Input-output calculus of international trade

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Abstract

Although significant progress has been made in capturing value added flows behind gross trade statistics, there is room for improving the computational and analytical foundations of the proposed frameworks. In pursuit of such improvements, this paper proposes a generalised framework for value added accounting and a new framework for gross exports accounting. It is shown that both frameworks can be modified to account for the cumulative or incremental trade costs in the world of global value chains. The proposed formulations are exposed to numerical tests with the data from the World Input-Output Database for 2010.

1. Introduction

As is known, gross trade statistics attribute the origin of traded products wholly to the exporting countries and, likewise, their destination to the importing countries. It is also known that, with the advent of the international fragmentation of production, imported products may be used as intermediate inputs in the production of exports. The product life cycle may therefore extend far beyond the national borders of the exporting and importing countries. Behind the observed gross trade flows between the reported origins and destinations, there is a web of the unobserved value added flows that link the producers and users of traded and even non-traded products. Indeed, "what you see in not what you get!" (a catchy expression from Maurer and Degain, 2010).

Researchers familiar with the input-output methods addressed the need of accounting the unobserved value added behind gross trade flows and proposed simple measures of vertical specialisation in trade based on national IO tables in late 1990s. The compilation of experimental inter-country (or multi-regional) IO tables prompted the development of new frameworks for a much more profound decomposition of international trade flows, or gross exports accounting. One of the latest efforts by Z. Wang, S.-J. Wei and K. Zhu allows for a breakdown of gross exports into sixteen value added components at the bilateral sector level. These components can be classified according to the origin of value added and its path to the destination.

Although significant progress has been made in capturing value added behind the reported gross trade statistics, there is room for improving the computational foundations and expanding the analytical applications of the proposed frameworks. In pursuit of such improvements, this paper proposes a generalised framework for value added accounting and a new framework for gross exports accounting. These are closely related to the previous work and are thought to be easily customisable for policy analysis.

Global value chains are modeled by means of block matrices which are fully consistent with the structure of the global IO tables with K countries and N economic sectors. This allows the user to simultaneously handle all bilateral flows from the country/sector of origin to the country (and in certain cases, sector) of destination. In fact, the paper clearly delineates two types of accounting frameworks: one focuses on the national and sectoral origin of exports, while another identifies their ultimate destination. The former reallocates bilateral exports to the flows of primary inputs embodied therein, or provides a decomposition backwards to the origin of those inputs. With the value added as the primary input, the framework leads to various familiar "trade in value added" or "value added in trade" measures. The latter traces bilateral exports forward to their destination as those are eventually embodied in partner and third countries' exports. This is a decomposition of "exports embodied in exports", which is particularly useful for counting the trade costs that are incurred incrementally along cross-border value chains. The utility of both frameworks will become apparent in the relevant sections of this paper.

The generalised, block-matrix-based approach to accounting also helped making the following advances. First, it allowed to link some measures known from the literature on trade in value added (VS, VS1, VAX) within simple accounting identities. Second, it enabled the derivation of two matrices of the inter-sectoral transfer of value added which, to the author's knowledge, did not explicitly appear in previous studies. These are the inter-sectoral flows of value added before the product leaves the exporting country and after it arrives at the importing country. Excluding these two matrices or respective matrix elements from the value added accounting equations at the bilateral sector level makes them incomplete. Third, it allowed to include the full sequence of valuation layers into the computations, leading to a consistent trade cost accounting with global value chains.

The paper is organised as follows. Section 2 briefly overviews the frameworks proposed so far and discusses their core conceptual differences. Section 3 explains the derivation of the generalised frameworks and relates them to the earlier results of Koopman et al. (2012) and Stehrer (2013). Section 4 develops specific applications of the generalised frameworks, including new trade cost accounting techniques. Section 5 tests some of the derived formulae with the real world production and trade data and briefly discusses the results. Finally, Section 6 concludes.

2. Overview of the existing frameworks

As a starting point, we should note that gross trade statistics report the observed direct flow of product produced by sector *i* from the exporting country *r* to the partner country *s*, as outlined in Figure 1. These traditional statistics that largely originate from customs records have their own merits and will never become obsolete. However, the information they provide is limited and inconsistent with the notion of the international fragmentation of production. In this way, a mobile phone exported from China to Russia is treated as a Chinese product consumed in Russia, though it is known that China may only contribute a tiny part of the total value (as in the case of the iPhone). At the other extreme, Russia exports chemicals or steel to Germany, but it is unknown who eventually consumes the products made from those intermediates and, hence, why exactly Germany demands those from Russia. Various accounting frameworks have been proposed so far to model those indirect, unobserved value chains.[1]

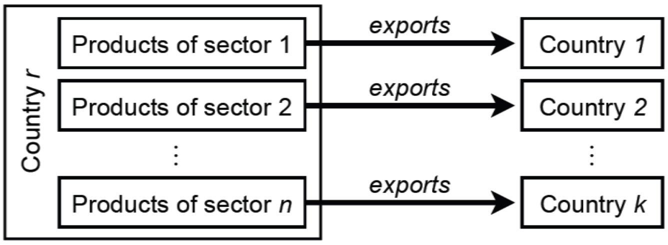

Figure 1: Simplified representation of gross trade statistics.

Most studies in this sub-area of research refer to Hummels et al. (1999) as the point of departure. Hummels and his co-authors did not provide a method for the complete decomposition of gross trade flows, but proposed the first measures of vertical specialisation in trade that became the building blocks for the subsequent research efforts and are still widely used for global value chains analysis.[2] These measures, known as VS and VS1, can be described for any single country as follows: (A) VS accounts for the import content of a country's exports, or "how much foreign value added is required to produce a unit of direct exports?"; (B) VS1 accounts for a country's domestic value added in partner exports, or "how much domestic value added is required to produce partner countries' exports, per unit of direct exports of the country in focus?".

VS depicts a country as a recipient of foreign value added to be further processed for exports, or its relative position with respect to the upstream value chain. VS1 depicts a country

[1] The rationale behind the use of the IO models is not discussed here. The reader may note that there are other methods to investigate the international fragmentation of production, including case studies and analyses of trade in intermediate goods. However, the IO-based methods are largely considered superior for macroeconomic analysis.

[2] Examples include OECD (2013b), OECD, WTO and UNCTAD (2013), UNCTAD (2013). Note that the measures of vertical specialisation appear in those publications under different names, e.g. "backward/forward participation" or "upstream/downstream component".

as a supplier of domestic value added to be used in partner exports, or its relative position with respect to the downstream value chain. VS therefore associates with the backward perspective and VS1 with the forward perspective in global value chain analysis.

Daudin et al. (2009) proposed an additional measure that is in fact a subset of VS1: domestic value added used in partner exports that ultimately returns home in final products. They called it VS1* and developed an inter-country IO table for their computations to correct the inaccuracies in the measures derived by Hummels et al. (1999) from the single-country IO tables.

Johnson and Noguera (2012)[3] are usually credited for the introduction of the measure of the value added content of bilateral trade, or “value added exports”. Their VAX measure counts the value added produced in a source country and finally absorbed in a destination country as a ratio to the gross exports at the sector and aggregate level. Johnson and Noguera generalised the computation procedures in an inter-country IO setting which paved the way for the subsequent gross exports accounting efforts. Their contribution is also intimately related to the measurement of the factor content of trade as in Trefler and Zhu (2010).

Accounting systems for the complete decomposition of bilateral and total gross trade into value added components appeared in a series of papers by R.Koopman, W.Powers, Z.Wang, S.-J.Wei, K. Zhu and R.Stehrer in 2010-2013. Koopman et al. (2010) integrated previous literature on vertical specialisation with newer literature on the value added content of trade. They developed a framework that was in many respects similar to that of Johnson and Noguera but shifted the focus to the complete decomposition of gross exports not limited to the exports of final products. In brief, Koopman et al. (2010) core contribution is (A) a consistent and relatively simple method of computation of true VS and VS1 values in an inter-country setting; (B) a decomposition that attributes all value added in a country’s exports to its sources and destinations (seven components).

Koopman et al. (2012) provided a “unified framework” that breaks up a country’s total gross exports into the sum of nine components. They show that the value added exports (VAX), VS, VS1, and VS1* are linear combinations of these components. Although not explicitly articulated, this methodology contained a conceptual deviation from their 2010 results (to be explained below).

Stehrer (2013) applied the framework of Koopman et al. (2012) at the bilateral level. This allowed for a detailed account of the role of third countries in bilateral value added trade. He also revisited the discussion in Stehrer (2012) on “trade in value added” and “value added in trade” which are two interrelated measurement concepts.

[3] The first draft manuscript of Johnson and Noguera dates back to 2008.

Wang et al. (2013) provide the most elaborate accounting framework with a breakdown of gross exports into sixteen components at the bilateral sector level. They also claim to have overcome some technical difficulties in previously proposed frameworks that hindered the interpretation of results.

A number of recent studies review and adapt the frameworks mentioned so far for specific analytical purposes. Kuroiwa (2014) applies the framework of Koopman et al. (2012) to derive a gross exports accounting equation for the special case of IDE-JETRO's Asian Input-Output Tables which, unlike the global inter-country IO tables, contain exogenous vectors of imports from and exports to the rest of the world. He then uses the equation to assess the technological intensity of China's exports. Kuboniwa (2014a, 2014b, 2014c) develops a theoretical discussion on the relationship between trade balances in value added and gross terms drawing on many of the previously discussed concepts.

Perhaps the most important application of some of these accounting frameworks to real data is the joint OECD-WTO Trade in Value Added database (OECD, 2013a).

Distinguishing two principal types of the accounting frameworks helps to understand the contributions mentioned so far and to structure readers' thoughts for further inquiry into global value chains. The frameworks of the first type model the unobserved flows that link the destination of exports and the sectors that initially contribute value added – see Figure 2. In other words, this is a backward decomposition that traces gross exports to the respective sectors of origin, or reallocates all observed bilateral export flows into the unobserved value added flows between origins and destinations. Koopman et al. (2012) and Stehrer (2013) are examples of this type of frameworks.

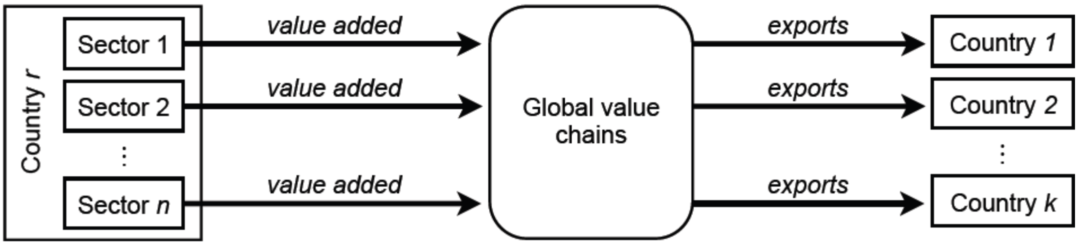

Figure 2: Simplified representation of a value added accounting framework.

In principle, this decomposition is capable of capturing the flows of any primary input or emission embodied in exports. The component flows may be classified according to the path to destination and/or use at destination. In case of value added, some of the component flows may be smaller or larger than the respective gross export flows, and do not necessarily sum to 100% of direct exports at bilateral level, which Wang et al. (2013) refer to as a shortcoming (in particular, of the VAX measure).

The frameworks of the second type model the use of exports throughout the value chain – see Figure 3. This is a forward decomposition that traces direct exports to their eventual country of destination. In this case, the observed bilateral export flows are reallocated into unobserved flows of embodied products through the downstream value chain. This framework is also capable of discerning the country of origin of value added contained therein but not its sectoral origin. Koopman et al. (2010) and Wang et al. (2013) exemplify this type of frameworks, though the latter partly draw on the first type.

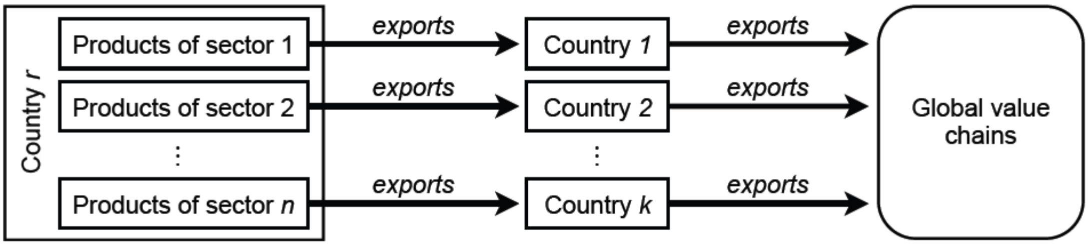

Figure 3: Simplified representation of a gross exports accounting framework.

One property is inherent to the frameworks of the second type. They decompose direct gross exports only one step forward. All component flows are therefore parts of the direct exports from the exporting country r to the partner country s, and their sum is necessarily confined to 100% of those direct exports.

It appears from the above that there is unlikely a single or unified accounting framework for measuring global value chains. Moreover, there is weak rationale in the search for such unified approach as both frameworks build on somewhat different concepts and are capable of addressing diverse policy issues. Value added accounting frameworks (Figure 2) are concerned with the sectoral origin of trade flows and may better suit the analyses of value creation, factor content, technological change, productivity, material inputs, embodied emissions and so on. Meanwhile, gross exports accounting frameworks (Figure 3) are concerned with the destination of traded products in the form of direct or embodied exports crossing national borders, which is required to analyse trade policy measures and trade costs in the global value chain environment.

The desirable features of the generalised frameworks that this paper intends to elaborate are as follows:

- handle all flows among countries/sectors simultaneously at the maximum disaggregate level, which effectively requires computing all component flows in the block matrix form;
- provide flexibility in terms of aggregation across countries and/or sectors;
- build on an intuitive matrix setup and minimise computations.

In line with the above, this paper first generalises the value added accounting framework as in Figure 2 and finds various useful identities which link it to previous literature on trade in value added. Next is the generalised version of the gross exports accounting framework as in Figure 3. Finally, the gross exports accounting framework is extended to model the whole value chain beyond just one step forward until the embodied exports reach their final destination as shown in Figure 4. This type of decomposition has not been discussed, to the author's knowledge, in previous literature.

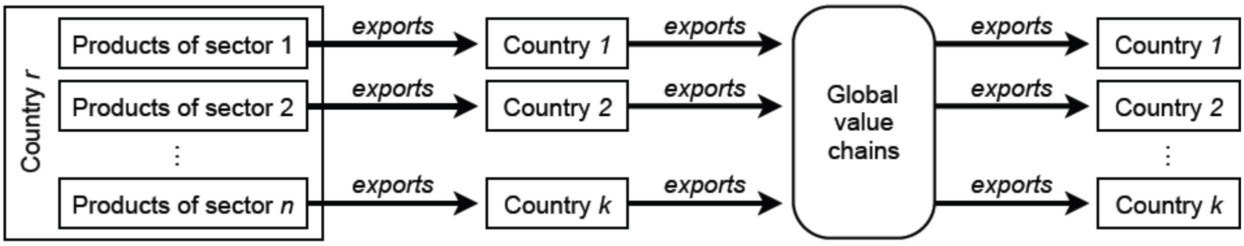

Figure 4: Simplified representation of a complete gross exports accounting framework.

3. Developing generalised accounting frameworks[4]

3.1. Notation and matrix setup

For a holistic value chain analysis, production and trade in the global economy are modeled by an inter-country IO table. The summary of notation is in Table 1.

The matrix representation of the inter-country (or multi-regional) table for K countries appears as follows:

$$\mathbf{Z}=\begin{bmatrix}\mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1k}\\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2k}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{Z}_{k1} & \mathbf{Z}_{k2} & \cdots & \mathbf{Z}_{kk}\end{bmatrix}\qquad \mathbf{F}=\begin{bmatrix}\mathbf{f}_{11} & \mathbf{f}_{12} & \cdots & \mathbf{f}_{1k}\\ \mathbf{f}_{21} & \mathbf{f}_{22} & \cdots & \mathbf{f}_{2k}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{f}_{k1} & \mathbf{f}_{k2} & \cdots & \mathbf{f}_{kk}\end{bmatrix}\qquad \mathbf{x}=\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ \vdots\\ \mathbf{x}_k\end{bmatrix}$$

$$\mathbf{m}(1)_{(\mathbf{Z})}=\begin{bmatrix}\mathbf{m}(1)_{(\mathbf{Z})1} & \mathbf{m}(1)_{(\mathbf{Z})2} & \cdots & \mathbf{m}(1)_{(\mathbf{Z})\mathbf{k}}\end{bmatrix}\qquad \mathbf{m}(1)_{(\mathbf{F})}=\begin{bmatrix}m(1)_{(F)1} & m(1)_{(F)2} & \cdots & m(1)_{(F)k}\end{bmatrix}$$

$$\mathbf{m}(2)_{(\mathbf{Z})}=\begin{bmatrix}\mathbf{m}(2)_{(\mathbf{Z})1} & \mathbf{m}(2)_{(\mathbf{Z})2} & \cdots & \mathbf{m}(2)_{(\mathbf{Z})\mathbf{k}}\end{bmatrix}\qquad \mathbf{m}(2)_{(\mathbf{F})}=\begin{bmatrix}m(2)_{(F)1} & m(2)_{(F)2} & \cdots & m(2)_{(F)k}\end{bmatrix}$$

$$\vdots \qquad\qquad \vdots$$

$$\mathbf{m}(g)_{(\mathbf{Z})}=\begin{bmatrix}\mathbf{m}(g)_{(\mathbf{Z})1} & \mathbf{m}(g)_{(\mathbf{Z})2} & \cdots & \mathbf{m}(g)_{(\mathbf{Z})\mathbf{k}}\end{bmatrix}\qquad \mathbf{m}(g)_{(\mathbf{F})}=\begin{bmatrix}m(g)_{(F)1} & m(g)_{(F)2} & \cdots & m(g)_{(F)k}\end{bmatrix}$$

$$\mathbf{v}=\begin{bmatrix}\mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_\mathbf{k}\end{bmatrix}$$

[4] Subsections 3.1-3.3 largely draw on Muradov, 2014.

In an economy of N economic sectors, each block element $\mathbf{Z}_{rs}$ is a N×N matrix, $\mathbf{f}_{rs}$ and $\mathbf{x}_r$ are N×1 vectors and $\mathbf{v}_s$ is a 1×N vector. **Z** is therefore a KN×KN matrix of intermediate demand, **F** is a KN×K matrix of final demand, **x** is a KN×1 column vector of total output and **v** is a 1×KN row vector of value added.

Table 1: Summary of notation used in this paper.

Index or matrix	Description
	Indices
r	index of the exporting country, $r \in K$
s	index of the partner country, $s \in K$
t	index of third countries $t \in K$
i	index of the selling (supplying) sector, $i \in N$
j	index of the purchasing (using) sector $j \in N$
g	index of the valuation layer, from 1 to 6 in this paper
K	number of countries
N	number of economic sectors (industries) in each country
G	number of valuation layers (margins, taxes/subsidies)
	Matrices and vectors
Z	KN×KN matrix of intermediate demand
F	KN×K matrix of final demand
x	KN×1 column vector of total output
$\mathbf{m}(g)_{(Z)}$	1×KN row vector of g-th margin or net tax on intermediate inputs
$\mathbf{m}(g)_{(F)}$	1×K row vector of g-th margin or net tax on final products
$\mathbf{M}(g)_{(Z)}$	KN×KN matrix of g-th margin or net tax on intermediate inputs
$\mathbf{M}(g)_{(F)}$	KN×K matrix of g-th margin or net tax on final products
v	1×KN row vector of value added
$\mathbf{V}_c$	KN×KN diagonal matrix of value added coefficients
$\mathbf{E}_{tot}$	KN×K block-diagonal matrix of total gross exports
$\mathbf{E}_{bil}$	KN×K block-off-diagonal matrix of bilateral gross exports
A	KN×KN matrix of technical coefficients
L	KN×KN Leontief global inverse
I	KN×KN (or appropriately sized) identity matrix
i	column vector of ones (summation vector) of appropriate size
$\mathbf{S}_n$	KN×K sector-wise aggregation matrix
T	KN×K matrix of bilateral import tariff rates

As the inter-country IO table is at basic prices, it must also account for the margins and taxes (less subsidies) on intermediate inputs and final products. Accordingly, $\mathbf{m}(g)_{(Z)}$ is the g-th 1×KN row vector of margins or net taxes on intermediate inputs and $\mathbf{m}(g)_{(F)}$ is the g-th 1×K row vector of margins or net taxes on final products. One can also treat $\mathbf{m}(g)_{(Z)}$ and $\mathbf{m}(g)_{(F)}$ as condensed valuation layers on intermediate and final output. There are G valuation layers that are discussed in more detail in subsection 4.2. Theoretically, the elements in $\mathbf{m}(g)_{(Z)}$ and $\mathbf{m}(g)_{(F)}$ should be zero for trade and transport margins, non-negative for taxes and non-positive for subsidies, or may take any values in a more general case of net taxes. However, international trade and transport margins are usually positive in the inter-country IO tables because the sector that supplies the relevant services is modelled exogenous to the system (Streicher and Stehrer, 2014).[5]

The fundamental accounting identities imply that the total sales by sector i for intermediate and final use equal total output, $\mathbf{Zi} + \mathbf{Fi} = \mathbf{x}$, and the purchases of intermediate and primary inputs at basic prices by sector j plus margins and net taxes on intermediate inputs equal total input (outlays) that must also be equal to total output, $\mathbf{i}'\mathbf{Z} + \sum_{g=1}^{G} \mathbf{m}(g)_{\mathbf{(Z)}} + \mathbf{v} = \mathbf{x}'$, where $\mathbf{i}$ is an appropriately sized summation vector.

