The effects of a production disruption in a linear programming input-output model

Wolfgang Koller
Email: koller@wu.ac.at

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Abstract

There exist various approaches to use input-output analysis for measuring the direct and indirect effects of a disruption to an economic system stemming from natural disasters, bottlenecks in primary factors and energy resources, strikes, and restrictions due to regulation. The present paper brings together linear programming and input-output-analysis to introduce various scenarios of linear restrictions to an economy where there is only one technology to produce each good (Leontief technology). Thus, while technological adaptation is not an option, other forms of adaptation, e.g., changing the composition of final demand and of the trade structure are considered. The effect of the disruption is computed as the difference between the base scenario and the solution of the linear program. The analysed models stand in a long tradition of linear-programming-input-output approaches. While in older models the production or the value added of the overall economy was employed as the objective function to be maximized, in our approach it is final demand, although a further restriction is imposed that final demand in any sector must not exceed the final demand before the disruption. The suitability and interpretation of various modelling approaches are discussed, both from a theoretical stand-point and with the help of application studies based on real world input-output tables. The applications deal with the cut-back of the production of a group of industries and with the effect of a shortage of various energy resources.

1 Introduction

Linear programming and input output analysis have been very closely related concepts from the beginning of their development. In the pioneering times of both concepts this link carried the name "activity analysis" and comprised important theoretical contributions to both fields, e.g. (Koopmans, 1951, 1953). The main purpose of activity analysis was to generalize input-output analysis by introducing the choice between different technologies or, in the words of Wood and Dantzig (1951), taking "into consideration the many different ways of doing things."

The present paper applies linear programming to input-output analysis in a much more limited manner. Our approach tries to keep as closely as possible to the original assumptions of input-output analysis. Foremost, it is assumed that there is only one technology to produce each commodity. Still, in the event of restraints imposed on the economy, e.g. stemming from supply bottlenecks or regulation, there is a choice to be made which can be modeled with the help of linear programming. First, this concerns the choice between various final demand opportunities, all of which cannot be satisfied under a certain restraint. Second, in an extension of our basic model, there is a choice between domestic production and importing.

The motivation of the paper is to provide an applicable and operable model of the impact of a limitation on the production in certain sectors of the economy on the economy as a whole. The application studies we have chosen show that we think of urgent issues such as peak oil and carbon reduction as possible application fields. The typical question to be answered by our model is: "How will the economy react to a bottleneck in the production of oil products or to a regulation of carbon emissions, assuming that technological adaptation is precluded?"

It should be stated that prices are not considered in the model, i.e. they are assumed as unchanged by the introduction of a bottleneck. Therefore, the model neglects the guiding effect in making the choices that is exerted by the price mechanism and could be thought of as beneficial at coping with a certain bottleneck even in the hypothetical situation without technological adaptation. Obviously, one could try to extend the approach in the direction of a general equilibrium model, but we think that even in the current form it can be helpful.

In the framework of static comparative analysis, our approach compares a situation before the introduction of a certain limitation with a situation after the introduction of this limitation. Basing this analysis on simple assumptions – essentially the same as those of the classical Leontief model – should facilitate the discussion of the policies linked to the bottleneck or regulation in question.

Modeling a bottleneck by combining linear programming and input-output analysis is not a new idea and there are some important predecessors in this. The following overview proceeds in chronological order. Though not explicitely focussing on the theme of a bottleneck, the chapter on "Choice in Interindustry Models" in the textbook by Chenery and Clark (1959) is a clear exposition of optimal use of limited primary resources such as labour, capital and labour. The objective function to be maximised is the total value of final output, i.e. a weighted sum of final output where the weights can be interpreted as prices. The simplest of various models presented considers no more activities that produce a commodity than there are commodities. This model, though the exposition is different, is analoguous to the basic model formulation to be presented by the present paper.

The focus of Schluter and Dyer (1976) is on the interpretation of a constrained input-output solution. They present a shortcut procedure to calculate a solution when an additional constraint is introduced into the problem. At the core of this procedure is a transition matrix, which also has an economic interpretation. The objective function in this approach is the sum of gross output rather than final output. As to the appropriate objective function, the authors state: "The choice of maximizing the sum of gross sector outputs in the objective function is not essential to the solution of the unconstrained model. Maximizing final demand or value added would yield the same solution as maximizing output. Since all sectors have positive value added and output, the value of the objective function is maximized when all final demand opportunities are utilized." However, since this does not apply to the constrained model, the question of the choice of objective function is essentially left open.

