P-Graph Approach for the Optimal Allocation of Human Resources to Economic Sectors in Crisis Conditions

K. B. Aviso[a,b], C. D. Cayamanda[a,b], A. M. R. Danga[b], M. A. B. Promentilla[a,b] , K. D. S. Yu[b,c], J. R. Santos[d] and R. R. Tan[a,b]

[a] *Chemical Engineering Department, De La Salle University, 2401 Taft Avenue, Malate, Manila, 1004, Philippines*
[b] *Center for Engineering and Sustainable Development Research, De La Salle University, 2401 Taft Avenue, Malate, Manila, 1004, Philippines*
[c] *School of Economics, De La Salle University, 2401 Taft Avenue, Malate, Manila, 1004, Philippines*
[d] *Department of Systems Engineering, The George Washington University*

Abstract

The impact of disasters manifests not only in the destruction of infrastructure and ecological resources but also in the form of human casualties. Economic losses resulting from disasters create ripple effects throughout the economy by virtue of the interdependencies between economic sectors -- and human resources will play a vital role in rebuilding the economy. The need to attend to the victims of disasters for example will require a large supply of trained medical and healthcare service providers. Post-disaster rebuilding efforts on the other hand, need a good technical project workforce. However, since humans are not immune to the impact of disasters, inoperabilities within the workforce will also occur during crisis conditions. P-graph is a graph theoretic methodology for the synthesis of networks; current applications have been restricted to process engineering applications such as chemical reaction mechanisms, process plant design and supply chain optimization. In this work we propose an extension of the P-graph methodology for economic systems. The input-output structure of the economy is represented into a P-graph model for optimizing the allocation of human resources between economic sectors. A case study using the input-output tables of the Philippines will be utilized to demonstrate the capabilities of the model.

Keywords: input-output, disasters, inoperability, P-graph, optimization

I. Introduction

The accelerated growth of human population, coupled with the onset of climate change, is expected to bring about an increased incidence of and casualties from natural disasters. These disasters can result in the destruction of infrastructure systems, reduction in resources, and the loss of lives, which will translate to economic losses. These losses create ripple effects across the economy by virtue of the interdependence between economic sectors such that the total impact may prove to be much greater than initially perceived. After the onset of disasters, the focus is in rebuilding the nation by addressing the physical and mental needs (Pynes and Tracy, 2007) of the victims and by reconstructing damaged infrastructure (Gotham and Greenberg, 2008) and it is expected that the government will provide for these needs even with limited resources. These recovery efforts entail human resources such as trained medical and health care service providers and technical project engineers, which may have been affected by the disaster as well. Santos et al (2014) highlights the critical role of the workforce during post-disaster recovery. Hence, it is important to identify a rational approach of prioritizing the workforce allocation between economic sectors in order to minimize the reduction in economic productivity.

The input-output (IO) (Leontief, 1936) framework is capable of representing the interdependence of economic structures such that it was utilized primarily for forecasting economic behavior and for policy design (Leontief, 1936; Miller and Blair, 2009). In fact, IO-based optimization models have been developed for identifying the rational allocation of resources during a crisis (Kananen et al., 1990; Haimes and Jiang, 2001; Jiang and Haimes, 2004). In this work, we propose the use of the Process graph (P-graph) model for identifying the optimal allocation of resources between economic sectors during crisis conditions. P-graph is a graph-theoretic method initially developed for process network synthesis (Friedler et al., 1992) which has been applied for various network optimization problems such as heat exchanger networks (Nagy et al., 2001), mass separation networks (Lee and Park, 1996) and supply chains. A recent review paper (Lam, 2013) highlights the development of P-graph in various fields of application.

The development of the model is discussed in the succeeding sections of this article. The rest of the paper is organized as follows. Section 2 gives the formal problem statement while Section 3 gives an overview of P-graph methodology in the context of the proposed application. The subsequent sections discuss two case studies to demonstrate how the IO framework can be represented as a P-graph model. The first case study is a simple 2-sector economy, which demonstrates the allocation of a scarce resource. The second case study utilizes the 11-sector IO table of the Philippines to show the optimal allocation of workforce during a pandemic. Finally, we summarize the key results and conclusions from this paper and suggest areas for future extensions.

