country it equals: $x_i = x_i^r$.

In Table 2, the sum of the corresponding rows in domestic and imported blocks equals industry output x_i. This can be written as an equation:

$$x_i = \sum_{j=1}^{n} \left(Xn_{ij} + IMn_{ij}\right) + KIo_i + IMki_i = \sum_{j=1}^{n} x_{ij} + y_i \qquad (i = 1, \dots, n) \qquad (8)$$

where n is the number of industries. The transition to a mathematical model of the national economy requires certain assumptions.

The basic assumption - is the linear dependence of input and output (total - domestic and imported*):*

$$\frac{x_{ij}}{x_i} = \frac{Xn_{ij} + IMn_{ij}}{x_i} = a_{ij}$$

Where a_{ij} denotes the coefficients of direct costs - the amount of i-th industry product per unit of j-th industry output. As the result, we have the system of equations, where the variables are outputs x_i and final uses y_i.

$$x_i = \sum_{j=1}^{n} a_{ij} x_j + y_i \qquad (9)$$

In matrix form, it looks:

$$X = AX + Y \qquad (10)$$

A denotes the matrix of technical coefficients - direct costs. It has the same sense as in the IRIO model. The solution of this system looks as follows:

$$X = (I - A)^{-1} Y = LY \qquad (11)$$

Where L denotes the matrix of the full costs. b_{ij} denotes full cost of i-th industry product per unit of j-th industry final uses. Matrices L in the IRIO model and L in the national IO model have the same sense. In fact, the IRIO model considers the world economy as an economy of a country. Industries in different countries are considered as independent industries in one country.

3. Areas of Application of IO models.

Input-Output models are very widely used in the world, especially in problems related to the assessment of interference of different industries, regions, countries' economies, predicting possible consequences of different events.

Like IO model of the national economy, the world IO models allow realize forecast calculations for the three scenarios.

1) Calculation of possible values of final demand of all sectors in all countries for the given gross output;
2) Calculation of possible values of gross output of all sectors in all countries for the given final demand;
3) The so-called mixed scenario - exogenously given part of gross output and part of the final demand. The rest x_i^r and $\bar{y}_i^r$ are determined endogenously.

Even these three scenarios represent a powerful tool for evaluating mutual influence of different countries' economies. But international experience in the world IO model application has several options from international assessment of emissions of pollutants, to evaluation of the impact of natural disasters.

In addition, very often it is necessary to have information on the extent of participation of a country in the world gross output. Estimation of the extent of a country dependence on the world's final demand or gross output is no less important. IO tables, and specifically the World IO model make it easy to calculate such data. For example, in the global mining and quarrying the following 5 countries had the highest volume of production in 2010:

1 – China – 558,9 млрд $ (11,7% of world production)
2 – USA – 423,7 млрд $ (8,9% of world production)
3 – Australia – 227,5 млрд $ (4,8% of world production)
4 – Russia – 198,9 млрд $ (4,2% of world production)
5 – Canada – 124,7 млрд $ (2,6% of world production)

IO Table makes it possible to evaluate the possible consequences of a crisis or disaster in a particular country or group of countries. For example, if, a crisis happens in the USA, so the final demand of the USA falls by 10%, then the Russian gross output will decrease by 0.2 %, final demand will also be reduced by 0.2 %. In this case, the gross output in Mexico will fall by 1,6%. That is, a crisis in the U.S. will have significant negative influence on Mexico’s economy but it will have no real influence on Russian economy. This conclusion, of course is not amazing, as Russia's trade volume with the United States is several times less than the trade between United States and Mexico.

If such a crisis happens in China with a decrease in final demand by 10%, then the production of Russia will decrease by 0.18 % and final demand by 0.2%. I.e. such a crisis in China will also have also insignificant effect on Russian economy. This is true until the volume of trade between considered countries does not change dramatically.

Everyone knows about the latest incident in Ukraine. Now Europe is going to impose sanctions on Russia. One can use world IO model to evaluate the effects of different scenarios of these sanctions. In particular, using the model of the world MRIO IO Tables, changing only trade share in the matrix G, one can try to predict how the sanctions will influence on the Russian economy. We analyzed three possible scenarios:

1) All European countries will reduce the consumption of Russian mining and quarrying product by 10%, providing alternative gas and mineral supplies from the rest of the world. It's quite probable scenario, since to find alternative suppliers to provide 10% of consumed minerals and gas is a solvable problem, even if it has to be done for one year.

