MULTINATIONAL ENTERPRISES IN MULTI-REGIONAL INPUT-OUTPUT ANALYSIS

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Abstract

Global Multi-Regional Input-Output (GMRIO) models have become indispensable tools for the value-added analysis of international trade and the consumption-based re-attribution of remote environmental impacts. The concept of global supply chains underlies both these areas of research: a 'new wave of globalisation' has seen the disintegration of production processes across national borders as enterprises have strategically outsourced and offshored parts of their business. 80% of world trade is estimated to involve multinational enterprises – either through intra-firm trade or international sourcing and marketing – and there is growing recognition that certain 'lead' enterprises govern extended international supply networks – either through collaborative relationships or by exerting power over captive suppliers. there is a pressing need to move from analysis at an aggregate industry-level to a more disaggregate enterprise-level. In response, this paper asks: can multinational enterprises be sensibly characterised within GMRIO models. A methodological approach, based on a stochastic disaggregation technique, is proposed for incorporating enterprises into a GMRIO model. In addition to reflecting the scale and structure of the world's leading companies, the approach deals with uncertainty introduced by incomplete information. As outside observers we are unlikely to know the true input-output structure of a given enterprise, but using Monte Carlo simulation and knowledge of the meta-constraints encoded by the original input-output data we can start to explore the probable role of large companies in the global economy. Further, by creating a framework whereby multiple enterprises can be simultaneously incorporated into the GMRIO system, double-counting issues can also be investigated. For example, simply summing individual enterprise carbon footprints risks double-counting a portion of emissions as the enterprises in question may fall within one another's supply chain. Finally, the description of methodological steps taken is supplemented with numerical experiments that aim to highlight the advantages, limitations and possible extensions of the overall approach.

1 THE ENTERPRISE IN INPUT-OUTPUT ANALYSIS

The literature is reviewed in this section with a view to understanding how enterprises, rather than aggregate industries, have been assessed using input-output (IO) models and techniques.

The use of IO in the assessment of enterprises has a long history covering three central approaches. More recently a hybrid approach, that combines these approaches, has emerged. For clarity of exposition, the following terms are used to distinguish between the four approaches:

1. `Inferred enterprise assessment` – using conventional IO models and an assumption that the enterprise behaves according to the sector average (Section 1.1).

2. `Isolated enterprise assessment` – based on Enterprise Input-Output (EIO) models that characterise the internal functions of an enterprise as a stand-alone system of IO interdependencies (Section 1.2).

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3. `Integrated enterprise assessment` – using augmented conventional IO models to provide a characterisation of an enterprise within a wider-economy (reviewed in Section 1.3).

4. `Nested enterprise assessment` – using a hybrid approach that sees an enterprise sub-system fully integrated into a wider economy (Section 1.4).

Each approach is reviewed in turn below.

1.1 *Inferred enterprise assessment*

Conventional IO models can be used to assess the performance[1] of an industry sector, from which the performance of an enterprise located within this sector can be inferred by scaling sector-level results to enterprise output and assuming sector average performance on behalf of the enterprise (Joshi, 1999). The suitability of this approach is limited by the degree of heterogeneity observed in many sectors[2]. As a result, applications of this method are usually limited to benchmarking enterprise performance against sector average performance, including for the purpose of enterprise carbon footprinting (Wiedmann *et al.*, 2009).

1.2 *Isolated enterprise assessment*

Conventional IO models and techniques provide a means of analysing an economy characterised by a system of interdependent sectors. As a field of enquiry Input-Output Analysis (IOA) can be placed at the intersection of national accounting practices and the study of macroeconomics. However, the concepts and mathematical techniques of IOA have also been applied at the level of the enterprise. In contrast to conventional IO, this separate but related field, referred to as Enterprise Input-Output (EIO), can be placed at the intersection of business accounting[3], enterprise management and the study of microeconomics.

The foundations of EIO

Pre-empting the early developments in EIO, Mattessich (1956) drew attention to the deep, but often overlooked, relationships between the strata of accounting (i. e., business, national and international balance of payments accounting) and the strata of economics (i. e., microeconomics, macroeconomics and foreign trade economics)[4], and indicated the particular relationship between IOA and the conventional business accounting system (Mattessich, 1957). Mattessich's vision paved the way for the first practical translation of a full business accounting system (i. e., including current, net fixed assets, equity and production accounts) into an open input-output framework (Richards, 1960). In Richards' EIO model, debits and credits to the interdependent accounts of an enterprise correspond to the inputs and outputs of industry sectors in a conventional IO framework. As capital formation was not specifically accounted for, the model provided a representation of

[1] A performance measure could include, for example, upstream or downstream economic impacts or the emissions embodied in intermediate inputs, etc.

[2] Supply and demand structures may vary greatly from one establishment to another within a given sector; when aggregated together an average structure emerges, but this may be radically different to that of a given constituent establishment.

[3] Business accounting encapsulates both financial and managerial or cost accounting.

[4] Richard Mattessich (an eminent founder of the modern conception of accounting theory) notes that while national and balance of payments accounting are readily encompassed within economics, technically the same accounting principles are used in these fields as in business accounting; and advises that in comparing accountancy (which emerged out of bookkeeping) and economics (that has its origins in philosophy) it must be realised that they, though clearly interwoven, represent different dimensions which have a common basis but spread out in different directions.

the monetary flows into and out of respective accounts during a given accounting period and thus a method for predicting changes (albeit under the limiting assumption of linearity) in the accounts given some exogenous perturbation.

However, a belief that only the production account of an enterprise can be appropriately transcribed into an input-output system (nonlinearity being the primary restriction for other account types) led Farag (1967, 1968), Livingstone (1969) and Stone (1969) to focus on the development of EIO models to assist in the planning and costing of production activities within a divisionalised enterprise[5]: in this case, production departments (rather than account types) correspond to industry sectors, external sales and inventory requirements replace final demand categories and primary inputs of raw materials, labour, overhead and profit take the place of sector average value-added inputs. Livingstone went a step further than Farag by illustrating how, by applying the input-output methodologies to the physical (rather than monetary) flows of production, the usefulness of EIO could be extended to enterprise logistics planning for anticipated changes in activity levels and in appraising performance for control purposes. Gambling & Nour (1970) recognised, however, that the adaption of monetary models to physical systems of mixed units was less than straightforward: in particular, a central assumption of monetary models, that the total value of inputs equal the total value of outputs, could not be readily applied to a physical mixed units system (for example, due to the difficulty of equating inputs in one physical unit with outputs in another). In an effort to provide a rigorous means of comparing the, by that time, various forms and applications of EIO models, Butterworth & Sigloch (1971) presented a generalisation of the multi-stage input-output model whereby all possible alternatives could be derived as special cases (including the Leontief model) and in addition observed the links to the more general area of linear programming.

From account- to process-oriented EIO

Following the foundational developments in EIO, wide-spread adoption of the methods by practicing accountants and managers of enterprises failed to occur[6], however, significant use within state-owned enterprises in China during the 1980s has been reported (Lin & Polenske, 1998). It was not until the work of Polenske and colleagues in the 1990s that academic interest in EIO truly resurfaced; the insight primarily driving this renewed interest was that EIO could be usefully deployed to evaluate environmental interventions of an enterprise, hence providing a tool for environmental management (Polenske, 1997)[7]. Central to Polenske's (1997) reformulation of the EIO model was a substitution of production departments for production processes and a focus on their underlying input-output relationships including by-products and waste (Lin & Polenske, 1998). This allowed linkages between the monetary and technological functions of an enterprise (or subsystem such as a plant or subsidiary) to be explicitly analysed, with the additional option of constructing the EIO model from the bottom-up using engineering-process information. Specifically, Lin & Polenske's (1998) expos-

[5] I. e., an enterprise that consists of a set of production departments under the following assumptions: each department produces only one homogenous product via a linear production function using a singular technology or production process; the output of each department can serve either the demands of another department, inventory requirements or external sales; all prices of inputs and outputs are known and given and that unit output price equals average cost; and, that the sum of departmental-level production across all departments is consistent with the total production of the enterprise as a whole.

[6] The formalisation of the accounting profession based on matrix algebra methods, meanwhile, continued apace of course culminating in the computer-based form of the now ubiquitous spread sheet.

[7] Polenske also conceived the usefulness of and means by which an EIO model could be embedded into an economy-wide IO model; an idea that was later adopted by way of practical application in Lenzen *et al.* (2010a).

ition of the process-orientated EIO model put forward the following potential extensions and applications:

1. As a process-analysis tool the model can help: trace flows of energy and materials between production units; estimate demand for energy, materials, labour etc., at different production levels; identify bottlenecks of production activities; evaluate alternative technologies or processes in terms of total input requirements and profitability.

2. Transformation of the model into a purely monetary model (using price and cost data) would allow: the tracing of financial flows and the study of the cost and profit structure of the enterprise, individual segments of the market or individual products; an examination of the impacts of exogenous changes, therefore serving as a basis for business planning.

3. Extension of the model to include waste-management processes and pollution control can assist managers to: examine the impacts of environmental regulation on the enterprise and to evaluate alternative compliance strategies; quantify costs of waste disposal; identify factors that influence the generation of waste and pollution; and, consider optimal strategies for enterprise-wide pollution control through the simultaneous evaluation of all pollutants.

Subsequently, process-orientated EIO models have been adopted for a variety of applications, for example: in the analysis of the economic, energy and environmental trade-offs of alternative cokemaking technologies in China (Polenske & McMichael, 2002); to compare the energy and material usage and the generation of waste and pollution between two tile manufacturers (Albino & Kühtz, 2004; Kühtz *et al.*, 2010); to forecast human resource requirements of an enterprise subject to exogenous changes in demand for output from and supply of inputs into the enterprise (Correa & Craft, 1999); and, to investigate the interactions between energy and material flows and monetary flows in a coal mine (Liang *et al.*, 2010). In addition, recognising that the modern enterprise is often characterised by multi-regional production activities, Li *et al.* (2008) introduced distinct regions into an EIO framework; an extension analogous to the Multi-Regional Input-Output (MRIO) model in conventional IOA.

In related research, Grubbstrom & Tang (2000) bridged the fields of EIO and material requirements planning by illustrating how timing properties of inputs into a production process can be captured by means of the Laplace transform within an overall EIO framework. Returning to an account-oriented EIO framework, Marangoni & Fezzi (2002) restructured the profit and loss account (integrating additional data from 70 management accounts) of a leading pharmaceutical subsidiary in order to simulate the consequences of various sales forecasts.

Taking EIO beyond the gates of the enterprise

By extending process-oriented EIO methods beyond the gates of the enterprise, Albino and colleagues have made two important additional contributions: firstly in the analysis of the network of production processes that characterise the supply chain of a final product (Albino *et al.*, 2002); and, secondly in the analysis of the network of production processes within an industrial district, i. e., a localised cluster of interdependent enterprises (Albino *et al.*, 2003). Material, energy and pollution flows are assessed to provide holistic measures of resource consumption and environmental impacts of the supply chain and industrial district, respectively. In both instances, the effects of alternative formulations of the system (e. g., the use of different technologies) and changes in demand for output can be evaluated, thus providing tools for the collective, inter-enterprise, environmental management of specific supply chains and industrial districts. It is with these developments that EIO merges with the disciplines of Life-Cycle Assessment (LCA) and Life-Cycle Costing (LCC).

A future role for EIO in Life-Cycle Sustainability Analysis

In general, LCA is a widely used method for addressing the environmental aspects of products and services that has developed from the analysis of simple products to more complicated systems and from product-level decision-making to economy-wide policy evaluation (Guinée *et al.*, 2011). Developments in the direction of more complicated systems included the use of process-oriented EIO models (Lave, 2006). However, within a life-cycle assessment context, EIO models, in common with other process-focused LCA methods, encounter a truncation issue when defining the boundary of analysis: i. e., the full life-cycle of a product, plant or enterprise cannot be captured. Hybrid approaches, linking process-level analysis with economy-wide IOA, have been developed to overcome this issue (Hendrickson *et al.*, 2006; Suh, 2004).

LCC can be considered as the monetary counterpart to environmentally focused LCA: whereby the total costs occurring during a products life-cycle are evaluated for purposes of business (e. g., outsourcing and ownership decisions) and supply chain (e. g., vulnerabilities to fiscal environmental regulation) management (Lindholm & Suomala, 2005). Settanni and colleagues have proposed and shown that EIO methods could provide a formalised computational structure for LCC (Settanni, 2008; Settanni *et al.*, 2011), while Nakamura & Kondo (2009) have linked LCC, LCA and IOA at a macro-economic level in the analysis of waste.

Most recently, the EIO-based developments in LCC has been extended with the aim of providing a computational structure for a unified Life-Cycle Sustainability Analysis (LCSA) including aspects of LCC, LCA and social LCA (Heijungs *et al.*, 2012).

Dealing with unavailable and uncertain data in EIO

The models and applications of EIO discussed above are all of a deterministic nature. That is, they require the collection or estimation of comprehensive data, which must subsequently be assumed accurate for the tool to be considered useful in decision-making. In reality, two problems are found: first the quantity and quality of data required can be considerable, hence the time taken and expense incurred in gathering the necessary data can also be considerable; and second, the required data may be uncertain in the sense that it is inherently unknowable, variable within the timeframe of analysis, or that resources are not available or in place to accurately measure it. Perhaps then the state of the art in EIO surrounds the treatment of uncertainty and data unavailability. Two important contributions are found in the literature. First, Lenzen & Lundie (2012) overcome the problem of limited data availability (in this case due to the scale of the application) by estimating EIO tables for 22 dairy product manufacturing sites. In contrast to standard procedures for constructing EIO tables involving full-survey of inputs and outputs of sub-systems (processes, departments etc.), the following steps were taken: data on inputs to and outputs from each site, as a whole, were measured; qualitative expert knowledge of what inputs and outputs are important to each sub-system was captured in a weighted binary matrix; this matrix was then scaled such that total inputs and outputs across all sub-systems balance with that of the overall site.

Second, within an EIO model for an LCC of a vertically-integrated multi-product manufacturing process, Settanni & Emblemsvåg (2010) take explicit account of uncertainties associated with: (a) the efficiencies of converting inputs into outputs from each of the system's processes (i. e., the ratio of main output to waste and other by-products); (b) the requirements for externally-purchased inputs; and, (c) the requirements for intra-system process outputs. In this stochastic EIO model, uncertainties are characterised by subjectively-defined triangular distributions, although the authors note that, in the absence of detailed information, a uniform distribution as defined by bounded intervals (where all values within the interval are assumed to be equally plausible) can be used. The model is solved by way of Monte Carlo simulation, where all uncertain values are allowed to vary simultaneously within a given iteration; each iteration then provides a unique and plausible characterisation of the manufacturing system. By adopting a stochastic rather than deterministic approach the inherent level of vagueness associated with certain aspects of a system can be retained within a structured input-output framework. The theoretical underpinnings of stochastic EIO are further elaborated by Li *et al.* (2012).

Summary

There is a diverse literature base dealing with isolated enterprise assessment within an IO framework characterised by EIO models. While EIO models exhibit the same restrictive assumptions of economy-wide IO models (e. g., that economies of scale, learning curves and productivity changes cannot be easily accounted for), their value to corporate environmental management in particular is clear. The premise of EIO has transformed from one of restructuring enterprise cost accounts, to the ordered representation of physical and monetary interdependencies between sub-systems of an enterprise, supply chain or area of localised production.

The application of isolated enterprise assessment approaches based on EIO models to the measurement of an enterprise's influence over its supply chain emissions faces the same limitation as process-based enterprise carbon footprinting methods: the full extent of global supply chains stemming from an enterprise are necessarily truncated due to the data intensity of the method and the easy availability of such data. However, the detailed depiction of the interdependencies between enterprises within an industrial district presents insights into how double-counting issues can be treated through the explicit representation of the activity of multiple enterprises within a holistic framework. Furthermore, the treatment of uncertainty associated with unavailable or vague information presents opportunities for the stochastic characterisation of groups of enterprises where the availability of detailed, and commercially sensitive, data may be limited.

EIO can be considered as a bottom-up modelling tradition. The logical next steps and perhaps the most exciting developments in the field are in relation to the embedding of EIO models within conventional, top-down, economy-wide IO models, the subject of Section 1.4. The following section details the literature that has viewed the enterprise from a different perspective: its role within the wider-economy.

1.3 *Integrated enterprise assessment*

The term *integrated enterprise assessment* is used here to describe work that has been carried-out with a view to understanding the role of an enterprise in the wider-economy, whereby the enterprise is integrated into a conventional IO framework through its characterisation as an additional sector. The motivation for this work is twofold: first, to assess the effect of the wider-economy upon the enterprise, which is of interest to the management of the enterprise in question (Tiebout, 1967); and second, to evaluate the total effect an enterprise has on the wider-economy, which is of interest, for example, to a regional analyst (Hewings, 1971). This field of enquiry is relatively small, but the contributions made in the literature raise important issues of particular interest to the work presented in Section 2. As such, each contribution is reviewed in some detail.