Define a KN×KN diagonal matrix of value added coefficients:

$$\mathbf{V_c} = \begin{bmatrix} \hat{\mathbf{v}}_{\mathbf{c,1}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{v}}_{\mathbf{c,2}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \hat{\mathbf{v}}_{\mathbf{c,k}} \end{bmatrix}$$

where each block $\hat{\mathbf{v}}_{\mathbf{c,s}}$ is a N×N diagonalised vector of value added coefficients with the elements $v_{c,j} = \frac{v_j}{x_j}$. In matrix notation, $\mathbf{V_c} = \hat{\mathbf{v}}\hat{\mathbf{x}}^{-1}$.

We will occasionally require a similar vector of margin coefficients applicable to intermediate inputs:

$$\mathbf{m}(g)_{\mathbf{c(Z)}} = \lfloor \mathbf{m}(g)_{\mathbf{c(Z),1}} \quad \mathbf{m}(g)_{\mathbf{c(Z),2}} \quad \cdots \quad \mathbf{m}(g)_{\mathbf{c(Z),k}} \rfloor$$

where $\mathbf{m}(g)_{\mathrm{c(Z),s}}$ is a 1×N row vector of margin coefficients with the elements $m(g)_{c(Z),j} = \frac{m(g)_j}{x_j}$. In matrix notation, $\mathbf{m}(g)_{\mathbf{c}(Z)} = \mathbf{m}(g)_{(Z)}\hat{\mathbf{x}}^{-1}$.

Two KN×K matrices are required to represent total gross exports and bilateral gross exports:

[5] We assume that the inter-country IO table here does not contain any statistical discrepancies, and the sum of intermediate purchases at basic prices, net taxes and margins on intermediate inputs and value added at basic prices is equal to sector output at basic prices.

$$\mathbf{E}_{tot} = \begin{bmatrix} \mathbf{e_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{e_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{e_k} \end{bmatrix}, \qquad \mathbf{E}_{bil} = \begin{bmatrix} \mathbf{0} & \mathbf{e_{12}} & \cdots & \mathbf{e_{1k}} \\ \mathbf{e_{21}} & \mathbf{0} & \cdots & \mathbf{e_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e_{k1}} & \mathbf{e_{k2}} & \cdots & \mathbf{0} \end{bmatrix}$$

In $\mathbf{E}_{tot}$, each N×1 block element $\mathbf{e}_r$ is equal to the sum of the international sales for intermediate and final use over all trading partners, $\mathbf{e}_{tot,r} = \sum_{s \neq r}^{K} \mathbf{Z}_{rs}\mathbf{i}_n + \sum_{s \neq r}^{K} \mathbf{f}_{rs}$. In $\mathbf{E}_{bil}$, each N×1 block element $\mathbf{e}_{rs}$ only accounts for the bilateral flows, $\mathbf{e}_{bil,rs} = \mathbf{Z}_{rs}\mathbf{i}_n + \mathbf{f}_{rs}$, $r \neq s$, and $\mathbf{i}_n$ is an N×1 vector of ones for the summation across sectors.

The Leontief inverse, which is key to the demand-driven input-output analysis, in case of the inter-country IO table is defined as follows:

$$(\mathbf{I - A})^{-1} = \begin{bmatrix} \mathbf{I} - \mathbf{A_{11}} & -\mathbf{A_{12}} & \cdots & -\mathbf{A_{1k}} \\ -\mathbf{A_{21}} & \mathbf{I} - \mathbf{A_{22}} & \cdots & -\mathbf{A_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{A_{k1}} & -\mathbf{A_{k2}} & \cdots & \mathbf{I} - \mathbf{A_{kk}} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{L_{11}} & \mathbf{L_{12}} & \cdots & \mathbf{L_{1k}} \\ \mathbf{L_{21}} & \mathbf{L_{22}} & \cdots & \mathbf{L_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L_{k1}} & \mathbf{L_{k2}} & \cdots & \mathbf{L_{kk}} \end{bmatrix} = \mathbf{L}$$

$\mathbf{A}_{rs}$ blocks are N×N technical coefficient matrices that relate intermediate inputs to total output $a_{ij} = \frac{z_{ij}}{x_j}$, and $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$. Leontief inverse $\mathbf{L}$ is a KN×KN multiplier matrix that allows total output to be expressed as a function of final demand: $\mathbf{x} = \mathbf{Ax} + \mathbf{Fi} = (\mathbf{I - A})^{-1}\mathbf{Fi} = \mathbf{LFi}$. An important property is that the value added coefficients and margin coefficients must column-wise add up to a row vector of ones: $\mathbf{i}'\mathbf{V_c}\mathbf{L} + \sum_{g=1}^{G} \mathbf{m}(g)_{\mathbf{c}}\mathbf{L} = \mathbf{i}'$.

The sector-wise aggregation matrix $\mathbf{S}_n$ is constructed from the N×1 summation vectors $\mathbf{i}$:

$$\mathbf{S}_n = \begin{bmatrix} \mathbf{i} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{i} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{i} \end{bmatrix}$$

We will often need to pre-multiply by $\mathbf{S}_n'$ which compresses a KN×K matrix into the K×K (country by country) dimension.

In addition to the usual matrix summation and multiplication, the frameworks in this paper will often require extracting diagonal or off-diagonal block elements from block matrices which is done with the modified "hat" operators $\wedge$ $\vee$. However, these must not apply to the elements within the blocks. For example:

$$\overset{\wedge}{\mathbf{A}} = \begin{bmatrix} \mathbf{A_{11}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A_{22}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A_{kk}} \end{bmatrix} \quad \text{and} \quad \overset{\vee}{\mathbf{F}} = \begin{bmatrix} \mathbf{0} & \mathbf{f_{12}} & \cdots & \mathbf{f_{1k}} \\ \mathbf{f_{21}} & \mathbf{0} & \cdots & \mathbf{f_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{f_{k1}} & \mathbf{f_{k2}} & \cdots & \mathbf{0} \end{bmatrix}$$

These operators may apply to a single matrix, or an expression in square brackets, or both. For example, in $\overset{\vee}{\left[\mathbf{V_c}\overset{\vee}{\mathbf{L}}\overset{\vee}{\mathbf{F}}\right]}$, the diagonal block elements are first removed from $\mathbf{L}$ and $\mathbf{F}$. $\mathbf{V_c}$, $\overset{\vee}{\mathbf{L}}$ and $\overset{\vee}{\mathbf{F}}$ are then multiplied and, finally, the diagonal block elements are removed from the product thereof.

The above setup and notation signify a departure from using simplified two/three country and two/three sector examples to explain the framework. Instead, this paper suggests using "zoom in" views on block matrices and single block elements that may well describe the nature and direction of flows (see examples in Appendix B).

3.2. Value added accounting framework

Note that, by definition of gross exports:

$$\mathbf{E}_{bil} - \mathbf{E}_{tot} = \mathbf{F} + \mathbf{Z}_{(KN\times K)} - \hat{\mathbf{x}}_{(KN\times K)} \tag{1}$$

where $\mathbf{Z}_{(KN\times K)}$ is the $\mathbf{Z}$ matrix condensed to the KN×K dimension (aggregated across the partner country *s*' sectors), and $\hat{\mathbf{x}}_{(KN\times K)}$ is a KN×K block-diagonalised vector of total output arranged in a similar way, to conform with the $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ matrix dimensions. The resulting matrix on both sides of (1) has bilateral trade flows in the off-diagonal block elements and total exports with the negative sign in the diagonal block elements.

Using that $\mathbf{Z}_{(KN\times K)} = \mathbf{A}\hat{\mathbf{x}}_{(KN\times K)}$, rewrite the right part of equation (1) as follows:

$$\mathbf{F} - \hat{\mathbf{x}}_{(KN\times K)} + \mathbf{Z}_{(KN\times K)} = \mathbf{F} - \hat{\mathbf{x}}_{(KN\times K)} + \mathbf{A}\hat{\mathbf{x}}_{(KN\times K)} = \mathbf{F} - (\mathbf{I} - \mathbf{A})\hat{\mathbf{x}}_{(KN\times K)} = \mathbf{F} - \mathbf{L}^{-1}\hat{\mathbf{x}}_{(KN\times K)}$$

Then multiply both sides of (1), including the rewritten right side, by the value added multiplier matrix $\mathbf{V_c L}$:

$$\mathbf{V_c L}\left(\mathbf{E}_{bil} - \mathbf{E}_{tot}\right) = \mathbf{V_c L}\left(\mathbf{F} - \mathbf{L}^{-1}\hat{\mathbf{x}}_{(KN \times K)}\right)$$

A simple rearrangement gives:

$$\mathbf{V_c L E}_{bil} = \mathbf{V_c L F} + \mathbf{V_c L E}_{tot} - \mathbf{V_c}\hat{\mathbf{x}}_{(KN \times K)} \tag{2}$$

$\mathbf{V_c L E}_{bil}$ can be treated as a "bilateral value added in trade matrix", following the definitions of Stehrer (2012). This matrix reallocates gross export flows to the value added flows of known country and sectoral origin. Each element corresponds to both direct and indirect flows of value added that originates in sector i of country r and "lands" in country s to satisfy the aggregate (intermediate plus final) demand in country s. Usually, the $\mathbf{V_c L E}_{bil}$ matrix does not feature the "value added in trade" calculations and does not explicitly appear in any of the existing frameworks. However, this matrix is useful to estimate the domestic value added embodied in direct and indirect gross trade flows. Note that the columns of $\mathbf{V_c L E}_{bil}$ add up to the total imports of country s (that is the sum of columns of $\mathbf{E}_{bil}$) less embodied valuation (see a more detailed discussion in subsection 4.1). The rows of $\mathbf{V_c L E}_{bil}$ sum to the domestic value added in total gross exports of country r, sectoral or aggregated.

The $\mathbf{V_c L F}$ matrix is much more familiar as it is equal to Johnson and Noguera's bilateral "value added exports" matrix used for their derivation of the VAX measure. So it can be called the "trade in value added" matrix. Each element here represents both direct and indirect flows of value added that originates in sector i of country r and "ends up" in country s to satisfy its final demand. The rows of $\mathbf{V_c L F}$ sum to the total value added (sectoral or aggregated) produced in country r. The columns of $\mathbf{V_c L F}$ sum to the total value added absorbed in country s.

$\mathbf{V_c L E}_{tot}$ is the "value added in total trade" matrix. Koopman et al., 2010 proposed to use it for the computation of the multilateral VS and VS1 measures, as the column sums of the off-diagonal elements give VS and their row sums give VS1 in monetary terms. Note also that the columns of $\mathbf{V_c L E}_{tot}$ add up to the total exports of country s (equal to the column sum of $\mathbf{E}_{tot}$) less embodied valuation (see a more detailed discussion in subsection 4.1). The rows of $\mathbf{V_c L E}_{tot}$ sum to the total exports of country r's value added (sectoral or aggregated) in gross trade flows.

$\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}$ is the block-diagonal matrix of sectoral value added.

As our primary interest is international trade, it is legitimate to consider the off-diagonal block elements only from equation (2). $\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}$ then disappears from this basic accounting relationship since it only has the diagonal block elements:

$$\left[\mathbf{V_c}\overset{\vee}{\mathbf{L}}\mathbf{E}_{bil}\right]=\left[\mathbf{V_c}\overset{\vee}{\mathbf{L}}\mathbf{F}\right]+\left[\mathbf{V_c}\overset{\vee}{\mathbf{L}}\mathbf{E}_{tot}\right] \tag{3}$$

The right side of equation (3) can be recognised as the sum of multilateral VAX and VS1 measures in monetary terms. This basic accounting relationship implies a straightforward interpretation: the value added that originates in sector i of country r and "lands" in country s via direct and indirect trade flows is equal to the value added that "ends up" in the final demand of country s plus the value added that is re-exported by country s to third countries. $\left[\mathbf{V_c}\overset{\vee}{\mathbf{L}}\mathbf{F}\right]$ is therefore a net term and $\left[\mathbf{V_c}\overset{\vee}{\mathbf{L}}\mathbf{E}_{tot}\right]=\mathbf{V_c}\overset{\vee}{\mathbf{L}}\mathbf{E}_{tot}$ is a double-counted term. This gives a basic decomposition of the bilateral value added in trade into two components that can be expressed as its shares. Recall that the default dimension of the resulting matrices is KN×K (country-sector by country).

Subtracting from both sides of equation (3) indirect bilateral flows of value added $\left[\mathbf{V_c}\overset{\vee}{\overset{\vee}{\mathbf{L}}}\mathbf{E}_{bil}\right]$, adding foreign value added in direct bilateral exports $\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\overset{\vee}{\mathbf{L}}\right]\mathbf{E}_{bil}$ (where $\mathbf{V}_{\mathbf{c}(N\times KN)}=[\hat{\mathbf{v}}_{\mathbf{c,1}}\quad \hat{\mathbf{v}}_{\mathbf{c,2}}\quad \cdots \quad \hat{\mathbf{v}}_{\mathbf{c,k}}]$) and an inter-sectoral transfer term $\left(\left[\mathbf{i}'\hat{\mathbf{V}}_{\mathbf{c}}\mathbf{L}\right]-\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\right)\mathbf{E}_{bil}$ rearranges the value added components so that they add up to the gross bilateral exports of products from sector i of country r to country s:

$$\mathbf{E}_{bil}=\left[\mathbf{V_c}\overset{\vee}{\mathbf{L}}\mathbf{F}\right]+\mathbf{V_c}\overset{\vee}{\mathbf{L}}\mathbf{E}_{tot}-\left[\mathbf{V_c}\overset{\vee}{\overset{\vee}{\mathbf{L}}}\mathbf{E}_{bil}\right]+\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\overset{\vee}{\mathbf{L}}\right]\mathbf{E}_{bil}+\left(\left[\mathbf{i}'\hat{\mathbf{V}}_{\mathbf{c}}\mathbf{L}\right]-\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\right)\mathbf{E}_{bil} \tag{4}$$

The inter-sectoral transfer term explains the difference between sectoral gross exports and sectoral value added contributed to those exports. It accounts for the flows of value added among the sectors of the exporting country r before exports take place. However, one should note that the sum on the right side of equation (4) will be somewhat smaller than bilateral gross exports because of the valuation vectors. A simplified form $\left(\mathbf{I}-\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\right)\mathbf{E}_{bil}$ may be used to ensure the exact equality of the results on both sides, at the expense of an implicit inclusion of the valuation terms $\sum_{g=1}^{G}\mathbf{m}(g)_{\mathbf{c}}\mathbf{L}$ (see Muradov, 2014 for a detailed derivation and an extended discussion of the inter-sectoral transfer terms).

An aggregation with respect to the exporting sectors i will drop the inter-sectoral term because at the country level, gross exports are equal to total value added contributed to those exports (again, less the valuation terms):

$$\mathbf{S}'_n\mathbf{E}_{bil} = \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{S}'_n\left[\check{\overline{\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}}}\right] + \mathbf{S}'_n\left[\widehat{\mathbf{V}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}}\right]\mathbf{E}_{bil} \tag{5}$$

Equation (5) translates the basic relationship (3) into a decomposition of bilateral gross exports at the country level, in a way largely similar to Koopman et al., 2012 and Stehrer, 2013. The components can now be expressed as ratios (rather than shares) to gross exports and some of these ratios may well exceed 1. This formulation also links various measures known from the literature on gross exports accounting or vertical specialisation in their monetary form: the first term on the right side is the bilateral VAX measure, the second is VS1, and the fourth is VS. The third term may be treated as a "reversed VS1" because it represents the exporter's value added that flows from third countries to partners, i.e. in a direction that is opposite to VS1. So, these four measures from (5) add up to the aggregate gross exports:

GROSS (BILATERAL) EXPORTS = VAX + VS1 – "reversed VS1" + VS

The framework is capable of uncovering additional detail in the path of value added from the origin to the destination. For this purpose, the $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]$ and $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\right]$ terms may be split into various components. The former can be expressed as follows:

$$\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] = \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \left[\check{\overline{\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}}}}\right] \tag{6}$$

The first term on the right side $\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}$ captures the value added that originates in sector i of country r and is embodied in the products made in country r for final demand in country s. The second term $\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}$ captures the value added that originates in sector i of country r and is embodied in the products made in country *s* for final demand in country *s*. The third term $\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]$ captures the value added that originates in sector *i* of country *r* and is embodied in the products made in third countries for final demand in country *s*. The principal distinction between these terms is therefore in the place where intermediate products are transformed into final products: in the exporting country *r*, partner country *s* or third countries.

Another manipulation will require modified operators on block matrices to split $[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}] = \mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}$:

$$\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} = \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} + \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] \tag{7}$$

where $\circ$ and $'$ signify, respectively, the block-by-block multiplication and block-by-block transposition. Within block elements, normal matrix multiplication rules hold, so that:

$$\mathbf{E}'_{bil} = \begin{bmatrix} \mathbf{0} & \mathbf{e_{12}} & \cdots & \mathbf{e_{1k}} \\ \mathbf{e_{21}} & \mathbf{0} & \cdots & \mathbf{e_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e_{k1}} & \mathbf{e_{k2}} & \cdots & \mathbf{0} \end{bmatrix}' = \begin{bmatrix} \mathbf{0} & \mathbf{e_{21}} & \cdots & \mathbf{e_{k1}} \\ \mathbf{e_{12}} & \mathbf{0} & \cdots & \mathbf{e_{k2}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e_{1k}} & \mathbf{e_{2k}} & \cdots & \mathbf{0} \end{bmatrix}$$

and

$$\check{\mathbf{L}}\circ\mathbf{E}'_{bil} = \begin{bmatrix} \mathbf{0} & \mathbf{L_{12}} & \cdots & \mathbf{L_{1k}} \\ \mathbf{L_{21}} & \mathbf{0} & \cdots & \mathbf{L_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L_{k1}} & \mathbf{L_{k2}} & \cdots & \mathbf{0} \end{bmatrix} \circ \begin{bmatrix} \mathbf{0} & \mathbf{e_{21}} & \cdots & \mathbf{e_{k1}} \\ \mathbf{e_{12}} & \mathbf{0} & \cdots & \mathbf{e_{k2}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e_{1k}} & \mathbf{e_{2k}} & \cdots & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{L_{12}e_{21}} & \cdots & \mathbf{L_{1k}e_{k1}} \\ \mathbf{L_{21}e_{12}} & \mathbf{0} & \cdots & \mathbf{L_{2k}e_{k2}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L_{k1}e_{1k}} & \mathbf{L_{k2}e_{2k}} & \cdots & \mathbf{0} \end{bmatrix}$$

The first term in (7), $\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}$, is a matrix of value added that originates in sector *i* of the exporting country *r* and returns home via gross exports from the partner country *s* ("reflected value added"). The second term, $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$, is a matrix of value added that originates in sector *i* of the exporting country *r* and is re-exported by the partner country *s* to third countries ("redirected value added").

Finally, compile new equations for a more profound decomposition of bilateral domestic value added in trade and gross export flows from (3), (6) and (7):

$$\underbrace{\left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{bil}\right]}_{\substack{\text{domestic VA in}\\ \text{direct and indirect}\\ \text{exports}}} = \overbrace{\underbrace{\mathbf{V}_{\mathbf{c}}\hat{\mathbf{L}}\check{\mathbf{F}}}_{\substack{\text{domestic VA in}\\ \text{direct exports of}\\ \text{final products}}} + \underbrace{\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\hat{\mathbf{F}}}_{\substack{\text{domestic VA in}\\ \text{exports of intermediates}\\ \text{for final use}}} + \underbrace{\left[\mathbf{V}_{\mathbf{c}}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]}_{\substack{\text{domestic VA in}\\ \text{indirect exports of}\\ \text{final products}}}}^{\text{bilateral exports of value added, net terms (VAX)}} + \overbrace{\underbrace{\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}}_{\substack{\text{domestic VA}\\ \text{that returns home}\\ \text{via gross exports}}} + \underbrace{\left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]}_{\substack{\text{domestic VA}\\ \text{re-exported to third countries}}}}^{\text{re-exports of value added, double counted terms (VS1*, VS1)}} \qquad (8)$$

$$\underbrace{\mathbf{E}_{bil}}_{\substack{\text{bilateral}\\ \text{gross}\\ \text{exports}}} = \overbrace{\underbrace{\mathbf{V}_{\mathbf{c}}\hat{\mathbf{L}}\check{\mathbf{F}}}_{\substack{\text{domestic VA in}\\ \text{direct exports of}\\ \text{final products}}} + \underbrace{\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\hat{\mathbf{F}}}_{\substack{\text{domestic VA in}\\ \text{exports of intermediates}\\ \text{fir final use}}} + \underbrace{\left[\mathbf{V}_{\mathbf{c}}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]}_{\substack{\text{domestic VA in}\\ \text{indirect exports of}\\ \text{final products}}}}^{\text{bilateral exports of value added, net terms (VAX)}} + \overbrace{\underbrace{\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}}_{\substack{\text{domestic VA}\\ \text{that returns home}\\ \text{via gross exports}}} + \underbrace{\left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]}_{\substack{\text{domestic VA}\\ \text{re-exported to third countries}}}}^{\text{re-exports of value added, double counted terms (VS1*, VS1)}} -$$
$$\overbrace{-\underbrace{\left[\mathbf{V}_{\mathbf{c}}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]}_{\substack{\text{domestic VA}\\ \text{re-directed from}\\ \text{third countries}}} + \underbrace{\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}}_{\text{foreign VA}}}^{\substack{\text{re-exports of value added, double counted terms}\\ \text{("reversed" VS1, VS)}}} + \overbrace{\underbrace{\left(\left[\mathbf{i}'\hat{\mathbf{V}}_{\mathbf{c}}\mathbf{L}\right] - \left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\right)\mathbf{E}_{bil}}_{\substack{\text{transfer of domestic and foreign VA from}\\ \text{the sectors of origin to the exporting sectors}}}}^{\substack{\text{intra-country flows of value added,}\\ \text{a technical term}}} \qquad (9)$$

The sector-wise aggregation of (9) to the K×K (or country by country) dimension drops the inter-sectoral term:

$$\mathbf{S}'_n\mathbf{E}_{bil} = \mathbf{S}'_n\mathbf{V}_{\mathbf{c}}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{S}'_n\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\hat{\mathbf{F}} + \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right] + \mathbf{S}'_n\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} + \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] -$$
$$-\mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right] + \mathbf{S}'_n\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil} \qquad (10)$$

Equation (10) yields the results that are identical to those in Stehrer, 2013 (see equation 9), and the only difference is that in the latter study, the double counted terms are split into final and intermediate components, using that $\mathbf{E}_{bil} = \check{\mathbf{F}} + \check{\mathbf{Z}}_{(KN\times K)} = \check{\mathbf{F}} + \left[\mathbf{A}\check{\hat{\mathbf{x}}}_{(KN\times K)}\right]$.