II. Problem Statement

The problem statement may be formally stated as follows: an economic I-O system is comprised of *n* sectors such that each sector produces only one type of product output. Thus, there are also *n* commodities. Each sector requires *m* resources and given a crisis event that creates a sudden shortage of the *k*th commodity or the *m*th resource, the problem is to determine the optimal allocation of commodity *k* or resource *m* in the system so as to minimize total reduction of gross domestic product (GDP) assuming perfect mobility of the *m*th resource within the system, implying that all the available resource can be freely distributed between sectors.

III. Process Graph Methodology

The Process graph (or P-graph) methodology is a graph-theoretic approach, which was initially used for solving Process Network Synthesis (PNS) problems, whose complexity is characterized by its combinatorial nature (Friedler et al., 1992). P-graphs are used in several applications such as optimization of process systems (Nagy et al., 2001), reduction of emissions and wastes (Lam et al., 2013), identification of reaction pathways (Lin et al., 2010), and identifying sustainable energy supply chains (Lam, 2013). P-graph is the graphical representation of matrix calculations such as mixed-integer linear and nonlinear programming (MILP and MINLP) problems. Thus, the similarity in structure of the P-graph model with the IO framework makes it possible to develop the IO models for risk assessment and resource allocation into a P-graph model.

The P-graph primarily consists of a set of processes or operating units referred to as O-type vertices, and a set of materials referred to as M-type vertices, which represent the flows of material goods or energy in the system (Varbanov, 2012). Materials are either consumed or produced by a processing unit; and the directed edge or arc (m_i, o_j) linking a material to an operational unit describes this relationship. The set of materials are further subdivided into the set of raw materials (RM), the set of intermediates (I) and the set of products (P). This subdivision is done to facilitate the generation of feasible solution structures. An example of a typical unit process as shown in Figure 1a can be converted into an equivalent P-graph model as depicted in Figure 1b. It is also possible to indicate the flow rates of the materials coming into and out of an operating unit by varying the flow rates associated with the arcs.

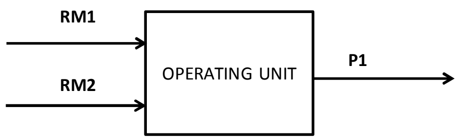

Figure 1a

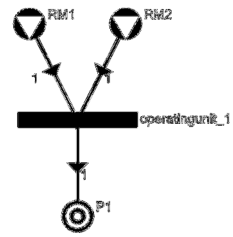

Figure 1b

Unlike traditional LP models, the P-graph model is able to identify not only the optimal solution but also all other feasible structures of the network. These feasible structures are identified using three main algorithms - maximal structure generation (MSG), solution structure generation (SSG) and the accelerated branch-and-bound (ABB) (Varbanov, 2012).

In this paper, we extend the P-graph approach to create a new visualization tool for the IO structure of an economy by representing each sector (n) as a processing unit, which utilizes several input materials from other sectors in the economy to produce a single product type (p).

IV. Case Study 1

The first case study considers a simplified economy adapted from Miller and Blair (2009), which consists of 2 sectors. The economic transactions of this hypothetical economy are shown in Table 1 together with the associated resource consumption of each sector. This resource may represent natural (e.g., land and water), as well as human resources. Data on economic transactions across the two sectors and their resource requirements are provided in Table 1. Subsequently, these data are used to derive the extended technical coefficients of the IO matrix as shown in Table 2. The last row in Table 2 represents the resource intensity of a sector and is derived by dividing the total resource input by the total output of that sector. Sector 1 for example, utilizes a total of 20 resource units to produce a total output of 1000 units. Sector 1 therefore has a resource intensity of 0.02. The corresponding P-graph representation of the technical coefficients matrix is shown in Figure 2a wherein the output of Sectors 1 and 2 are represented as M1 and M2 respectively. The net product outputs of M1 and M2 correspond to the final demands of Sector 1 and Sector 2 respectively. There is an associated price per unit of M1 and M2 produced such that the total GDP of the system is equivalent to the total price of the net flow rate of products M1 and M2. Considering a disruptive event which results in a 10% loss in the availability of a resource, the problem is in the optimal allocation of the scarce resource between the two economic sectors in order to minimize the over-all GDP loss. Furthermore, the allocation of resources is constrained such that the total outputs of the sectors are not to exceed and not to deviate by more than 15% from the total output of the unperturbed system. In addition, the adjusted final demand of the individual sectors should not exceed the final demand of the unperturbed system. The optimal network is obtained by using the ABB algorithm in the P-graph model and the result is as shown in Figure 2b with a total GDP of 1,855.77 corresponding to a 9.47% reduction from the original GDP of 2,050.