Critique of inferred enterprise assessment

Focusing on the first motivation, Tiebout (1967) notes that *inferred enterprise assessment*[8] provides an enterprise with two relatively limited capabilities: (a) the identification of possible areas of additional market potential by comparing the enterprise's current marketing position with that of the industry as a whole (i. e., by inspecting the sales (row) coefficients of the enterprise's respective industry, general aggregate industry customers are identified; if the enterprise does not already supply specific customers located within such industries, then a competitor may well be doing so); and, (b) conventional industry-level impact analysis can be directly mapped to the enterprise (i. e., should a 10% increase in the final demand for automobiles require, via inter-industry linkages, a 5% increase in total output from the steel-industry, then a steelmaking enterprise could infer that its total output would also be required to increase by 5%).

Tiebout views these capabilities as being primarily limited by two factors: (a) the technical coefficients of a given industry always represent industry averages no matter how disaggregated the IO table is; and, (b) IO tables are constructed on the basis of similarity of input purchases rather than output sales. Hence, for an enterprise to infer anything from an IO table, it must assume a sales and purchasing structure in common with the industry average. While the assumption of an industry average purchasing structure becomes more satisfactory the more disaggregated the IO table, this is not necessarily the case for the assumption of industry average sales structure. By way of extreme example, consider an automobile industry consisting of just two manufactures producing identical vehicles, where one sells only to final demand households while the other specialises in sales to commercial customers (e. g., corporate fleet vehicles). Should one of these manufacturers wish to make an inferred assessment from an IO table, it would find an industry average purchasing structure identical to its own but a radically different sales structure, with the likely result of misleading analysis of downstream effects (e. g., changes in final demand) or market potential.

Tentative steps towards integrated enterprise assessment

In response to the identified limitations of *inferred enterprise assessment*, Tiebout (1967) proposes a simple approach: represent the enterprise as a new row of technical coefficients in the IO table by dividing known enterprise sales to different industry sectors by each sectors reported total output. The input (purchasing) side of the problem is, however, dismissed as being of no particular interest to the enterprise. Tiebout subsequently proposes that requirements from the enterprise induced by downstream changes in the economy can be observed. For example, a unit change in final demand for a given industry's output can be traced back to the enterprise by multiplying the row of enterprise technical coefficients by the relevant column of the original Leontief inverse matrix. Although the author suggests that the enterprise is introduced into the IO model, this is in fact not the case; the enterprise technical coefficients are simply taken as an exogenous multiplier intensity vector, much as an emissions intensity vector is used in environmentally-extended IOA. The problem with this approach is that the role of the enterprise is already accounted for within the Leontief inverse matrix (i. e., the enterprise has not been stripped out of the industry of which it was originally a part of), hence double-counting of economic effects are likely to ensue, particularly if the enterprise accounts form a significant share of its aggregate industry.

Hewings (1971) takes inspiration from Tiebout's approach in proposing a method for: (a) evaluating the impact of a new enterprise on the wider-economy (i. e., the second motivation noted above), whereby the new en-

[8] Inferred enterprise assessment is alternatively described by Tiebout as "the industry of which you are a part" approach.

terprise is characterised as an additional row *and* column in the technical coefficients matrix and an expanded Leontief inverse calculated; and, (b) evaluating the impact of the removal of an enterprise on the wider-economy, whereby the transactions of the enterprise are stripped out of the industry rows and columns of which it was a part. As (Morrison, 1973) points out, in both cases the total output of each industry in the table would be affected and hence the technical coefficients matrix would need to be re-calculated, as would the Leontief inverse. Modern computational resources render such concerns as trivial, but at the time of these developments the emphasis was evidently upon the minimisation of computational tasks. Morrison's other concern is in regard to the stability of technical coefficients faced with the prospect of the arrival or removal of an enterprise, thus calling into question the static nature of IO models for the analysis of inherently dynamic responses.

Greytak (1972) also criticises Tiebout's and Hewings' proposals focusing on three possible factors that could cause the enterprise-level calculation of technical requirements to diverge from strict comparability to industry-level coefficients. The factors identifies by Greytak are outlined below and are supported by more recent literature and guidelines:

1. `Treatment of the trade sector` – IO tables show the inter-linkages between the producers and consumers of products; the monetary value of products traversing such linkages are therefore shown as direct transactions, while in reality these transactions are likely to involve an intermediary wholesale or retail trade establishment (Miller & Blair, 2009). The trade sector is still depicted in the IO table but only by its role as a provider of marginal services (e. g., the marketing and temporary storage of products)[9]. Transport sectors are similarly treated as marginal service providers. An enterprise is likely to be able to identify who its immediate customers are, but if the enterprise is particularly dependent upon trade establishments for the distribution of its products, then it may be difficult to identify who ultimately consumes its products. Hence, the calculation of the enterprise's sales (row) coefficients, such that they are comparable to industry sales coefficients, may be problematic.

2. `Issue of product mix` – Many enterprises, particularly large corporations, produce multiple products[10]. These products may fall into different industry and/or commodity classifications (e. g., a steelmaker may also produce fabricated metal products). In compiling IO tables, strict accounting conventions are adopted to accommodate for such situations[11]. This presents a particular problem when removing an enterprise from an IO table, as the manner in which it was originally captured within the table may be unknown.

[9] For example, Clause 6.146 of the System of National Accounts 2008 states: "Although wholesalers and retailers actually buy and sell goods, the goods purchased are not treated as part of their intermediate consumption when they are resold with only minimal processing such as grading, cleaning, packaging, etc. Wholesalers and retailers are treated as supplying services to their customers by storing and displaying a selection of goods in convenient locations and making them easily available for customers to buy. Their output is measured by the total value of the trade margins realized on the goods they purchase for resale" (United Nations *et al.*, 2008).

[10] For example, Clause 5.2 of the System of National Accounts 2008 states: "The majority of enterprises by number engages in only one sort of production. The majority of production, though, is carried out by a relatively small number of large corporations that undertake many different kinds of production, there being virtually no upper limit to the extent of diversity of production in a large enterprise..." (United Nations *et al.*, 2008).

[11] Clause 5.2 continues: "...If enterprises are grouped together on the basis of their principal activities, at least some of the resulting groupings are likely to be very heterogeneous with respect to the type of production processes carried out and also the goods and services produced. Thus, for analyses of production in which the technology of production plays an important role, it is necessary to work with groups of producers that are engaged in essentially the same kind of production. This requirement means that some institutional units must be partitioned into smaller and more homogeneous units..." (United Nations *et al.*, 2008).

3. `Distinguishing sales` – Intermediate transactions between industries in an IO table represent an industry's consumption of purchased products other than fixed assets[12]. Purchases of fixed assets are conventionally recorded within the final demand category of fixed capital formation, while consumption of fixed assets is recorded as a value-adding activity (Miller & Blair, 2009). At the enterprise-level this distinction corresponds to purchases made on current account and capital account, allowing purchasing (column) coefficients to be specified (Nations, 2000). This causes a problem in estimating sales coefficients: the enterprise may know the volume of sales to each industry but it may not necessarily know whether the products are sold for intermediate consumption or as fixed assets.

None of the issues raised by Greytak are insurmountable (for example, Katz & Burford (1981) propose a technique for estimating enterprise output coefficients when input coefficients are known), but they do highlight the need to understand how a specific IO table has been constructed before it can be sensibly modified through the introduction of an enterprise. Unfortunately, no further applications of *integrated enterprise assessment* are found in the literature before the development of the nested hybrid approach discussed in Section 1.4. However, comparable methods have been developed in the field of IO-based LCA, whereby a single product, rather than an enterprise, is represented as an additional sector in an IO framework.

Comparable approaches in IO-based LCA

Joshi (1999) presents a range of different models of varying complexity for IO-based LCA of products. The simplest approach is comparable to *inferred enterprise assessment*, whereby the product under analysis is assumed to be well approximated by the industry average. A slightly more complex approach that sees the product represented as a new hypothetical sector is directly comparable to Hewings method. Joshi notes that this approach assumes that the original technical coefficients matrix is unaffected by the introduction of the new sector, which if the product under analysis is already included in the IO table cannot be the case.

A third model is subsequently proposed by Joshi (1999) that overcomes this problem: the sector already containing the product of interest (in aggregate form) can be disaggregated into two sectors, one representing the product and the other representing the sector less the product, with strict constraints tying corresponding coefficients together. Specification of the product sector coefficients automatically determines the value of the remaining industry coefficients. It is surprising that this approach has not been adopted in the *integrated enterprise assessment* literature as it provides a satisfactory solution to the issue of double-counting with relatively little additional computational work required. The process of IO sector disaggregation is then of particular interest to the accurate representation of an enterprise within the wider-economy.

General disaggregation techniques

The level and implications of sectoral and regional aggregation in IO tables has received significant attention in the literature (for example: Andrew *et al.*, 2009; Katz & Burford, 1981; Lenzen, 2000; Su & Ang, 2010; Wiedmann *et al.*, 2007; Williams *et al.*, 2009): as the level of aggregation increases, the

[12] For example: Clause 6.213 of the System of National Accounts 2008 states: "Intermediate consumption consists of the value of the goods and services consumed as inputs by a process of production, excluding fixed assets whose consumption is recorded as consumption of fixed capital"; in addition, Clause 6.214 states: "...intermediate consumption also does not include costs incurred by the gradual using up of fixed assets owned by the enterprise: the decline in their value during the accounting period is recorded as consumption of fixed capital" (United Nations *et al.*, 2008).

assumption of homogenous sector inputs and outputs becomes less valid. All IO tables necessarily exhibit some degree of aggregation, but the extent to which the associated 'aggregation bias' effects a particular analysis is difficult to quantify (Kymn, 1990; Miller & Blair, 2009). Efforts to measure aggregation bias have therefore focused on observing the effects of further aggregating an existing IO table. For example, Katz & Burford (1981) find that the output multipliers of industry sectors in an original IO table exhibit a wider range of values compared to those measured in a more aggregated version of the table. Similarly, Andrew *et al.* (2009) investigate the role of regional aggregation by measuring the effect of systematically disabling the regional resolution of a full MRIO model, finding that the 'domestic technology assumption' frequently used in Single-Region Input-Output (SRIO) studies inaccurately estimates the carbon footprint for many countries.

The implications of aggregation bias are of particular importance when linking economic IO data with satellite environmental accounts. If the two data sources (economic and environmental) exhibit dissimilar levels of aggregation then the analyst is faced with two options (Lenzen, 2011): aggregate the more disaggregate data source such that it concords directly with the more aggregate data source; or, disaggregate the more aggregate data source such that it concords directly with the more disaggregate data source. In common with earlier studies, Lenzen (2011) identifies that the aggregation option leads to an undesirable loss of information, but also notes that the alternative disaggregation option requires additional information that may involve labour-intensive collection or that may simply be unavailable to the analyst[13]. Lenzen's analysis of the relative merits of the aggregation and disaggregation options, within the context of environmental multipliers derived from randomly generated and subsequently aggregated and disaggregated IO tables, found that the disaggregation option is superior even when the disaggregation process is based on a minimum of additional information.

In addressing the problem of further labour- and time-costs associated with gathering additional information required to disaggregate IO tables, Wolsky (1984) developed a technique for prioritising information collection efforts according to whether or not the as yet uncollected data would materially affect a particular analysis. Specifically, Wolsky's technique concerns the disaggregation of a single sector into two new sectors by expanding an existing IO table through the introduction of an additional row and column based on known (or estimated) total output weights of the two new sectors. The technique centres on the identification of constraints that bound all unknown parameters (that, if known, would fully define the disaggregated IO table) according to the following steps:

1. Construct an initial estimate of the disaggregated IO table – the *augmented matrix* – by assuming that the new sectors are essentially identical to the original sector, both in terms of technology (i. e., input coefficients are the same) and output sales structure (i. e., output coefficients take a fixed proportion of the original sector, according to the new sector output weights).

2. Define a *distinguishing matrix* parameterised by a set of, as yet unknown, independent variables[14] such that the sum of the augmented and distinguishing matrices would yield the exact disaggregated IO table.

[13] If information is lost when original data is aggregated, then reconstruction of the original data through the reverse process of disaggregation must somehow involve the retrieval of the lost information. Should this information be rendered unavailable, then a degree of uncertainty is introduced into the system.

[14] The unknown independent variables, or distinguishing parameters, fall into three categories: those that manifest the difference in technology between the two new sectors; those that manifest the difference in output sales structure between the two new sectors; and, those that manifest the intra-aggregate exchanges between the two new sectors.

3. Define a *system of inequalities*, in terms of the unknown independent variables, that provide upper and lower bounding constraints on the unknown, but interdependent, elements of the distinguishing matrix.

4. Reduce the system of inequalities to yield the upper and lower *bounds* of the unknown independent variables.

5. Prioritise the collection of pertinent data according to the magnitude of the bounding range on values of the unknown variables (i. e., where the range is very small defer effort to the estimation of variables where the range is large); when sufficient resources are available to estimate all unknown variables, the calculated bounds can still serve as a check on the validity of collected data.

Although pioneering, the problem with Wolsky's approach is four-fold (Gillen & Guccione, 1990): first, what constitutes a negligibly small bounding range of an unknown variable is unclear; second, although the number of system unknowns is reduced though the identification of independent variables (relative to the number of originally unknown technical coefficients), this does not necessarily translate to a reduction in the data collection effort required[15]; third, IO table-wide constraints are ignored (including the constraint that the sum of a sector's inputs must be less than unity and the constraint that the sum of a sector's intermediate outputs must be such that final demand for the sector's output is non-negative); and finally, the technique is not easily scalable in terms of disaggregation of a sector into more than two new sectors.

In response to these problems, Lindner *et al.* (2012) extended Wolsky's basic disaggregation technique in three directions: first, by generalising the technique such that an original sector can be disaggregated into any number of new sectors; second, by incorporating IO table-wide constraints; and third, by using a random walk algorithm[16] to identify the bounding range of distinguishing parameters. This work was motivated by a recognition that it is not always possible to gather the necessary information needed to uniquely define a disaggregated IO table, hence the uncertainty associated with the full range of possible solutions to the disaggregation problem must be investigated. Lindner and colleagues illustrated the value of the technique by disaggregating the Chinese electricity sector into three new sectors representing renewables, subcritical coal and other fossil fuels, respectively, finding that the emissions intensity factors of the new sectors could be twice that of the initial estimate provided by the augmented matrix.

Summary

Integrated enterprise assessment is a relatively underdeveloped field of research. Studies dealing specifically with the characterisation of enterprises have been taken as far as the introduction of an enterprise as an additional row and column within an economy-wide IO table, but without altering the original IO table. For the case where the production activity of the enterprise is already captured in the aggregate IO data, this approach could lead to significant levels of double-counting in the estimation of multipliers, particularly for large enterprises that constitute a significant share of sector-level production activity. Several important considerations relating to the practical application of the approach have been raised in the literature,

[15] For example, to estimate the difference in demand of a particular input between the two new sectors presumably first requires the demand for the input to be estimated for each sector individually, i. e., the relevant technical coefficients still need to be estimated (Gillen & Guccione, 1990).

[16] Stated simply, the random walk algorithm iteratively steps through the 'solution space' defined by inequality constraints on the distinguishing parameters, randomly selecting valid values for each parameter in turn until the algorithm arrives at a complete solution, hence yielding a sample distinguishing matrix. The procedure is repeated until the required number of sample matrices have been constructed and evaluated.

namely: the special treatment of the trade sector in IO tables; the issue of enterprise product-mix; and, the distinction of sales destined for intermediate consumption and those destined for capital formation.

Although not dealing directly with enterprises, other related studies have presented disaggregation techniques that could be applied to appropriately adjust an original IO table following the isolation of an enterprise from an aggregate sector. If applied to enterprise-level analysis, such techniques could provide a means of addressing double-counting issues. These studies further indicate that even with limited information about a particular enterprise, stochastic modelling approaches can be used to gain an understanding of the likely range of pertinent measures.

The further development of *integrated enterprise assessment* approaches to include characterisations of multiple enterprises, based on deterministic or stochastic disaggregation techniques, holds promise for the suitable measurement of the influence that a group of enterprises may have over their supply chain emissions. Applied to MRIO models with global coverage, such approaches would allow global supply chains to be described while providing for the explicit treatment of double-counting.

1.4 *Nested enterprise assessment*

Nested enterprise assessment can be considered as a hybrid approach combining the principles of *isolated* and *integrated enterprise assessment*. The approach originated in the field of LCA, under the name of integrated hybrid LCA, as a means of overcoming the truncation error encountered when a boundary is drawn around a process-level system under analysis (Heijungs & Suh, 2002; Suh, 2004). Prior to its development other so-called hybrid methods included tiered hybrid analysis (involving complementary, but essentially separate, IOA and LCA analyses) and IO-based hybrid analysis (the sector disaggregation approach discussed in Section 1.3); but it was only with the development of integrated hybrid LCA that a detailed sub-system of interdependent production processes could be fully nested in an economy-wide IO framework (Heijungs *et al.*, 2006; Suh & Huppes, 2005). The nested approach has the particular advantage of providing an analysis framework that not only retains the detailed process interactions within the sub-system but that also allows overall interdependencies between the subsystem and the rest of the economy to be evaluated (Hendrickson *et al.*, 2006).