The above derivation of the gross exports accounting equation reveals that it is in fact a decomposition of value added not direct exports flows. That's the reason why the terms for the two-way bilateral trade between the partner country and third countries – captured by $\left[\mathbf{V}_{\mathbf{c}}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]$, $\left[\mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_{\mathbf{c}}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$ and $\left[\mathbf{V}_{\mathbf{c}}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]$ – appear in this formula for bilateral gross exports, which may first seem counter-intuitive. This is also the reason why the range of the individual components in (10) and (11) expressed as the ratios to gross exports is not confined to 0-100%, so a normalisation with respect to gross exports will give ratios rather than shares, as stressed by Wang et al., 2013.

A remarkable property of the $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$ and $\left[\check{\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}}\right]$ terms is that their difference gives the balance of trade in the exporting country's value added between the partner country and third countries. This difference is zero after aggregating across all partner countries.

The flexibility of the framework allows for the aggregation of all the above equations across the exporting country sectors or partner countries or both. For example, aggregate the basic accounting relationship to show how domestic value added splits between final use and re-exports by all partner countries:

$$[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}]\mathbf{i}=[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}]\mathbf{i}+[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}]\mathbf{i}$$

$$\mathbf{S}'_n[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}]\mathbf{i}=\mathbf{S}'_n[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}]\mathbf{i}+\mathbf{S}'_n[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}]\mathbf{i}$$

The results above are, respectively, in the KN×1 (exporting country-sector) and K×1 (exporting country) dimension.

Of particular interest is the aggregation of the gross exports decomposition at the country level:

$$\mathbf{S}'_n\mathbf{E}_{bil}\mathbf{i}=\mathbf{S}'_n[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}]\mathbf{i}+\mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i}-\mathbf{S}'_n\left[\check{\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}}\right]\mathbf{i}+\mathbf{S}'_n\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}\mathbf{i} \tag{11}$$

It can be shown that, at the aggregate country level, $\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i}$ is equal to $\mathbf{S}'_n\left[\check{\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}}\right]\mathbf{i}$, which means that the balance of trade in the exporting country r's value added among all partners is zero (for an explicit proof, see Muradov, 2014). $\mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i}-\mathbf{S}'_n\left[\check{\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}}\right]\mathbf{i}$ can therefore be replaced with $\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i}$, and equation (11) may be rewritten in terms of the measures known from the literature on trade in value added and vertical specialisation:

GROSS (TOTAL) EXPORTS = VAX + VS1* + VS

3.3. Extended value added accounting framework

The discussion has so far focused on the decomposition of value added flows that originate in sector *i* of the exporting country *r* and “end up” or “land” in the partner country *s*. The resulting flows are therefore disaggregated at origin but (implicitly) aggregated at destination, hence the default KN×K, or country-sector by country dimension. An extension to the KN×KN (country-sector by country-sector) dimension would capture value added created in sector *i* of country *r* and embodied in products of sector *j* consumed or re-exported by partner country *s*. Such change of perspective is not a trivial exercise and requires a redefinition of various matrices.

Worth mentioning is that decompositions of value added at destination rather than at origin have been suggested on an *ad hoc* basis in the literature on trade in value added. Koopman et al., 2010 propose to aggregate across the exporting country sectors and disaggregate the partner country sectors in a matrix similar to the “value added in total trade” $\mathbf{V_c L E}_{tot}$ matrix in this paper. They treat it as a “sectoral measure of value-added trade in global value chains” (see formula 12 in Koopman et al., 2010 for the two-country case). Meng et al., 2012 briefly discuss a similar type of disaggregation applied to their sectoral “trade in value added” measure that they use to derive an alternative, TiVA-based version of the revealed comparative advantage indicators (equations 12-13 in Meng et al., 2012).

Below, to consistently address the implicit aggregation in the previously used measurements, we redefine $\mathbf{F}$, $\hat{\mathbf{x}}_{(KN\times K)}$, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ matrices from the KN×K to the KN×KN dimension:

$$\mathbf{F}_{(KN\times KN)} = \begin{bmatrix} \hat{\mathbf{f}}_{11} & \hat{\mathbf{f}}_{12} & \cdots & \hat{\mathbf{f}}_{1k} \\ \hat{\mathbf{f}}_{21} & \hat{\mathbf{f}}_{22} & \cdots & \hat{\mathbf{f}}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{f}}_{k1} & \hat{\mathbf{f}}_{k2} & \cdots & \hat{\mathbf{f}}_{kk} \end{bmatrix}, \qquad \hat{\mathbf{x}}_{(KN\times KN)} = \begin{bmatrix} \hat{\mathbf{x}}_1 & 0 & \cdots & 0 \\ 0 & \hat{\mathbf{x}}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\mathbf{x}}_k \end{bmatrix},$$

$$\mathbf{E}_{tot(KN\times KN)} = \begin{bmatrix} \hat{\mathbf{e}}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{e}}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \hat{\mathbf{e}}_k \end{bmatrix}, \qquad \mathbf{E}_{bil(KN\times KN)} = \begin{bmatrix} \mathbf{0} & \hat{\mathbf{e}}_{12} & \cdots & \hat{\mathbf{e}}_{1k} \\ \hat{\mathbf{e}}_{21} & \mathbf{0} & \cdots & \hat{\mathbf{e}}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{e}}_{k1} & \hat{\mathbf{e}}_{k2} & \cdots & \mathbf{0} \end{bmatrix}$$

The above conversion of $\mathbf{F}$, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ and $\mathbf{x}$ is for computational purpose only, to keep the sectoral dimension of results, and does not involve any meaningful interpretation.

Now $\mathbf{Z}$, $\mathbf{F}_{(KN\times KN)}$, $\mathbf{E}_{tot(KN\times KN)}$, $\mathbf{E}_{bil(KN\times KN)}$ and $\hat{\mathbf{x}}_{(KN\times KN)}$ are all KN×KN matrices. Owing to the above specification, all blocks in $\mathbf{F}_{(KN\times KN)}$, $\mathbf{E}_{tot(KN\times KN)}$, $\mathbf{E}_{bil(KN\times KN)}$ and $\hat{\mathbf{x}}_{(KN\times KN)}$ contain either diagonal elements only or zeros except **Z** where the blocks contain nonnegative values in all or many of the elements. For the equation (1) to hold in the KN×KN dimension, one more term is required to offset the presence of the off-diagonal elements in each block of **Z**:

$$\mathbf{Z^*} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1k} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{k1} & \mathbf{Z}_{k2} & \cdots & \mathbf{Z}_{kk} \end{bmatrix} - \begin{bmatrix} \widehat{[\mathbf{Z}_{11}\mathbf{i}]} & \widehat{[\mathbf{Z}_{12}\mathbf{i}]} & \cdots & \widehat{[\mathbf{Z}_{1k}\mathbf{i}]} \\ \widehat{[\mathbf{Z}_{21}\mathbf{i}]} & \widehat{[\mathbf{Z}_{22}\mathbf{i}]} & \cdots & \widehat{[\mathbf{Z}_{2k}\mathbf{i}]} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{[\mathbf{Z}_{k1}\mathbf{i}]} & \widehat{[\mathbf{Z}_{k2}\mathbf{i}]} & \cdots & \widehat{[\mathbf{Z}_{kk}\mathbf{i}]} \end{bmatrix}$$

Then the equation (1) in the KN×KN dimension is as follows:

$$\mathbf{E}_{bil(KN\times KN)} - \mathbf{E}_{tot(KN\times KN)} = \mathbf{F}_{(KN\times KN)} + \mathbf{Z} - \hat{\mathbf{x}}_{(KN\times KN)} - \mathbf{Z}^* \tag{12}$$

The same manipulation applies as in subsection 3.2, and the difference is that one more term appears on the right side:

$$\mathbf{V_c L E}_{bil(KN\times KN)} = \mathbf{V_c L F}_{(KN\times KN)} + \mathbf{V_c L E}_{tot(KN\times KN)} - \mathbf{V_c}\hat{\mathbf{x}}_{(KN\times KN)} - \mathbf{V_c L Z}^* \tag{13}$$

Finally, removing the diagonal block elements yields the basic accounting relationship in the KN×KN or full country-sector by country-sector dimension:

$$\left[\mathbf{V_c}\check{\mathbf{L}\mathbf{E}}_{bil(KN\times KN)}\right] = \left[\mathbf{V_c}\check{\mathbf{L}\mathbf{F}}_{(KN\times KN)}\right] + \left[\mathbf{V_c}\check{\mathbf{L}\mathbf{E}}_{tot(KN\times KN)}\right] - \left[\mathbf{V_c}\check{\mathbf{L}\mathbf{Z}}^*\right] \tag{14}$$

All subsequent equations for the value added accounting should include the last term from (14). In the first two matrices on the right side of equation (14), each element should be interpreted as the value added originating in sector *i* of country *r* embodied in the product of sector *j* used by country *s* for domestic consumption or re-exports. The last term accounts for the re-allocation of the value added originating in sector *i* of country *r* resulting from the inter-sectoral flows of intermediates for which country *s* is responsible. Appendix D in Muradov, 2014 explores the properties of this matrix in more detail.

Usual aggregation options are available. A pre-multiplication of (14) and any related equation by the sector-wise aggregation matrix $\mathbf{S}'_n$ will condense the results to the K×KN, or country by county-sector dimension. Then each element should be interpreted as the value added originating from (all sectors of) country *r* embodied in the product of sector *j* used by country *s* for domestic consumption or re-exports. The aggregation of (14) and related equations across the recipient country's sectors to the KN×K dimension, i.e. a post-multiplication by $\mathbf{S}_n$, will make the last inter-sectoral term equal zero and will revert to the framework discussed in the previous subsection.

One should also note that a rearrangement of (14) into the gross exports decomposition in the KN×KN dimension will be difficult to interpret because of the presence of zeros in the off-diagonal elements of each block in $\mathbf{E}_{bil(KN\times KN)}$. However, an aggregated version in K×KN dimension can be interpreted in terms of the total value added components embodied in the exported products received at the partner side (more details may be found in Muradov, 2014)

3.4. Gross exports accounting framework

By definition, bilateral gross exports comprise the cross-border flows of intermediate and final products:

$$\mathbf{E}_{bil} = \check{\mathbf{Z}}_{(KN\times K)} + \check{\mathbf{F}}$$

Exports of intermediates can be expressed as a function of the partner country's total output $\check{\mathbf{Z}}_{(KN\times K)} = \check{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)}$. Meanwhile, total output $\hat{\mathbf{x}}_{(KN\times K)}$ is the sum of intermediates for domestic use, final products for domestic use and total exports, which in the KN×K block-diagonalised form can be written as:

$$\hat{\mathbf{x}}_{(KN\times K)} = \hat{\mathbf{Z}}_{(KN\times K)} + \hat{\mathbf{F}} + \mathbf{E}_{tot}$$

Insert the decomposed $\hat{\mathbf{x}}_{(KN\times K)}$ into $\check{\mathbf{Z}}_{(KN\times K)} = \check{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)}$ and then into $\mathbf{E}_{bil} = \check{\mathbf{Z}}_{(KN\times K)} + \check{\mathbf{F}}$ to obtain:

$$\mathbf{E}_{bil} = \check{\mathbf{A}}\hat{\mathbf{Z}}_{(KN\times K)} + \check{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{F}}$$

Now, gross bilateral exports are the sum of (A) direct exports of intermediates for domestic intermediate use by partner, (B) direct exports of intermediates for domestic final use by partner, (C) direct exports of intermediates for exports by partner and (D) direct exports of final products.

The eventual use of the exported intermediates described by the first term $\check{\mathbf{A}}\hat{\mathbf{Z}}_{(KN\times K)}$ remains undetermined, i.e. these can either be embodied in domestic final use by partner or in partner exports. Accordingly, subsequent manipulations decompose this term until it is completely allocated to domestic final use and exports.

Using that $\hat{\mathbf{Z}}_{(KN\times K)} = \hat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} = \hat{\mathbf{A}}\left(\hat{\mathbf{Z}}_{(KN\times K)} + \hat{\mathbf{F}} + \mathbf{E}_{tot}\right)$ leads to an infinite series of inter-industry interactions:

$$
\begin{aligned}
\mathbf{E}_{bil} &= \check{\mathbf{A}}\hat{\mathbf{Z}}_{(KN\times K)} + \check{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{F}} = \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} + \check{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{F}} = \\
&= \check{\mathbf{A}}\hat{\mathbf{A}}\left(\hat{\mathbf{Z}}_{(KN\times K)} + \hat{\mathbf{F}} + \mathbf{E}_{tot}\right) + \check{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{F}} = \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} + \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\hat{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{F}} = \\
&= \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{A}}\left(\hat{\mathbf{Z}}_{(KN\times K)} + \hat{\mathbf{F}} + \mathbf{E}_{tot}\right) + \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\hat{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{F}} = \\
&= \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} + \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\hat{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{A}}\hat{\mathbf{F}} + \check{\mathbf{A}}\mathbf{E}_{tot} + \check{\mathbf{F}} = \dots
\end{aligned}
$$

Compiling and rearranging the terms after n → ∞ rounds of interactions results in:

$$
\begin{aligned}
\underset{n\to\infty}{\mathbf{E}_{bil}} &= \check{\mathbf{A}}\left[\hat{\mathbf{A}}\right]^n \hat{\mathbf{x}}_{(KN\times K)} + \left(\check{\mathbf{A}}\left[\hat{\mathbf{A}}\right]^n + \dots + \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{A}} + \check{\mathbf{A}}\hat{\mathbf{A}} + \check{\mathbf{A}}\right)\hat{\mathbf{F}} + \left(\check{\mathbf{A}}\left[\hat{\mathbf{A}}\right]^n + \dots + \check{\mathbf{A}}\hat{\mathbf{A}}\hat{\mathbf{A}} + \check{\mathbf{A}}\hat{\mathbf{A}} + \check{\mathbf{A}}\right)\mathbf{E}_{tot} + \check{\mathbf{F}} = \\
&= \check{\mathbf{A}}\left[\hat{\mathbf{A}}\right]^n \hat{\mathbf{x}}_{(KN\times K)} + \check{\mathbf{A}}\left(\left[\hat{\mathbf{A}}\right]^n + \dots + \hat{\mathbf{A}}\hat{\mathbf{A}} + \hat{\mathbf{A}} + \mathbf{I}\right)\hat{\mathbf{F}} + \check{\mathbf{A}}\left(\left[\hat{\mathbf{A}}\right]^n + \dots + \hat{\mathbf{A}}\hat{\mathbf{A}} + \hat{\mathbf{A}} + \mathbf{I}\right)\mathbf{E}_{tot} + \check{\mathbf{F}} = \\
&= 0 + \check{\mathbf{A}}\left(\mathbf{I} - \hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}} + \check{\mathbf{A}}\left(\mathbf{I} - \hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \check{\mathbf{F}}
\end{aligned}
$$

The elements in $\check{\mathbf{A}}\left[\hat{\mathbf{A}}\right]^n \hat{\mathbf{x}}_{(KN\times K)}$ are approaching zero because the column sums of **A** and $\hat{\mathbf{A}}$ are less then 1 in a monetary IO table.

It's worth noting that, due to the known property of the block-diagonal matrices, $\left(\mathbf{I} - \hat{\mathbf{A}}\right)^{-1}$ is equal to a block-diagonal matrix of local Leontief inverses:

$$
\left(\mathbf{I} - \hat{\mathbf{A}}\right)^{-1} = \begin{bmatrix} \mathbf{I} - \mathbf{A_{11}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} - \mathbf{A_{22}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} - \mathbf{A_{kk}} \end{bmatrix}^{-1} = \begin{bmatrix} (\mathbf{I} - \mathbf{A_{11}})^{-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & (\mathbf{I} - \mathbf{A_{22}})^{-1} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & (\mathbf{I} - \mathbf{A_{kk}})^{-1} \end{bmatrix}
$$

The equation obtained above reallocates the direct exports of sector i from the exporting country r according to their eventual use by the direct partner s:

$$\mathbf{E}_{bil} = \underbrace{\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}}}_{\substack{\text{intermediates eventually transformed} \\ \text{by partner into final products for} \\ \text{domestic use}}} + \underbrace{\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}}_{\substack{\text{intermediates eventually used} \\ \text{by partner for exports}}} + \underbrace{\check{\mathbf{F}}}_{\substack{\text{final products directly} \\ \text{exported to partner for} \\ \text{domestic use}}} \qquad (15)$$

Note that exports in this type of decomposition embody value added from all sectors and all countries of origin. The component matrices represent flows of products (not value added) and are necessarily confined to direct gross exports. In other words, value chains are confined to the national borders. Each component flow can be expressed as a share of direct gross exports and will not exceed 100%.

This decomposition is conceptually close to those in Koopman et al., 2010 and Wang et al., 2013, though differs in the way of identifying the eventual use of direct exports. The approach of Wang et al., 2013 can also help splitting bilateral gross exports into domestic and foreign value added: $\mathbf{E}_{bil} = \left[\mathbf{v_c}\hat{\mathbf{L}}\right]' \circ \mathbf{E}_{bil} + \left[\mathbf{v_c}\check{\mathbf{L}}\right]' \circ \mathbf{E}_{bil}$, where $\mathbf{v_c} = \mathbf{i}'\mathbf{V_c}$ is a row vector of value added coefficients and the $\circ$ and $'$ operators need to apply element-wise unlike those in the previous formulations in this paper. However, the sectors of origin of value added remain aggregated.

In the decomposition above, it is still unknown where the re-exported term $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ is destined for. The next exercise will trace this flow to the next tiers of the value chain and allocate it according to the eventual use.

The $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ term needs disaggregating according to the next country of destination, or second-tier partner. Given that $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ is a KN×K matrix that shows the flows among the exporting countries r and the first-tier partners s, our exercise requires extending the matrix to the third dimension KN×K×K. Then it will show the flows from the exporter r through the first-tier partner s to the second-tier partner t. This is visualised in Figure 5.

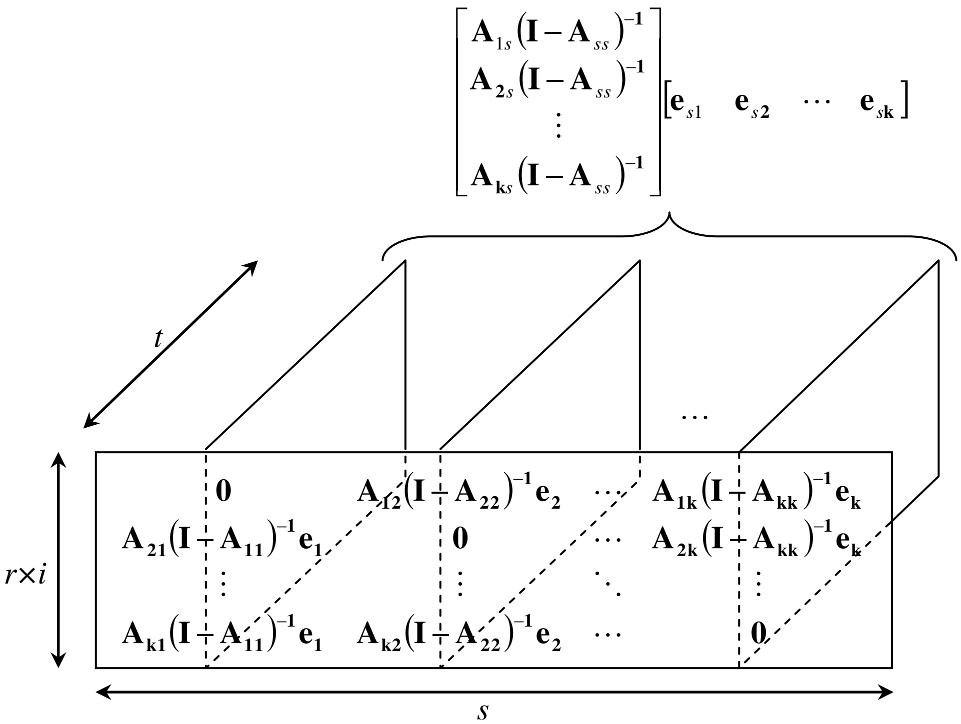

Figure 5: Transformation of the $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ matrix into a 3rd-order tensor.