2) All European countries will reduce the consumption of fossil, produced by Russian mining and quarrying industry, by 30%, providing alternative gas and mineral supplies from the rest of the world. This problem is more difficult to realize because, for example, the supply of gas in such volumes require some infrastructure that is extremely difficult to provide earlier than in 3-5 years.

3) All European countries will simply stop the consumption of fossil mined by Russia, providing a complete replacement of suppliers from the rest of the world. To ensure complete replacement of the supplier, it is required quite voluminous investments, for example, in the construction of a new gas pipeline or the development of shale gas, which still have been

unprofitable. Therefore it is unlikely that it will take less than 10 years to implement this scenario.

All European countries are the EU 21 countries that consume the products of the Russian mining and quarrying industry more than $ 100 million a year. These are Austria, Belgium, Bulgaria, Czech Republic, Germany, Spain, Estonia, Finland, France, Great Britain, Greece, Hungary, Italy, Latvia, Lithuania, Netherlands, Poland, Romania, Slovakia, Slovenia, and Sweden.

The matrix of technical coefficients A ' will remain unchanged in all three cases, but the matrix of trade coefficients G will change. Table 3 shows the changes in trade coefficients for Russia and the rest of the world in these countries, depending on the scenario of sanctions.

Our aim is to assess the implications for Russia no matter how European countries provide alternative supplies. Analysis of the impact of sanctions on all of the countries that take part in it - is a topic for another study.

In the first case, the total output of Russia will decreases by 0.6%

Final demand will decrease by 0.5%.

In the second case output will decrease by 1.3%.

Final demand will decrease by 1.6%.

In the third - by 4.4%

Final demand will decrease by 5.2%.

It is clear from the results, that if one can implement the toughest sanctions scenario, Russia will incur significant losses. Another issue that is of great interest for another study - how it is possible to replace Russian gas and mineral supplies by other suppliers. All possible scenarios can be calculated using the World IO models.

Note also that due to trade shares matrix in the MRIO model, we can take into account the changes in gas supplies between the countries, and the impact of these changes on the matrix B. In this case, for the construction of such a forecast, we do not need to know the exact details of who in what volumes will deliver. To change the proportion of trade shares for corresponding countries is enough.

If we tried to use the IRIO model to calculate scenarios of sanctions, then we would have got a lot of problems. When the volumes of trade between countries changes, elements of the Leontief inverse (matrix L) change too. It is practically impossible to estimate how they change without accurate information, detailed by all industries and countries in the IO table. Accordingly, the calculations, based on the original Leontief inverse would cause a greater error. Even greater error would have appeared in the calculations, based on national IO model. Though it is quite possible, if there is an access to accurate information about exports and imports of all the countries, which take part in the sanctions scenarios.

4. Calculation of Value Added

Recently, the production process has become extremely complicated, multi-step. Factories producing certain materials, parts or components of different goods are scattered across the globe. Product price conditioned by the cost of materials, which are produced in different countries, form a chain of value added – a global value chain.

In this regard, the gross figures do not allow us to see the actual picture of the production of value added. Therefore special methods for the assessment of imported and domestic value added component in the product are required.

Consider two types of approaches:

A) Approaches based on NIOT.

National approach is based on NIOT with divided domestic and import consumption. The Federal State Statistics Service of the Russian Federation calculate product - by - product IO tables. For our calculations we used the Russian national industry - by - industry input-output tables for 35 sectors (NIOT) from WIOD.

We consider the method of the Russian economist L. Strizhkova [2]. It relies on the economic law of value transition. Since the intermediate consumption included in the cost of production can be decomposed into a gross value added (GVA) and intermediate inputs infinitely, the limit of the total GVA in the cost of production is equal to the valuation of the product itself.