Wang and Miller (1995) develop a linear-programming/input-output model to analyze the economic impact of a transportation and energy supply bottleneck on the economy of Taiwan. A particular feature of their approach is that they consider specific rules for the rationing scheme to be applied as an answer to the bottleneck. Should the restriction be placed totally on final demand, should it be placed totally on industrial production or should it be placed partially placed on both of these uses? A factor r to be specified by the planning authority determines the proportion of the restriction to be placed on industrial production, $(1-r)$ thus being placed on final demand. This feature makes the approach of Wang and Miller quite different from our own. In our approach the restriction is placed as far as possible on final demand, possibly introducing minimum final demand restrictions. Nevertheless, the paper by Wang and Miller is a direct predecessor of ours, in particular with respect to its application studies. As to the choice of the objective function, Wang and Miller formulate their models both with the sum of value added and with the sum of gross sector outputs as the objective function to be minimized. They say that the choice is up to the planning authority facing a bottleneck scenario and applying the model.

Another paper linking linear programming and input-output analysis is Rose et al. (1997), which analyzes the regional economic impact of an earthquake. At the core of this model is a linear program similar to the other ones discussed, with the sum of final demand as the objective function. The earthquake is assumed to disrupt the electricity lifelines of the region, thus causing an electricity bottleneck. The simulation approach goes considerably beyond the basic model, as the 36 different service areas are affected differently by the earthquake and other aspects such as resilience are taken into account.

What all the contributions cited above have in common is the assumption that there is only one technology to produce each commodity. In the short discussion of the contributions we have put some emphasis on the choice of the appropriate objective function. Though the sum of value added and the sum of final demand are equivalent, we favour the sum of final demand for several, related reasons.

First, it conforms to a teleological interpretation of the economy as a system whose function is to provide us with the commodities we consume (thus, the other function, which is to provide us with the opportunities to earn the income we need to be able to buy these commodities, is viewed as secondary).

Second, it is consistent with the demand driven view of the input-output model and puts the correct emphasis on the double nature of "final demand". What is usally called so, means final demand opportunities on the one hand and final output on the other. In the traditional input-output model it is implicitely assumed that there are no capacity constraints and the two meanings of "final demand" coincide. A linear programming/input-output model makes the distintion between those meanings explicit, as final demand opportunities (for the n commodities) are given exogenous, i.e. in the form of a linear restriction of the linear program, and (the sum of) final output is to be maximised.

Thirdly, it puts the right emphasis on the double nature of "value added". By this we mean that in the present context we want to stress the primary input aspect, not the income aspect of what is denoted by "value added" in input-output tables.

The paper proceeds as follows. The next section presents the models, going from simple formulations to more elaborated ones. Particularly, we ask how the linear programming/input-output model should be extended in order to consider an open economy, where imports might be substituted for domestic production in case of a resource constraint. This aspect has not been dealt with in previous input-output research, to the best knowledge of the author. Furthermore other restrictions are introduced which encompass minimum final demand and behavioral assumptions about trade balance. The ensuing section contains two application studies based on Austrian data, one on the impact of a production constraint in mineral oil products and the other on the impact of limiting the carbon emissions of the sectors underlying the European emission trading system. Here it will be demonstrated how a progressive tightening of the bottleneck brings about a more than proportional reduction of the final demand able to be satisfied. Finally there is a section with conclusions and remarks about further useful extensions of the approach.

2 Modeling approach

2.1 Notation and Definitions

We define the following vectors (all of length n):

$\mathbf{q} = (q_i)$	production
$\mathbf{m} = (m_i)$	imports
$\mathbf{f} = (f_i)$	final demand (including consumption, capital formation and exports)
$\mathbf{c} = (c_i)$	final consumption
$\mathbf{g} = (g_i)$	gross capital formation
$\mathbf{h} = (h_i)$	final demand, excluding exports
$\mathbf{x} = (x_i)$	exports
$\mathbf{z} = (z_i)$	intermediate demand
$\mathbf{p} = (p_i)$	intermediate demand for domestic goods
$\mathbf{s} = (s_i)$	pollution
$\mathbf{m}_f, \mathbf{m}_p, \ldots$	imports into final demand, into intermediate demand, etc.