The result is a thee-dimensional matrix, or a 3rd-order tensor where the third dimension is constructed by computing the outer product of the s^{th} column in $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}$ and s^{th} row in $\mathbf{E}_{bil}$:

$$\begin{bmatrix} \mathbf{A_{1s}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1} \\ \mathbf{A_{2s}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1} \\ \vdots \\ \mathbf{A_{ks}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{e_{s1}} & \mathbf{e_{s2}} & \cdots & \mathbf{e_{sk}} \end{bmatrix} = \begin{bmatrix} \mathbf{A_{1s}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1}\mathbf{e_{s1}} & \mathbf{A_{1s}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1}\mathbf{e_{s2}} & \cdots & \mathbf{A_{1s}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1}\mathbf{e_{sk}} \\ \mathbf{A_{2s}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1}\mathbf{e_{s1}} & \mathbf{A_{2s}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1}\mathbf{e_{s2}} & \cdots & \mathbf{A_{2s}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1}\mathbf{e_{sk}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A_{ks}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1}\mathbf{e_{s1}} & \mathbf{A_{ks}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1}\mathbf{e_{s2}} & \cdots & \mathbf{A_{ks}}\left(\mathbf{I}-\mathbf{A_{ss}}\right)^{-1}\mathbf{e_{sk}} \end{bmatrix}$$

These KN×K matrices are perpendicular to $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ and their row sums are equal to the s^{th} column of $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$. So the tensor contraction along the third dimension results in reverting to the KN×K $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ matrix.

In principle, the s^{th} row in $\mathbf{E}_{bil}$ may be replaced with the sum of the rows in the component matrices from $\mathbf{E}_{bil}=\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}}+\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}+\check{\mathbf{F}}$. Then the re-exported term may be disaggregated again into the fourth dimension (KN×K×K×K) and so on, which may lead to a series of high-dimensional tensors.

In order to keep the data in a manageable form for the decomposition to the next tiers, we opt for the tensor contraction along the second dimension, that is the first-tier partners *s*:

$$\sum_{s=1}^{K}\begin{bmatrix} \mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\\ \mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\\ \vdots \\ \mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\end{bmatrix}\begin{bmatrix} \mathbf{e}_{s1} & \mathbf{e}_{s2} & \cdots & \mathbf{e}_{sk}\end{bmatrix} =$$

$$= \begin{bmatrix} \sum\limits_{s=1}^{K}\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \sum\limits_{s=1}^{K}\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \sum\limits_{s=1}^{K}\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \\ \sum\limits_{s=1}^{K}\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \sum\limits_{s=1}^{K}\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \sum\limits_{s=1}^{K}\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \\ \vdots & \vdots & \ddots & \vdots \\ \sum\limits_{s=1}^{K}\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \sum\limits_{s=1}^{K}\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \sum\limits_{s=1}^{K}\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{0} & \mathbf{A}_{12}(\mathbf{I}-\mathbf{A}_{22})^{-1} & \cdots & \mathbf{A}_{1k}(\mathbf{I}-\mathbf{A}_{kk})^{-1} \\ \mathbf{A}_{21}(\mathbf{I}-\mathbf{A}_{11})^{-1} & \mathbf{0} & \cdots & \mathbf{A}_{2k}(\mathbf{I}-\mathbf{A}_{kk})^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{k1}(\mathbf{I}-\mathbf{A}_{11})^{-1} & \mathbf{A}_{k2}(\mathbf{I}-\mathbf{A}_{22})^{-1} & \cdots & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{0} & \mathbf{e}_{12} & \cdots & \mathbf{e}_{1k} \\ \mathbf{e}_{21} & \mathbf{0} & \cdots & \mathbf{e}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e}_{k1} & \mathbf{e}_{k2} & \cdots & \mathbf{0} \end{bmatrix} = \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$$

This operation results in a KN×K $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$ matrix where the country of origin is still *r* while the country of destination is *t*, or the second-tier partner. Replace $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ in equation (15) with $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$:

$$\underbrace{\mathbf{E}_{bil}}_{\text{1st+2nd tier}} = \underbrace{\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}}}_{\substack{\text{1st tier}\\ \text{from r to s}}} + \underbrace{\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}}_{\substack{\text{2nd tier}\\ \text{from r to t=s}}} + \underbrace{\check{\mathbf{F}}}_{\substack{\text{1st tier}\\ \text{from r to s}}} \tag{16}$$

The second term on the right side now captures the intermediate exports from sector *i* of country *r* that are embodied in all exports to country *s* (which also appears as *t* at the next tier) via third countries. As a result, we disaggregate the second-tier partners at the expense of aggregating the first-tier partners. Importantly, the term on the left side in (16) no longer represents direct bilateral exports. Instead, it accounts for the cumulative exports to the first- and second-tier partners.

Using equation (15) again decomposes the bilateral exports to the second-tier partners:

$$\underbrace{\mathbf{E}_{bil}}_{\text{1st+2nd tier}} = \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}} + \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil} + \check{\mathbf{F}} =$$

$$= \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}} + \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\left(\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}} + \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \check{\mathbf{F}}\right) + \check{\mathbf{F}} =$$

$$= \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}} + \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}} + \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\check{\mathbf{F}} + \check{\mathbf{F}}$$

Replace again $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ with $\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$ and allocate the second-tier total exports to the third-tier bilateral exports:

$$\underbrace{\mathbf{E}_{bil}}_{\text{1st+2nd+3rd tier}} = \underbrace{\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}}}_{\text{1st tier from r to s}} + \underbrace{\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}}}_{\text{2nd tier from r to s}} + \underbrace{\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}}_{\text{3rd tier from r to s}} + \underbrace{\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\check{\mathbf{F}}}_{\text{2nd tier from r to s}} + \underbrace{\check{\mathbf{F}}}_{\substack{\text{1st tier}\\ \text{from r to s}}}$$

In this way, further decomposing and reallocating exports along the value chain to the n[th] tier results in:

$$\underbrace{\mathbf{E}_{bil}}_{\text{1st+...+nth tier}} = \sum_{1}^{n}\left(\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{n}\hat{\mathbf{F}} + \sum_{1}^{n}\left(\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{n-1}\check{\mathbf{F}} + \left(\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{n}\mathbf{E}_{tot} =$$

$$= \sum_{0}^{n}\left(\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{n}\hat{\mathbf{F}} - \hat{\mathbf{F}} + \sum_{0}^{n}\left(\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{n}\check{\mathbf{F}} - \left(\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{n}\check{\mathbf{F}} + \left(\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{n}\mathbf{E}_{tot}$$

As the decomposition proceeds to an infinitely remote n[th] $\rightarrow \infty$ tier, the re-exported term approaches zero and is eventually reallocated between intermediates and final products for domestic use:

$$\underbrace{\mathbf{E}_{bil}}_{n\rightarrow\infty \text{ tiers}} = \left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}\hat{\mathbf{F}} - \hat{\mathbf{F}} + \left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}\check{\mathbf{F}} - 0 + 0 =$$

$$= \left(\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1} - \mathbf{I}\right)\hat{\mathbf{F}} + \left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}\check{\mathbf{F}}$$

This is a way to trace bilateral exports – products not value added (!) – throughout the whole value chain to the ultimate destination where they end up in partner final demand. The term on the left side can be treated as the cumulative bilateral exports $\mathbf{E}_{cum}$ where the elements are smaller or larger than direct bilateral exports, subject to the mode of partner integration into the value chain:

$$\mathbf{E}_{cum} = \underbrace{\left(\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1} - \mathbf{I}\right)\hat{\mathbf{F}}}_{\substack{\text{Exported and re-exported intermediates} \\ \text{eventually transformed into final products} \\ \text{for domestic use}}} + \underbrace{\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}\check{\mathbf{F}}}_{\text{Exported and re-exported final products}} = \left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}\mathbf{F} - \hat{\mathbf{F}} \qquad (17)$$

Equation (17) is not a decomposition of actual trade flows. Rather, it should be understood as a way to compute the cumulative bilateral exports $\mathbf{E}_{cum}$ where each element describes the amount of product by sector i of country r that is eventually used for final demand in country s, delivered by mode of direct or indirect exports.

An important property is that total cumulative exports to all destinations are equal to total direct gross exports:

$$\mathbf{E}_{cum}\mathbf{i} = \mathbf{E}_{bil}\mathbf{i}$$

$\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}$ is a new global multiplier matrix for the “exports embodied in exports” accounting above. It may be simplified to:

$$\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1} = \left(\mathbf{I}-\hat{\mathbf{A}}\right)\mathbf{L} \qquad (18)$$

which shows its relationship to the standard Leontief global inverse. The above also shows that $\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}$ exists as long as does $\mathbf{L}$.

The derivation of the equation of cumulative bilateral exports is also possible with the use of an alternative transformation at each tier:

$$\underbrace{\mathbf{E}_{bil}}_{\text{1st+2nd tier}} = \check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}}+\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}+\check{\mathbf{F}}=$$

$$=\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{F}}+\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}+\check{\mathbf{F}}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}+\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}=$$

$$=\underbrace{\mathbf{E}_{bil}}_{\text{1st tier}}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}+\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$$

The continuous substitution of $\mathbf{E}_{bil}$ to an infinitely remote $n^{th}\rightarrow\infty$ tier will yield:

$$\begin{aligned}\mathbf{E}_{cum} &= \left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}\mathbf{E}_{bil}-\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}\mathbf{E}_{tot}+\mathbf{E}_{tot}=\\ &=\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}\mathbf{E}_{bil}-\left(\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}-\mathbf{I}\right)\mathbf{E}_{tot}\end{aligned} \tag{19}$$

Cumulative bilateral exports can therefore be expressed as a function of either final demand or bilateral and total gross exports.

3.5. Comparing the value added accounting and the gross exports accounting frameworks

The derivation of the value added accounting framework in subsection 3.2-3.3 and the gross exports accounting framework in subsection 3.4 has already exposed their conceptual differences. Table 2 provides a summary of features of both frameworks for a clearer understanding of their analytical capabilities.

Table 2: Comparison of the value added accounting framework and the gross exports accounting framework.

Feature	Value added accounting framework	Gross exports accounting framework
Sector in focus	sector that contributes value added in the country of origin	sector that produces exports in the country of origin
Flow in focus	value added (embodied, unobserved)	exports (direct and embodied, observed and unobserved)
Type of decomposition	backward, to the origin of value added	forward, to the eventual destination of exports
Multipliers	Leontief inverse: $\mathbf{L}$	modified Leontief inverse: $\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\hat{\mathbf{A}}\right)^{-1}\right)^{-1}=\left(\mathbf{I}-\hat{\mathbf{A}}\right)\mathbf{L}$

Handling double-counted terms	isolates double-counted terms, accounts for net terms in $\mathbf{V_c LF}$ and related matrices	by default, does not isolate double-counted terms (only does so with the introduction of value added multipliers)[6]
Handling border crossing	value chain is not confined to national borders (because of global multipliers)	value chain is confined to national borders (because of partly localised multipliers)

To sum up, the value added accounting framework translates the aggregate demand for exports at destination and its components into the value added at origin. The Leontief inverse captures the global value chain and ignores national borders. Note that the value added coefficients may be disaggregated or replaced with any primary input coefficients with only minor loss of generality. This type of accounting framework therefore suits the analyses of any embodied primary inputs in international trade.

The gross exports accounting framework translates the exports at origin into the use at eventual destination. The modified Leontief inverse delimits the value chains within national borders from the international value chain and as such is capable of sequentially handling bilateral exports. This is essential for the analysis of trade costs and trade policy measures which apply to exports not value added flows. From subsection 4.2, it will become apparent that the value added accounting framework treats trade cots as primary inputs at origin, while the gross exports accounting framework treats them as valuation layers at each successive border crossing.

4. Specific applications

4.1. Trade balances in value added terms[7]

In an explanatory note to their joint work programme on trade in value added, the OECD and WTO identified the global trade imbalances as one of the principal policy drivers for developing new statistics on international trade. “When bilateral trade balances are measured in gross terms, the deficit with final goods producers (or the surplus of exporters of final products) is exaggerated because it incorporates the value of foreign inputs” (OECD and WTO, 2012). The accurate data on the true sources of trade imbalances is expected to better inform the policymakers, mitigating the risk of protectionist responses. The note also suggests that the

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[6] $\mathbf{S}'_n \mathbf{V_c LF}$ term from the value added accounting framework and $\mathbf{S}'_n \left(\left[\mathbf{v_c} \hat{\mathbf{L}} \right]' \circ \mathbf{E}_{cum} \right)$ from the gross exports accounting framework yield similar results (apart from the diagonal blocks), that is the net bilateral exports of value added, without the double-counted terms.

[7] This subsection partly draws on Muradov, 2014.

overall trade balance of a country with the rest of the world is equal in gross and value added terms.

Stehrer (2012, 2013) and Kuboniwa (2014b, 2014c) provide algebraic proofs and numerical illustrations of the properties of trade balances in value added terms. This paper employs the generalised value added accounting framework and offers another compact proof with a special account of the – usually omitted – valuation terms.

First, recall that, owing to the structure of the global IO table, the column sums of the value added multipliers and valuation multipliers are equal to ones: $\mathbf{i}'\mathbf{V}_\mathbf{c}\mathbf{L}+\sum_{g=1}^{G}\mathbf{m}(g)_\mathbf{c}\mathbf{L}=\mathbf{i}'$. Post-multiply both sides by the matrices of total exports and bilateral exports:

$$\mathbf{i}'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{tot}+\sum_{g=1}^{G}\mathbf{m}(g)_\mathbf{c}\mathbf{L}\mathbf{E}_{tot}=\mathbf{i}'\mathbf{E}_{tot}$$

$$\mathbf{i}'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}+\sum_{g=1}^{G}\mathbf{m}(g)_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}=\mathbf{i}'\mathbf{E}_{bil}$$

The above is the block-matrix formulation of the properties of the “value added in trade” matrices that were briefly noted in subsection 3.2. The columns in $\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{tot}$ add up to the total exports of country *s* (identical to the sum of columns in $\mathbf{E}_{tot}$) less the valuation embodied in total exports $\sum_{g=1}^{G}\mathbf{m}(g)_\mathbf{c}\mathbf{L}\mathbf{E}_{tot}$, while the columns in $\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}$ add up to the total imports of country *s* (the sum of columns in $\mathbf{E}_{bil}$) less the valuation embodied in total imports $\sum_{g=1}^{G}\mathbf{m}(g)_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}$. It is also true that the column sums of $\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{tot}$ are equal to the total value added in total exports, and the column sums of $\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}$ are equal to the total value added in total imports. The balance of value added in total trade (double-counted term) therefore differs from the gross trade balance by the balance of embodied valuation which tends to be small:

$$\mathbf{i}'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{tot}-\mathbf{i}'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}=\mathbf{i}'\mathbf{E}_{tot}-\mathbf{i}'\mathbf{E}_{bil}-\left(\sum_{g=1}^{G}\mathbf{m}(g)_\mathbf{c}\mathbf{L}\mathbf{E}_{tot}-\sum_{g=1}^{G}\mathbf{m}(g)_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}\right) \tag{20}$$

Then rearrange and pre-multiply equation (2) by the summation vector, moving to the 1×K dimension:

$$\mathbf{i}'\mathbf{V_c}\mathbf{LE}_{tot} - \mathbf{i}'\mathbf{V_c}\mathbf{LE}_{bil} = \mathbf{i}'\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)} - \mathbf{i}'\mathbf{V_c}\mathbf{LF} \qquad (21)$$

A careful review of the terms on the right side will reveal that the diagonal block elements are the only elements in the columns of $\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}$ and in the respective column sums $\mathbf{i}'\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}$. These can be interpreted as the total domestic value added generated in country s for final demand in s and elsewhere. The columns of $\mathbf{V_c}\mathbf{LF}$ sum to the total value added of whatever origin absorbed in country s. The right side of equation (21) therefore gives the difference between the total value added generated and the total value added absorbed in country s, i.e. a 1×K vector of trade balances in net value added terms. This certifies that the overall trade balance in net value added terms is equal to the overall gross trade balance less the balance in embodied valuation (see equation 20). This also succinctly confirms and elaborates the earlier results of Stehrer (2012, 2013) and Kuboniwa (2014b, 2014c) who don't explicitly consider the valuation layers.

The framework allows for a quick calculation of the bilateral trade balances as the differences between the relevant bilateral K×K matrices and their transposes. For the gross trade balances, this can be expressed as:

$$\mathbf{S}'_n\mathbf{E}_{bil} - (\mathbf{S}'_n\mathbf{E}_{bil})'$$

and for the bilateral balances of value added in gross trade:

$$\mathbf{S}'_n\mathbf{V_c}\mathbf{LE}_{bil} - (\mathbf{S}'_n\mathbf{V_c}\mathbf{LE}_{bil})'$$

and, finally, for the bilateral balances of trade in value added:

$$\mathbf{S}'_n\mathbf{V_c}\mathbf{LF} - (\mathbf{S}'_n\mathbf{V_c}\mathbf{LF})'$$

It is therefore evident that these three types of bilateral trade balances for a pair of countries are not equal unless under very special conditions. With the use of the value added accounting equations in the country by country dimension from subsection 3.2, these trade balances can be decomposed and those special conditions may be specified.

4.2. Trade cost accounting in the world of global value chains

A change in trade policy measures – and, hence, a change in trade costs – is designed to affect observed trade flows. However, in the world of global value chains, it cannot but affect the multitude of the unobserved flows of value added in the form of intermediate inputs. The discussion of the magnification of trade costs due to the multi-stage production seems to be as old as the discussion of the vertical specialisation in trade. Hummels et al. (1999) suggest that an important driving force behind vertical specialisation has been the trade barrier reduction. "Because the good-in-process crosses multiple borders, tariffs and transportation costs are incurred repeatedly. Hence, reductions in trade barriers yield a multiplied reduction in the cost of producing a good sequentially in several countries" (Hummels et al., 1999).

Yi (2003) develops a theoretical model that "delivers both magnified and nonlinear trade responses to tariff reductions". Yi (2010) investigates the magnification effect in more detail and classifies it into two distinct sources. The first is the border effect: the goods produced at various stages in different countries cross national borders while they are in process and incur trade costs multiple times. The second is the vertical specialisation effect: the import tariff applies to the customs value of gross exports as though imported goods were wholly produced in the exporting country, while they may actually carry the value added of third countries. Obviously, these two effects are not entirely separate: vertical specialisation occurs when intermediate products cross multiple borders.

Koopman et al. (2010) were the first to measure the magnified trade costs in a multi-country setting. Their illustrative calculation covered the bilateral international transportation margins and import tariffs faced by the exporting country in 2004, based on the GTAP database.

OECD and WTO (2012) stress that measuring trade in value added sheds new light on market access and trade disputes, given today's trade reality. Their document refers to a case where imported goods embody the domestic value added that returns home. Then tariffs and other trade policy measures (e.g. anti-dumping duties) would ultimately apply to domestic producers. Hence the need to estimate the cumulative costs of trade barriers.

To the author's knowledge, the first consistent formulation of the "cumulative tariffs" appeared in Rouzet and Miroudot (2013). The cumulative tariff, instead of considering input and output protection at a specific stage of production, traces the total cost of all tariffs incurred along the production process.

Before applying the frameworks developed in the previous section of this paper to generalize the accounting of cumulative trade costs, we have to make additional comments on the setup and terminology.

Trade policy measures are understood as any measures enforced by a national government (or a supra-national authority) that condition or subsidise the exportation or importation of goods and services. These measures usually add to the valuation of traded products, but not all of these can be easily quantified. While statistics usually exist for export and import tariffs and selected non-tariff barriers, only occasional estimates may be available for monetary equivalents of technical regulations, standards etc.

Trade costs at large include trade policy measures and the costs of market services that need to be purchased for the access to external markets. The latter are mainly trade and transport margins, but also information costs, communication costs and so on. Reliable estimates only exist for trade and transport margins.

The inter-country IO table is a convenient framework for the consistent accounting of the international trade costs. To properly record trade costs in an IO table, we should adhere to the national accounting conventions. The System of National Accounts (SNA) and related IO manuals do not explicitly discuss trade costs as these are perhaps rather vague from the national accounting perspective. However, those trade costs that change the valuation of products from basic to purchasers' prices are represented as the valuation layers in the IO tables. These include taxes less subsidies on products, trade and transport margins. Only the trade and transport sectors are treated as the "pass-through" sectors, so the information and other similar costs, the costs of compliance with technical regulations and standards should be understood as the purchases of intermediate inputs from the relevant supplying sectors.

In an inter-country IO table, the representation of the valuation layers is somewhat more complex than in a national IO table because taxes/subsidies and transport charges apply at both origin and destination. Accordingly, in between the basic price at origin and the purchasers' price at destination, there are FOB and CIF prices. FOB price is the price of a good at the border of the exporting country, or the price of a service delivered to a non-resident, including transport charges and trade margins up to the point of the border, and including any taxes less subsidies on the goods exported (Eurostat, 2008, p.164). CIF price is the price of a good delivered at the border of the importing country, or the price of a service delivered to a resident, before the payment of any import duties or other taxes on imports or trade and transport margins within the country (Eurostat, 2008, p.164). This paper therefore proposes that an ideal inter-country IO table is furnished with at least six valuation layers as Figure 6 shows.