Equations of the formation and the use of value (the sum of the column and the amount of rows in the table of the national MPS) one obtains the following:

$$y_i = KIo_i - IMpp_i$$

$$IMpp = IM * X$$

Where $IMpp_i = \sum_{j=1}^{n} IMn_{ij} = \sum_{j=1}^{n} \sum_{s=1}^{m} x_{ij}^{sr}$ – is the vector of intermediate consumption of imports. IM denotes the matrix of import intermediate consumption with elements IMn_{ij}.

In addition, we have IO equations:

$$X = B * Y = (E - A)^{-1} Y$$

Hence we obtain:

$$X = (E - A + IM)^{-1} KIo$$

Now if the vector of final domestic uses Kio is diagonalized, then we obtain the matrix of gross output decomposition by final domestic uses of industries:

$$X_{KIo} = (E - A + IM)^{-1} \widehat{KIo} = \hat{B} * \widehat{KIo}$$

Where the sum in the j-th column shows the full cost of domestic products of all types, used for providing the existing volume of final domestic uses of j-th industry product KIo_j,, and the sum of the i-th row always equals the gross output of the i-th industry x_i.

Multiplying the matrix of direct import costs IM by the matrix of output decomposition X_KIo, we obtain the matrix of the import consumption structure $\boldsymbol{IMf}$ with the elements imf_{ij}:

$$imf_{ij} = \sum_{k=1}^{n} (im_{ik} * \ b'_{kj}) * KIo_j$$

$$IMf = IM * X_{KIo}$$

Where b'_{kj} – are the elements of the matrix $\hat{B} = (E - A + IM)^{-1}$

Summing up the elements of this matrix, we obtain the import content of value added in final domestic uses imf.

$$imf = \sum_{i=1}^{n} \sum_{j=1}^{n} imf_{ij}$$

But as the vector consisting of the sum of rows of the matrix X_{KIo} equals the vector of outputs X, and $IMf = IM * X_{KIo}$ is a linear expression, we find that the sum of lines IMf gives us the vector $IMpp = IM * X$. Thus, it appears that the import content of value added in the cost of the final domestic product is the sum of total imports in intermediate consumption by all industries of the country.

$$\boldsymbol{imf} = \sum_{i=1}^{n} \sum_{j=1}^{n} \boldsymbol{im_{ij} X_j} = \sum_{i=1}^{n} \sum_{j=1}^{n} \boldsymbol{IM}\text{H}_{\boldsymbol{ij}}$$

At the same time, the import value added component of imported products, received in final uses, is considered equal to the total value of imported goods. Actually, the same is true for imports in intermediate consumption. Thus, a full evaluation of the imported value added used in the country's economy is

$$\boldsymbol{t_{TiVA.M}} = \sum_{i=1}^{n} \sum_{j=1}^{n} \boldsymbol{IM}\text{H}_{\boldsymbol{ij}} + \sum_{i=1}^{n} \boldsymbol{IMki_i}$$

Where $t_{TiVA.M}$ denotes import value added in the economy of a country.

To make the application of the method possible one should have national Input-Output tables with separate domestic and imported consumption, as well as information on imports, exports of a country.

The advantages of this approach include such points as:

- Details: any necessary details relating to the assessment of imported and domestic value added as part of products produced by the domestic industry can be calculated using this method.

- The minimum required information: enough to have the information from only one country and its relations with the outside world.

The main drawback of this approach is that:

- A distribution of exports is not taken into consideration. Method will provide acceptable accuracy only for those countries that do not participate in the multiple re-export process. I.e. The country exports its products (eg, raw materials), which enters the intermediate consumption of another country, then , the latter delivers its products (products for further production, such as machines), created on the basis of the purchased raw materials, into the intermediate consumption in the first country. If we evaluate the value-added by Strizhkova's method, the import content of the value added is higher than the actual one, because the method does not take into account the added value created in the first country, transferred into the second country, and then returned back to the first one.

2) Approaches, based on WIOT

Another approach based on world input-output tables is proposed by Stehrer in [3]. The total value added for all the countries is calculated there. Then, only those components are taken into account, which relate to the production, located outside of the particular country.

To implement this approach, it is necessary to have information on all the countries and their trading relationships, ie to have the World Input-Output Tables (WIOT). This approach allows taking into account the important details of cross-country trade relations, such as a re-export.