For final demand and its categories we make a distinction between domestic and import variables. E.g., we denote by $\mathbf{f}_d$ the final demand for domestic goods and by $\mathbf{m}_f$ the imports into final demand, correspondingly for final consumption and gross capital formation. However, we assume that there are no re-exports, thus $\mathbf{x} = \mathbf{x}_d$, $\mathbf{m}_f = \mathbf{m}_h$ and $\mathbf{m}_x = \mathbf{0}$, and no distinction is necessary in that case. $\mathbf{m}_p$ denotes intermediate demand for imported goods. The following relationships hold:

$$\mathbf{f} = \mathbf{c} + \mathbf{g} + \mathbf{x} = \mathbf{h} + \mathbf{x}$$
$$\mathbf{q} = \mathbf{p} + \mathbf{f}_d$$
$$\mathbf{z} = \mathbf{p} + \mathbf{m}_p$$
$$\mathbf{f} = \mathbf{f}_d + \mathbf{m}_f$$
$$\mathbf{m} = \mathbf{m}_f + \mathbf{m}_p$$

We define the following matrices in the conventional way:

$\mathbf{Z} = (z_{ij})$	matrix of intermediate input flows
$\mathbf{P} = (p_{ij})$	matrix of intermediate input flows of domestic goods

$\mathbf{M} = (m_{ij})$ matrix of intermediate imports
$\mathbf{A} = (a_{ij}) = (z_{ij}/q_j)$ matrix of technical input coefficients
$\mathbf{A}_d = (a_{ij}^d) = (p_{ij}/q_j)$ matrix of domestic input coefficients
$\mathbf{A}_m = (a_{ij}^m) = (m_{ij}/q_j)$ matrix of import input coefficients
$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = (l_{ij})$ Leontief inverse matrix
$\mathbf{L}_d = (\mathbf{I} - \mathbf{A}_d)^{-1} = (l_{ij}^d)$ Leontief inverse matrix for domestic production

The models we develop aim at a comparison of a base scenario, in which no bottleneck is assumed to exist, and a scenario that has a limitation on production or on other variables. The variables before the introduction of the limitation are symbolized by the superscript 0 and the restrictions on variables are symbolized by the superscript $*$. Solutions of the models in the bottleneck-scenario are symbolized by $\bullet$. We formulate our models in such a way that an uneffective bottleneck gives as solution the variables as observed in the base scenario. Using the notation just introduced, this means that the following property is fulfilled:

Property 1 *If for all variables y in the system that can be restricted due to a bottleneck we have $y^* = y^0$ then for all variables w in the system $w^\bullet = w^0$.*

Furthermore we use the following conventions: $\mathbf{e}$ is a vector of ones. Transposition is denoted by $'$. Vectors and matrices are written in bold and the sum of a vector is denoted by the same character but written in italics, e.g. $m = \mathbf{e}'\mathbf{m}$.

2.2 Model I

We consider an economy with a Leontief technology with n sectors. In model I we assume that there are no imports. In order to develop the models step by step we start with the introduction of a limitation on production. An example, as used in our application later, is a limitation on the production of mineral oil products.

The limitation can concern the production of one or of more than one sectors. Let us first consider the case where only one sector, say sector i, is restrained in its production. The limitation on the production of commodity i has impacts on how much an economy can produce of each other commodity. When the limitation is absolute, i.e. $q_i^* = 0$, it is to be expected that no other commodity can be produced, since in modern economies the interrelatedness is such that directly or indirectly the production of every commodity requires all other commodities as inputs. If the limitation is partial, $0 < q_i^* \leq q_i^0$, the question is what the economy will decide to produce and to what extent final demand can be satisfied. In our first model we assume that the final demand in the constrained scenario does not exceed the final demand in the unconstrained scenario in any sector. That is, we assume that final demand opportunities are given exogenously. Still, under the constraint the economy strives for an optimum with respect to the sum of final demand. Thus model Ia is given by the following linear program.