Only a single example of applied nested enterprise assessment has been found in the literature: the work carried out by Lenzen *et al.* (2010a) in their study of the University of Sydney. To capture both the internal interdependencies among business units within the university and the external interdependencies with the wider-economy, a monetary bi-regional input-output model is developed. The university and its business units (structurally considered as sectors within the framework) form one region, while a conventional IO table of the Australian economy forms the other. Inter-region transactions (i. e., the university's intermediate requirements from the wider-economy and the wider-economy's intermediate requirements from the university) are also represented forming an overall four-quadrant IO table. Respective final demand and value-added components for each region are also characterised. Details on how the university's accounts have been restructured into an IO framework and mapped to the Australian IO table sector classifications are given, illustrating the main limitation of the approach: the extent of detailed cost data required to characterise the internal and external interdependencies of a large establishment is considerable. However, the procedure followed to 'strip-out' the university from its original aggregated presence within the Australian IO table is unclear: the study indicates that the IO table was "netted by subtracting the turnover of the university". This suggests that the technical coefficients of the sector of which the university was a part have remained unchanged, thus implying the assumption that either the university's supply and demand structure is the same as the sector average or that the turnover of the university is very small relative to the sector total output. If this is the case and either assumption is deemed unsatisfactory, then adoption of the disaggregation techniques discussed in Section 1.3 would help ensure no double-counting of economic effects occur during analysis.

In demonstrating the value of *nested enterprise assessment* not only for financial analysis, but also, through extension, to the analysis of environmental burdens and social outcomes, Lenzen and colleagues have opened up a new avenue of research that is likely to expand in the coming years as the pressure for corporate environmental and social management intensifies and broadens in scope beyond the gates of the enterprise.

In a similar manner to the developments proposed for *integrated enterprise assessment* in the previous section, nested approaches could, in principle, be suitably developed and applied to the evaluation of the influence a group of enterprises may have over their supply chain emissions. Nested approaches present the additional advantage of enabling the detailed characterisation of important supply chains and interdependencies between the enterprises under investigation. However, the data intensity of a nested approach, particularly for a large number of enterprises, is likely to restrict the applicability of the approach. As such, the approach can be considered most suited to the detailed measurement of the influence of an individual enterprise.

2 MEASURING ENTERPRISE SUPPLY CHAIN EMISSIONS

This section addresses the following question:

> *How can the global supply chain emissions stemming from the intermediate consumption activity of (a) an individual enterprise and (b) a group of interdependent enterprises be estimated using publicly available data?*

The review of enterprise-level IO methods revealed that:

1. The system boundaries imposed in process-based methods – including conventional enterprise carbon footprinting methods and isolated enterprise assessment methods – curtail supply chains stemming from the intermediate consumption activity of an enterprise. The full complexity of global supply chains is therefore not captured by these approaches.

2. Hybrid approaches could yield effective enterprise-level carbon footprints but are too data-intensive for use with publicly available data or the assessment of the emissions stemming from a large group of enterprises.

3. Inferred enterprise assessment methods risk double-counting supply chain emissions, impose the unsatisfactory assumption that an enterprise conforms to the industry average, and can not be applied to enterprises, or groups of enterprises, that span multiple industries and regions.

4. Appropriate (deterministic or stochastic) disaggregation techniques, that ensure the removal of double-counted terms, have yet to be applied to the integrated enterprise assessment approach. Integrated enterprise assessment has itself yet to be applied to enterprise carbon footprinting or extended for use with global MRIO models.

5. Existing disaggregation techniques do not allow for the concurrent disaggregation of multiple IO sectors.

In order to exhaustively measure the supply chain emissions stemming from an individual enterprise and groups of enterprises, a Multi-Enterprise

Multi-Regional Input-Output (MEMRIO) model is developed in this chapter that selectively draws on the above mentioned techniques. In particular, existing disaggregation techniques, typically used to decompose highly aggregated sectors into sub-sectors, are extended to allow multi-national and/or multi-industry enterprises, and groups of such enterprises, to be characterised within a global IO framework. Monte Carlo simulation techniques are then used to stochastically estimate uncertain data.

2.1 *A multi-enterprise multi-region model*

In the MEMRIO model, interactions between national industries in the global economy are captured through the use of MRIO tables. Enterprises are characterised first by decomposing overall production activity into enterprise segments – that correspond to the regional and industrial classifications used in an MRIO table – and second by introducing these segments in to the model through the disaggregation of the corresponding MRIO sectors. Interactions between enterprise segments and the wider-economy are treated stochastically to reflect uncertainty over enterprise inputs and outputs, thereby permitting analysis based on limited, publicly available, data. The stochastic component of the model is governed by a system of constraints that (a) retains the overall balanced structure of inputs and outputs encoded in MRIO data, and (b) reflects user assumptions over the extent to which an enterprise may deviate from sector average performance. Monte-Carlo simulation is used to generate a sample set of disaggregated tables, each representing a plausible configuration for how focal enterprises are integrated in the global economy. Statistical enterprise assessments can then be performed over the set of sample tables.

The research question is addressed in two stages. First, the Total Consumption Attribution (TCA) method, developed in Skelton *et al.* (2011), is adapted for use at the enterprise-level and applied to each sample table compiled using the MEMRIO model. Applied to individual enterprises, the TCA method calculates the global supply chain emissions stemming from the intermediate consumption activity of the enterprise. Applied to enterprise groups, the TCA method calculates the global supply chain emissions stemming from the overall intermediate consumption activity of the group of enterprises. The risk of overestimating the supply chain emissions of a group of enterprises – by not taking into account the fact that constituent enterprises may fall within one another's supply chains – is ameliorated implicitly by the TCA method through the removal of double-counted terms. These consumption-based accounts of emissions reflect the *maximum potential influence* an enterprise, or an enterprise group, has over global supply chain emissions. However, each sample table yields a slightly different TCA result.

Second, enterprise, or enterprise group, TCA distributions are then calculated across the set of sample tables. The global supply chain emissions stemming from intermediate consumption activity can then be reported as a range of *likely* values.

The following section provides a brief summary of the MEMRIO model before giving a more detailed description of the main model procedures[17]. Several important terms used in the model description are defined in Table 1. The application of the TCA method at the enterprise-level is discussed in Section 2.3. In Sections 2.4–2.6 the MEMRIO model is run for hypothetical enterprises. Results from these tests are used to further explain how the model functions and provide insights into how user assumptions over the uncertainty associated with enterprise inputs and outputs can be reflected in the

[17]The MEMRIO model was developed using the Matlab programming language and consists of a main script and a set of functions. A complete reproduction of the Matlab code can be provided on request

model. Finally, an assessment of factors affecting overall model uncertainty is made in Section 2.7.

2.2 *Overview of the MEMRIO model*

The MEMRIO model, as its name suggest, starts with an existing MRIO model. Following Dietzenbacher (2005) it is assumed that the MRIO model satisfies the condition that final demand is non-negative with at least one positive element. This condition is sufficient to guarantee the central requirement of the open IO model: that the model yields a non-negative total output[18]. The model also requires enterprise segment data[19], which must conform to the condition that a segment's total output does not exceed the total output of the sector to which it is classified. By extension, the sum of total outputs of all segments ascribed to the same sector also cannot exceed the total output of the sector itself.

The MRIO model and all enterprise segment data are initially loaded into the model environment. A model set-up procedure is run with the primary purpose of organising and keeping track of how and where enterprise segments are introduced into the MRIO framework. The following six model procedures are executed sequentially:

1. `Construct the default table` – The first step involves the construction of an initial estimate of the MEMRIO table (the *default* table). The MRIO table is disaggregated to include an additional row and column for each enterprise segment. New coefficients are specified by applying the simplifying assumption that enterprise segments have the same IO structure as their parent MRIO sectors (i. e., that they exhibit *sector average performance*).

2. `Adjust the default table` – It is assumed that intra-enterprise transactions are negligible (i. e., that an enterprise segment does not buy from or sell to itself or other segments belonging to the enterprise). The default table therefore requires adjustment to set intra-enterprise coefficients to zero. This is done in such a way as to maintain the balanced IO structure of the default table. The output of this step is the *adjusted* table.

3. `Specify stochastic coefficients` – When the assumption of sector average performance is relaxed, enterprise coefficients are unknown. However, the majority of coefficients would have a negligible affect on the supply chain emissions of the enterprise. An emission multiplier cut-off parameter (specified by the user) is used to determine which coefficients are treated stochastically in the model (*floating* coefficients). Coefficients falling below the cut-off are fixed at their adjusted table values. In addition, enterprise value-added coefficients are deemed to be floating.

4. `Specify model constraints` – Upper and lower bounding constraints are placed on each floating coefficient according to parameters set by the user to reflect the estimated uncertainty in coefficient values. The general IO table requirements that total inputs must equal total outputs and that final demand must be non-negative places further system-wide constraints on floating coefficients.

5. `Construct sample tables` – By randomly sampling individual floating coefficients, a Monte-Carlo simulation is used to construct a set of *sample* tables. Each sample table represents a plausible configuration of how the enterprise is integrated within the global economy.

[18] This condition recognises that it is not strictly necessary for all technical coefficients to take a value less than one for an IO model to be viable.

[19] The issues, and uncertainties, involved in specifying enterprise segments are discussed in Section 2.7

TERM	DESCRIPTION
Model component	A *model component* refers to a scalar, vector, matrix or dataset that constitutes part of the MRIO or MEMRIO model. Model components that take the form of vectors or matrices are indicated as such. Wherever a model component type is unidentified it should be assumed to be a scalar component.
Aggregate and disaggregate model components	MRIO model components are referred to as *aggregate* model components since they have yet to be disaggregated though the introduction of enterprise segments. Following the introduction of enterprise segments into the system, MEMRIO model components are referred to as *disaggregate* model components.
Regions, industries, and sectors	MRIO models represent the economy as interdependent *industries* spread across multiple *regions*. Each region contains the same number of industries (N_I). The term *sector* is used to refer to a specific industry located within a particular region, which corresponds to a specific row or column within MRIO and MEMRIO model components. An MRIO model containing N_R regions therefore represents $N_R \times N_I = N_S$ sectors.
Segments, residual sectors, and entities	The characterisation of an enterprise's activity within a particular sector is referred to as a *segment*, and corresponds to a specific row or column within a MEMRIO model component. When a segment is disaggregated from an original sector, the production activity remaining within the sector is referred to as the *residual sector*. Collectively, sectors, residual sectors and segments are referred to as *entities* within MEMRIO model components. That is, each row or column in a MEMRIO model component corresponds to a particular entity: it may be an unaffected sector or, if part of the disaggregation process, an enterprise segment or a residual sector. The size of the MEMRIO model is therefore defined by the number of entities (N_E).

Table 1: Nomenclature used in the METCA model description.

TERM	DESCRIPTION
Model component blocks	Sub-components of model components are referred to as *blocks*. For example, when an element of the MRIO technical coefficients matrix is disaggregated it corresponds to multiple elements in the MEMRIO technical coefficients matrix: this group of elements is referred to as a block.
Production recipe	The *production recipe* of an MRIO sector or MEMRIO entity refers to the column of technical coefficients that describe the proportion of inputs from other sectors or entities required to production a unit of output.
Floating technical & value-added coefficient	Technical and value-added coefficients ascribed to enterprise segments and residual sectors are defined within the MEMRIO model as either being *fixed* or *variable*. Variable coefficients are referred to as *floating coefficients*.
Emissions multipliers	The *emissions multipliers* of a given MRIO sector are the emissions embodied in each of the sector's intermediary inputs required to produce a unit of output. The sum of these multipliers plus the direct emissions per unit output of the sector itself gives the *total emissions multiplier* of the sector.

Table 1 continued: Nomenclature used in the METCA model description.

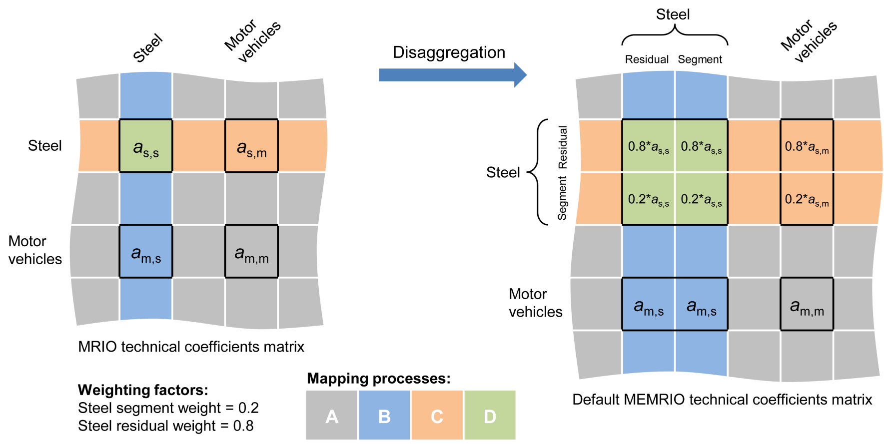


Figure 1: Illustration of the default table construction process.

6. `Estimate supply chain emissions` – The TCA method (introduced in Skelton *et al.* (2011)) is applied to estimate enterprise supply chain emissions in each sample table. The resulting enterprise TCA distribution is then statistically analysed to provide a range in which the supply chain emissions of the actual enterprise is likely to fall.

Each procedure is discussed in more detail below.

Constructing the default table

The first modelling step involves the construction of an initial estimate of the MEMRIO technical coefficients matrix. This initial estimate is referred to as the *default table*. The construction procedure builds on Wolsky's (1984) approach (discussed in Section 1.3) for disaggregating a single sector into two new sectors by applying the simplifying assumption that both enterprise segments and residual sectors have the same IO structure as their parent MRIO sectors. That is, unit output of an enterprise segment requires the same *production recipe* of inputs as the original MRIO sector from which it was disaggregated. To ensure that the default table remains balanced, output coefficients are adjusted according to entity weighting factors.

For example, Figure 1 provides an illustration of the disaggregation process. The intersection of a steel sector and a motor vehicles sector in an original MRIO table is shown on the left-hand-side of the diagram. Four technical coefficients are of interest: intra-sector requirements of both the steel sector ($a_{s,s}$) and the motor vehicles sector ($a_{m,m}$); the steel sector's requirement for motor vehicle inputs ($a_{m,s}$); and, the motor vehicle sector's requirement for steel inputs ($a_{s,m}$). An enterprise segment is introduced into the steel sector with a weighting factor of 0.2 (i. e., the enterprise segment produces 20% of the original steel sector's output, with the residual sector left to produce the remaining 80%), requiring an additional row and column to be inserted in the MEMRIO table as shown on the right-hand-side of the diagram. Four mapping processes are used to specify the MEMRIO technical coefficients:

1. `Mapping process A` – Not all MRIO coefficients are affected by the disaggregation process. Those that are not affected are mapped directly to the MEMRIO table, as is the case for the intra-sector requirement of the motor vehicles sector $a_{m,m}$.

2. `Mapping process B` – Since the inputs to both the enterprise segment and the residual sector are assumed to be the same as those of the original steel sector, the input coefficients of the steel sector are mapped directly to the residual sector in the MEMRIO table and duplicated across to the new column representing the enterprise segment. For

example, the enterprise segment's requirement for motor vehicles is simply $a_{m,s}$. MRIO coefficients undergoing column disaggregation correspond to MEMRIO *blocks* of dimension $1 \times$ *number of column entities*.

3. `Mapping process C` – The combined outputs of the enterprise segment and the residual sector to other sectors must correspond to the outputs of the original steel sector required by other sectors. Output coefficients are therefore mapped similarly to input coefficients but are, in addition, adjusted according to the enterprise and residual weighting factors. For example, say the motor vehicles sector requires 0.1 units of input from the steel sector ($a_{s,m} = 0.1$), in the MEMRIO table this would correspond to a requirement of 0.08 units ($0.8 \times a_{s,m}$) from the residual steel sector and 0.02 units ($0.2 \times a_{s,m}$) from the enterprise segment. MRIO coefficients undergoing row disaggregation correspond to MEMRIO *blocks* of dimension *number of row entities* $\times$ 1.

4. `Mapping process D` – MRIO coefficients undergoing both column and row disaggregation are mapped through a combination of mapping processes B and C, as is the case for the intra-sector requirement of the steel sector $a_{s,s}$. Here, the residual sector and the enterprise are assumed to require the same (intra-sector) steel inputs as the original steel sector (mapping process B), but this requirement is now provided by both the enterprise and the residual sector, again in accordance to their weighting factors (mapping process C). MRIO coefficients undergoing both column and row disaggregation correspond to MEMRIO *blocks* of dimension *number of row entities* $\times$ *number of column entities*.

Adjustments to the default table

The construction of the default table implies the assumption that enterprises engage in intra-enterprise[20] transactions in much the same way as sectors engage in intra-sector transactions. That is, it is assumed that an enterprise will require a certain amount of input from itself in order to produce output. Several cases need to be considered to assess the suitability of this assumption:

1. An enterprise with activities only in a single sector may indeed use some of its output as an input to its production processes. However, this is typically accounted for internally and therefore not captured in the national accounts data from which MRIO models are derived.

2. An enterprise with activities spanning several countries but within a single industry may have relatively autonomous operations across its different regions of activity, hence it may only report minor intra-enterprise transactions.

3. A vertically integrated enterprise with activities spanning several different industries may report significant intra-enterprise transactions. For example, an oil company which refines a major share of its extracted crude oil, may report the market price of crude oil purchased by business units in the refining industry from units in the extraction industry. Compliers of national accounts aim to capture these intra-enterprise transactions that cut across different industries.

The validity of the assumption that enterprises engage in intra-enterprise transactions is therefore dependent on the nature of the enterprises introduced into the model. On one hand it may be appropriate to set all intra-enterprise transactions to zero, but on the other, intra-enterprise transactions could be the most significant transactions made by certain enterprise segments.