When we have information about all the countries, we can introduce the assumption that the value added of the i-th industry in the r-th country is proportional to the output of the industry in this country: $Z_j^s = v_j^s x_j^s$. So when the i-th industry of the r-th country supplies products, valued at x_{ij}^{rs} to the j-th branch of the s-th country, $v_i^r x_{ij}^{rs}$ is the value added, which is contained in the products.

The key equation – is the equations of IO models, where A denotes the matrix of technical coefficients (direct costs per unit of output), and B - the matrix of full costs per unit of final demand:

$$\boldsymbol{X} = \boldsymbol{AX} + \boldsymbol{f} = \boldsymbol{Lf}$$

Where f denotes the *m*x*n* column vector of final demand. Vector f equals a sum of *m*x*n* column vectors of final demand for all the countries:

$$\boldsymbol{f} = \sum_{s=1}^{m} \boldsymbol{f}^{s}$$

The vector of output X^k, needed to provide the final demand of the k-th country f^k equals:

$$\boldsymbol{X}^{k} = \boldsymbol{L}\boldsymbol{f}^{k}$$

Of course:

$$\boldsymbol{X} = \sum_{k=1}^{m} \boldsymbol{X}^{k}$$

The import share of VA sufficient to provide the final demand f^k of the i-th country equals:

$$\boldsymbol{t^k_{TiVA.M}} = \boldsymbol{v^{-k} X^k} = \boldsymbol{v^{-k} L f^k}$$

Where $\boldsymbol{v^{-k}}$ denotes the value added coefficients vector with zeros for country *r* and non-negative entries for the other countries:. $\boldsymbol{v^{-k}}{}^{\boldsymbol{k}}_{\boldsymbol{i}} = 0$; $\boldsymbol{v^{-k}}{}^{\boldsymbol{r}}_{\boldsymbol{i}} = \boldsymbol{v^r_i}$ for $\boldsymbol{r \neq k}$**.** Thus, we consider only that part of the VA that is produced outside the country k.

$$\boldsymbol{t^k_{TiVA.M}} = \boldsymbol{v^{-k} L f^k}$$

Consider the example of the three countries. Without loss of generality, we calculate the import value added for the first country:

$$\boldsymbol{v^{-1}} = [\boldsymbol{0}, \boldsymbol{v^2}, \boldsymbol{v^3}] \quad \boldsymbol{L} = \begin{pmatrix} \boldsymbol{L^{11}} & \boldsymbol{L^{12}} & \boldsymbol{L^{13}} \\ \boldsymbol{L^{21}} & \boldsymbol{L^{22}} & \boldsymbol{L^{23}} \\ \boldsymbol{L^{31}} & \boldsymbol{L^{32}} & \boldsymbol{L^{33}} \end{pmatrix} \qquad \boldsymbol{f^1} = \begin{bmatrix} \boldsymbol{\overline{Y}^{11}} \\ \boldsymbol{\overline{Y}^{21}} \\ \boldsymbol{\overline{Y}^{31}} \end{bmatrix}$$

$$\begin{aligned} \boldsymbol{t^1_{TiVA.M}} &= \boldsymbol{v^{-1} X^1} = \boldsymbol{v^{-1} L \overline{Y}^1} \\ &= (\boldsymbol{v^2 L^{21}} + \boldsymbol{v^3 L^{31}})\boldsymbol{\overline{Y}^{11}} + (\boldsymbol{v^2 L^{22}} + \boldsymbol{v^3 L^{32}})\boldsymbol{\overline{Y}^{21}} + (\boldsymbol{v^2 L^{23}} + \boldsymbol{v^3 L^{33}})\boldsymbol{\overline{Y}^{31}} \end{aligned}$$

The first term refers to the value added generated in the 2 and 3 countries to meet the demand for domestic products in the first country. Second and third terms represent the value added generated in the 2 and 3 countries to meet the final demand in the first country to import products from the 2 and 3 countries.