Layers 1-4 in Figure 6 apply to the international trade transactions (off-diagonal blocks) of the inter-country IO table only, while layers 5 and 6 apply to both international trade and domestic transactions (all blocks) thereof. For an exhaustive trade cost analysis, it is important to separate taxes (subsidies) at destination that apply to imports only and to all products irrespective of their origin. As SNA (2008) explains, "imported goods on which all the required taxes on imports have been paid when they enter the economic territory may subsequently become subject to a further tax, or taxes, as they circulate within the economy" (para 7.91). This is an important distinction of Figure 6 above from Figure 1 in Streicher and Stehrer (2014) upon which it is drawn. Note also that the valuation layers in Figure 6 may be disaggregated to provide more detail, e.g. the taxes less subsidies layer may be split into taxes and subsidies, trade and transport margins into trade margins and transport margins.

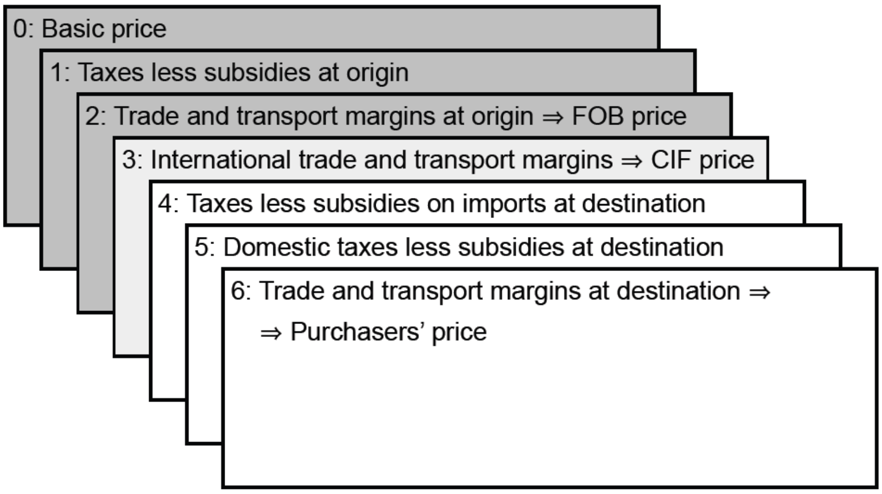

Figure 6. The desired sequence of valuation layers in an inter-country IO table. Author's adaptation of Figure 1 from Streicher and Stehrer (2014).

For a generalised representation, we assume that there are G valuation layers. Each g-th valuation layer may be described by two matrices

$$\mathbf{M}(g)_{(\mathbf{Z})} = \begin{bmatrix} \mathbf{M}(g)_{(\mathbf{Z}),11} & \mathbf{M}(g)_{(\mathbf{Z})\mathbf{m},12} & \cdots & \mathbf{M}(g)_{(\mathbf{Z}),1\mathbf{k}} \\ \mathbf{M}(g)_{(\mathbf{Z}),21} & \mathbf{M}(g)_{(\mathbf{Z}),22} & \cdots & \mathbf{M}(g)_{(\mathbf{Z}),2\mathbf{k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}(g)_{(\mathbf{Z}),\mathbf{k}1} & \mathbf{M}(g)_{(\mathbf{Z}),\mathbf{k}2} & \cdots & \mathbf{M}(g)_{(\mathbf{Z}),\mathbf{kk}} \end{bmatrix}$$

$$\mathbf{M}(g)_{(\mathbf{F})} = \begin{bmatrix} \mathbf{m}(g)_{(\mathbf{F}),11} & \mathbf{m}(g)_{(\mathbf{F})\mathbf{m},12} & \cdots & \mathbf{m}(g)_{(\mathbf{F}),1\mathbf{k}} \\ \mathbf{m}(g)_{(\mathbf{F}),21} & \mathbf{m}(g)_{(\mathbf{F}),22} & \cdots & \mathbf{m}(g)_{(\mathbf{F}),2\mathbf{k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{m}(g)_{(\mathbf{F}),\mathbf{k}1} & \mathbf{m}(g)_{(\mathbf{F}),\mathbf{k}2} & \cdots & \mathbf{m}(g)_{(\mathbf{F}),\mathbf{kk}} \end{bmatrix}$$

of which $\mathbf{M}(g)_{(\mathrm{Z})}$ is the KN×KN matrix of g-th margin (tax/subsidy) on intermediate inputs, and $\mathbf{M}(g)_{(\mathrm{F})}$ is the KN×K matrix of g-th margin (tax/subsidy) on final products. These matrices are condensed to row vectors that usually appear below the **Z** matrix and account for the change in valuation of the intermediate inputs from purchasers' to basic prices:

$$\mathbf{m}(g)_{(\mathbf{Z})} = \begin{bmatrix} \mathbf{m}(g)_{(\mathbf{Z})1} & \mathbf{m}(g)_{(\mathbf{Z})2} & \cdots & \mathbf{m}(g)_{(\mathbf{Z})\mathbf{k}} \end{bmatrix}$$

$$\mathbf{m}(g)_{\mathbf{(F)}} = \lfloor m(g)_{\mathbf{(F)1}} \quad m(g)_{\mathbf{(F)2}} \quad \cdots \quad m(g)_{\mathbf{(F)k}} \rfloor$$

where $\mathbf{m}(g)_{\mathbf{(Z)}} = \mathbf{i}'\mathbf{M}(g)_{\mathbf{(Z)}}$ is a 1×KN vector of total g-th margin (tax/subsidy) paid by the sectors column-wise on their intermediate inputs, and $\mathbf{m}(g)_{\mathbf{(F)}} = \mathbf{i}'\mathbf{M}(g)_{\mathbf{(F)}}$ is a 1×K vector of total g-th margin (tax/subsidy) paid by the final users on final products. An important note is that the $\mathbf{m}(g)_{(Z)}$ and $\mathbf{m}(g)_{(F)}$ vectors for trade and transport margins should theoretically be zero vectors because the respective valuation matrices reallocate margins to the trade and transport sectors from other sectors. However, the international trade and transport sector is usually modeled exogenous to the system and the rows of international trade and transport margins have positive values. This problem is discussed at length in Streicher and Stehrer (2014).

Dividing the $\mathbf{M}(g)_{(Z)}$ valuation matrix column-wise by the vector of total output $\mathbf{x}$ yields the matrix of valuation coefficients applicable to intermediate inputs:

$$\mathbf{M}(g)_{\mathbf{c(Z)}} = \mathbf{M}(g)_{\mathbf{(Z)}}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} \mathbf{M}(g)_{\mathbf{c(Z),11}} & \mathbf{M}(g)_{\mathbf{c(Z),12}} & \cdots & \mathbf{M}(g)_{\mathbf{c(Z),1k}} \\ \mathbf{M}(g)_{\mathbf{c(Z),21}} & \mathbf{M}(g)_{\mathbf{c(Z),22}} & \cdots & \mathbf{M}(g)_{\mathbf{c(Z),2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}(g)_{\mathbf{c(Z),k1}} & \mathbf{M}(g)_{\mathbf{c(Z),k2}} & \cdots & \mathbf{M}(g)_{\mathbf{c(Z),kk}} \end{bmatrix}$$

where each element for a pair of countries *r, s* $m(g)_{c(Z),ij} = \dfrac{m(g)_{ij}}{x_j}$ accounts for the part of total output (identical to total input) of sector *j* in *s* that must be spent to pay g-th margin (tax) on intermediate input sourced from sector *i*. Summing along the columns of the above matrix leads to a 1×KN row vector of margin coefficients $\mathbf{m(g)}_{c(Z)}$:

$$\mathbf{m}(g)_{\mathbf{c(Z)}} = \mathbf{i}'\mathbf{M}(g)_{\mathbf{c(Z)}} = \lfloor \mathbf{m}(g)_{\mathbf{c(Z),1}} \quad \mathbf{m}(g)_{\mathbf{c(Z),2}} \quad \cdots \quad \mathbf{m}(g)_{\mathbf{c(Z),k}} \rfloor$$

In the above setup, as is usual in the IO analysis, the rows of margins and net taxes on intermediate inputs are treated as primary inputs, similar to but separate from value added. As such, the valuation coefficients can enter the matrix calculations in the same way as the value added coefficients. Then the embodied valuation terms $\mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{LF}$ and $\mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{LE}_{bil}$ will count the total margins paid to satisfy final or aggregate external demand. The sum of elements in the 1×K vector $\mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{LF}$ must be equal to the sum of elements in the g-th valuation vector $\mathbf{m}(g)_{(Z)}$. The sum of elements in the 1×K vector $\mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{LE}_{bil}$ will exceed the sum of elements in $\mathbf{m}(g)_{(Z)}$ because of the double counting, as in the case of value added.

The above logic shows that "magnification" may be a confusing term because margins or taxes on products are not actually magnified but are double counted from the input-output perspective. The following is a more straightforward explanation of this point.

The basic price reflects the purchase of intermediates at purchasers' prices and value added at basic prices including other taxes on production which are not related to products (Eurostat, 2008, p.92). This means that the producer always purchasers the intermediate inputs at purchasers' prices which include all valuations, then produces output at basic prices and sells this output as intermediate inputs to next-stage producer at purchasers' prices. The margins and taxes incurred at the current production stage become part of intermediate consumption at the next production stage and so on. So the actual margins and net taxes are only paid once and do not magnify. However, the margins and net taxes may be double-counted in certain applications. This is a complete analogy with the value added accounting. Moreover, the common sense implies that the total amount of margins/taxes actually paid should remain unchanged, irrespective of the number of production stages in the global economy.

That said, margins/taxes can still be double counted which may be useful to illustrate the accumulation of trade costs along value chains. Margin/taxes can also affect the prices of output through the multi-stage production which is revealed with the use of the Leontief cost-push price model.

Price model

Let $\mathbf{p}$ be the column vector of index prices of industry output as in the standard Leontief price model (see Miller and Blair, 2009). The equilibrium condition requires that the price of industry output is entirely explained by the prices of intermediate and primary inputs:

$$\mathbf{p}'\hat{\mathbf{x}} = \mathbf{p}'\mathbf{Z} + \sum_{\mathbf{g=1}}^{\mathbf{G}} \mathbf{m}(g)_{\mathbf{(Z)}} + \mathbf{v}$$

where $\mathbf{x}$ and $\mathbf{Z}$ should be interpreted with the revised quantity terms (Miller and Blair, 2009). Post-multiplying by $\hat{\mathbf{x}}^{-1}$ leads to:

$$\mathbf{p}' = \mathbf{p}'\mathbf{A} + \sum_{\mathbf{g=1}}^{\mathbf{G}} \mathbf{m}(g)_{\mathbf{c(Z)}} + \mathbf{v}_{\mathbf{c}}$$

and solving for $\mathbf{p}$ yields:

$$\mathbf{p}' = \sum_{\mathbf{g=1}}^{\mathbf{G}} \mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{L} + \mathbf{v}_{\mathbf{c}}\mathbf{L} \tag{22}$$

In the price model without an exogenous change of the primary input coefficients, the index prices $\mathbf{p}$ will be equal to 1. Then the $\mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{L}$ and $\mathbf{v}_{\mathbf{c}}\mathbf{L}$ multipliers will give the shares of valuation (margins, net taxes) and value added in the equilibrium prices. In other words, each j,s^{th} element in the $\mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{L}$ vector corresponds to the part of equilibrium price of the output of industry j in country s that accounts for the margins/taxes incurred by industry j in country s and other industries along the downstream value chain. Note that, in line with the Leontief price model, $\mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{L}$ should be interpreted as the cost-push multipliers that translate an initial primary input coefficient or a change thereof into an index price of output or its change.

We introduce additional notation here to show that the $\mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{L}$ multipliers are the core element of Rouzet and Miroudot (2013) formula of "cumulative tariffs". Let $\boldsymbol{\tau}_{\mathrm{rs}}$ denote a N×1 vector of bilateral import tariff rates (expressed as decimals, or percentages divided by 100). Then the KN×KN matrix of bilateral import tariff rates on intermediate inputs is:

$$\mathbf{T}_{(KN\times KN)} = \begin{bmatrix} \boldsymbol{\tau}_{\mathbf{11}}\mathbf{i}' & \boldsymbol{\tau}_{\mathbf{12}}\mathbf{i}' & \cdots & \boldsymbol{\tau}_{\mathbf{1k}}\mathbf{i}' \\ \boldsymbol{\tau}_{\mathbf{21}}\mathbf{i}' & \boldsymbol{\tau}_{\mathbf{22}}\mathbf{i}' & \cdots & \boldsymbol{\tau}_{\mathbf{2k}}\mathbf{i}' \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\tau}_{\mathbf{k1}}\mathbf{i}' & \boldsymbol{\tau}_{\mathbf{k2}}\mathbf{i}' & \cdots & \boldsymbol{\tau}_{\mathbf{kk}}\mathbf{i}' \end{bmatrix}$$

Post-multiplication by a row vector of ones **i'** replicates the $\boldsymbol{\tau}_{\mathrm{rs}}$ vector N times to conform with the dimension of **Z**. In $\mathbf{T}_{(KN\times KN)}$, tariff rates are specific for the sector of origin but are the same for the sector of destination. Then the Rouzet and Miroudot (2013) version of cumulative tariffs can be simply written as:

$$\mathbf{T}_{(KN\times KN)cum} = \mathbf{T}_{(KN\times KN)} + \left[\mathbf{m}(\tau)_{\mathbf{c(Z)}}\mathbf{L}\right]'\mathbf{i}' \tag{23}$$

where $\mathbf{m}(\tau)_{\mathbf{c(Z)}}$ is the row vector of import tariff coefficients.[8] The cumulative tariff in this formulation consists of the direct bilateral tariff rate plus a uniform tariff-price multiplier that differentiates across exporting countries but is the same for all partner countries.[9]

[8] The original formulation of Rouzet and Miroudot (2013), using the notation of this paper, is as follows:

Either employed as a stand-alone multiplier vector, or in the matrix version of Rouzet and Miroudot (2013), $\mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{L}$ accounts for the cumulative impact of margins/taxes, paid by country *r* on its intermediate inputs, on the price of gross exports from country *r* to country *s*, but ignores the ultimate sectoral and national origin of the inputs that carried those margins/taxes. $\mathbf{m}(g)_{\mathbf{c(Z)}}\mathbf{L}$ multipliers are particularly useful for simulations with the use of the standard assumptions of the Leontief cost-push price model.

Application of the value added accounting framework

The next exercise applies the value added accounting framework using the standard Leontief demand-pull quantity model for another version of cumulative bilateral margins/taxes.

For illustrative purpose, split again bilateral gross exports into the exports of intermediate and final products:

$$\mathbf{E}_{bil} = \overset{\vee}{\mathbf{Z}}_{(KN\times K)} + \overset{\vee}{\mathbf{F}}$$

The respective direct bilateral g-th valuation layer is given by:

$$\mathbf{M}(g)_{\mathbf{(E)}} = \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{\mathbf{(F)}}$$

The above margins/taxes change the valuation of direct exports.

Following the logic of sequential production stages, the exports of intermediate and final products require intermediate inputs at the previous stage:

$$\mathbf{A}\overset{\vee}{\mathbf{Z}}_{(KN\times K)} + \mathbf{A}\overset{\vee}{\mathbf{F}}$$

This involves the corresponding valuation at the previous stage:

$$\mathbf{T}_{(KN\times KN)cum} = \mathbf{T}_{(KN\times KN)} + \left[\sum_{n=0}^{\infty} \mathbf{i}'\left(\mathbf{A}\circ\mathbf{T}_{(KN\times KN)}\right)\mathbf{A}^{\mathbf{n}}\right]' \mathbf{i}'.$$

Given that $\mathbf{A}\circ\mathbf{T}_{(KN\times KN)} = \mathbf{M}(\tau)_{\mathbf{c}(Z)}$ and $\sum_{n=0}^{\infty}\mathbf{A}^{\mathbf{n}} = \mathbf{L}$, this formula can be re-written in the form of equation (23).

[9] Rouzet and Miroudot (2013) formula requires a careful interpretation. Given the elaboration in this subsection, it is not clear what is the result of the summation of direct bilateral tariff rates and the cost-push multipliers that explain the cumulative effect of the price change on primary inputs through the entire downstream value chain.

$$\mathbf{M}(g)_{c(\mathbf{Z})}\overset{\vee}{\mathbf{Z}}_{(KN\times K)}+\mathbf{M}(g)_{c(\mathbf{Z})}\overset{\vee}{\mathbf{F}}$$

The above changes the valuation of direct intermediate inputs. Each element in either matrix after multiplication counts how much g-th margin/tax needs to be paid on the product of sector *i* in the first-tier supplying country *r* to produce the necessary amount of intermediate or final exports to country *s*.

Next stage intermediate inputs (backwards) are:

$$\mathbf{AA}\overset{\vee}{\mathbf{Z}}_{(KN\times K)}+\mathbf{AA}\overset{\vee}{\mathbf{F}}$$

And the corresponding valuation is:

$$\mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\overset{\vee}{\mathbf{Z}}_{(KN\times K)}+\mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\overset{\vee}{\mathbf{F}}$$

This above changes the valuation of embodied intermediate inputs two tiers back. Each element in either matrix counts the amount of g-th margin/tax payable on the second-tier supplier products. This decomposition can be continued backwards to an infinitely remote tier. Compiling the valuation of intermediate inputs at all tiers will result in:

$$\underset{n\to\infty}{\mathbf{M}(g)_{(\mathbf{Z},KN\times K)}}=\mathbf{M}(g)_{(\mathbf{Z},KN\times K)}+\mathbf{M}(g)_{\mathbf{c}(\mathbf{Z})}\overset{\vee}{\mathbf{Z}}_{(KN\times K)}+\mathbf{M}(g)_{\mathbf{c}(\mathbf{Z})}\mathbf{A}\overset{\vee}{\mathbf{Z}}_{(KN\times K)}+$$
$$+\mathbf{M}(g)_{\mathbf{c}(\mathbf{Z})}\mathbf{AA}\overset{\vee}{\mathbf{Z}}_{(KN\times K)}+\ldots+\mathbf{M}(g)_{\mathbf{c}(\mathbf{Z})}\mathbf{A}^{n}\overset{\vee}{\mathbf{Z}}_{(KN\times K)}=\mathbf{M}(g)_{(\mathbf{Z},KN\times K)}+\mathbf{M}(g)_{\mathbf{c}(\mathbf{Z})}\mathbf{L}\overset{\vee}{\mathbf{Z}}_{(KN\times K)}$$

Similarly, the cumulative valuation of the final products will yield:

$$\underset{n\to\infty}{\mathbf{M}(g)_{(\mathbf{F})}}=\mathbf{M}(g)_{(\mathbf{F})}+\mathbf{M}(g)_{\mathbf{c}(\mathbf{Z})}\overset{\vee}{\mathbf{F}}+\mathbf{M}(g)_{\mathbf{c}(\mathbf{Z})}\mathbf{A}\overset{\vee}{\mathbf{F}}+\mathbf{M}(g)_{\mathbf{c}(\mathbf{Z})}\mathbf{AA}\overset{\vee}{\mathbf{F}}+\ldots+\mathbf{M}(g)_{\mathbf{c}(\mathbf{Z})}\mathbf{A}^{n}\overset{\vee}{\mathbf{F}}=$$
$$=\mathbf{M}(g)_{(\mathbf{F})}+\mathbf{M}(g)_{\mathbf{c}(\mathbf{Z})}\mathbf{L}\overset{\vee}{\mathbf{F}}$$

Compiling all terms together will give a generalised formula for the cumulative accounting of trade costs corresponding to the g-th valuation layer:

$$\mathbf{M}(g)_{(\mathbf{E})cum} = \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{\mathbf{c(Z)}}\mathbf{L}\check{\mathbf{Z}}_{(KN\times K)} + \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{\mathbf{c(Z)}}\mathbf{L}\check{\mathbf{F}} = \\ = \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{\mathbf{c(Z)}}\mathbf{LE}_{bil} \tag{24}$$

The $\mathbf{M}(g)_{\mathbf{c}(Z)}\mathbf{LE}_{bil}$ term involves the double-counting of embodied valuation. For the specific case of import tariffs, the above can be re-written as:

$$\mathbf{M}(\tau)_{(\mathbf{E})cum} = \check{\mathbf{Z}}_{(KN\times K)} \circ \mathbf{T} + \check{\mathbf{F}} \circ \mathbf{T} + \left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right)\mathbf{LE}_{bil} = \mathbf{E}_{bil} \circ \mathbf{T} + \left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right)\mathbf{LE}_{bil} \tag{25}$$

where $\mathbf{T}$ is the matrix of bilateral import tariff rates in the country-sector by country (KN×K) dimension, and ∘ signifies the usual element-by-element multiplication. Read this equation as follows: cumulative tariffs (in monetary terms) are equal to the direct tariffs on bilateral exports plus the tariffs embodied in bilateral exports throughout the entire value chain. An important distinction to the tariff-price multiplier featuring the Rouzet and Miroudot (2013) formula is that the $\left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right)\mathbf{LE}_{bil}$ term is not uniform across producing countries. It accounts for tariffs as the embodied primary inputs payable on the products of sector i in country r irrespective of whether r is direct or n-th tier supplier. So it traces cumulative tariffs backwards to the origin of the products subject to those tariffs. To put it more explicitly, it captures the tariffs payable on inputs at origin and records these as embodied inputs at destination. One important drawback of this measure is therefore that it can't capture the indirect valuation of services.[10]

Finally, the element-by-element ratios of cumulative tariffs (or margins and net taxes, in general) to gross bilateral exports translate the estimates in monetary terms into percentages which is more convenient for trade policy analysis, e.g. comparison with direct tariff rates:

$$\mathbf{T}_{cum} = \mathbf{M}(\tau)_{(\mathbf{E})cum} \circ / \mathbf{E}_{bil} = \mathbf{T} + \left[\left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right)\mathbf{LE}_{bil}\right] \circ / \mathbf{E}_{bil} \tag{26}$$

where ∘/ is the element-by element division. For brevity, $\mathbf{T}_{cum}$ will be referred to as the "cumulative tariffs".