Table 1: Transactions Matrix of the Hypothetical Economy

	Sector 1	**Sector 2**	**Final Demand**	**Total Output**
Sector 1	150	500	350	1000
Sector 2	200	100	1700	2000
Resource	20	80	100*	n/a

*Total resource drawn from the environment by the economy

Table 2: Technical Coefficients Matrix

	Sector 1	**Sector 2**
Sector 1	0.15	0.25
Sector 2	0.20	0.05
Resource	0.02	0.04

The 10% reduction in available resources corresponds to a total of 90 units of resource available for allocation. The optimal result is in allocating 21% (18.72 units) of the resource to Sector 1 and the remaining 79% (71.28 units) to Sector 2 as shown in Figure 2b. The total output of Sector 1 is 935.89, wherein 140.38 units are utilized by Sector 1 itself (arc from M1 to Sector 1). This leaves 795.51 units of M1 available, of which 445.51 units go to Sector 2 (arc from M1 to Sector 2) and the remaining 350 units is the final demand for Sector 1. The same interpretation can be made for the distribution of M2. Based on the results of the optimization, the final demand for Sector 1 is fully met and remains at 350 units just like in the unperturbed system while that of Sector 2 is reduced by 11.43%. Thus, only the final demand of Sector 2 is affected by the reduction in available resource. However, both total outputs are reduced; by 6.40% in Sector 1 and 10.90% in Sector 2.

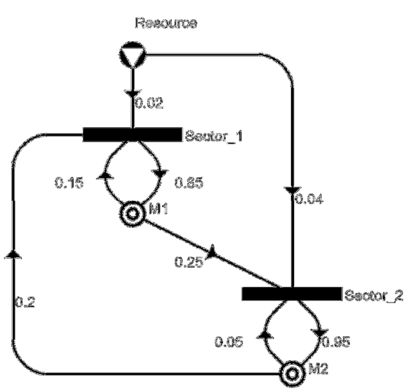

Figure 2a: P-graph representation of the two-sector economy

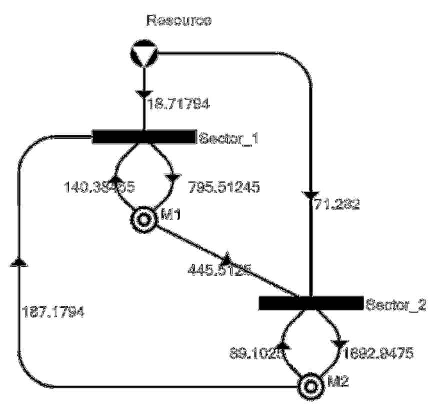

Figure 2b: Optimal network of Perturbed two-sector economy

V. Case Study 2

The second case study utilizes the 11-sector IO table of the Philippines (PNSCB, 2006) for identifying the optimal allocation of human resources when there is a spread of a pandemic which results in a reduction in workforce. In this case study, it is assumed that the productivity of an economic sector varies linearly with the number of workers it employs. During crisis conditions, it is expected that there will be an increased need for particular professions such as health care service providers and engineers. However, the model developed here does not differentiate between the expertise of those employed in the different economic sectors. Instead, it is assumed that the workforce is a resource with complete mobility within the economic system and can therefore be allocated freely between sectors. The total final demand and total output in million pesos for each economic sector are shown in Table 3 together with the total number of workers in each economic sector expressed in number of persons per million pesos of economic activity (DOLE, 2013).