[20] Intra-enterprise transactions include both intra- and inter-segment transactions between all segments belonging to a given enterprise.

Based on these considerations, the specification of intra-enterprise transactions is treated exogenously to the sampling procedure for unknown coefficients. For the purposes of the current model description it is assumed that intra-enterprise transactions are negligibly small. The default table therefore requires adjustment to set intra-enterprise coefficients to zero. However, simply setting intra-enterprise coefficients to zero would affect the underlying IO structure of the default table. Returning to the example illustrated in Figure 1, if the intra-enterprise coefficient contained in the steel intra-sector block (corresponding to mapping process D) were set to zero, then overall steel inputs to the enterprise, and overall enterprise outputs to the steel sector, would be under-reported. Hence, whole blocks containing intra-enterprise transactions must be adjusted to retain the underlying IO structure.

The adjustment procedure is inspired by Wolsky's (1984) idea of a *distinguishing* matrix parameterised by independent variables used to relate a default – or *augmented* – matrix with an under-defined disaggregated matrix. Considering the case of a single sector disaggregated into a single enterprise segment (s) and a residual sector (r) and using Wolsky's formulation, the intra-sector coefficients block can be fully described by

$$\begin{pmatrix} A_{rr} & A_{rs} \\ A_{sr} & A_{ss} \end{pmatrix} = \delta \begin{pmatrix} w_s & -w_r \\ w_s & -w_r \end{pmatrix} + \sigma \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} + \varepsilon \begin{pmatrix} w_s & -w_r \\ -w_s & w_r \end{pmatrix} + \begin{pmatrix} H_{rr} & H_{rs} \\ H_{sr} & H_{ss} \end{pmatrix} \tag{1}$$

where, A_{ij} are unknown coefficients, H_{ij} are default coefficients, w_i are weighting factors, δ and σ are independent variables that determine the difference between segment and residual sector demand for input from, and supply of output to, the disaggregated sector, respectively, and ε is a further independent variable that determines intra-aggregate exchanges.

To achieve the objective of resetting intra-enterprise coefficients to zero while maintaining the default IO structure between segments and residual sectors implies that only the intra-aggregate exchange variable ε can be used. Hence, the adjusted coefficients block, corresponding to the case above, can be fully described by

$$\begin{pmatrix} J_{rr} & J_{rs} \\ J_{sr} & J_{ss} \end{pmatrix} = \varepsilon \begin{pmatrix} w_s & -w_r \\ -w_s & w_r \end{pmatrix} + \begin{pmatrix} H_{rr} & H_{rs} \\ H_{sr} & H_{ss} \end{pmatrix} \tag{2}$$

where, J_{ij} are adjusted coefficients and the matrix containing weighting factors is referred to as the *intra-aggregate multipliers*. For this most simple of cases, the adjustment procedure is straightforward: determine the intra-aggregate exchange variable by solving the linear equality expressed in Equation 3 and calculate the adjusted technical coefficients block using Equation 2.

$$J_{ss} = \varepsilon w_r + H_{ss} = 0 \tag{3}$$

The adjustment procedure becomes more involved for the case of a single sector disaggregated into two segments (s1 and s2) and a residual sector (r). In this case, two intra-aggregate exchange variables must be specified (Equation 4), and the corresponding set of simultaneous linear equations (Equations 5 and 6) solved, to calculate the adjusted intra-sector coefficients block.

$$\begin{aligned} \begin{pmatrix} J_{rr} & J_{rs1} & J_{rs2} \\ J_{s1r} & J_{s1s1} & J_{s1s2} \\ J_{s2r} & J_{s2s1} & J_{s2s2} \end{pmatrix} &= \varepsilon_1 \begin{pmatrix} w_{s1} \frac{w_r}{1-w_{s1}} & -(1-w_{s1}) \frac{w_r}{1-w_{s1}} & w_{s1} \frac{w_r}{1-w_{s1}} \\ -w_{s1} & (1-w_{s1}) & -w_{s1} \\ w_{s1} \frac{w_{s2}}{1-w_{s1}} & -(1-w_{s1}) \frac{w_{s2}}{1-w_{s1}} & w_{s1} \frac{w_{s2}}{1-w_{s1}} \end{pmatrix} \\ &+ \varepsilon_2 \begin{pmatrix} w_{s2} \frac{w_r}{1-w_{s2}} & w_{s2} \frac{w_r}{1-w_{s2}} & -(1-w_{s2}) \frac{w_r}{1-w_{s2}} \\ w_{s2} \frac{w_{s1}}{1-w_{s2}} & w_{s2} \frac{w_{s1}}{1-w_{s2}} & -(1-w_{s2}) \frac{w_{s1}}{1-w_{s2}} \\ -w_{s2} & -w_{s2} & (1-w_{s2}) \end{pmatrix} + \begin{pmatrix} H_{rr} & H_{rs1} & H_{rs2} \\ H_{s1r} & H_{s1s1} & H_{s1s2} \\ H_{s2r} & H_{s2s1} & H_{s2s2} \end{pmatrix} \end{aligned} \tag{4}$$

$$J_{s1s1} = \varepsilon_1(1 - w_{s1}) + \varepsilon_2 w_{s2}\frac{w_{s1}}{1 - w_{s2}} + H_{s1s1} \quad (5)$$

$$J_{s2s2} = \varepsilon_1 w_{s1}\frac{w_{s2}}{1 - w_{s1}} + \varepsilon_2(1 - w_{s2}) + H_{s2s2} \quad (6)$$

The computational procedure developed to adjust the default table provides a generalised approach for specifying intra-aggregate multiplier matrices. This allows adjustments to be made to default coefficient blocks of any given size. Adjustments are made to every default coefficient block containing an intra-enterprise coefficient. The resulting *adjusted table* is taken as the reference point for the subsequent stochastic analysis.

Specifying stochastic coefficients

To construct the *default* and *adjusted* tables it was assumed that enterprise segments and residual sectors exhibit the same supply and demand structure as the original sector from which they were disaggregated. This assumption of *average performance* is relaxed for the construction of sample tables, reflecting the reality that enterprise segments will typically differ, perhaps significantly so, from aggregate sector performance. For example, a motor vehicle manufacturer specialising in the production of high-performance vehicles may require less input of basic metal products per unit output than the sector average, with high-technology inputs and in-house value-adding processes instead accounting for a major share of end-product value.

The first step in constructing a sample table is to identify *floating coefficients*. Floating coefficients are defined as technical and value-added coefficients that become nondetermined when the assumption of average performance is relaxed. In principle, this includes all supply (row) and demand (column) technical coefficients and all value-added coefficients of disaggregated entities. However, the emissions multipliers associated with many MRIO coefficients are negligibly small: meaning that corresponding sampled coefficients would have a negligible bearing on the calculation of enterprise supply chain emissions. A further consideration is the computational requirements of the sampling process. Calculation time grows dramatically as the number of floating coefficients increases.

These considerations are reflected in the model through the use of a *user-control parameter* that specifies an emissions multiplier cut-off value: only technical coefficients corresponding to emissions multipliers with a value higher than the cut-off are assumed to be *floating*. For example, a cut-off value of 0.01 implies emissions multipliers M_{ij} that represent less than 1% of a sector's total emissions multiplier m_j to be negligibly small; corresponding blocks of MEMRIO coefficients A_{ij} are fixed at adjusted table values. The emissions multiplier cut-off is used to control the scale of the sampling procedure (i. e., the stochastic component of the model) and hence the overall computational requirements.

Specifying a system of constraints

Following the identification of floating coefficients, the next step involves the specification of a system of equality and inequality constraints that determines the envelope of possible values these coefficients can take. Four types of constraints are considered:

1. `Coefficient bounding constraints` – In principle, each floating technical coefficient has a lower bound equal to zero – reflecting the IO model requirement for positive transactions (i. e., producing sectors sell products to purchasing sectors) – and an upper bound equal to one – reflecting the requirement that inputs must be equal to outputs. However, inaccuracies from procedures used to balance MRIO tables can, although rarely, result in technical coefficients with a value greater than one. Relaxation of this upper bound constraint also implies value-added coefficients can take negative values to maintain overall balance

DIMENSIONAL PARAMETER	SYMBOL	VALUE
Number of MEMRIO *blocks* containing floating technical coefficients	N^{block}	1
Number of MEMRIO *columns* containing floating coefficients	N^{col}	3
Number of MEMRIO *rows* containing floating coefficients	N^{row}	3
Number of floating *technical* coefficients	N^{tech}	9
Number of floating *value-added* coefficients	N^{va}	$N^{col} = 3$
Total number of floating coefficients	N^{float}	$N^{tech} + N^{va} = 12$

Table 2: Example dimensional parameters.

between inputs and outputs. Within the model, a means of controlling upper and lower coefficient bounds is required to reflect the estimated extent to which enterprise segments can differ from sector average performance. This is achieved using two user-control parameters: one to control upper and lower bounds on floating technical coefficients B^A and the other to control upper and lower bounds floating value-added coefficients B^u.

2. `Column sum constraints` – For each purchasing (column) entity, the sum of technical and value-added coefficients must be equal to one, again reflecting the IO model requirement that total inputs must be equal to total outputs. Since a proportion of technical coefficients are fixed, this column constraint can be restated: the sum of floating technical and value-added coefficients must be equal to one less the sum of fixed technical coefficients.

3. `Row sum constraints` – For each producing (row) entity, the sum of intermediate transactions must be less than or equal to its total output, reflecting the IO model requirement for non-negative final demand transactions. Since a proportion of technical coefficients and corresponding intermediate transactions are fixed, this row constraint can be restated: the sum of floating intermediate transactions must be less than or equal to total output less the sum of fixed intermediate transactions.

4. `Coefficient block constraints` – The weighted sum of each MEMRIO technical coefficients block must be equal to its parent MRIO coefficient. That is, the disaggregation process must be reversible.

Steps taken to specify the system of constraints are illustrated below for the case of a single sector (t) disaggregated into two segments (s1 and s2) and a residual sector (r). For the purposes of this example, it is assumed that floating technical coefficients are found only in the coefficient block $\begin{pmatrix} A_{rr} & A_{rs1} & A_{rs2} \\ A_{s1r} & A_{s1s1} & A_{s1s2} \\ A_{s2r} & A_{s2s1} & A_{s2s2} \end{pmatrix}$ corresponding to sector t's intra-sector demand $\mathcal{A}_{tt}$. In addition, each of the three disaggregation entities has a floating value-added coefficient $\begin{pmatrix} u_r & u_{s1} & u_{s2} \end{pmatrix}$. Several dimensional parameters can also be specified (Table 2). For this simple example, each of the four constraint types are considered in turn below.

`Coefficient bounding constraints` – The upper and lower bounding constraints on the 12 floating coefficients are expressed by

$$\begin{pmatrix} \mathbf{b}_A^{lower} \\ \mathbf{b}_u^{lower} \end{pmatrix} = \mathbf{b}^{lower} \leqslant \xi = \begin{pmatrix} \xi_A \\ \xi_u \end{pmatrix} \leqslant \mathbf{b}^{upper} = \begin{pmatrix} \mathbf{b}_A^{upper} \\ \mathbf{b}_u^{upper} \end{pmatrix} \tag{7}$$

where, ξ is the floating coefficients vector (of size $N^{float} \times 1$) consisting of technical coefficients ξ_A and value-added coefficients ξ_u, $\mathbf{b}^{lower}$ is the lower bounds vector (of size $N^{float} \times 1$) consisting of technical coefficient lower bounds $\mathbf{b}_A^{lower}$ and value-added coefficient lower bounds $\mathbf{b}_u^{lower}$, and $\mathbf{b}^{upper}$ is the upper bounds vector (of size $N^{float} \times 1$) consisting of technical coefficient upper bounds $\mathbf{b}_A^{upper}$ and value-added coefficient upper bounds $\mathbf{b}_u^{upper}$. In the example case, these vectors are given by

$$\xi_A = \begin{pmatrix} A_{rr} \\ A_{rs1} \\ A_{rs2} \\ A_{s1r} \\ A_{s1s1} \\ A_{s1s2} \\ A_{s2r} \\ A_{s2s1} \\ A_{s2s2} \end{pmatrix} \qquad \xi_u = \begin{pmatrix} u_r \\ u_{s1} \\ u_{s2} \end{pmatrix} \tag{8}$$

$$\mathbf{b}_A^{lower} = \begin{pmatrix} J_{rr} \\ J_{rs1} \\ J_{rs2} \\ J_{s1r} \\ J_{s1s1} \\ J_{s1s2} \\ J_{s2r} \\ J_{s2s1} \\ J_{s2s2} \end{pmatrix} - B^A \begin{pmatrix} J_{rr} \\ J_{rs1} \\ J_{rs2} \\ J_{s1r} \\ J_{s1s1} \\ J_{s1s2} \\ J_{s2r} \\ J_{s2s1} \\ J_{s2s2} \end{pmatrix} \qquad \mathbf{b}_u^{lower} = \begin{pmatrix} \nu_r \\ \nu_{s1} \\ \nu_{s2} \end{pmatrix} - B^u \begin{pmatrix} \nu_r \\ \nu_{s1} \\ \nu_{s2} \end{pmatrix} \tag{9}$$

$$\mathbf{b}_A^{upper} = \begin{pmatrix} J_{rr} \\ J_{rs1} \\ J_{rs2} \\ J_{s1r} \\ J_{s1s1} \\ J_{s1s2} \\ J_{s2r} \\ J_{s2s1} \\ J_{s2s2} \end{pmatrix} + B^A \begin{pmatrix} J_{rr} \\ J_{rs1} \\ J_{rs2} \\ J_{s1r} \\ J_{s1s1} \\ J_{s1s2} \\ J_{s2r} \\ J_{s2s1} \\ J_{s2s2} \end{pmatrix} \qquad \mathbf{b}_u^{upper} = \begin{pmatrix} \nu_r \\ \nu_{s1} \\ \nu_{s2} \end{pmatrix} + B^u \begin{pmatrix} \nu_r \\ \nu_{s1} \\ \nu_{s2} \end{pmatrix} \tag{10}$$

where, A_{ij} are floating technical coefficients, u_j are floating value-added coefficients, J_{ij} are adjusted technical coefficients, ν_j are default value-added coefficients, and B^A and B^u are user-control parameters. B^A determines floating technical coefficient bounds as a percentage of the corresponding value in the adjusted table. For example, $B^A = 0.1$ would yield bounds $\pm 10\%$ of adjusted table values. Similarly, B^u determines floating value-added coefficient bounds as a percentage of the corresponding value in the default table[21].

`Column sum constraints` – Floating coefficient column sum constraints are expressed in the form of inequalities[22] by

$$\mathbf{INEQ}_{col\_A}\, \xi \leqslant \mathbf{ineq}_{col\_A} \tag{11}$$

where, ξ is again the floating coefficients vector, $\mathbf{INEQ}_{col\_A}$ is a matrix of multipliers (of size $N^{col} \times N^{float}$) pertaining to floating coefficient column sum inequalities, and $\mathbf{ineq}_{col\_A}$ is the corresponding vector of inequality constraints (of size $N^{col} \times 1$). The inequality multiplier matrix and constraints vector for the example are given by

$$\mathbf{INEQ}_{col\_A} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \tag{12}$$

$$\mathbf{ineq}_{col\_A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} c_r \\ c_{s1} \\ c_{s2} \end{pmatrix} \tag{13}$$

where, c_j are the column sum of fixed technical coefficients for each disaggregation entity.

[21] Value-added coefficients are unaffected by the default table adjustment procedure, hence default table values are used. In addition, although the adjustment procedure set intra-enterprise transactions to zero (i.e., in the example both A_{s1s1} and A_{s2s2} are set to zero), the model does not consider these coefficients to be strictly *fixed*. For computational convenience (and flexibility for model extensions) they are instead included in the floating coefficients system, but with lower and upper bounds that effectively constrain the coefficient to zero

[22] Strictly, column sum constraints that include value-added coefficients are equalities. However, rounding errors arising from model calculations can lead to inaccurate conflicts when column sum constraints are defined as equalities. The use of inequalities provides a more flexible environment in which value-added coefficients are effectively treated as having two components: a component that is randomly sampled along with floating technical coefficients (the reason why this is desirable is discussed in the following section) and a further component that is determined as a result of the sampling procedure. That is, inequality constraints ensure sampled coefficients do not violate the requirement that total inputs are not *greater than* total outputs, while the resulting difference between a column entity's inputs and total output relates to the non-sampled component of value-added.