[10] Since this formula captures the tariffs at origin, and the direct tariffs on services are zero, the indirect (embodied) tariffs on services will also be zero. Meanwhile, in the Rouzet and Miroudot (2013) formula, the cumulative tariffs on services (before any sector aggregation) will be uniform across partner countries and will not show the variation of value chains in the bilateral country setting. This problem is addressed by the next model that employs the gross exports accounting framework.

Application of the gross exports accounting framework

Capturing the valuation terms along a forward decomposition to the eventual destination of exports provides a useful alternative. We will derive it first for the specific case of import tariffs.

Within the gross exports accounting framework, the tariffs payable on gross bilateral exports to the first-tier partner are equal to:

$$\underbrace{\mathbf{M}(\tau)_{(E)}}_{\text{1st tier}} = \mathbf{E}_{bil} \circ \mathbf{T} = \underbrace{\left(\check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \hat{\mathbf{F}} \right) \circ \mathbf{T}}_{\substack{\text{1st tier} \\ \text{from r to s}}} + \underbrace{\left(\check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \mathbf{E}_{tot} \right) \circ \mathbf{T}}_{\substack{\text{1st tier} \\ \text{from r to s}}} + \underbrace{\check{\mathbf{F}} \circ \mathbf{T}}_{\substack{\text{1st tier} \\ \text{from r to s}}}$$

$\check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \mathbf{E}_{tot}$ passes along the value chain to the second-tier partners and is subject to the import tariffs payable at the next border. It can be shown that $\check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \left(\mathbf{E}_{bil} \circ \mathbf{T} \right)$ captures this transition to the next tier. The new term now represents import tariffs paid on first-tier partner exports to second-tier partner which are in fact a part of the initial exports from the country of origin. Combine the direct (first-tier) and second-tier tariffs within a single equation:

$$\underbrace{\mathbf{M}(\tau)_{(E)}}_{\text{1st+2nd tier}} = \underbrace{\mathbf{E}_{bil} \circ \mathbf{T}}_{\substack{\text{1st tier} \\ \text{from r to s}}} + \underbrace{\check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \left(\mathbf{E}_{bil} \circ \mathbf{T} \right)}_{\substack{\text{2nd tier} \\ \text{from r to t=s}}}$$

Obviously, the equation above contains double-counted terms and therefore leads to the incremental valuation. Then the total tariffs paid at the first, second and third tiers are:

$$\underbrace{\mathbf{M}(\tau)_{(E)}}_{\text{1st+2nd+3rd tier}} = \underbrace{\mathbf{E}_{bil} \circ \mathbf{T}}_{\substack{\text{1st tier} \\ \text{from r to s}}} + \underbrace{\check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \left(\mathbf{E}_{bil} \circ \mathbf{T} \right)}_{\substack{\text{2nd tier} \\ \text{from r to s}}} + \underbrace{\check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \left(\mathbf{E}_{bil} \circ \mathbf{T} \right)}_{\substack{\text{3rd tier} \\ \text{from r to s}}}$$

The continuous addition of the import tariffs paid on a "melting" part of the initial exports passing along the downstream value chain to an infinitely remote $n^{th} \rightarrow \infty$ tier will result in:

$$\underbrace{\mathbf{M}(\tau)_{(E)}}_{n \rightarrow \infty \text{ tiers}} = \mathbf{M}(\tau)_{(E)inc} = \sum_{0}^{n} \left(\check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \right)^{n} \left(\mathbf{E}_{bil} \circ \mathbf{T} \right) = \left(\mathbf{I} - \check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \right)^{-1} \left(\mathbf{E}_{bil} \circ \mathbf{T} \right) \quad (27)$$

Each element in the resulting KN×K matrix counts all tariffs (in monetary terms) payable on the product of sector *i* in country *r* at the border of country *s* irrespective of whether *s* is direct or n-th tier partner. Like the cumulative measure of tariffs $\mathbf{M}(\tau)_{(E)cum}$ derived from the value added accounting framework above, the $\mathbf{M}(\tau)_{(E)inc}$ term involves double counting of the import tariffs paid. But it does so in a different way: it incrementally captures the tariffs payable at (the border of) destination and records these as exports at origin. It is not an estimate of the tariffs actually paid because, clearly, the nth tier partner passes the tariffs paid on n-1st exports to the n+1st tier partner. $\mathbf{M}(\tau)_{(E)inc}$ may therefore be treated as an incremental resistance term, while $\mathbf{M}(\tau)_{(E)cum}$ is an embodied or cumulative tariffs term. Likewise, $\mathbf{M}(\tau)_{(E)cum}$ corresponds to a notion of "tariffs embodied in exports", and $\mathbf{M}(\tau)_{(E)inc}$ to "exports embodied in tariffs".

The implicit tariff rates in this case are as follows:

$$\mathbf{T}_{inc} = \mathbf{M}(\tau)_{(E)inc} \circ / \mathbf{E}_{bil} = \left[\left(\mathbf{I} - \check{\mathbf{A}} \left(\mathbf{I} - \hat{\mathbf{A}} \right)^{-1} \right)^{-1} \left(\mathbf{E}_{bil} \circ \mathbf{T} \right) \right] \circ / \mathbf{E}_{bil} \tag{28}$$

where ∘/ is the element-by element division. For brevity, $\mathbf{T}_{inc}$ will be referred to as the "incremental tariffs".

Lastly, a generalisation of this incremental tariff accounting requires that $\mathbf{E}_{bil} \circ \mathbf{T}$ be replaced with a g-th valuation layer on bilateral gross exports $\mathbf{M}(g)_{(\mathbf{E})} = \mathbf{M}(g)_{(\mathbf{Z},KN \times K)} + \mathbf{M}(g)_{(\mathbf{F})}$.

5. Data and results

5.1. Data

A number of global inter-country IO databases have recently become available, building on varying philosophy of construction, offering different coverage and contents. Most of these were reviewed in the 2013 special issue of *Economic Systems Research* (2013, Vol.25, No.1). WIOD, Eora, EXIOPOL, OECD ICIO model and various MRIO versions of GTAP datasets contain inter-country IO tables that are compatible with the matrix setup in subsection 3.1. However, none of those contain the full sequence of valuation layers as shown in Figure 6. At best, Eora discerns four valuation layers: subsidies on products, taxes on products, trade margins and transport margins, but does not separate subsidies, taxes and margins at origin, at destination and in international transit. WIOD records the information on valuation that is needed to change the national SUTs from purchasers' prices to basic prices, but does not utilise it to produce consistent valuation layers for the symmetric world table. Worth noting is that only Eora and EXIOPOL re-price imports from CIF prices recorded at destination into basic prices at origin (observed by Bouwmeester et al., 2014, p.520).

To test the frameworks elaborated in the previous sections, this paper employs the World Input-Output Database (WIOD). The WIOD database is the outcome of a project funded by the European Commission and implemented by a consortium of 11 international partners. It contains a series of national and inter-country SUTs and IOTs supplemented by sets of socio-economic and environmental indicators for 1995-2011. WIOD covers 27 European Union member states, 13 other major non-European economies plus estimates for the rest of the world. The classification used in WIOD discerns 35 industries and 59 products, based on NACE rev.1 and CPA, respectively (Timmer et al., 2012; Dietzenbacher et al., 2013). The WIOD project is recognised for benchmarking the ICIO data against the updated national account aggregates, ensuring accuracy in handling international merchandise and services trade statistics and has been widely used for the quantitative research of various implications of global value chains.[11]

The reasonable balance between country and sector detail, the transparency of the compilation procedures and the availability of the underlying SUTs makes WIOD tables rather convenient source of data for our computation exercises. An important drawback is that the international trade transactions in WIOD remain at FOB prices, and, hence, include export taxes less subsidies, trade and transport margins paid at origin, on top of basic prices. The reason is that the data on international flows of intermediates and final products in WIOD are taken from the national use tables for imports where the FOB price is treated as basic price. Moreover, the information from the valuation layers in national SUTs is not useful for re-pricing imports into the basic prices of the exporting country, because for the calculation of the margin and tax rates by product, WIOD compilers assumed that they did not apply to exports (Dietzenbacher et al., 2013, p.80). Further complication arises because of the non-uniform price concepts used in the national accounting practices. For example, the national SUTs for the USA in WIOD contain tables of margins and net taxes where all elements are zero, and the use at basic prices is equal to the use at purchasers' prices. So, the customisation of WIOD data leading to the full sequence of valuation layers for the purpose of this paper appears to be a complex procedure and will likely result in an arduous modification of the whole inter-country IO table.

Meanwhile, two valuation layers may be readily compiled creating only minor inconsistencies with the original world IO tables in WIOD – these are the matrices of

[11] Available at http://www.wiod.org.

international trade and transport margins and the matrices of import taxes at destination. The compilation of the matrices of international trade and transport margins for 2010 involved the following manipulations:

- using the UN Comtrade data on total bilateral gross exports and imports among 40 WIOD countries in 2010 to obtain a uniform aggregate CIF/FOB ratio;
- applying the uniform CIF/FOB ratio (= 1.068347) to the international trade blocks of the WIOD international use tables (goods only), following the approach of Lenzen et al. (2012) in the construction of Eora;
- running the standard RAS balancing procedure on the resulting matrix of margins, using the aggregate bilateral international trade and transport margins from the WIOD international use tables as constraints;[12]
- transforming the rectangular matrix of international trade and transport margins (of dimension country-product × country-industry) into an input-output matrix (country-industry × country-industry) using the Eurostat model D (fixed product sales structure assumption); the block-columns for the rest of the world (RoW) are now missing because the use table for RoW is not available;
- applying the uniform CIF/FOB ratio to the block columns corresponding to RoW as the importing country in the original world IO table (including the intra-RoW trade present in the "domestic" blocks of RoW); this yields an estimate of the international trade and transport margins payable on exports to the RoW.

The result is the matrices of international trade and transport margins on international flows of intermediate and final products that are entirely consistent with the original world IO table except RoW as importer. In the WIOD world IO tables, total international trade and transport margins on RoW imports are zero, while they are non-zero in the estimates obtained above. An immediate solution is to offset the emergence of these non-zero margins by adding an appropriate row with the negative signs as a statistical discrepancy term.

The following is a brief description of the compilation of the bilateral import tariff matrices for 2010:

- extracting the bilateral import tariff data from the UN TRAINS[13] at ISIC Rev.3 two-digit level for 40 WIOD countries in 2010 (MFN and preferential rates with the ad valorem equivalents of the non-ad valorem rates);
- computing the actual tariff rates as the simple averages of the MFN and preferential rates, assuming that the preference utilisation is 50%;

[12] In fact, the first step in RAS, i.e. computing the column vector of r scalers block-wise, was sufficient to balance the full matrix.

[13] UN Comtrade and UN TRAINS were accessed via WITS.

- applying bilateral import tariff rates to goods in the WIOD international use table (re-priced CIF with the international trade and transport margins); the tariff rates differentiate across partner countries but are uniform across the purchasing industries in each partner country;
- transforming the rectangular matrix of import tariffs paid (in monetary terms, of dimension country-product × country-industry) into an input-output matrix (country-industry × country-industry) using the Eurostat model D; the block-columns for RoW are still missing;
- creating a "proxy" rest-of-world reporter in UN TRAINS, covering ~60% of trade between RoW and WIOD countries; extracting data on bilateral import tariff rates at ISIC Rev.3 two-digit level between WIOD countries and the "proxy" rest-of-world region and on intra-RoW international transactions (MFN and preferential rates with the ad valorem equivalents of the non-ad valorem rates, preference utilisation assumed at 50%);
- aggregating bilateral import tariff rates from ISIC Rev.3 into WIOD 35 industry classification using the additional data on bilateral tariff line imports at ISIC Rev.3 two-digit level;
- applying the obtained tariff rates to the imports by RoW from WIOD countries and intra-RoW transactions in the original world IO table (re-priced CIF with the respective international trade and transport margins); this yields an estimate of the import tariffs payable on exports to RoW.

The result is the matrices of import tariffs payable on the international flows of intermediate and final products. These matrices cannot be benchmarked on WIOD data, so they are only partially consistent with the original world IO table. For example, the taxes less subsidies on products, including import taxes, are zero in the USA (see the remark above), while in the resulting valuation layer they are non-zero and are unlikely to be offset by the net taxes on domestic products. Again, this is a problem inherent to sourcing the primary data from the national accounts. Statistical discrepancy terms may be introduced where necessary (below the row of value added) to balance the output in the world IO table.

The two valuation layers have been compiled as described above for the year 2010, and the WIOD industry-by-industry IO table for the same year was used to test the frameworks of this paper.

5.2. Results: value added and gross exports accounting

The computations using the value added accounting framework and the gross exports accounting framework result in a large array of data consisting of a series of matrices. The default dimension is KN×K (country-sector × country), and each matrix contains 58,835 data points. Each data point corresponds to a flow of value added or product from sector i of the exporting country r to the partner country s. There are various ways in which these data can be aggregated and visualised, and one familiar method involves deriving the relative VS and VS1 measures from the "value added in total trade" $\mathbf{V_c L E}_{tot}$ matrix. For a brief analysis in this subsection, we will extract as much information as possible in monetary terms from various matrices obtained and will focus on selected countries or country pairs. We will be able to infer the mode of country participation in global value chains while simultaneously accounting for the magnitude of the flows.

The choice of China and Russia is thought to serve our purpose well: China is often subject to similar analyses, given its remarkable export performance, while Russia represents another interesting case of the only large natural resource exporter covered by the WIOD database.

As a starting point, we will rank top ten export and import partners of China and Russia using four different measurement concepts and compare the results. In Figures 7-8, the leftmost chart (Figures 7.1 and 8.1) shows the direct gross exports $\mathbf{E}_{bil}$. Next (Figures 7.2 and 8.2) is the cumulative exports $\mathbf{E}_{cum}$, calculated with the equation (17) or (19). This chart treats partners as the final destinations of exported products, irrespective of how they reach these partners – directly or in the form of embodied inputs. The third chart (Figures 7.3 and 8.3) quantifies the value added that originates in the exporting country and reaches partners in whatever form and for whatever purpose. So, this is the domestic value added in gross exports, based on the $\mathbf{S}'_n \mathbf{V_c L E}_{bil}$ matrix. Finally, the fourth measurement concept (Figures 7.4 and 8.4) combines the previous two and captures the domestic value added that flows from the exporting country to partners only for partner final use ($\mathbf{S}'_n \mathbf{V_c L F}$ [14]). This is the net measurement while three previous concepts involve double counting of intermediates that cross national borders multiple times.

The change of measurement concept introduces very little variation in the list of China's top ten export partners. China's exports from the global value chain perspective is largely similar and proportional to its observed gross exports.

The same exercise to rank Russia's top ten export partners reveals a more intriguing result. The principal destination of Russia's direct gross exports is Italy. However, Italy is not the partner that finally absorbs the most of Russia's exports. This is the USA. But both USA and Italy lag slightly behind China which is the top destination for the value added of Russian origin. So, the three double-counting measurement concepts yield three different estimates, while the net measurement confirms that the most important destination where Russia's value added is finally absorbed is the USA.

[14] To account for a country indirect exports to itself, the diagonal blocks were taken from the $\mathbf{S}'_n \mathbf{V_c L \check{F}}$ matrix.

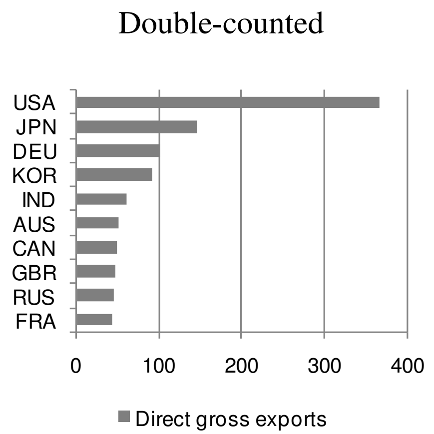

Figure 7.1: Top ten export partners of China: direct gross exports

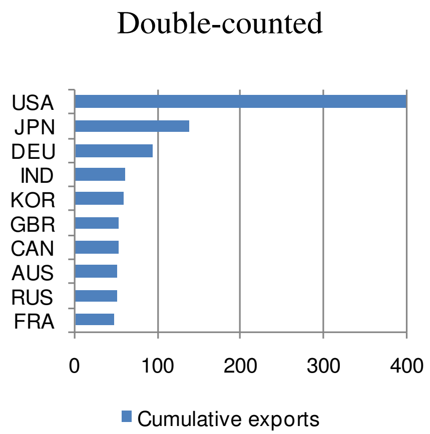

Figure 7.2: Top ten export partners of China: cumulative exports

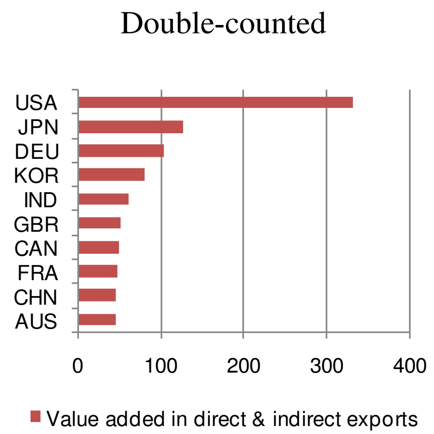

Figure 7.3: Top ten export partners of China: value added in direct and indirect gross exports

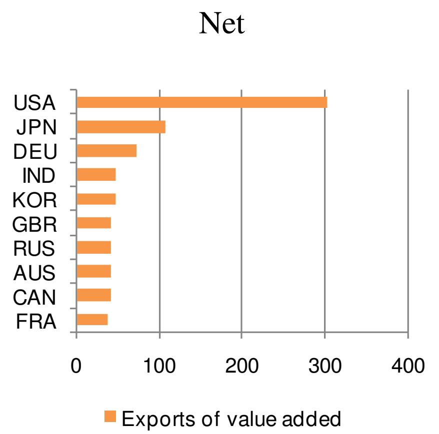

Figure 7.4: Top ten export partners of China: exports of value added

Figure 7: Top ten export partners of China in 2010 by four measurement concepts (US $ billion, Rest of the World not shown).
Source: WIOD database, author's calculations.

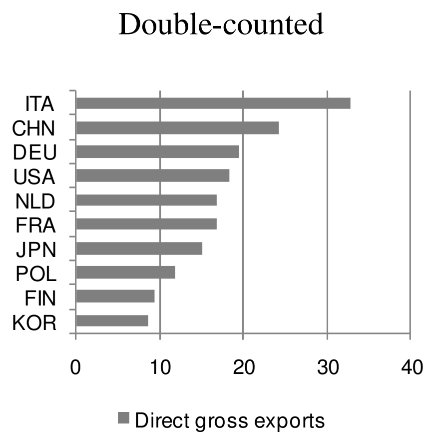

Figure 8.1: Top ten export partners of Russia: direct gross exports

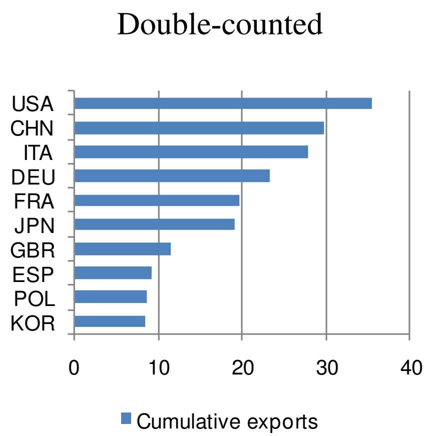

Figure 8.2: Top ten export partners of Russia: cumulative exports

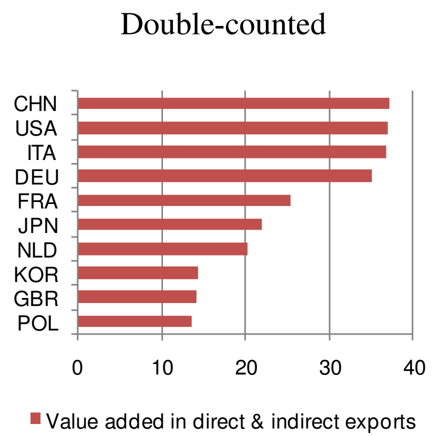

Figure 8.3: Top ten export partners of Russia: value added in direct and indirect gross exports

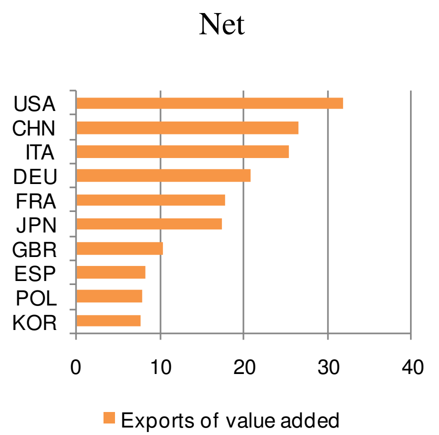

Figure 8.4: Top ten export partners of Russia: exports of value added

Figure 8: Top ten export partners of Russia in 2010 by four measurement concepts (US $ billion, Rest of the World not shown).
Source: WIOD database, author's calculations.