Model Ia

maximize

$$\mathbf{e}'\mathbf{f} \tag{1}$$

subject to

$$\mathbf{f} \leq \mathbf{f}^0 \tag{2}$$

$$\mathbf{l}_i\mathbf{f} \leq q_i^*, \tag{3}$$

where row-vector $\mathbf{l}_i$ is the i-th row of $\mathbf{L}$. This models reveals the basic mechanism of the impact of a limitation in the production in other sectors. As the limitation gets more severe, i.e. if we let the ratio q_i^*/q_i^0 decrease, more and more sectors are consecutively restricted in their deliveries to final demand. As long as the limitation is "small" only the final demand for commodity i is restricted. With a further decreasing ratio q_i^*/q_i^0 a point is reached where no commodity i can be delivered to final demand. As this ratio decreases further, there is not enough production of commodity i to fulfill the intermediate demand for commodity i coming from other sectors. Then the linear program determines which commodity is the next one to be restricted in its deliveries to final demand. The first commodity to be restricted in its final demand is the one with the highest direct and indirect requirements of commodity i. Generally, the sequence of commodities whose final demand is restriced is given by the size of total requirement coefficients.

The generalisation of model Ia allows for limitations in the production of more that one sector. The following formulation considers a simultanenous limitation in the production of all sectors.

Model Ib

maximize

$$\mathbf{e}'\mathbf{f} \tag{4}$$

subject to

$$\mathbf{f} \leq \mathbf{f}^0 \tag{5}$$

$$\mathbf{L}\mathbf{f} \leq \mathbf{q}^*, \tag{6}$$

where $\mathbf{q}^* \leq \mathbf{q}^0$ is the production constraint. It may seem a useful simplification to drop the linear constraint $\mathbf{f} \leq \mathbf{f}^0$ in the above model formulation, since by the constraint $\mathbf{L}\mathbf{f} \leq \mathbf{q}^*$ the solution for $\mathbf{f}$ is implicitely constrained. However, due to the reasons discussed in the introduction we always include the direct restriction on $\mathbf{f}$. Certainly, these two alternative models are not the same.

A further generalisation of model Ib can also take acount of the availability of primary resources or the maximal tolerance in pollution. Though the model can easily be formulated as to deal simultaneously with several primary resources and pollution emissions, we introduce only one pollutant. Also, for simplicity, the constraint is on the sum of pollution over all sectors, not on each individual sector.

Model Ic

maximize

$$\mathbf{e}'\mathbf{f} \tag{7}$$

subject to

$$\mathbf{f} \leq \mathbf{f}^0 \tag{8}$$

$$\mathbf{L}\mathbf{f} \leq \mathbf{q}^* \tag{9}$$

$$\mathbf{r}'\mathbf{L}\mathbf{f} \leq s^*, \tag{10}$$

where $\mathbf{r}$ is the vector of pollution coefficients, $\mathbf{r} = (s_i/q_i)$.

2.3 Model II

Modern input-output tables make a distinction between domestically produced commodities and imports. Having available tables for domestically produced inputs and imported inputs, it is not immediately clear without further assumptions which tables are to form the basis for the calculation of the input coefficient matrix to be used. Will production require domestic inputs and imported inputs always in the same proportions? Furthermore, will a certain final demand impulse be satisfied by imports or by domestic production?

These questions are usually approached by choosing between the assumptions of competitive avd non-competitive imports. In model II we assume that all imports are non-competitive. In model III all imports are competitive.

The assumption of non-competitive imports amounts to saying that imported commodities are different from domestic commodities even though identically classified. Thus there is no way to substitute imports for domestic production. Consequently, the domestic input-output table forms the basis for the calculation of the input coefficient matrix and Leontief inverse. Model IIa is derived from model Ic by adapting the used variables accordingly.

Model IIa

maximize

$$\mathbf{e}'\mathbf{f}_d \tag{11}$$

subject to

$$\mathbf{f}_d \leq \mathbf{f}_d^0 \tag{12}$$

$$\mathbf{L}_d\mathbf{f}_d \leq \mathbf{q}^* \tag{13}$$

$$\mathbf{r}'\mathbf{L}_d\mathbf{f}_d \leq s^*, \tag{14}$$

The model makes only limited predictions about the impact of a production or pollution constraint on imports. Let $\mathbf{f}_d^*$ be the solution of model II. Then the intermediate imports are given as $\mathbf{A}_m\mathbf{L}_d\mathbf{f}_d^*$.