`Row sum constraints` – Floating intermediate transaction row sum constraints are expressed in the form of inequalities by

$$\mathbf{INEQ}_{\text{row\_Z}}\,\xi \leqslant \mathbf{ineq}_{\text{row\_Z}} \tag{14}$$

where, ξ is the floating coefficients vector, $\mathbf{INEQ}_{\text{row\_Z}}$ is a matrix of multipliers (of size $N^{\text{row}} \times N^{\text{float}}$) pertaining to floating intermediate transaction row sum inequalities, and $\mathbf{ineq}_{\text{row\_Z}}$ is the corresponding vector of inequality constraints (of size $N^{\text{row}} \times 1$). The inequality multiplier matrix and constraints vector for the example are given by

$$\mathbf{INEQ}_{\text{row\_Z}} = \begin{pmatrix} x_r & x_{s1} & x_{s2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_r & x_{s1} & x_{s2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_r & x_{s1} & x_{s2} & 0 & 0 & 0 \end{pmatrix} \tag{15}$$

$$\mathbf{ineq}_{\text{row\_Z}} = \begin{pmatrix} x_r \\ x_{s1} \\ x_{s2} \end{pmatrix} - \begin{pmatrix} r_r \\ r_{s1} \\ r_{s2} \end{pmatrix} \tag{16}$$

where, x_j in the inequality multiplier matrix are column entity total outputs, x_i in the inequality constants vector are row entity total outputs, and r_i are the row sum of fixed intermediate transactions for each disaggregation entity. Column and row sum constraints are concatenated into a single system of inequalities as expressed by

$$\begin{pmatrix} \mathbf{INEQ}_{\text{col\_A}} \\ \mathbf{INEQ}_{\text{row\_Z}} \end{pmatrix} \xi = \mathbf{INEQ} \cdot \xi \leqslant \mathbf{ineq} = \begin{pmatrix} \mathbf{ineq}_{\text{col\_A}} \\ \mathbf{ineq}_{\text{row\_Z}} \end{pmatrix} \tag{17}$$

`Coefficient block constraints` – Floating technical coefficient block constraints are expressed in the form of equalities by

$$\mathbf{EQ}_{\text{block}}\,\xi = \mathbf{eq}_{\text{block}} \tag{18}$$

where, ξ is the floating coefficients vector, $\mathbf{EQ}_{\text{block}}$ is a matrix of multipliers (of size $N^{\text{block}} \times N^{\text{float}}$) pertaining to floating coefficient block equalities, and $\mathbf{eq}_{\text{block}}$ is the corresponding vector of equality constraints (of size $N^{\text{block}} \times 1$). The equality multiplier matrix and constants vector for the example are given by

$$\mathbf{EQ}_{\text{block}} = \begin{pmatrix} w_r & w_{s1} & w_{s2} & w_r & w_{s1} & w_{s2} & w_r & w_{s1} & w_{s2} & 0 & 0 & 0 \end{pmatrix} \tag{19}$$

$$\mathbf{eq}_{\text{block}} = \mathcal{A}_{tt} = \mathbf{i}' \begin{pmatrix} w_r A_{rr} & w_{s1} A_{rs1} & w_{s2} A_{rs2} \\ w_r A_{s1r} & w_{s1} A_{s1s1} & w_{s2} A_{s1s2} \\ w_r A_{s2r} & w_{s1} A_{s2s1} & w_{s2} A_{s2s2} \end{pmatrix} \mathbf{i} \tag{20}$$

where, w_j are column entity weighting factors, $\mathcal{A}_{tt}$ is the MRIO coefficient corresponding the sole MEMRIO block containing floating coefficients, and $\mathbf{i}$ is a summation vector. The re-aggregation formula for the disaggregated block is shown on the right-hand-side of Equation 20, which illustrates the rational behind the equality multiplier matrix. Additional blocks of floating coefficients would introduce further rows of equality multipliers and corresponding MRIO technical coefficient.

Constructing sample tables

Having split the MEMRIO model into deterministic and constrained-stochastic parts, the central modelling procedure involves the construction of a set of sample MEMRIO tables using a Monte-Carlo simulation approach. Each *sample table* is constructed by randomly sampling floating coefficients within the imposed system of constraints. When an individual floating coefficient is sampled its value becomes fixed within the current sample table. This affects the system of constraints imposed on subsequently sampled coefficients: the envelope of possible values a floating coefficient can take becomes increasingly restricted as more coefficients are sampled. Consequently the system of constraints must be adjusted after each coefficient has been sampled. When all floating coefficients have been sampled the sample table becomes fully defined and represents just one possible solution to the stochastic

MEMRIO model. Sample tables can be stored for later analysis or analysed within the Monte-Carlo loop. The system of constraints must be reset to initial values before the next sample table can be constructed.

The order in which floating coefficients are sampled is important. For example, if the coefficients in a given column are sampled sequentially from top to bottom, then a bias could be introduced into the set of sampled tables: coefficients near the top of the column would exhibit a wider distribution of values then those at the bottom. To ensure that no bias is introduced into the system, a random permutation of the floating coefficient sampling order is used[23]. A linear optimisation routine is used to sample floating coefficients, which consists of the following steps:

1. Construct an objective function that identifies the coefficient to be sampled as the object for linear optimisation.

2. Use linear optimisation to estimate the *minimum* value the coefficient can take within the current system of constraints[24].

3. Make the *negative* of the coefficient the object for linear optimisation.

4. Use linear optimisation to estimate the *maximum* value the coefficient can take.

5. Randomly sample the coefficient from the range of possible values delineated by the estimated minimum and maximum permissible values, assuming a probability density function of continuous uniform distribution (i. e., all values within the range have an equal likelihood of being sampled).

6. Introduce the sampled coefficient into the current system of constraints in the form of an equality.

The steps taken to sample a floating coefficient are illustrated below by continuing with the example of a single sector (t) disaggregated into two segments (s1 and s2) and a residual sector (r). Say the random permutation of the sampling order determined that floating coefficient A_{s1s2} is the first to be sampled. The *minimum* value linear optimisation problem and its required objective function $\mathbf{obj}^{\min}_{A_{s1s2}}$ are expressed by

$$\mathbf{obj}^{\min}_{A_{s1s2}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{21}$$

$$A^{\min}_{s1s2} = \min_{\xi} \mathbf{obj}^{\min}_{A_{s1s2}} \cdot \xi \quad \text{subject to} \quad \begin{cases} \mathbf{INEQ} \cdot \xi \leqslant \mathbf{ineq} \\ \mathbf{EQ}_{\text{block}}\, \xi = \mathbf{eq}_{\text{block}} \\ \mathbf{b}^{\text{lower}} \leqslant \xi \leqslant \mathbf{b}^{\text{upper}} \end{cases} \tag{22}$$

and the *maximum* value linear optimisation problem and its required objective function $\mathbf{obj}^{\max}_{A_{s1s2}}$ are expressed by

$$\mathbf{obj}^{\min}_{A_{s1s2}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix} \tag{23}$$

$$A^{\max}_{s1s2} = \min_{\xi} \mathbf{obj}^{\max}_{A_{s1s2}} \cdot \xi \quad \text{subject to} \quad \begin{cases} \mathbf{INEQ} \cdot \xi \leqslant \mathbf{ineq} \\ \mathbf{EQ}_{\text{block}}\, \xi = \mathbf{eq}_{\text{block}} \\ \mathbf{b}^{\text{lower}} \leqslant \xi \leqslant \mathbf{b}^{\text{upper}} \end{cases} \tag{24}$$

where, ξ is the floating coefficients vector, **INEQ** and **ineq** are the inequality multiplier matrix and constants vector, respectively (Equation 17), $\mathbf{EQ}_{\text{block}}$ and $\mathbf{eq}_{\text{block}}$ are the block equality multiplier matrix and constants vector, respectively (Equation 18), and $\mathbf{b}^{\text{upper}}$ and $\mathbf{b}^{\text{lower}}$ are the upper and lower

[23] Random permutations of the floating coefficients sampling order are specified using in-built functionality of the Matlab language that draws from a uniformly distributed pseudorandom number generator.

[24] Linear optimisation is performed using inbuilt linear programming functionality of the Matlab language. Specifically, a medium-scale active-set linear programming algorithm, within the *linprog* function, is used, although the model could be readily modified to accommodate an alternative algorithm.

coefficient bounds vectors (Equation 7). The range of possible values A_{s1s2} can take is then given by

$$A^{min}_{s1s2} \leqslant A_{s1s2} \leqslant A^{max}_{s1s2} \tag{25}$$

When a value is randomly sampled from the range the coefficient becomes fixed within the current sample table A^{fixed}_{s1s2}. The induced change in the system of constraints caused by defining A^{fixed}_{s1s2} is captured by specifying the following additional system of equalities

$$\mathbf{EQ}_{coeff}\, \xi = \mathbf{eq}_{coeff} \tag{26}$$

where, $\mathbf{EQ}_{coeff}$ and $\mathbf{eq}_{coeff}$ are the *coefficient* equality multiplier matrix and constants vector, respectively. Following the sampling of the first floating coefficient A_{s1s2}, the equality multiplier matrix and constants vector are given by

$$\mathbf{EQ}_{coeff} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{27}$$

$$\mathbf{eq}_{coeff} = A^{fixed}_{s1s2} \tag{28}$$

For each additional coefficient that is sampled, a further row is added to the *coefficient* equality multiplier matrix and the constants vector. The final stage is to concatenate all equalities into a single system of equalities as expressed by

$$\begin{pmatrix} \mathbf{EQ}_{block} \\ \mathbf{EQ}_{coeff} \end{pmatrix} \xi = \mathbf{EQ} \cdot \xi = \mathbf{eq} = \begin{pmatrix} \mathbf{eq}_{block} \\ \mathbf{eq}_{coeff} \end{pmatrix} \tag{29}$$

The updated system of equalities is taken as input to the next sampling routine.

Model checks

Model checks are performed during the construction of the MEMRIO *default*, *adjusted* and *sample* tables. All tables are assessed against four test criteria to ensure that they have been constructed correctly:

1. `Non-negative coefficient test` – A general requirement of IO tables is that all technical coefficients are non-negative. Any coefficients found to be less than zero would indicate that an error has occurred during the construction of the table.

2. Column sum test – The column sum of technical coefficients plus value-added coefficients must be equal to one, ensuring that total inputs equal total outputs.

3. Non-negative final demand test – A fundamental requirement of IO tables is that final demand is non-negative with at least one positive element (Dietzenbacher, 2005).

4. Re-aggregation test – Re-aggregation of a MEMRIO table according to the MRIO sector classification must yield the original MRIO table.

Default and adjusted tables are recalculated if they fail any of these test. Sample tables that fail the tests are flagged as invalid within the Monte-Carlo simulation.

2.3 *Estimating enterprise supply chain emissions*

The MEMRIO model provides a means of assessing the role of individual enterprises, or a group of enterprises, within the context of the global economy. Using the model the overall production activity of a multi-national and/or multi-industry enterprise is decomposed into enterprise segments that correspond to the regional and industrial classifications used in an MRIO model. The MEMRIO model has been designed to depend only on publicly available enterprise data. It is assumed that this data is sufficient to accurately estimate enterprise total output (revenue) from production activity in each segment[25], but is insufficient to estimate enterprise segment *production recipes* and detailed sales structures. That is, enterprise technical coefficients corresponding to purchases from and sales to MRIO sectors cannot be uniquely specified. To accommodate this lack of detailed data, the MEMRIO model treats enterprise technical coefficients stochastically: sample MEMRIO tables are constructed that present plausible configurations of how an enterprise is integrated within the wider economy. Measures of interest can then be statistically analysed across the set of sample tables.

Specifically, the MEMRIO model was developed to assess enterprise supply chain emissions. This is achieved using the TCA method developed in Skelton *et al.* (2011). The total consumption attribution of an enterprise (e) with multiple segments characterised in a MEMRIO sample table is given by

$$\mathrm{TCA_e} = \mathbf{f}^*(\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{A}_{:\mathrm{e}}\mathbf{x}_\mathrm{e} + \mathbf{f}_\mathrm{e}\mathbf{x}_\mathrm{e} \tag{30}$$

where $\mathbf{f}^*$ is a sub-vector of the emissions intensity row vector where the elements ascribed to enterprise segments have been stripped out, $\mathbf{I}$ is an appropriately sized identity matrix, $\mathbf{A}^*$ is a sub-matrix of the sampled MEMRIO technical coefficients matrix where rows and columns ascribed to enterprise segments have been stripped out, $\mathbf{A}_{:\mathrm{e}}$ is another sub-matrix of the sampled MEMRIO matrix where the rows ascribed to enterprise segments and columns ascribed to all other entities have been stripped out (i. e., ':' denotes all entities except enterprise segments), and $\mathbf{f}_\mathrm{e}$ and $\mathbf{x}_\mathrm{e}$ are sub-vectors of the emissions intensity row vector and total output column vector, respectively, where elements not belonging to the enterprise have been stripped out.

The supply chain emissions perspective of an enterprise estimates indirect emissions from all supply chains stemming from the enterprise and the direct emissions of the enterprise itself: it provides a measure of global emissions embodied in the total output of goods and services produced by the enterprise. The application of the TCA method across a set of sample tables yields a distribution of possible values. This distribution can then be statistically analysed to give a value-range in which the actual enterprise TCA is likely to fall. Section 2.4 provides a detailed explanation of TCA distributions provided by the MEMRIO model by analysing results for a hypothetical enterprise.

The MEMRIO model includes three *user-control parameters* that allow an analyst to control the scale of the stochastic part of the model. Section 2.5 considers how the model responds to changes in user-control parameters, again by drawing on results for the same hypothetical enterprise reported in Section 2.4. The aim of this exercise is not to provide a comprehensive sensitivity analysis of the model to user-control parameters (since model response to these parameters is dependent on the nature of the enterprise under analysis), but rather to develop an understanding of the underlying mechanisms that translate changes in parameter value into observed changes in enterprise TCA distributions.

The estimation of the supply chain emissions of a *group* of interdependent enterprises is discussed in Section 2.6. Results are analysed for a hypothetical group of three enterprises, with the aim of highlighting the importance of how the TCA method implicitly removes the risk of double-counting emissions, which arises from enterprises falling within one another's supply chains.

The analysis of hypothetical enterprises in Sections 2.4–2.6 use the Global Trade Analysis Project (GTAP) Version 8 GMRIO model as input into the MEMRIO model.

[25]The feasibility of this assumption is considered in Section 2.7

USER-CONTROL PARAMETER	SYMBOL	VALUE
Emissions multiplier *supply* cut-off parameter	C^s	1
Emissions multiplier *demand* cut-off parameter	C^d	0.01
Floating *technical* coefficient bounding parameter	B^A	0.5
Floating *value-added* coefficient bounding parameter	B^u	0.5
Number of sample MEMRIO tables	N	500

Table 3: Base case specification of user-control parameters for Enterprise 1.

2.4 *Distributions of enterprise supply chain emission*

The application of the TCA method across a set of sample MEMRIO tables gives a stochastic estimate of enterprise supply chain emissions. This section outlines the stages involved in constructing a set of sample tables for a hypothetical enterprise and the subsequent statistical analysis of the enterprise's TCA distribution. The aim of this section is two-fold: to provide an example of the type of results generated by the MEMRIO model and how they can be evaluated; and to provide an understanding of the underlying mechanisms that lead to a distribution in TCA values across a set of sample tables.

For the purposes of this exercise, a hypothetical enterprise (enterprise 1) is specified as an automobile manufacturer characterised by a single segment ascribed to the German Motor Vehicles & Parts sector in the GTAP-GMRIO model[26]. The total output (revenue) of the segment is $50 bn, which accounts for 13% of the segment's parent MRIO sector's total output. In addition, user-control parameters taken as inputs into the model were specified according to Table 3.

Both floating coefficient bounding parameters (B^A and B^u) were set to 0.5, implying that all technical and value-added coefficients that are treated stochastically in the model must be sampled from a range of ±50% of their adjusted table values. To simplify the analysis, the emissions multiplier cut-off parameter is split into two parameters: one to control the segment and residual sector demand (input) coefficients (C^d), the other to control their supply (output) coefficients (C^s). The supply and demand cut-off's were set to 1 and 0.01, respectively. This forces the model to only specify floating coefficients in segment and residual sector *columns*: those with corresponding MRIO emissions multiplier's greater than 1% of the total emissions multiplier of the German Motor Vehicles & Parts sector. All segment and residual sector *row* coefficients are fixed at adjusted table values.