If the same method is applied to the MRIO model, then the formula is almost the same:

$$\boldsymbol{t^k_{TiVA.M}} = \boldsymbol{v^{-k} X^k} = \boldsymbol{v^{-k} B \overline{Y}^k}$$

Except the moment that direct cost matrix B and vector $\boldsymbol{\overline{Y}^k}$ in the MRIO model differ from L and $\boldsymbol{f^k}$ accordingly. In the same example, for 3 countries:

$$\boldsymbol{\overline{Y}^1} = \begin{bmatrix} \boldsymbol{\overline{Y}^{11}} + \boldsymbol{\overline{Y}^{21}} + \boldsymbol{\overline{Y}^{31}} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{t^1_{TiVA.M}} = \boldsymbol{v^{-1} X^1} = \boldsymbol{v^{-1} B \overline{Y}^1} = \boldsymbol{v^2 B^{21}}(\boldsymbol{\overline{Y}^{11}} + \boldsymbol{\overline{Y}^{21}} + \boldsymbol{\overline{Y}^{31}}) + \ \boldsymbol{v^3 B^{31}}(\boldsymbol{\overline{Y}^{11}} + \boldsymbol{\overline{Y}^{21}} + \boldsymbol{\overline{Y}^{31}})$$

The first term refers to the value added generated in the 2-nd country to meet the demand for domestic and imported products in the first country. The second term refers to the third country.

The main advantage of this approach is that it takes into account all flows of value added, including re-export and re-import. However, in this case, to calculate the specific details, such as the distribution of import value added to the final and intermediate consumption, we need to use data from the national table. Although, if we have access to the global table, the information contained in the national table, is also available to us.

5. Mathematical and empirical comparison of methods.

1) Mathematical comparison

Let’s consider 2 key expressions for estimating the import part of the value added. There are 2 approaches::

Russian:

$$X_{KIo} = \hat{B} * \widehat{KIo}$$

WIOD:

$$X^k = B\bar{Y}^k$$

If we write out vectors X^k by columns and gather them in the matrix $\hat{X}$, then we get the same meaning as that of the X_{KIo}. I.e. principles of determining the structure of the output distribution are the same.

$$\hat{X} = B\hat{Y}$$

But at the same time, the vector of final demand differs from the vector of domestic final uses, as the vector of domestic final uses *KIo* includes exports of domestic products and the vector of final demand f, does not include exports.

The key difference between the methods is a way to determine the structure of industries’ value added. There is no possibility to calculate the vector of value added coefficients outside the country $\boldsymbol{v^{-k}}$**, i**n the national IO model. In the approach, based on world IO models this vector plays a decisive role.

$$\boldsymbol{t^k_{TiVA.M} = v^{-k}X^k = v^{-k}B\bar{Y}^k}$$

Thus, the approach based on the global IO models allows one to calculate how much of value added that is necessary for final demand of the particular country, is produced by foreign countries But at the same time, the value added, necessary to provide the country’s exports, which is a part of the country’s final uses, is not considered.

And the approach based on the national IO model allows us to calculate how much of value added must be received in the product form to provide an appropriate intermediate consumption within the country needed to provide the final demand for the products in the particular country.

2) Empirical comparison

We calculated the value-added of Russian products at 2010 year and got the following results:

- According to the method, based on national IO model, value added of imported goods amounted $ 280 billion. 93.6 of them were used in the intermediate consumption and 186.4 - in final demand.

- According to the method based on the IRIO model, value added of imported products in Russia amounted $ 243.8 billion.

- According to the method, based on the MRIO model, value added of imported products in Russia amounted to $ 240.2 billion.

It’s impossible to miss that the methods, based on the world IO models, give less value added, than the method, based on the national IO model. This is natural, because in the method, based on the national model, entire cost of imported products is considered as imported value added.

Moreover, the method, based on the world IO models, considers re-export of the value added produced in Russia, which, as it turns out, forms a significant percentage of the total value added imported into Russia.

6. Conclusions.

This article provides a comparative analysis of methods for assessing the import value added in the economy, as well as a comparative analysis of input-output model , which is based on the methods described .

Current approaches based on the national balance sheet, are obviously not accurate enough, since they do not take into account a re-export. In today's globalization, such neglect cause a significant error.