It is not unlikely that the same phenomen that causes a restriction on the domestic production also affects the international production. Thus a restriction on domestic production is often accompanied by a restriction on imports. Also, the capacity to import a certain good might be limited by technical reasons and transportation costs. Therefore, a harmless extension of the model would include a restriction on imports:

Model IIb

maximize

$$\mathbf{e}'\mathbf{f}_d \tag{15}$$

subject to

$$\mathbf{f}_d \leq \mathbf{f}_d^0 \tag{16}$$

$$\mathbf{L}_d\mathbf{f}_d \leq \mathbf{q}^* \tag{17}$$

$$\mathbf{r}'\mathbf{L}_d\mathbf{f}_d \leq s^* \tag{18}$$

$$\mathbf{A}_m\mathbf{L}_d\mathbf{f}_d \leq \mathbf{m}_p^*, \tag{19}$$

However, imports to final demand are unconcerned by the models IIa and IIb. A severe restriction in the production of an economy might well influence the quantities of imports, even when holding the assumption of non-competitiveness of imports. An economy that does not produce enough is restricted in its capacity to export and may lack the financial means to import. At least in the long run a negative trade balance is not viable. Thus it seems practical to extend the model in that direction.

The extension of the model accounts for the (long run) availability of financial resources of the economy to import. We assume that there is a certain ratio γ between the trade balance and GDP under which the economy must not fall in the long run. For example, we could take for γ the value of 0 or the value observed in the base year:

$$\gamma = \frac{x^0 - m^0}{f_d^0 - m_p^0} = \frac{x^0 - m^0}{h^0 + x^0 - m^0}$$

Furthermore, we modify the objective function of the model. We now differentiate between exports, $\mathbf{x}$, and all other final demand, $\mathbf{h}$. Exports are considered instrumental for attaining the means to import. Therefore, they are not included in the objective function of the model. In this model the economy strives for an maximum in $\mathbf{e}'\mathbf{h} = \mathbf{e}'\mathbf{h}_d + \mathbf{e}'\mathbf{m}_h$. We do not wish to analyse the impact that a restriction on production or any other bottleneck has on the structure of final demand imports but on the sum of imports to final demand. Therefore the following model does not contain the vector $\mathbf{m}_h$ but makes use only of the variable $m_h = \mathbf{e}'\mathbf{m}_h$. In analogy to the restriction $\mathbf{h}_d \leq \mathbf{h}_d^0$ we introduce the restriction $m_h \leq m_h^0$, meaning that as the opportunities for final demand for domestic goods also the opportunities for final demand for imported goods are limited.

Model IIc

maximize

$$\mathbf{e}'\mathbf{h}_d + m_h \tag{20}$$

subject to

$$\mathbf{h}_d \leq \mathbf{h}_d^0 \tag{21}$$

$$m_h \leq m_h^0 \tag{22}$$

$$\mathbf{L}_d\mathbf{h}_d + \mathbf{L}_d\mathbf{x}_d \leq \mathbf{q}^* \tag{23}$$

$$\mathbf{r}'\mathbf{L}_d\mathbf{h}_d + \mathbf{r}'\mathbf{L}_d\mathbf{x} \leq s^* \tag{24}$$

$$\mathbf{A}_m\mathbf{L}_d\mathbf{h}_d + \mathbf{A}_m\mathbf{L}_d\mathbf{x} \leq \mathbf{m}^* \tag{25}$$

$$(\gamma\mathbf{e}' + (1-\gamma)\mathbf{e}'\mathbf{A}_m\mathbf{L}_d)\mathbf{h}_d + (1-\gamma)(\mathbf{e}'\mathbf{A}_m\mathbf{L}_d - \mathbf{e}')\mathbf{x} + m_h \leq 0, \tag{26}$$

where equation 26 is a transformation of

$$\frac{x-m}{f_d+m_p} \geq \gamma.$$

Note that in equation 25 we write $\mathbf{m}^*$ on the right hand side (in contrast, in equation 19 it was $\mathbf{m}_p^*$). This modification seems consistent with the modified objective function in model IIc. It also reflects that in this model the economy has the flexibility to use a limited quantity of import supply of commodity i as input for production or for final demand, depending what is best with respect to the objective function.