Construction of the adjusted table

A *default* MEMRIO table **H** is first constructed by disaggregating the MRIO table. This involves splitting the row and column of the German Motor Vehicles & Parts sector into a row and column representing the enterprise segment (s) and a row and column representing the residual sector (r). For the newly disaggregated entities, default technical coefficients are specified by assuming sector average performance. The intersection of the segment row and column represents the only intra-enterprise transaction within the MEMRIO model. The *adjusted* table **J** is therefore constructed by re-balancing

[26]The hypothetical enterprise was selected on the basis that the narrative relating to the diverse and emissions-intensive supply chains stemming from automobile manufacture is relatively easy to communicate. For example, the production of an automobile requires inputs of metal products from first-tier suppliers, these suppliers need steel inputs from second-tier suppliers, who in turn purchase electricity from third-tier suppliers. Germany was selected on the basis that it has a prominent automotive industry dominated by several large manufacturers.

the intra-sector coefficients block so as to set the intra-enterprise transaction to zero. The results of this procedure are given by

$$
\begin{aligned}
\mathbf{H}_{rs} &= \begin{pmatrix} H_{rr} & H_{rs} \\ H_{sr} & H_{ss} \end{pmatrix} = \begin{pmatrix} 0.172 & 0.172 \\ 0.024 & 0.024 \end{pmatrix} & \sum_{col} \mathbf{H}_{rs} &= \begin{pmatrix} 0.196 & 0.196 \end{pmatrix} \\
\mathbf{Z}^{H}_{rs} &= \begin{pmatrix} Z^{H}_{rr} & Z^{H}_{rs} \\ Z^{H}_{sr} & Z^{H}_{ss} \end{pmatrix} = \begin{pmatrix} 62194 & 8615 \\ 8615 & 1193 \end{pmatrix} & \sum_{row} \mathbf{Z}^{H}_{rs} &= \begin{pmatrix} 70809 \\ 9808 \end{pmatrix} \\
\mathbf{J}_{rs} &= \begin{pmatrix} J_{rr} & J_{rs} \\ J_{sr} & J_{ss} \end{pmatrix} = \begin{pmatrix} 0.169 & 0.196 \\ 0.027 & 0 \end{pmatrix} & \sum_{col} \mathbf{J}_{rs} &= \sum_{col} \mathbf{H}_{rs} \\
\mathbf{Z}^{J}_{rs} &= \begin{pmatrix} Z^{J}_{rr} & Z^{J}_{rs} \\ Z^{J}_{sr} & Z^{J}_{ss} \end{pmatrix} = \begin{pmatrix} 61001 & 9808 \\ 9808 & 0 \end{pmatrix} & \sum_{row} \mathbf{Z}^{J}_{rs} &= \sum_{row} \mathbf{Z}^{H}_{rs}
\end{aligned} \tag{31}
$$

where, $\mathbf{H}_{rs}$ is the default intra-sector coefficients block containing the intra-enterprise transaction (H_{ss}), $\mathbf{Z}^{H}_{rs}$ is the corresponding default intermediate transactions block, $\mathbf{J}_{rs}$ is the adjusted intra-sector coefficients block, and $\mathbf{Z}^{J}_{rs}$ is the corresponding adjusted intermediate transactions block. The results confirm that the desired outcome was achieved: the intra-enterprise transaction has been set to zero while maintaining the overall supply and demand structure of the disaggregated entities.

Specification of stochastic coefficients

The emissions multiplier *demand* cut-off C^d was pre-specified as 0.01. This implies that segment and residual sector *column* technical coefficients are considered to be *floating* only if their corresponding MRIO emissions multiplier is greater than 1% of the total emissions multiplier of the German Motor Vehicles & Parts sector.

The total emissions embodied in a unit of output from the German Motor Vehicles & Parts sector is calculated to be 293 t CO_2. 13 emission multipliers individually accounted for more than 1% of this value[27]. These multipliers correspond to a total of 28 MEMRIO technical coefficients: 12 multipliers each map to a pair of MEMRIO coefficients (i. e., following mapping process B as illustrated in Figure 1) and one multiplier maps to a block of four coefficients (i. e., following mapping process D). To simplify the analysis, the emissions multiplier *supply* cut-off C^s was set to one, implying that all segment and residual sector row coefficients (i. e., those following mapping process C) are fixed. Hence, the only MEMRIO columns containing floating technical coefficients are those corresponding to the residual sector and the enterprise segment. Therefore, only two value-added coefficients are specified to be floating. A total of 30 floating coefficients are treated stochastically in the construction of each sample table.

Distribution of sampled coefficients

A total of 500 sample tables were constructed and validated against the four test criteria outlined in Section 2.2. However, the linear optimisation routine reported errors during the construction of 8 sample tables. The errors relate to overly stringent constraints caused by finite computational precision. In such cases, the optimisation routine minimises the worst case constraint violation. In the case of the MEMRIO model, errors introduced by computational precision are negligible compared with the uncertainty associated with IO data. All 500 sample table could therefore be safely taken forward for analysis. However, given the small number of samples reporting optimisation errors, and to avoid the potential risk of including erroneous results, sample tables with reported errors are excluded from the analysis.

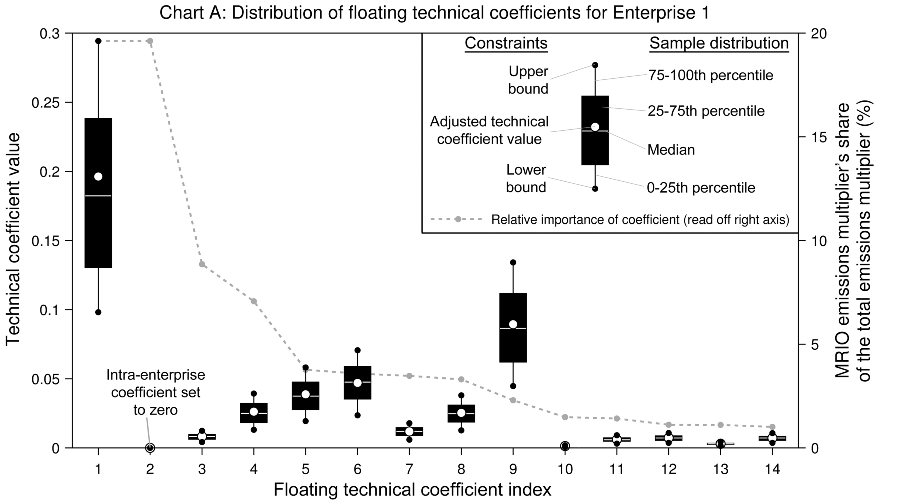
[27]Collectively, these 13 emission multipliers account for 58% of the total emissions multiplier, indicating that even with so few coefficients deemed to be floating (only 0.18% of the possible 7353), a major proportion of the enterprise's supply chain emissions will be accessed stochastically within the MEMRIO model.

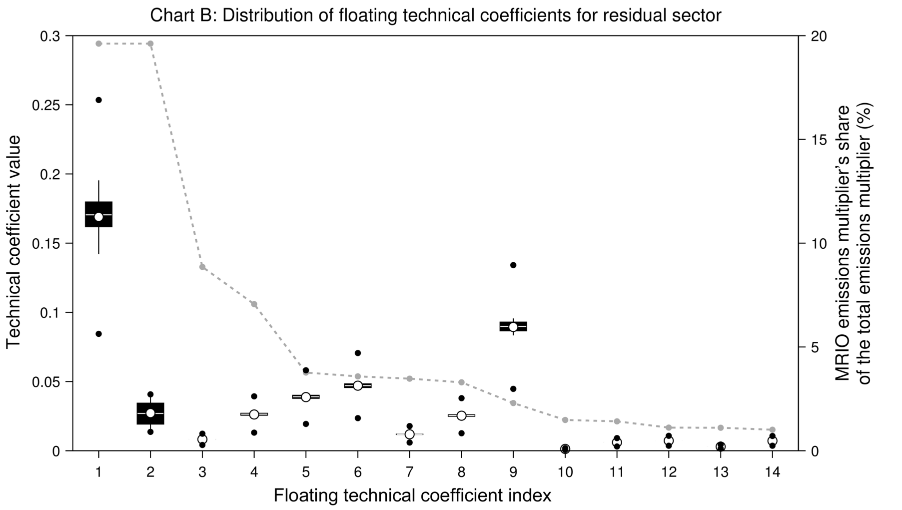
Figure 2 conveys four pieces of information about each of the 30 floating coefficients and their sampled values across the set of 492 sample tables:

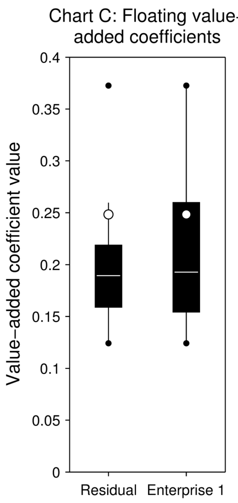
1. The value of the coefficient in the adjusted table (indicated by a white dot).
2. The lower and upper bounding constraints placed on the coefficient (indicated by black dots).
3. The distribution of the sampled values (indicated using a box plot: values falling between the 25th and 75th percentiles are shown by the width of the black box, values falling between the 0th and 25th percentiles and between the 75th and 100th percentiles are shown by vertical black lines and the median sample value is shown by a white horizontal line).
4. The relative importance of each coefficient as measured by its corresponding MRIO emissions multiplier's share of the column entity's total emissions multiplier (indicated by a grey dot along a dashed line plot and used to determine the order technical coefficients take along the x-axis).

The figure is divided into three charts. Chart A includes the floating technical coefficients that relate to the inputs to the enterprise. These are numbered 1-14, corresponding to the supplier sectors detailed in the key (located at the bottom right of the figure). Chart B includes floating technical coefficients for the same 14 supplier sectors, this time relating to inputs to the residual sector. Finally, Chart C includes the two floating value-added coefficients, one for the enterprise and one for the residual sector. Inspection of Figure 2 allows the following observations to be made:

1. The relative contributions of input sectors to the overall embodied emissions per unit output of the German Motor Vehicles & Parts sector is given by the grey line plot (read off the right-hand axis). Important contributors include the German electricity, transport, and metals industries. However, the most important contribution (almost 20%) comes from intra-sector inputs. For example, a motor vehicle manufacturer may purchases sub-components from a motor vehicle parts manufacturer. Such transactions manifest as intra-sector transactions due to the level of aggregation in the MRIO model.
2. The relative embodied emissions contributions of the input sectors do not necessarily correspond to their relative importance as suppliers to the sector (as measured by their technical coefficients). For example, the German Business Services sector (represented by technical coefficient 9) has a high technical coefficient relative to its share of embodied emissions and the German Electricity sector (represented by technical coefficient 3) has a low technical coefficient relative to its share of embodied emissions. This can be explained by the high embodied emissions intensity of the electricity sector and the low embodied emissions intensity of the business services sector.
3. The enterprise's requirement from the residual German Motor Vehicles & Parts sector (coefficient 1, Chart A) is likely to predominantly determine the distribution of the enterprise's sample TCAs given such a high and wide range of sample coefficient values and the importance of this input in terms of embodied emissions.
4. In some cases the sample range (represented by the box plots) does not cover the full range permitted by the individual constraints placed on the coefficient (indicated by black dots). This is the case for the residual sector's coefficients (shown in Chart B), and occurs primarily due to the difference in size of the enterprise and the residual sector.







Floating technical coefficient	Region	Industry	Disaggregation entity
1	Germany	Motor vehicles & parts	Residual sector
2	Germany	Motor vehicles & parts	Enterprise 1
3	Germany	Electricity	-
4	Germany	Road & rail transport	-
5	Germany	Metal products	-
6	Germany	Machinery & equipment	-
7	Germany	Ferrous metals	-
8	Germany	Chemicals, rubber & plastic products	-
9	Germany	Business services	-
10	Greece	Sea transport	-
11	Germany	Non-ferrous metals	-
12	USA	Motor vehicles & parts	-
13	Germany	Air transport	-
14	Czech Republic	Motor vehicles & parts	-

Figure 2: Distributions of sampled floating coefficients.

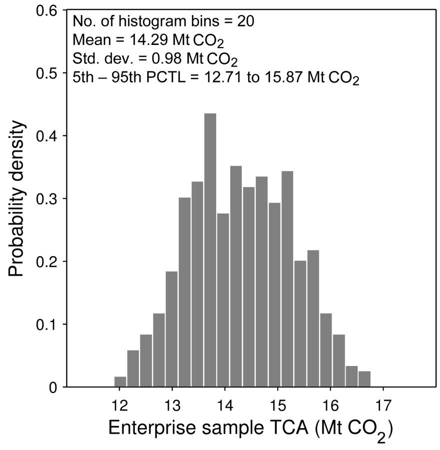


Figure 3: Distribution of hypothetical enterprise TCAs.

The residual sector has an 87% share of the aggregate sector's total output, therefore a relatively large change in an enterprise coefficient will lead to a relatively small change in the corresponding residual sector coefficient as a result of the coefficient block constraints imposed on the model.

5. The interdependence of coefficient sample values is further complicated by the presence of system-wide (column sum and row sum) constraints. This is most notably observed in the distribution of the residual sector's value-added coefficient (shown in Chart C). The value of this coefficient is not directly tied to the enterprise's value-added coefficient, so that fact that the sample does not exploit the full range permitted by the coefficient's upper and lower bounding constraints can only be explained by the fact that wider model constraints are binding.

Distribution of enterprise TCAs

The distribution of the enterprise's TCA across the set of sample MEMRIO tables is shown in Figure 3 as a probability density histogram. The sample mean ($\bar{x}$) and standard deviation (s) of the distribution are given by

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{TCA}_i \tag{32}$$

$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (\mathrm{TCA}_i - \bar{x})^2} \tag{33}$$

where, TCA_j is the enterprise's total consumption attribution estimated using sample table i, and n is the total number of sample tables.

The enterprise's TCA distribution, with user-control parameters specified according to Table 3, is found to have a mean of 14.29 Mt CO_2, a standard deviation of 0.98 Mt CO_2, and it can be concluded that the supply chain emissions of the enterprise are likely to fall between 12.7 and 15.9 Mt CO_2 (5th–95th percentiles). The following section considers how this *base-case* distribution is affected by the choice of user-control parameters.

2.5 *Model response to user-control parameters*

User-control parameters affect the scale of the stochastic component of the MEMRIO model. At the extremes, setting both emissions multiplier cut-off parameters (C^d and C^d) to 1 forces the model to fix all coefficients at their adjusted table values, thereby rendering the model fully deterministic, while setting these parameters to zero forces the model to treat all disaggregated coefficients stochastically (i. e., all coefficients in the segment and residual sector columns and rows would be considered to be floating). Having specified which coefficients are to be modelled stochastically, the bounding parameters (B^A and B^u) are used to control the extent to which individual coefficients can deviate from their adjusted table values. At the extremes, setting both bounding parameters to zero would again render the model fully deterministic since upper and lower bounds would converge to adjusted table values, while setting these parameters to 1 would allow floating coefficients to range from zero to +100% of adjusted table values.

The model response to user-control parameters is evaluated in this section by systematically adjusting the emissions multiplier demand cut-off (C^d) and both bounding parameters (B^A and B^u) for the case of the hypothetical automobile manufacturer introduced in the previous section. The parameter values used in the previous section are taken as a *base-case* scenario. Over a further six MEMRIO model runs, the values of C^d, B^A, and B^u are individually halved and then doubled with respect to these base-case values, as outlined in Table 4 The aim of this exercise is to develop an understanding of the underlying mechanisms that translate changes in user-control parameters into observed effects on the hypothetical enterprise TCA distribution. The extent to which these findings help inform about the general model response to user-control parameters is also considered.

Response to cut-off parameter C^d

Figure 4 shows the effect of halving (model run 2) and doubling (model run 3) the emissions multiplier demand cut-off parameter C^d on the enterprise TCA distribution. Charts A, B and C show the enterprise TCA distribution for the base case, and model runs 2 and 3, respectively. For each model run, the probability density distribution of observed values is represented by a 20-bin histogram. For each model run the distribution sample mean, standard deviation, full range, and 5th–95th percentile range are reported in Table 4. A smooth probability density function (PDF) is also fitted to aid the comparison of distributions shown together in Chart D[28].

In the case of the hypothetical automobile manufacturer, Figure 4 shows that the TCA distribution is insensitive to the choice of cut-off parameter value: the sample mean remains almost constant, increasing by 0.07% when the parameter is halved and decreasing by 0.28% when doubled; the sample standard deviation increases slightly in both cases, by 1% and 4%, respectively; halving the parameter decreases the sample range by 3.49% but increases the 5th-95th percentile range by 2.85%, when doubled both ranges increase by 5.34% and 4.43%, respectively. Effects of this magnitude are within the variability expected across multiple model runs of the base-case scenario.

Figure 5 helps to explain this apparent insensitivity to the cut-off parameter. The figure shows how the number of emissions multipliers falling above the cut-off increases as the parameter value tends to zero (read off the right-hand axis), and how this affects the share of the total emissions multiplier that is stochastically captured in the MEMRIO model (read off the left-hand axis).

When the cut-off parameter is doubled ($C^d = 0.02$) five emissions multipliers that were accounted for in the base-case scenario drop below the cut-off value. Together, these five multipliers account for 6.8% of the total emissions multiplier. For the enterprise, the five multipliers correspond to coefficients 10–14 in Chart A of Figure 2. Inspection of Figure 2 Chart A shows that these coefficients have relatively low and narrow sample value ranges compared to coefficients 1–9. This helps to explain why the removal

[28]The smooth PDF is estimated using the built-in kernel smoothing functionality of the Matlab language.

MODEL RUN	ADJUSTMENT	C^d	B^A	B^u	SAMPLE MEAN ($\bar{x}$)	SAMPLE STD. DEV. (s)	SAMPLE RANGE	SAMPLE 5TH–95TH PCTL. RANGE
1	base-case	0.01	0.5	0.5	14.29	0.98	11.89–16.77	12.71–15.87
2	halve C^d	0.005	0.5	0.5	14.30	0.99	12.08–16.78	12.73–15.99
3	double C^d	0.02	0.5	0.5	14.26	1.02	11.78–16.91	12.57–15.87
4	halve B^A	0.01	0.25	0.5	14.40	0.57	12.54–16.03	13.48–15.30
5	double B^A	0.01	1	0.5	13.90	1.62	9.53–18.01	11.27–16.45
6	halve B^u	0.01	0.5	0.25	14.19	0.83	12.28–16.07	12.79–15.48
7	double B^u	0.01	0.5	1	14.24	1.12	11.01–18.32	12.43–16.01

Table 4: Response of enterprise TCA distributions to user-control parameters.