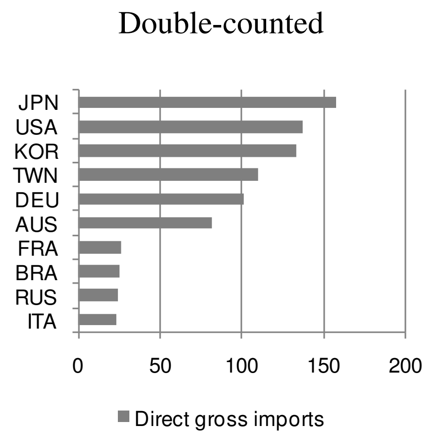

Figure 9.1: Top ten import partners of China: direct gross imports

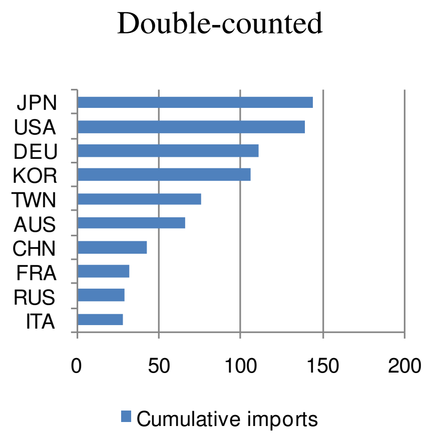

Figure 9.2: Top ten import partners of China: cumulative imports

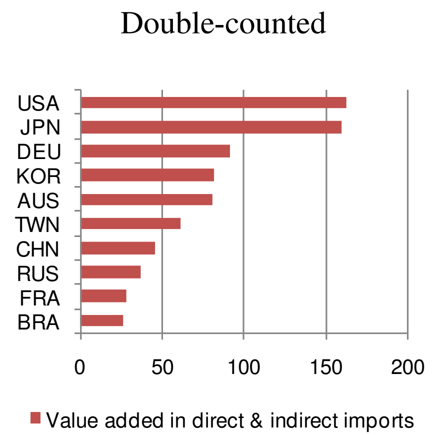

Figure 9.3: Top ten import partners of China: value added in direct and indirect gross imports

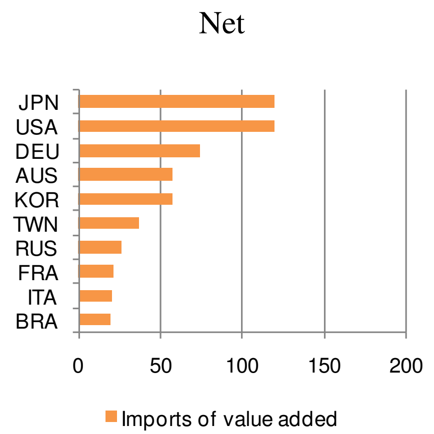

Figure 9.4: Top ten import partners of China: imports of value added

Figure 9: Top ten import partners of China in 2010 by four measurement concepts (US $ billion, Rest of the World not shown). Source: WIOD database, author's calculations.

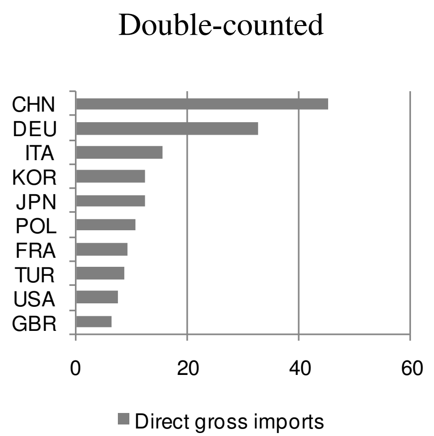

Figure 10.1: Top ten import partners of Russia: direct gross imports

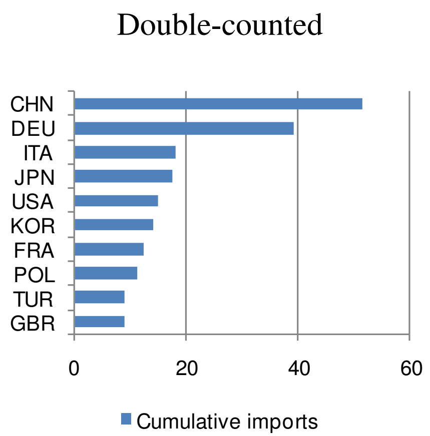

Figure 10.2: Top ten import partners of Russia: cumulative imports

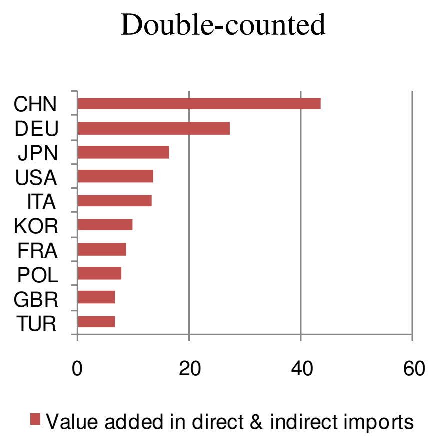

Figure 10.3: Top ten import partners of Russia: value added in direct and indirect gross imports

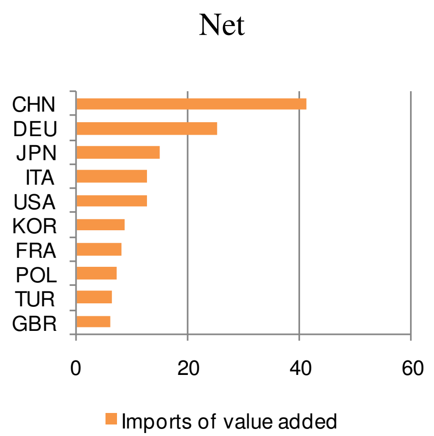

Figure 10.4: Top ten import partners of Russia: imports of value added

Figure 10: Top ten import partners of Russia in 2010 by four measurement concepts (US $ billion, Rest of the World not shown). Source: WIOD database, author's calculations.

Now, consider ranking top ten import partners in a similar way as shown in Figures 9-10. For China, the alteration from Figure 9.1 to Figure 9.3 is not dramatic but clearly visible. Germany rises to the third place while Korea and Taiwan descend by one or two positions as the source of products finally absorbed in China and the source of value added received in China. Noteworthy is that China emerges as the 7th most important source of imports for itself, surpassing France, Russia or Brazil. The net measurement, however, denies China a place within its own top ten importers.

Meanwhile, the order of Russia's top import partners experiences only minor shift from one measurement concept to another. Perhaps the most significant is the rise of the USA from 9th to 4th-5th position in the list.

From the above, we can infer that China is mostly integrated into global value chains via its imports, or the linkage backward to the upstream value chain. Russia's integration is that via exports, or the linkage forward to the downstream value chain. China is closer to the end of the value chain, largely serving the US market. Russia is at the beginning of the value chain, also serving the US market, but via a chain of intermediate partners most of which are apparently in Europe.

Countries predominantly integrated into the backward, upstream value chain tend to generate value added in trade less than their total gross exports, while the opposite is usually true for those countries that are predominantly integrated into the forward, downstream value chain. So, China's value added in total global exports is equal to 0.97 of its total gross exports, and this ratio for Russia is 1.43.

Another way to look at this feature is to contrast the three double-counting measurements of exports at the sector level. Figure 11 shows the distribution across all 41 WIOD countries of China's exports of electrical and optical equipment (WIOD sector c14, ISIC Rev.3 / NACE Rev.1 30+31+32+33) which accounts for 36.6% of China's total gross exports.[15] It's apparent that the final destination of exports largely conforms with their direct destination. Only Korea, Mexico, Taiwan and the Czech Republic consume significantly less electronic and optical equipment from China than they directly receive. They re-export a sizable portion of that equipment as embodied inputs. However, value added in exports generated by the electrical and optical equipment sector in China is much less than the direct or cumulative exports of the products thereof. There are two underlying reasons: first, the electrical and optical equipment made in China embodies foreign value added (~9% of this sector gross exports) and, second, it embodies value added from other domestic sectors (~65%). Figure 14 will explicitly show this.

[15] The full list of WIOD countries and sectors is in Appendix A.

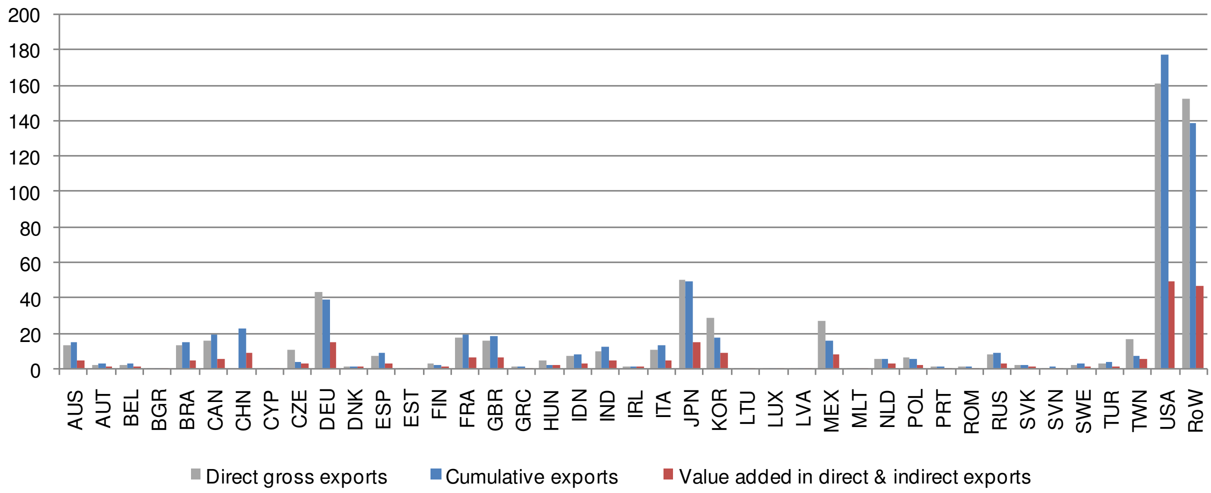

Figure 11: China's exports of "Electrical and Optical Equipment" (WIOD c14; ISIC Rev.3 / NACE Rev.1 30 to 33) by three measurement concepts in 2010, US $ billion.
Source: WIOD database, author's calculations.

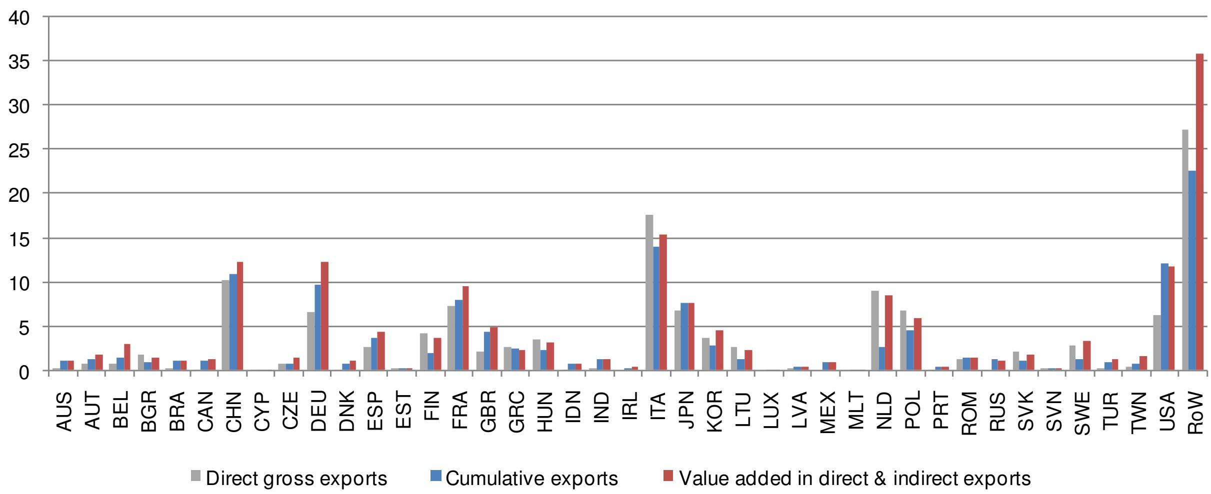

Figure 12: Russia's exports of "Mining and Quarrying" (WIOD c2; ISIC Rev.3 / NACE Rev.1 C or 10 to 14) by three measurement concepts in 2010, US $ billion.
Source: WIOD database, author's calculations.

The largest exporting sector in Russia is mining and quarrying (WIOD sector c2, ISIC Rev.3 / NACE Rev.1 C=10+11+12+13+14) that contributes 35.0% of total gross exports. As may be expected, the three measurements render an entirely different pattern, compared to China – see Figure 12. The variation between the exports to direct destination and the exports to final destination is significant, and such countries as the Netherlands, Sweden, Finland, Lithuania, Poland appear to be the important transit points for the products of Russia's mining and quarrying sector on their way along the value chain. The value added in exports generated by this sector in Russia in many cases exceeds direct bilateral exports and, in some cases, cumulative exports, too. Little foreign value added content (1.4%) and the intense circulation through third country exports explain this finding.

Pursuing our inquiry at the sector level, we will focus on China's exports to the USA and Russia's exports to Germany. This will exemplify the application of the accounting frameworks discussed so far.

Figure 13 is designed to partly explain the transition from direct to cumulative exports (from $\mathbf{E}_{bil}$ to $\mathbf{E}_{cum}$). The upper chart is the decomposition of direct bilateral exports (equation 15) where the red portions of the columns designate the exported intermediates that eventually leave the partner as embodied inputs to next destinations. Note that the production chains are confined to the national borders here.

The USA appear as predominantly the final market for China's exports, most of which are in the form of final products and a smaller part is intermediates eventually transformed into final products at destination, i.e. in the USA. The middle chart does not contain the re-exported term (in red) as it has been completely reallocated among the final destinations (equation 17), including all WIOD countries. Likewise, the re-exports of embodied intermediates from other countries may be finally reallocated to the USA, as is the case for many sectors. The middle chart for the China–USA pair bears a strong resemblance to the upper one, because the USA are not heavily involved in trading made-in-China intermediates with third countries. Finally, the lower chart brings direct and cumulative exports into comparison, without components thereof. It follows that the cumulative exports from China to the USA, after the infinite series of reallocations of re-exported intermediates, is a little larger than direct gross exports.

In a similar decomposition of Russia's exports to Germany, the re-exported term (in red) dominates the upper chart. Bilateral exports is largely driven by the products of mining and quarrying (c2) and basic metals (c12), of which 38% and 73%, respectively, are embodied in Germany's exports to next destinations. The middle chart unveils that basic metals of Russian origin eventually "leak" from Germany to other countries, while mining and quarrying products, on the contrary, accumulate in Germany for eventual final use. The intense inflows and outflows of made-in-Russia intermediates lead to a substantial reshuffle of Russia's cumulative exports to Germany, compared to direct gross exports – see the lower chart of Figure 13.

Figure 14 decomposes the same bilateral export flows using the value added accounting framework. In contrast to Figure 13, value chains here are treated as entirely global value chains, not confined to country borders. The columns represent the flows of value added from/to specific sectors that sum to bilateral gross exports. As indicated earlier, this should not be understood as the actual decomposition of gross exports. Rather, gross exports serve as a benchmark to handle and classify value added flows.

The upper chart in Figure 14 is the decomposition of value added in exports at origin (equation 5, dimension: country-sector by country, KN×K). The columns quantify the flows of value added from the sector of origin in China (left chart) or Russia (right chart) received, respectively, in the USA and Germany for aggregate use. The columns are split into various components according to either mode of delivery or use at destination, or both. So, China's exports to the USA in terms of domestic value added are confirmed to largely serve final domestic demand (marked in yellow). However, an important determinant of gross exports of many sectors in China is the net exports of other sectors value added (marked in blue). As has already been observed, electrical and optical equipment – the largest exporting sector in China – receives value added from other domestic sectors which equals ~65% of its gross exports worldwide. The same ratio applies to this sector exports to the USA. In case of textiles and machinery not elsewhere specified, the value added originating in domestic sectors other than the direct exporting sectors is estimated, respectively, at 51% and 47% of the bilateral gross exports to the US market. This is the way many sectors participate in the China–USA value chain indirectly via other sector exports, which is typical for China's agriculture, mining and quarrying and service sectors. So, the value added generated by agriculture in China and exported indirectly to the USA is 13 times more that the direct agricultural exports to the USA, and that of mining and quarrying is 55 times more. The foreign value added content of China's direct exports to the USA seems to be insignificant in this chart. Note, however, that aggregating across all sectors will cancel out the inter-sectoral term (blue), and the presence of foreign value added will become more pronounced. The indirect flows of value added from China's sectors to the USA via third countries (dark red) appear to be minimal. To sum up, China's exports to the USA rely on an intense, predominantly domestic value chain where intermediates are processed for final demand in the US market.

The lower chart is the decomposition of value added at destination (an extension of equation 14, dimension: country by country-sector, K×KN). It involves a change of perspective: whereas the upper chart focuses on the sectors that generate value added, the lower chart focuses on the sectors that deliver products where that value added is embodied. In other words, the upper chart disregards the sector-wise use of value added, and the lower chart disregards the sector origin of value added. The interpretation of the components is the same except for the inter-sectoral term (marked in blue), which now accounts for trading value added among the sectors at the partner side. Let's use the electrical and optical equipment again as an example. The value added generated by all China's sectors ending up in the domestically consumed electrical and optical equipment in the USA (yellow) is equal to 48% of its bilateral gross exports. The value added of the same origin found in the US exports of electrical and optical equipment (red) is 4%, and that in the US imports of electrical and optical equipment from third countries (dark red) is 11% thereof. Finally, 28% of bilateral gross exports are the value added of Chinese origin embodied in electrical and optical equipment and received indirectly in the USA from other US processing sectors. Foreign value added also accounts for 28% thereof. A more illustrative case is public administration and related services (WIOD sector c31). The direct exports of these services from China to the USA is zero. The value added generated in the public administration sector in China and embodied in the US aggregate demand is minimal (see the upper chart). However, the public administration services consumed domestically in the USA contain value added from China nearly equal to that embodied in textiles and textile products. This is almost entirely explained by the corresponding inter-sectoral transfer term that captures the value added originating in China and flowing to the public administration sector via other US domestic sectors.

A similar decomposition of Russia–Germany bilateral value added in trade signals that the respective value chain extends far beyond bilateral linkages. The value added generated by mining and quarrying in Russia that is used in Germany's exports is equal to 71% of bilateral gross exports of mining and quarrying products. The value added of the said origin that flows indirectly to Germany via third countries is almost identical to the corresponding gross exports, while the value added from other Russia's sectors is 12% and foreign value added is only 2%. So, the only sector where exports, before reaching Germany, absorb non-negligible amount of other domestic sector value added is basic metals. The overall domestic fraction of the value chain is small, while the indirect fraction linking Germany and third countries is significant.

Moving to the decomposition at destination reshuffles the picture again, as may naturally be expected, given the structure of Russia's exports. Now, the value added sourced from all Russian sectors and embodied in the domestically consumed mining and quarrying products in Germany amounts to 15% of their bilateral gross exports. Russia's value added provided by this sector in Germany to other domestic sectors is equal to 81% thereof. The case of transport equipment (c15) resembles that of public administration in the China–USA pair: negligible direct exports and non-negligible Russia's value added content of the made-in-Germany transport equipment for both domestic use and exports, sourced indirectly via other German sectors. Overall, the domestic processing chain at the partner side is significant, and so remain the indirect linkages to third countries.

The decompositions in Figures 13 and 14 are thought to provide a useful toolbox for an exhaustive analysis of bilateral production and trade linkages. This may well inform various policy deliberations by answering such questions as: why the partner demands given sector exports? are there significant indirect unobserved linkages? what happens before, or after, a product is exported from a given country to a partner? and so on.

Figure 13.1: Decomposition of China's exports to the US.