Model IIc could be extended in various directions in order to adapt it to a specific application area. For example, one could introduce the assumption that the opportunities to export are limited, thus adding the restriction $\mathbf{x} \leq \mathbf{x}^0$ or $\mathbf{x} \leq \mathbf{x}^*$ to the model. Clearly, other details can easily be modified.

In view of the application we intend in this paper and considering that we have already brought into our models aspects of long-run viability we want to introduce only one more extension of the model. When there is a severe production limitation in sector i the models presented so far will react by curbing the final demand for commodity i and possibly even set it to zero. This is not realistic, since many commodities are crucial for the support of the economy, society and human life. There must be for almost all commodites a minimum delivery to final demand. In this respect it is useful to distinguish between final consumption, $\mathbf{c}$, and gross capital formation, $\mathbf{g}$, and to allow for a distinction between the various goods involved. For example, it might be easier to reduce final consumption for cars than for food. Or, depending on the application context, it might be decided that a certain minimum quantity of building materials must be available for gross capital formation in order to support the production capacity of the economy. Leaving open the practical question of how these minimum levels are to be specified we state that the minimum level of final demand (without exports) is given as:

$$\mathbf{h}_d^\diamond = \hat{\boldsymbol{\delta}}_c\mathbf{c}_d^0 + \hat{\boldsymbol{\delta}}_g\mathbf{g}_d^0,$$

where $\boldsymbol{\delta}_c$ and $\boldsymbol{\delta}_g$ denote the percentages of $\mathbf{c}_d^0$ and $\mathbf{g}_d^0$, respectively, that have to be satisfied at the least and the symbol $\hat{}$ denotes diagonalisation of a vector. Similarly, a minimum level of imports to final demand, $m_h^\diamond$ can be specified. Incorporating these extensions we have the final model:

Model IId

maximize

$$\mathbf{e}'\mathbf{h}_d + m_h \tag{27}$$

subject to

$$\mathbf{h}_d \leq \mathbf{h}_d^0 \tag{28}$$
$$m_h \leq m_h^0 \tag{29}$$
$$\mathbf{h}_d \geq \mathbf{h}_d^\diamond \tag{30}$$
$$m_h \geq m_h^\diamond \tag{31}$$
$$\mathbf{L}_d\mathbf{h}_d + \mathbf{L}_d\mathbf{x} \leq \mathbf{q}^* \tag{32}$$
$$\mathbf{r}'\mathbf{L}_d\mathbf{h}_d + \mathbf{r}'\mathbf{L}_d\mathbf{x} \leq s^* \tag{33}$$
$$\mathbf{A}_m\mathbf{L}_d\mathbf{h}_d + \mathbf{A}_m\mathbf{L}_d\mathbf{x} \leq \mathbf{m}^* \tag{34}$$
$$(\gamma\mathbf{e}' + (1-\gamma)\mathbf{e}'\mathbf{A}_m\mathbf{L}_d)\mathbf{h}_d + (1-\gamma)(\mathbf{e}'\mathbf{A}_m\mathbf{L}_d - \mathbf{e}')\mathbf{x} + m_h \leq 0, \tag{35}$$

In contrast to the models presented before, model IId can turn out to be infeasable. In that case, the limitation on production is so drastic that the minimum levels of final demand as given by equation 31 cannot be maintained.

2.4 Model III

In model III we assume that all imports are competitive. For two reasons this leads us to a considerable simplification of the linear programme to be specified. First, there is no technological restriction to substitute imported intermediate inputs for domestically produced intermediate inputs and vice versa. The relevant technology is solely defined by the matrix $\mathbf{A}$, while the matrices $\mathbf{A}_d$ and $\mathbf{A}_m$ contain no relevant information in the context of model III. Second, a given final demand $\mathbf{h}$ can be met by domestic production or by imports in largely discretionary proportions, i.e., as long as technological and other limitations are complied with. The aim of the economy is to satisfy as much final demand as possible. Accordingly, the objective function is $\mathbf{e}'\mathbf{h}$. But it is beyond the scope of the model to determine the import shares in $\mathbf{h}$. For these two reasons, we include $\mathbf{m}$ but are not interested in making the distinction between $\mathbf{m}_p$ and $\mathbf{m}_h$. Model III is the analogue of model IId under the assumption of competitive imports. In a preliminary form it can be written as follows:

Model III, preliminary

maximize

$$\mathbf{e}'\mathbf{h} \tag{36}$$

subject to

$$\mathbf{h} \leq \mathbf{h}^0 \tag{37}$$

$$\mathbf{h} \geq \mathbf{h}^\diamond \tag{38}$$

$$\mathbf{Lh} + \mathbf{Lx} - \mathbf{Lm} \leq \mathbf{q}^* \tag{39}$$

$$\mathbf{r}'\mathbf{Lh} + \mathbf{r}'\mathbf{Lx} - \mathbf{r}'\mathbf{Lm} \leq s^* \tag{40}$$

$$\mathbf{m} \leq \mathbf{m}^* \tag{41}$$

$$(1-\gamma)\mathbf{e}'\mathbf{m} + \gamma\mathbf{e}'\mathbf{h} - (1-\gamma)\mathbf{e}'\mathbf{x} \leq 0 \tag{42}$$

Let $\mathbf{h}^\bullet$, $\mathbf{m}^\bullet$ and $\mathbf{x}^\bullet$ denote vectors of final demand (without exports), imports and exports that solve model III. Then the corresponding vector of production is given as $\mathbf{q}^\bullet = \mathbf{Lh}^\bullet + \mathbf{Lx}^\bullet - \mathbf{Lm}^\bullet$. Though the solution for $\mathbf{h}^\bullet$, $\mathbf{m}^\bullet$ and $\mathbf{x}^\bullet$ might not be unique, $\mathbf{q}^\bullet$ is. Inspection of the model in its preliminary form reveals that if $\mathbf{h}^\bullet$, $\mathbf{m}^\bullet$ and $\mathbf{x}^\bullet$ form a solution then $\mathbf{h}^\bullet$, $\mathbf{m}^{\bullet\bullet} = \mathbf{m}^\bullet + \Delta$ and $\mathbf{x}^{\bullet\bullet} = \mathbf{x}^\bullet + \Delta$ also form a solution as long as the linear restrictions of the model are fulfilled. Therefore Property 1 is not fulfilled by the preliminary form of the model.

As a remedy we introduce the concept of nonnegative net exports, $\bar{\mathbf{x}} = (\bar{x}_i)$, and nonnegative net imports, $\bar{\mathbf{m}} = (\bar{m}_i)$, whose elements are defined as follows:

$$\bar{x}_i = \begin{cases} x_i - m_i & \text{if } x_i \geq m_1 \\ 0 & \text{otherwise} \end{cases} \tag{43}$$

$$\bar{m}_i = \begin{cases} m_i - x_i & \text{if } m_i \geq x_1 \\ 0 & \text{otherwise} \end{cases} \tag{44}$$

Note that net exports are given as nonnegative net exports minus nonnegative net imports, $\mathbf{x} - \mathbf{m} = \bar{\mathbf{x}} - \bar{\mathbf{m}}$. Model III, in its final form, is given as:

Model III

maximize

$$\mathbf{e}'\mathbf{h} \tag{45}$$

subject to

$$\mathbf{h} \leq \mathbf{h}^0 \tag{46}$$

$$\mathbf{h} \geq \mathbf{h}^\diamond \tag{47}$$

$$\mathbf{Lh} + \mathbf{L}\bar{\mathbf{x}} - \mathbf{L}\bar{\mathbf{m}} \leq \mathbf{q}^* \tag{48}$$

$$\mathbf{r}'\mathbf{Lh} + \mathbf{r}'\mathbf{L}\bar{\mathbf{x}} - \mathbf{r}'\mathbf{L}\bar{\mathbf{m}} \leq s^* \tag{49}$$

$$\bar{\mathbf{m}} \leq \mathbf{m}^* \tag{50}$$

$$(1-\gamma)\mathbf{e}'\bar{\mathbf{m}} + \gamma\mathbf{e}'\mathbf{h} - (1-\gamma)\mathbf{e}'\bar{\mathbf{x}} \leq 0 \tag{51}$$

3 Application and Conclusions

These sections of the paper will be contained only in the full paper.

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