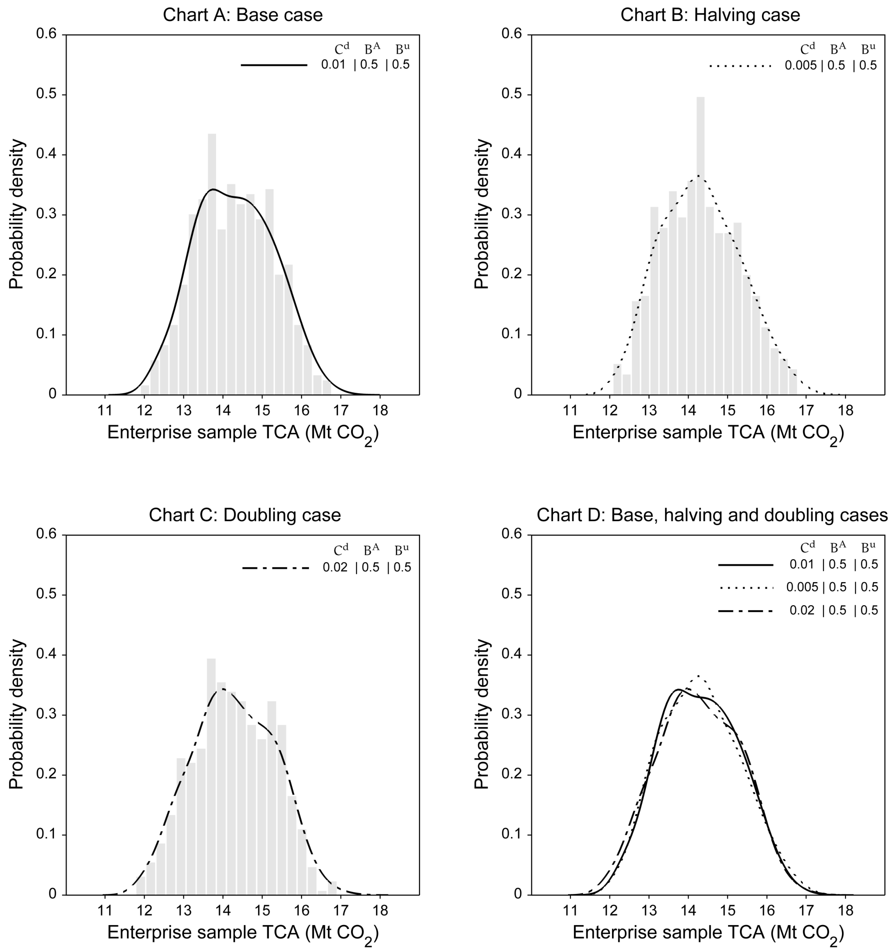


Figure 4: Response of enterprise TCA distribution to cut-off parameter C^d.

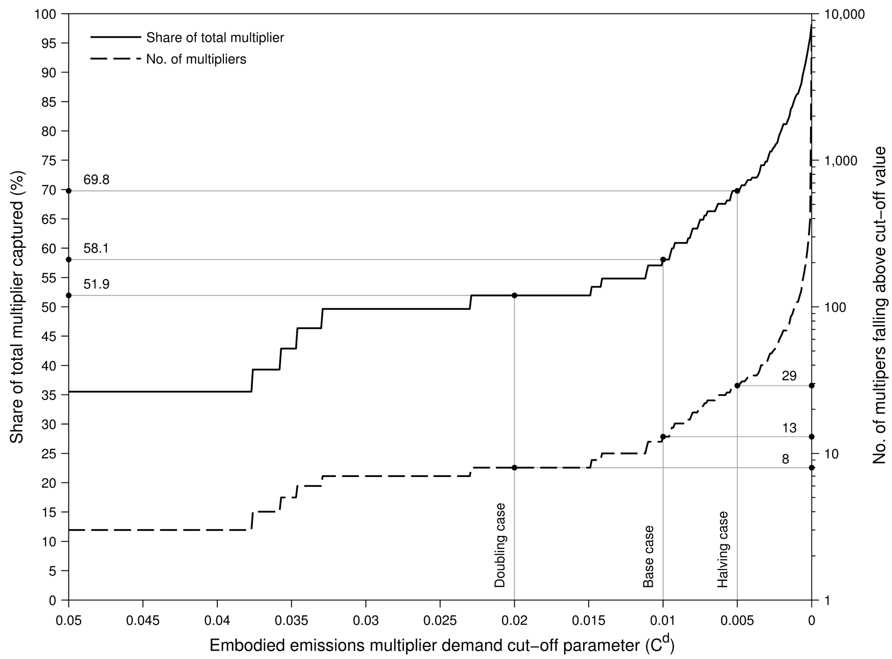


Figure 5: Interpreting insensitivity to cut-off parameter C^d.

of coefficients 10–14 from the stochastic component of the model has a negligible effect on the enterprise TCA distribution. If the cut-off parameter were to be set even higher at 0.5, then coefficients 4–9 would also be fixed at their adjusted table values. Further inspection of Figure 2 Chart A shows that these coefficients have relatively high and wide sample value ranges, indicating that setting the cut-off parameter at 0.5 would significantly reduce the spread of the enterprise TCA distribution.

When the cut-off parameter is halved ($C^d = 0.005$) 16 additional coefficients are treated stochastically in the model. It would be reasonable to anticipate that this would significantly increase the spread of the enterprise TCA distribution. However, the sample standard deviation and the 5th–95th percentile range increase only slightly and the full sample range actually decreases. These results can be partly explained by the fact that as the number of floating coefficients increases, the likelihood of randomly sampling coefficients that yield the minimum or maximum possible TCA value is reduced. That is, the effects of individual sample coefficient values are more likely to cancel each other out within a given sample table[29]. A larger set of sample tables is therefore required to capture the full extent of the sample distribution. Decreasing the cut-off parameter further still is very unlikely to significantly alter the distribution.

Overall, the base-case estimate that the enterprise's supply chain emissions are likely to fall between 12.7 and 15.9 Mt CO_2 is shown to be robust for cut-off parameter values between 0 and 0.02. Hence there is no a priori reason, other than computational resource considerations, for selecting a cut-off value between this range. Cut-off parameter values greater 0.02 are, however, likely to reduce the width of the range in which the enterprise's supply chain emissions are estimated to fall. The point at which estimates of enterprise supply chain emissions become sensitive to the choice of cut-off parameter depends greatly on the input structure of the MRIO sectors in which enterprise segments are located. The inspection of emissions multipliers (e. g., Figure 5) and coefficient sample value distributions (e. g., Figure 2) can assist the analyst in selecting an appropriate cut-off parameter value that balances the trade-off between a robust distribution estimate and the computational resources required to perform the analysis.

Response to bounding parameter B^A

Figure 6 shows the effect of halving (model run 4) and doubling (model run 5) the floating technical coefficient bounding parameter B^A on the enterprise TCA distribution. Charts A, B and C show the enterprise TCA distribution for the base case, and model runs 4 and 5, respectively. Chart D provides a comparison of the distributions using smooth PDFs. For each model run the distribution sample mean, standard deviation, full range, and 5th–95th percentile range are reported in Table 4.

The floating technical coefficient bounding parameter does not affect the number of coefficients that are treated stochastically in the model. Hence, the same coefficients as those shown in Figure 2, for the base-case scenario, are sampled within the Monte-Carlo simulation for both model run 4 and 5. The bounding parameter does, however, have the effect of changing the sample value distributions of each individual technical coefficient. The base-case parameter value of 0.5 places upper and lower bounds on technical coefficients of $\pm 50\%$ of the adjusted table value. Halving the parameter tightens coefficient constraints to $\pm 25\%$, while doubling the parameter relaxes constraints to $\pm 100\%$.

Adjusting the technical coefficient bounding parameter in this way has a statistically significant effect on the enterprise TCA distributions. Halving the parameter decreases the spread of the distribution: the sample standard deviation, value range, and 5th–95th percentile range decrease by 42%,

[29]A similar effect occurs in the estimation of national carbon footprints....

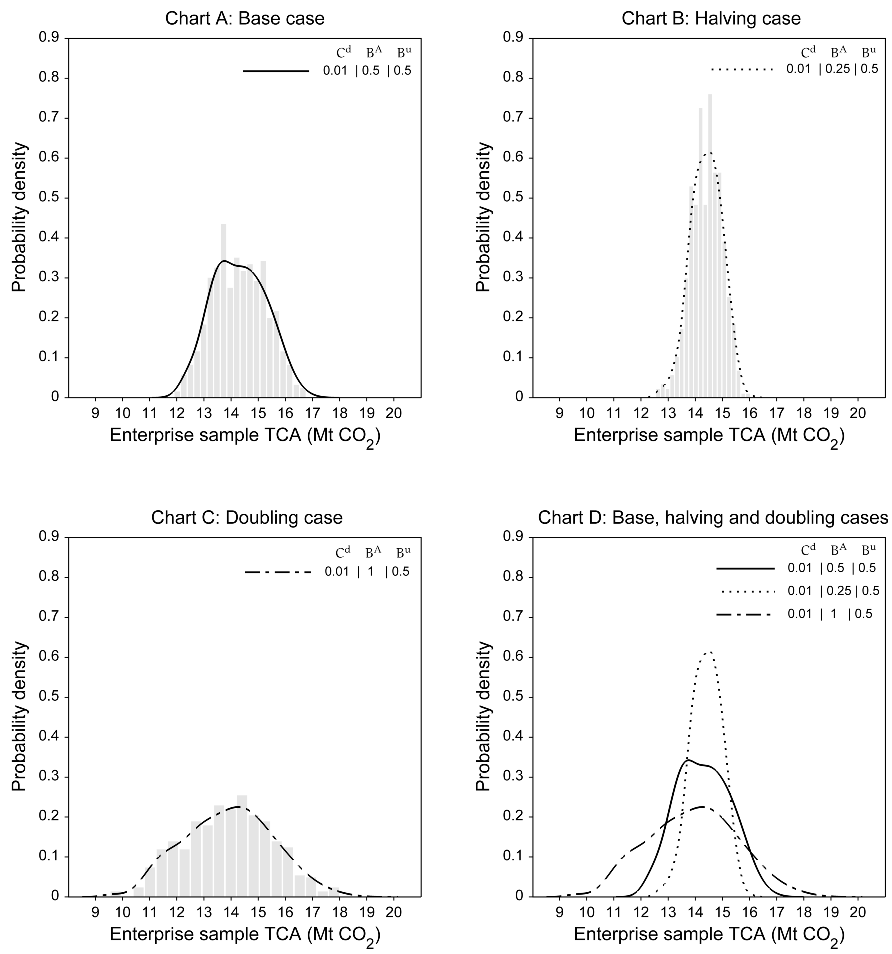


Figure 6: Response of enterprise TCA distribution to technical coefficient bounding parameter B^A.

29%, and 43%, respectively. Conversely, when the parameter is doubled the spread of the distribution increases: the same measures increase by 65%, 74%, and 64%, respectively.

The observed effects on the enterprise TCA distribution are explained by dramatic shifts in bounding constraint range placed on several floating coefficients. For example, Figure 2 Chart A indicates that the enterprise's requirement for residual sector inputs (coefficient 1) has a bounding range of 0.01–0.29. When the bounding parameter is halved this range shrinks to 0.15–0.25, and expands to 0–0.39 when the parameter is doubled. For the later case, if coefficient 1 is sampled at close to zero, then the coefficient would make a negligible contribution to the enterprise's TCA; but if it is sampled close to the upper bound of 0.39, then the coefficient would account for a major share of the TCA. Coefficients 4–9 similarly exhibit significant shifts in their bounding constraint range between the halving and doubling scenario.

The sample mean decreased slightly by 2.7% when the bounding parameter was doubled. Although this observed effect is relatively small, it may be explained by the interplay of system-wide constraints. For example, if the enterprise's requirement for residual sector inputs were to be the first coefficient sampled and it took a value near its upper bound, then this would tighten the constraints on the other enterprise input coefficients due to the specification of column sum constraints. Since residual sector inputs to the enterprise are particularly emissions-intensive, this sampling order would lead to a high enterprise TCA. However, the sampling order is randomly (uniformly) specified for each sample table. With more coefficients likely to be sampled before the residual sector requirement than after it, and with all floating coefficients able to take higher values when the bounding parameter is doubled, this has the effect of constraining the residual sector requirement to slightly lower values thus causing the sample mean of the distribution to decrease.

Overall, as the bounding parameter tends to zero, the spread of the distribution tightens and the sample mean tends toward the enterprise TCA estimated using the adjusted table (14.52 Mt CO_2). As the bounding parameter increases, up to a point, the spread of the distribution increases and the sample mean decreases slightly. Beyond a certain point however, the lower bounds on coefficients will be overridden by the requirement for non-negativity, while upper bounds will be overridden by the other model equality and inequality constraints.

Response to bounding parameter B^u

Figure 7 shows the effect of halving (model run 6) and doubling (model run 7) the floating value-added coefficient bounding parameter B^u on the enterprise TCA distribution. Charts A, B and C show the enterprise TCA distribution for the base case, and model runs 6 and 7, respectively. Chart D provides a comparison of the distributions using smooth PDFs. For each model run the distribution sample mean, standard deviation, full range, and 5th–95th percentile range are reported in Table 4.

The floating value-added bounding parameter has the effect of changing the sample value distributions of the two valued-added coefficients shown in Figure 2 Chart C. The base-case parameter value of 0.5 places upper and lower bounds on these coefficients that are ±50% of the adjusted table value. Halving the parameter tightens coefficient constraints to ±25%, while doubling the parameter relaxes constraints to ±100%. The effect on the sample mean is not statistically significant.

Halving the parameter decreases the spread of the distribution: the sample standard deviation, value range, and 5th–95th percentile range decrease by 15%, 22%, and 15%, respectively. Conversely, when the parameter is doubled the spread of the distribution increases: the same measures increase by 14%, 50%, and 13%, respectively. This observed effect is explained by the column

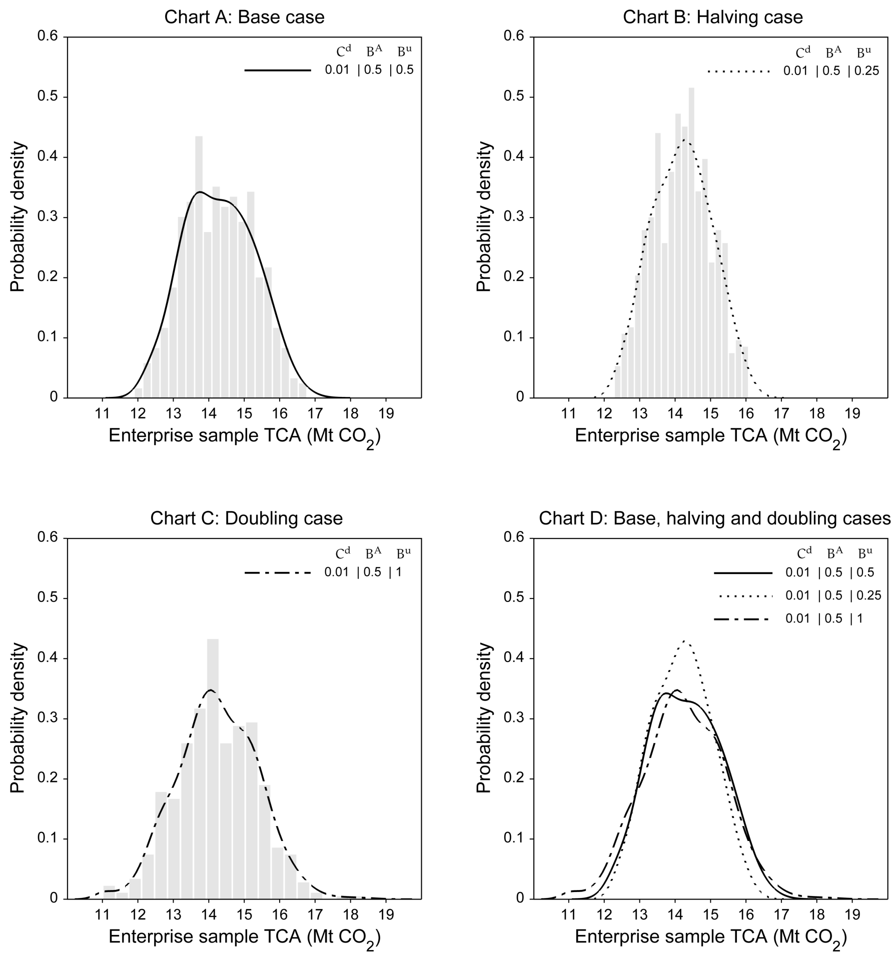


Figure 7: Response of enterprise TCA distribution to value-added coefficient bounding parameter B^u.

sum constraints that are imposed in the model. For example, halving the parameter has the effect of reducing the likelihood that floating technical coefficients can take values significantly above or below their respective adjusted coefficient values, leading to a reduction in the spread of the enterprise TCA distribution.

2.6 *Enterprise group supply chain emissions*

The MEMRIO model was not only developed to assess individual enterprise but also groups of interdependent enterprises. The estimation of the supply chain emissions of an enterprise group must account for the fact that constituent enterprises may fall within one another's supply chains. Failure to do so would lead to the double-counting of emissions and the risk of severely overestimating the supply chain emissions of the group as a whole. For example, consider an automobile manufacturer supplied by an auto-parts maker. If the supply chain emissions of each enterprise were estimated separately and simply added together, then emissions released by the auto-parts company in producing parts for the automobile manufacturer would be counted twice. Furthermore, since a portion of the auto-parts supply chain (including, steel producers and electricity providers, etc.) also forms part of the automobile manufacturer's supply chain, then emissions released all along common supply chains would be also be counted twice. Such double-counting must be stripped-out to evaluate the combined influence these two enterprises have over their collective supply chain emissions.