Approaches based on world IO models, provide a greater accuracy, since they take into account a re-export products of all the countries. However, they are also not perfect, since the calculation does not take into account all the components (export) of final uses of a country.

The only significant advantage of methods based on the national model is the minimum information necessary for calculations, as it is not always possible to obtain data on other countries. If we have an access to the global IO tables, the national IO model as part of the world is also available. Therefore it is better and preferable to use methods based on the world models.

Moreover, in the modern era of information it will be impossible without the use of global Input-Output models soon. Models that allow interactively calculate tconsequences of any life, political or economic events in the world. In this respect, the MRIO model is much more convenient and easier to use, since it contains less coefficients and allows one to simulate many situations , without dipping into the minor details.

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Table 1

World input-output table

x_{11}^{11}	… x_{1j}^{11}	… x_{1n}^{11}	… … …	x_{11}^{1s}	… x_{1j}^{1s}	… x_{1n}^{1s}	… … …	x_{11}^{1m}	… x_{1j}^{1m}	… x_{1n}^{1m}	$\bar{y}_1^{11}$	… $\bar{y}_1^{1s}$	… $\bar{y}_1^{1m}$	x_1^1
x_{21}^{11}	… x_{2j}^{11}	… x_{2n}^{11}	… … …	x_{21}^{1s}	… x_{2j}^{1s}	… x_{2n}^{1s}	… … … .	x_{21}^{sr}	… x_{2j}^{rs}	… x_{2n}^{rs}	$\bar{y}_2^{11}$	… $\bar{y}_2^{1s}$	… $\bar{y}_2^{1m}$	x_2^1
…	…	…		…	…	…					…	…	…	
x_{i1}^{r1}	… x_{ij}^{r1}	… x_{in}^{r1}	… … …	x_{i1}^{rs}	… x_{ij}^{rs}	… x_{in}^{rs}	… … …	x_{i1}^{rs}	… x_{ij}^{rs}	… x_{in}^{rs}	$\bar{y}_i^{rs}$	… $\bar{y}_i^{rs}$	… $\bar{y}_i^{rs}$	x_i^r
…	…	…		…	…	…					…	…	…	
x_{n1}^{rs}	… x_{nj}^{rs}	… x_{nn}^{rs}	… … …	x_{n1}^{rs}	… x_{nj}^{rs}	… x_{nn}^{rs}	… … …	x_{n1}^{rs}	… x_{nj}^{rs}	… x_{nn}^{rs}	$\bar{y}_n^{rs}$	… $\bar{y}_n^{rs}$	… $\bar{y}_n^{rs}$	x_n^r
…	…	…	… … …	…	…		… … …				…	…		
x_{11}^{m1}	… x_{1j}^{m1}	… x_{1n}^{m1}	… … …	x_{11}^{ms}	… x_{1j}^{ms}	… x_{1n}^{ms}	… … …	x_{11}^{mm}	… x_{1j}^{mm}	… x_{1n}^{mm}	$\bar{y}_1^{m1}$	… $\bar{y}_1^{ms}$	… $\bar{y}_1^{mm}$	x_1^m
x_{21}^{m1}	… x_{2j}^{m1}	… x_{2n}^{m1}	… … …	x_{21}^{ms}	… x_{2j}^{ms}	… x_{2n}^{ms}	… … …	x_{21}^{mm}	… x_{2j}^{mm}	… x_{2n}^{mm}	$\bar{y}_2^{m1}$	… $\bar{y}_2^{ms}$	… $\bar{y}_2^{mm}$	x_2^m
…	…	…		…	…	…					…	…	…	
x_{i1}^{m1}	… x_{ij}^{m1}	… x_{in}^{m1}	… … …	x_{i1}^{ms}	… x_{ij}^{ms}	… x_{in}^{ms}	… … …	x_{i1}^{mm}	… x_{ij}^{mm}	… x_{in}^{mm}	$\bar{y}_i^{m1}$	… $\bar{y}_i^{ms}$	… $\bar{y}_i^{mm}$	x_i^m
…	…	…		…	…	…					…	…	…	
x_{n1}^{m1}	… x_{nj}^{m1}	… x_{nn}^{m1}	… … …	x_{n1}^{ms}	… x_{nj}^{ms}	… x_{nn}^{ms}	… … …	x_{n1}^{mm}	… x_{nj}^{mm}	… x_{nn}^{mm}	$\bar{y}_n^{m1}$	… $\bar{y}_n^{ms}$	… $\bar{y}_n^{mm}$	x_n^m
Z_1^1	… Z_j^1	… Z_n^1	… … …	Z_1^s	… Z_j^s	… Z_n^s	… … …	Z_1^m	… Z_j^m	… Z_n^m				