Table 3. Final Demand, Total Output and Associated Workforce in 2000 (PNSCB, 2006)

	Sector	Final Demand (in Million PhP)	Total Output (in Million PhP)	Workforce (persons per million PhP)
1	Agriculture, Fishery and Forestry	236,057	686,745	17.90
2	Mining and Quarrying	(151,600)	37,788	5.60
3	Manufacturing	1,501,120	3,299,139	0.90
4	Construction	269,486	289,360	7.20
5	Electricity, Gas and Water	68,669	196,704	0.70
6	Transportation, Communication and Storage	316,864	514,091	5.40
7	Trade	511,748	803,519	9.20
8	Finance	185,136	300,570	1.40
9	Real Estate and Ownership of Dwellings	256,105	290,760	8.20
10	Private Services	426,180	659,932	6.50
11	Government Services	442,004	442,004	4.70

We consider the case where the economy suffers a 10% reduction in the available workforce. This loss will translate to reduced levels of efficiency across economic sectors. The linear relationship between the available workforce and the productivity of an economic sector is utilized to identify the optimal allocation of workers between economic sectors in order to minimize GDP losses. It is further assumed that the GDP in any sector during a crisis cannot be greater than the base case scenario and that reductions in total output for each individual sector should not exceed 15%. The optimal solution for a 10% reduction in workforce, results in an overall final demand reduction of 6.28%. The individual percentage reductions in final demand and workforce are shown in Table 4. The optimization results in the decline in total GDP, but the final demand of the Manufacturing; Electricity, Gas and Water; Transportation, Communication and Storage; Finance; Real Estate and Ownership of Dwellings and Government Services sectors were unaffected by the reduction in available workforce. Furthermore, the total workforce allocated to the Mining and Quarrying and the Government Services sectors is equivalent to the unperturbed system. On the other hand, Agriculture, Fishery and Forestry; Construction and Trade sectors experienced 15% reduction in workforce each, which is the largest reduction among all the sectors. These three sectors had the highest workforce per PhP output ratio. The Manufacturing; Electricity, Gas and Water and Finance sectors experienced no reduction in final demand despite a reduction in their workforce allocation which may have been due to the fact that these three sectors had the lowest workforce per PhP output. However, the total outputs of these three sectors were reduced.

Table 4. Results for Perturbed System in Case Study 2

	Sector	Final Demand (in Million PhP)	Total Output (in Million PhP)	% Reduction in Final Demand	% Reduction in Workforce Allocation
1	Agriculture, Fishery and Forestry	153,398	583,695	35%	15%
2	Mining and Quarrying	(145,104)	37,700	4.33 %	0 %
3	Manufacturing	1,501,131	3,211,239	0 %	2.66%
4	Construction	226,560	245,905	15.91%	15%
5	Electricity, Gas and Water	68,659	191,609	0 %	2.59%
6	Transportation, Communication and Storage	316,875	493,915	0 %	3.93%
7	Trade	400,382	682,975	21.76 %	15%
8	Finance	185,177	292,828	0 %	2.59%
9	Real Estate and Ownership of Dwellings	256,130	288,894	0 %	0.66%
10	Private Services	401,344	624,240	5.82 %	5.40%
11	Government Services	442,000	442,000	0 %	0 %

VI. Conclusions

During crisis conditions, it is important to identify the appropriate strategy for allocating limited resources in order to facilitate the recovery of an economic system. The workforce availability which may be reduced due to the spread of a pandemic or as a result of lives lost during a calamity must be strategically allocated to the economic sectors in order to minimize the reduction in total final demand. In this work, a P-graph model, which is traditionally used for process synthesis problems, was successfully developed to represent the IO structure of an economic system. Furthermore, the optimal allocation of resources between the economic sectors was obtained using the ABB algorithm of the P-graph model.

Future work will look into integrating the effect of workforce expertise on human resource allocation during a crisis by differentiating between worker skill and expertise. Modifying resource mobility can also be considered since particular sectors may require particular skills. Furthermore, the generation and evaluation of alternative feasible structures for resource allocation should be considered and tradeoffs in final demand, total output and resource allocation reduction between these alternative structures should be investigated.

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