By disaggregating enterprise segments from MRIO sectors, the MEMRIO model enables the estimation of enterprise group supply chain emissions free from double-counting, no matter how many supplier-tiers separate constituent enterprises. This is achieved simply by applying the TCA method (Equation 30) with respect to all enterprise segments associated with a group of enterprises. The TCA method is introduced in Skelton *et al.* (2011) and implicitly tackles the double-counting issue through the removal of feedback loops and intra-group transactions. Returning to the example, the TCA method, as applied to the dual-enterprise group, would effectively sever the direct supply chain link between the two enterprises. In addition, any feedback loops – whereby the output of either enterprise gets taken into each other's extended supply chains – would be removed. The importance of removing double-counted components is demonstrated below through the assessment of a hypothetical group of three enterprises.

Two additional enterprises (enterprise 2 and 3) characterised by single segments are introduced into the German Motor Vehicles & Parts sector alongside the hypothetical automobile manufacturer (enterprise 1) assessed in Section 2.4. The segment total outputs of enterprise 2 and 3 are specified as \$40 bn and \$30 bn (the total output of enterprise 1 is specified as \$50 bn). A single model run, with a sample size of 500, is performed with the same user-control parameters as the base-case scenario in Section 2.5 ($C^s = 1$, $C^d = 0.01$, $B^A = 0.5$, and $B^u = 0.5$). TCA distributions for each individual enterprise are shown in Figure 8 Charts A–C. The TCA distribution of the three enterprises treated as a group is shown in Chart D. Smooth PDFs are fitted to the distributions in Charts A–D and compared against one another in Chart E. In addition, the TCA of individual enterprises calculated for each sample MEMRIO table were added together to create a 'sum of enterprise TCA' distribution, which is also shown in Chart E as a smooth PDF.

Again, 13 emissions multipliers fall above the demand cut-off parameter. However, as there are now three enterprise segments located within the same sector, these multipliers correspond to a total of 64 floating technical coefficients (including 16 coefficients from the intra-disaggregated sector

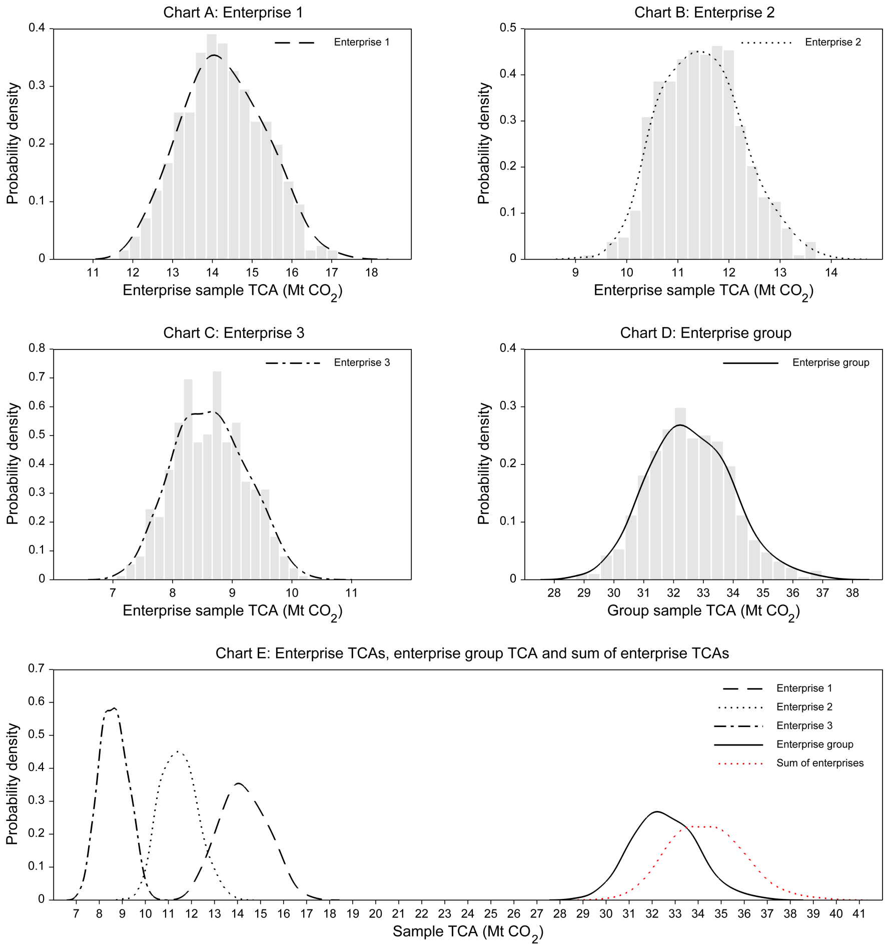


Figure 8: TCA distributions for three enterprises in the same sector.

block alone[30]) and 4 floating value-added coefficients (one for each of the enterprises and one for the residual sector). The difference in enterprise TCA distributions is explained simply by the different total outputs of the three enterprises (12.7%, 9.7%, and 7.3% of the German Motor Vehicles & Parts sector total output, respectively).

The distribution of the group TCA (Chart D) strips out duplicated emissions sources found across the set of individual enterprise TCAs. For example, a given sample may determine that Enterprise 1 has a significant requirement for inputs from Enterprise 2, causing a proportion of the emissions embodied in Enterprise 2's total output to also become embodied in the output of Enterprise 1. Hence simply adding the TCA of each enterprise together would result in a degree of double-counting. The error caused by double-counting is shown in Chart E where the group TCA distribution is shown alongside the 'sum of enterprise TCA' distribution. The distribution sample mean is 6% lower when the double-counting of emissions is removed in the group calculation.

The risk of over estimating the supply chain emissions of a group of enterprises is dependent on the extent to which constituent enterprises fall within one another's supply chains. The inputs to the MEMRIO model, as described in this chapter, do not detail specific supply chain linkages between enterprises. Instead, the inputs and outputs of enterprise segments, including inter-enterprise transactions, are treated stochastically in the model. The TCA distribution of an enterprise group captures a spectrum of plausible configurations for supply chain overlap and ultimately provides a value range in which the actual supply chain emissions of the group is likely to fall. Model extensions are discussed in the following section that allow known supply chain links to be encoded in the model.

2.7 *Discussion & conclusion*

A 'new wave of globalisation' has seen the fragmentation of production across enterprises and national borders, leading to the widespread emergence of global supply chains that are typically coordinated by large, lead enterprises (Milberg & Winkler, 2013; Gereffi & Lee, 2012). Within this context, the MEMRIO model provides a foundation for the future assessment of lead enterprises in the global economy. Interactions between national industries in the global economy are captured in the model through the use of MRIO tables. Enterprises, that are characterised as a set of sector-classified segments, are introduced through the disaggregation of national industries. Interactions between enterprise segments and the wider economy are treated stochastically to reflect uncertainty over enterprise inputs and outputs, thereby permitting analysis based on limited, publicly available, data. The stochastic component of the model is governed by a system of constraints that (a) retains the overall balanced structure of inputs and outputs encoded in MRIO data, and (b) reflects user assumptions over the extent to which an enterprise may deviate from sector average performance. Monte-Carlo simulation is used to generate a sample set of disaggregated tables, each representing a plausible configuration for how focal enterprises are integrated in the global economy. Statistical enterprise assessments can then be performed over the set of sample tables.

Specifically, the MEMRIO model was developed to address the question of *how can the global supply chain emissions stemming from the intermediate consumption activity of (a) an individual enterprise and (b) a group of interdependent enterprises be estimated using publicly available data?*. This is achieved through the application of the TCA method across a set of sample MEMRIO tables and the subsequent statistical analysis of enterprise TCA distributions. In the case

[30] In constructing the adjusted technical coefficient matrix, only the intra-enterprise transactions were set to zero, inter-enterprise transactions are permitted within the MEMRIO model (e.g., Enterprise 1 can requires inputs from Enterprise 2 and 3).

of an individual enterprise, the TCA method estimates global emissions that are embodied in the goods and services sold by that enterprise, while in the case of a group of interdependent enterprises, the TCA method estimates global emissions that are embodied in the products sold externally by the enterprise group. The risk of overestimating the supply chain emissions of a group of enterprises – by not taking into account the fact that constituent enterprises may fall within one another's supply chains – is ameliorated implicitly by the TCA method through the removal of double-counted terms. These measures of supply chain emissions represent the quantity of potentially abatable emissions that an enterprise, or a group of enterprises, has influence over.

In this section, three important areas of model uncertainty are considered: sources of uncertainty associated with underlying MRIO data that are compounded in MEMRIO model results; additional sources of uncertainty associated with the specification of enterprise segments; and, the uncertainty over enterprise inputs and outputs that is analysed stochastically within the model. These three areas of uncertainty, and related options for model refinement, are discussed in turn below.

MRIO model uncertainty

The MEMRIO model requires a pre-existing MRIO model. The sources of uncertainty associated with MRIO models fall into two main categories:

1. `Errors in underlying source data` – Errors and omissions in original survey data, the imputation of missing data, and table balancing procedures generate inaccuracies in national IO tables (Lenzen, 2000; Hertwich & Peters, 2009). The aggregation of economic activity according to broad industries and the assumption that economic transactions correspond proportionally to the physical flow of products introduce further uncertainty (Yamakawa & Peters, 2009; Wilting, 2012). In addition, errors are also found in the bilateral trade data required to construct MRIO tables (Lenzen *et al.*, 2004).

2. `Model construction uncertainties` – The harmonisation of national tables and bilateral trade data into an integrated MRIO table introduces several additional sources of uncertainty. Sectoral and spatial aggregation bias and temporal discrepancies are introduced by dissimilar national classification schemes, the unavailability of tables for some regions, and the use of tables produced for different time period, respectively (Lenzen *et al.*, 2004; Peters *et al.*, 2011). The choice over methods used to harmonise valuation systems, adjust currencies, estimate trade flow matrices, and ultimately balance the MRIO table also add layers of uncertainty (Lenzen *et al.*, 2010b, 2012).

Previous studies have used Monte-Carlo simulation techniques to assess how the various sources of uncertainty associated with IO tables manifest as overall uncertainty in IO multipliers (Lenzen, 2000; Yamakawa & Peters, 2009; Lenzen *et al.*, 2010b; Wilting, 2012). Results indicate that there is a cancelling-out effect caused by the matrix inversion involved in calculating IO multipliers. This suggests that although the sources of uncertainty in MRIO models are numerous, they may not have a significant affect on the distributions of enterprise supply chain emissions estimated using the MEMRIO model. Nevertheless, the MEMRIO model could be extended to account for MRIO coefficient uncertainty within the existing Monte-Carlo simulation loop[31].

[31]A model extension such as this would benefit from Wiedmann *et al.*'s (2011) called for the compilers of national IO tables and MRIO models to estimate standard errors in their data

Uncertainty in the specification of enterprise segments

The description of the MEMRIO model in Section 2.2 began with known enterprise segment data. However, the process of specifying an enterprise as a set of segments is not straightforward and can introduce a significant source of model uncertainty. The specification of enterprise segments requires overall enterprise production activity to be mapped to the sector classification scheme of the MRIO model taken as input into the MEMRIO model. In principle, the information required to do this can be derived from publicly available sources such as corporate annual reports (specifically, consolidated financial statements) and databases that aggregate such information[32].

For the case where the activity of an enterprise is entirely located in a single sector, two issues need to be addressed: the enterprise must be correctly mapped to the relevant MRIO sector and reported enterprise revenue must be adjusted to the MRIO valuation system. The former can be addressed with reference to the standard classification system used in the compilation of National Accounts – the International Standard Industrial Classification of all Economic Activities (ISIC) Revision 4 (United Nations *et al.*, 2008) – and the concordance tables that map ISIC codes to MRIO model-specific sectors. The latter requires knowledge of the MRIO valuation system and the accounting standards followed by the enterprise. Enterprises follow slightly different accounting standards depending on where they are headquartered. This has implications for what is included (e. g., sales and export taxes, shipping costs, cost of returned goods, etc.) in an enterprise's consolidated revenue. It is conventional for MRIO tables to use basic prices. If the accounting conventions followed by the enterprise lead to a reported revenue that does not correspond to basic prices, then appropriate adjustments are required: for example, sales taxes (e. g., VAT) may need to be deducted from the reported revenue.

For the case where the activity of an enterprise spans multiple MRIO sectors, an additional issue concerning the aggregation of enterprise financial data in consolidated statements needs to be addressed. For example, an automobile manufacturer with factories spread across Europe may report a single revenue figure for the overall enterprise, rather than a breakdown of revenue by region. This has major implications for the ease with which enterprise activity can be mapped to MRIO sector. However, International Accounting Standard 14 (ISA14) establishes principles for reporting financial information about the different types of goods and services an enterprise produces and the different geographical areas in which it operates, with the aim of helping users of financial statements make more informed judgements about the enterprise as a whole (International Accounting Standards Board, 2005). Enterprises following ISA14 (i. e., most publicly listed enterprises) present revenue figures for business segments and for geographical segments[33]. For example, in its 2009 annual report, BP disclosed revenue by two geographic segments – US and non-US – and by three business segments – Exploration & Production, Refining & Marketing, and Other Business & Corporate – (BP, 2009). ISA14 also stipulates that inter-segment transactions should be reported where such transactions represent a significant share of the overall revenue realised by individual business or geographic segments.

[32] For example, *Mergent Online* provides a 10 year archive of over 300,000 annual reports from globally listed companies. For enterprises with operations in the US, the US Securities and Exchange Commission (SEC) provides an additional source of enterprise information through compulsory enterprise filings.

[33] The International Accounting Standards Board (2005) define a business segment as a "distinguishable component of an [enterprise] that is engaged in providing an individual product or service or a group of related products or services and that is subject to risks and returns that are different from those of other business segments", and a geographical segment as a "distinguishable component of an [enterprise] that is engaged in providing products or services within a particular economic environment and that is subject to risks and returns that are different from those of components operating in other economic environments".

Knowledge of business and geographic segments can help in the specification of enterprise segments required by the MEMRIO model. However, the accounting standards do not stipulate the resolution at which business and geographic segments should be reported. Furthermore, business and geographic segments are reported separately[34]. This mismatch in reported and required segment resolution presents an important source of uncertainty in the MEMRIO model.

The MEMRIO model could be extended to account for the uncertainty associated with the specification of enterprise segments. For example, segment total outputs could be sampled from estimated bounds during the Monte-Carlo simulation[35]. A more sophisticated development would be to stochastically model the process of mapping enterprise activity to specific MRIO sectors. For example, it might be known that an enterprise is active in a single industry but regionally spread across Europe; plausible configurations of segments located in different European countries could be sampled.

Uncertainty over enterprise inputs and outputs

The inputs to and outputs from an enterprise are treated stochastically in the MEMRIO model. The overall uncertainty associated with these coefficients can be specified using three user-control parameters. Section 2.5 provided important insights into how the model responds to the choice of control parameter. The emissions multiplier cut-off parameter is used to control the number of coefficients that are treated stochastically, while the two bounding parameters control the user-estimated uncertainty of these stochastic coefficients.

Ideally, the cut-off parameter would be disregarded so that all enterprise input and output coefficients are included in the Monte-Carlo simulation. However, by imposing an appropriate cut-off value, a user can minimise the computational requirements of the model without significantly affecting model results. This is achieved by assessing relevant emissions multipliers (e. g., Figure 5) and the adjusted table values of corresponding technical coefficients (e. g., Figure 2). An option for model development could be to automate this trade-off decision.

Ideally, the choice of bounding parameter would be informed by secondary data or expert option. However, for the case where the uncertainty in technical and value-added coefficients is unknown, it is recommended that a sensitivity analysis of model results to bounding parameters be performed. For a given enterprise, the inspection of consolidated financial statements, required to specify enterprise segments, is likely to also provide an indication of the value-added by the enterprise. Value-added includes, for example, compensation of employees and taxes on production and approximates to the difference between the cost of sales and overall enterprise revenue. Such knowledge would allow value-added coefficients to be deterministically defined or specified with relatively tight bounding constraints.

Some users may have assess to additional information that could be used to refine the model for a particular analysis. For example, significant intra-enterprise transactions may be disclosed by an enterprise; known values can be specified during the construction of the adjusted table instead of assuming such transactions are negligible. Similarly, known transactions within an enterprise group can be specified. Access to detailed cost and sales data of an enterprise would allow all major enterprise coefficients to be deterministically defined; the stochastic component of the model could then be shifted to higher-order supplier tiers where data is unavailable. Finally, information

[34] For example, BP's annual report does not specify what share of revenue gained from Exploration & Production pertains to US or non-US operations (BP, 2009).

[35] Similarly, enterprise segment emissions intensities could also be sampled: the model currently assumes that segments have the same emissions intensities as their parent MRIO sectors, when in fact significant variability may exist within a sector.

about the heterogeneity of different industries can allow for the use of sector-specific bounding parameters. For example, if inputs to the steel industry are known to be relatively homogeneous across different steel makers, then tighter bounds can be set for that particular sector.

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