Table 2

National Input-Output Table with divided domestic and imported consumption

Xn_{11}	Xn_{12}	…	Xn_{1j}	…	Xn_{1n}	$\bar{y}_1$	E_1	x_1
Xn_{21}	Xn_{22}	…	Xn_{2j}	…	Xn_{2n}	$\bar{y}_2$	E_2	x_2
	…		…	…			…	
Xn_{i1}	Xn_{i2}	…	Xn_{ij}	…	Xn_{in}	$\bar{y}_i$	E_i	x_i
	…		…	…			…	
Xn_{n1}	Xn_{n2}	…	Xn_{nj}	…	Xn_{nn}	$\bar{y}_n$	E_n	x_n
IMn_{11}	IMn_{12}	…	IMn_{1j}	…	IMn_{1n}	$IMki_1$		
IMn_{21}	IMn_{22}	…	IMn_{2j}	…	IMn_{2n}	$IMki_2$		
	…		…	…		…		
IMn_{i1}	IMn_{i2}	…	IMn_{ij}	…	IMn_{in}	$IMki_i$		
	…		…	…		…		
IMn_{n1}	IMn_{n2}	…	IMn_{nj}	…	IMn_{nn}	$IMki_n$		
Z_1	Z_2	…	Z_j	…	Z_n			

Table 3

Forecast Scenarios

		Trade shares of Russia (RUS) & the Rest of the World (RoW)							
		No Sanctions		Sanctions 10%		Sanctions 30%		Sanctions 100%	
№	Список стран:	RUS	RoW	RUS	RoW	RUS	RoW	RUS	RoW
1	Austria	0,10	0,16	0,09	0,17	0,07	0,19	0	0,26
2	Belgium	0,03	0,16	0,03	0,17	0,02	0,17	0	0,20
3	Bulgary	0,42	0,18	0,38	0,22	0,30	0,31	0	0,61
4	Czech Republic	0,18	0,16	0,17	0,17	0,13	0,21	0	0,34
5	Germany	0,14	0,29	0,13	0,30	0,10	0,33	0	0,43
6	Spain	0,06	0,66	0,05	0,67	0,04	0,68	0	0,72
7	Estonia	0,23	0,03	0,21	0,05	0,16	0,10	0	0,26
8	Finland	0,46	0,14	0,41	0,18	0,32	0,27	0	0,59
9	France	0,18	0,45	0,16	0,46	0,13	0,50	0	0,63
10	Great Britain	0,03	0,37	0,02	0,37	0,02	0,38	0	0,40
11	Greece	0,28	0,50	0,25	0,53	0,20	0,59	0	0,78
12	Hungary	0,72	0,03	0,65	0,10	0,51	0,25	0	0,75
13	Italy	0,19	0,62	0,17	0,64	0,13	0,68	0	0,81
14	Lithuania	0,81	0,07	0,73	0,15	0,57	0,32	0	0,88
15	Latvia	0,57	0,04	0,51	0,10	0,40	0,21	0	0,61
16	Netherlands	0,21	0,47	0,19	0,49	0,14	0,53	0	0,67
17	Poland	0,29	0,15	0,26	0,18	0,20	0,24	0	0,44
18	Romania	0,18	0,19	0,16	0,21	0,13	0,24	0	0,37
19	Slovakia	0,57	0,05	0,51	0,10	0,40	0,22	0	0,61
20	Slovenia	0,14	0,17	0,13	0,19	0,10	0,22	0	0,32
21	Sweden	0,18	0,18	0,17	0,19	0,13	0,23	0	0,36
Change in Russian GDP		**0%**		**-0,5%**		**-1,6%**		**-5,2%**	