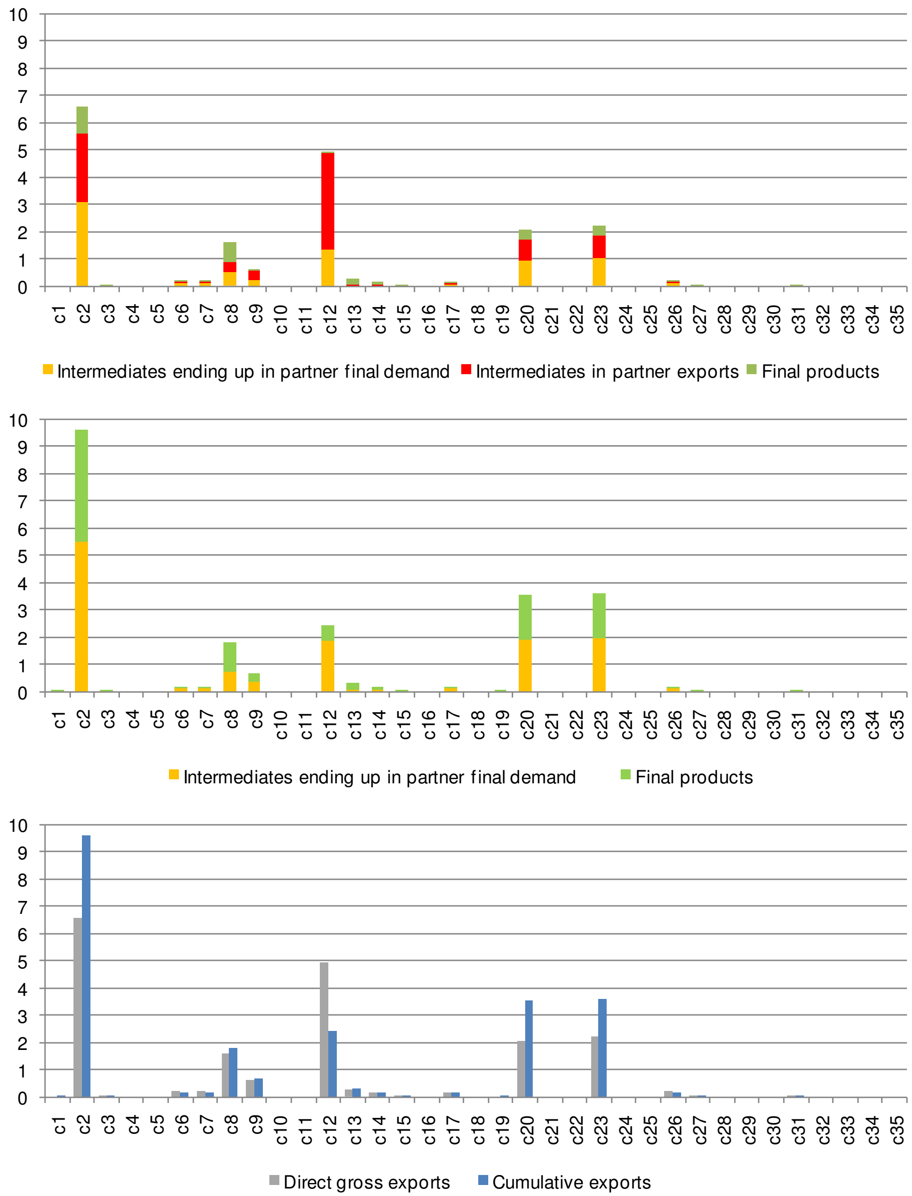

Figure 13.2: Decomposition of Russia's exports to Germany.

Figure 13: Bilateral trade decomposed with the gross exports accounting framework, 2010, US $ billion.
Source: WIOD database, author's calculations.

Figure 14.1: Decomposition of China's value added in exports to the US.

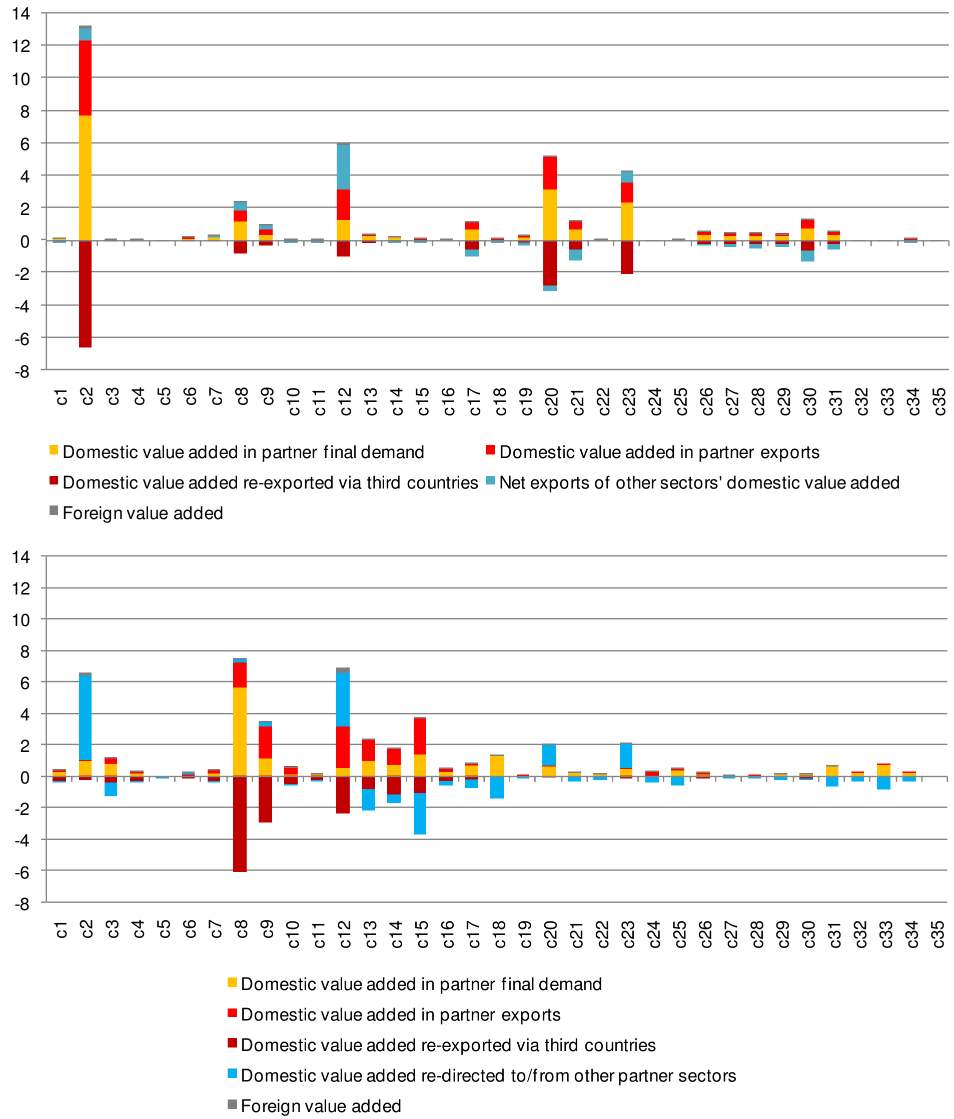

Figure 14.2: Decomposition of Russia's value added in exports to Germany.

Figure 14: Bilateral trade decomposed with the value added accounting framework, 2010, US $ billion. Source: WIOD database, author's calculations.

5.3. Results: trade cost accounting

In subsection 4.2, various techniques have been elaborated to show that trade costs may virtually flow through global value chains in largely the same way as value added. Here we will briefly document how these techniques work with only one valuation layer compiled in the WIOD table format for 2010 that is the import taxes at destination (import duties).

First, Figure 15 reports the results of the application of the Leontief price model. The columns quantify the multipliers that attribute a part of the equilibrium price of China's or Russia's sector output to the cumulative import tariffs paid on intermediate inputs (equation 22 or 23). Note that the measurement units on the vertical axis signify the percentage of the equilibrium price of total output not an import tariff rate. We will resort to individual sector examples again for the interpretation. Assume that the basic price of textiles and textile products made in China (for whatever use) is US$ 100; then 0.8 cents is the monetary value of all import tariffs incurred directly and indirectly throughout the production of those textiles. In case of Russia, it is 1.8 cents. Of US$ 100 which is the basic price of electrical and optical equipment made in China, 1.3 cents have to be paid as the cumulative import tariffs along the whole upstream value chain. In Russia, it only amounts to 0.6 cents. Service suppliers are also subject to tariffs because they may use a variety of imported intermediate goods for their production. Overall, the import tariff factor does not dramatically inflate the price of output in either China or Russia. At maximum, it account for 1.9 cents out of US$ 100 price of Russia's transport equipment.

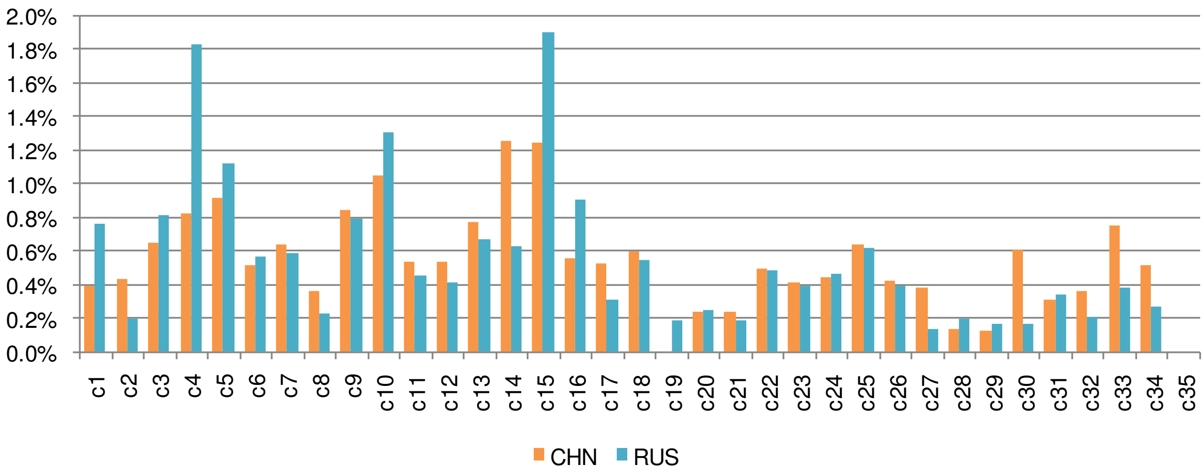

Figure 15: Import tariff multipliers from the Leontief price model, per cent of the equilibrium price of total output, 2010.

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations.

The tariff-price multipliers from Figure 15 are uniform for a given sector output, irrespective of its use. They account for both the importing country tariffs directly applied to imported intermediate inputs and the indirect tariffs applied by third countries at the previous production stages. A simple matrix manipulation[16] may split the last two components for analytical purpose. The indirect fraction of the import tariff multipliers appear mostly insignificant, but more pronounced in case of China, which is natural, given its integration into the preceding, upstream value chain. This means, for example, that, if China introduces a duty-free regime for all partners and all products, other things being equal (including quantities, as required by the Leontief price model), then 0.2 cents out of US$ 100 would still have to be paid to cover the import tariffs by third countries at the previous production stages.

Further, three measurements are available for bilateral import tariffs: direct tariffs paid on gross trade flows ($\mathbf{T}$), cumulative import tariffs as embodied inputs ($\mathbf{T}_{cum}$, equation 26) and incremental tariffs as import resistance term ($\mathbf{T}_{inc}$, equation 28). The last two measures account for global value chains from somewhat different perspectives as explained in subsection 4.2. The results of all three measurements for 2010 are summarised in Figure 16. Figure 16.1 reports the import tariffs applied, that are the import duties paid on all imports to WIOD countries as percentages of their total gross imports. The EU member countries apply very low import tariffs that do not exceed 1.7% in all measurement concepts. The cumulative tariffs are higher than the direct tariffs, and the incremental tariffs are in between the direct and cumulative tariffs for the European countries. Brazil, Korea and Russia stand out for the highest direct tariffs (6.4-6.9%), cumulative tariffs (6.9-7.6%) and incremental tariffs (7.8-9.6%).

Figure 16.2 reports the import tariffs faced, that are the import duties paid at destination on all exports of WIOD countries as percentages of their total gross exports. Now the incremental resistance term for the European exporters is consistently higher than other measurements, exceeding the directly measured import tariffs faced by nearly one percentage point in many cases. But overall, the import tariffs faced are still low for the EU members, reaching the maximum of 3.1% that is the incremental resistance to Italy's exports. Exports of Brazil, Japan, Korea, Indonesia and Turkey are subject to the incremental import tariffs of 5.1-6.6%, whereas the respective estimate of cumulative tariffs is at comparable level and that of direct tariffs is about one percentage point lower.

The estimates in Figures 16.1-16.2 suggest that the overall tariff protection and the related costs to exporters are low even with a due account of the multi-stage production. These estimates are in fact computed as the weighted implicit tariff rates, so they disguise the variation among

[16] This requires splitting the vector of import tariff coefficients $\mathbf{m}(\tau)_{\mathbf{c(Z)}}$ into two vectors, of which the first only contains entries for China (Russia), and another does so for the remaining WIOD countries.

sectors that is worth exploring. Moreover, it is difficult to develop preference for any single measure of tariffs consistent with global value chains from these visualisation exercises.

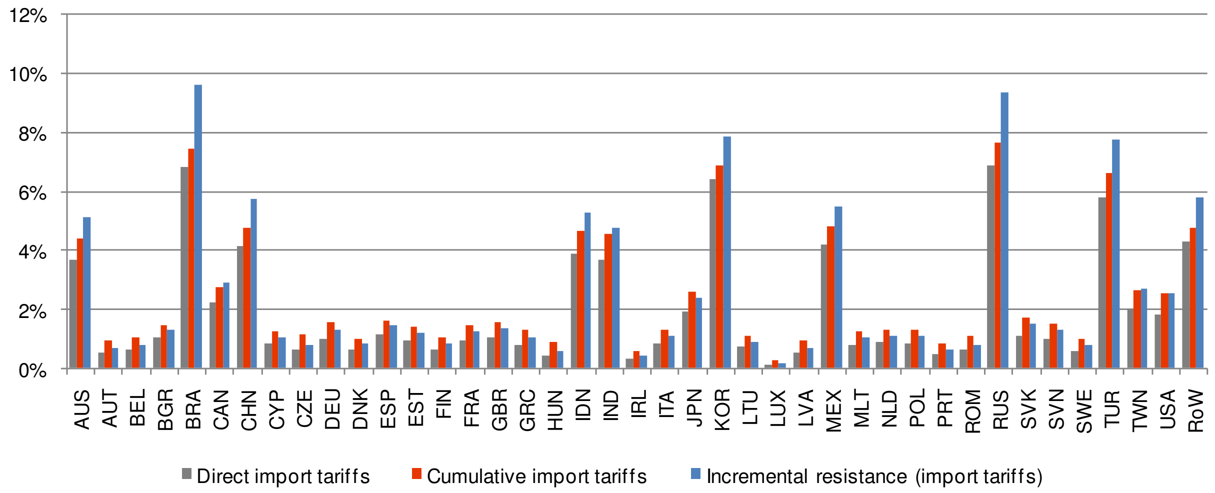

Figure 16.1: Import tariffs applied by importing country.

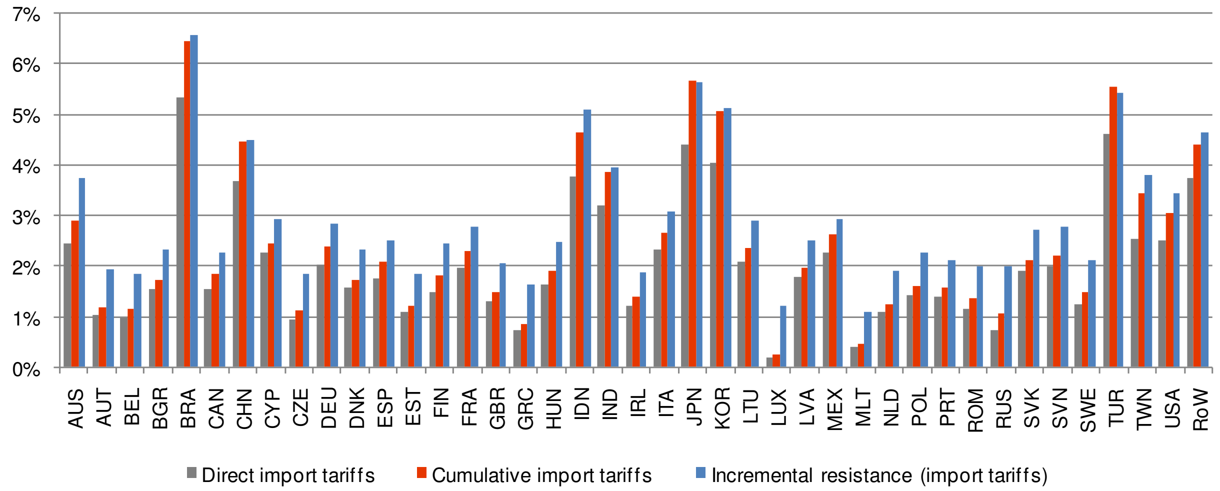

Figure 16.2: Import tariffs faced by exporting country.

Figure 16: Import tariffs by importing and exporting country and by three measurement concepts, 2010, per cent of gross bilateral exports.
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations.

We will therefore focus again on China's (Russia's) total exports and bilateral exports to the USA (Germany) by sector and explore the tariffs faced. As Figure 17.1 suggests, though the principal China's exporting sector – electrical and optical equipment – faces relatively low tariffs by all measurements (1.6-2.3%), other sectors with non-negligible exports such as textile and textile products, leather and footwear, food and beverages are subject to the import tariff rates of

10-12%. Tariff rates are the highest for the products of agriculture exported from China – 15.2% direct and 16.0-16.8% cumulative/incremental. The difference between the cumulative or incremental and the direct measurements appear to be small. In line with the outcome of the value added decompositions in the previous subsection, we can infer that the reason is the weak downstream value chain extending from China forward to partners.

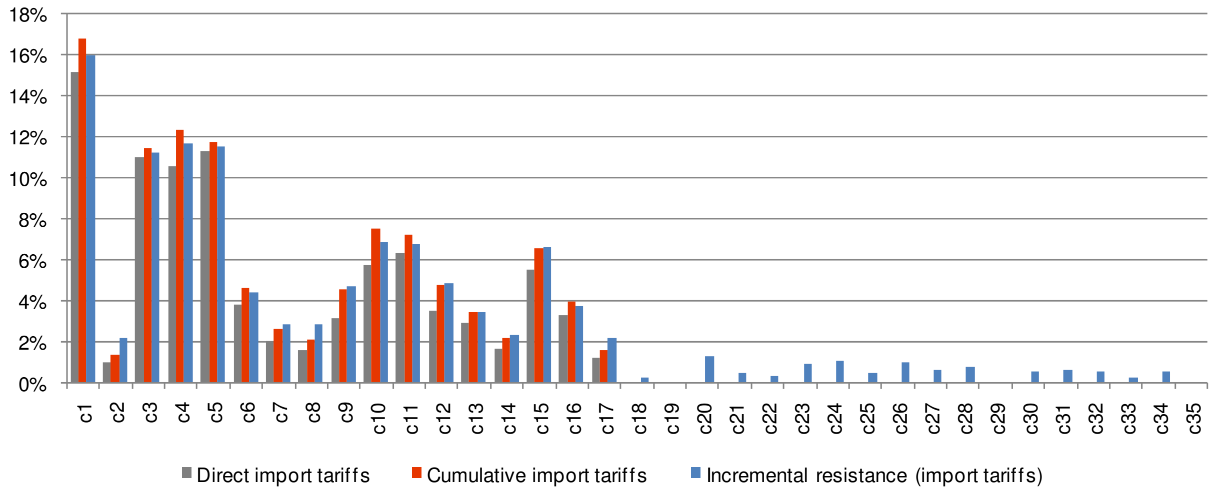

Figure 17.1: Import tariffs faced by China's exports worldwide.

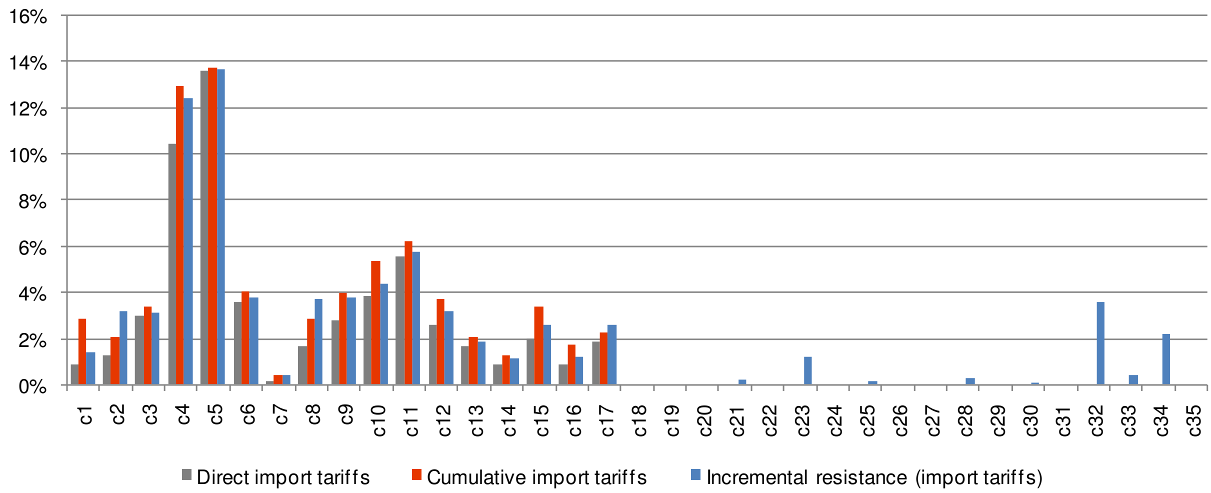

Figure 17.2: Import tariffs faced by China's exports in the USA.

Figure 17: Import tariffs faced by China's exports by three measurement concepts, 2010, per cent of gross bilateral exports.

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations.

Note: some results for services have been suppressed because of division by zero (no direct gross exports).

Figure 17.2 reports the same measurements for China's bilateral exports to the USA. The product structure of bilateral trade seems to influence the results, reducing the estimates for some sectors (agriculture) and raising them for the other (leather and footwear). Interestingly, the education services exported from China are incrementally taxed 0.5% worldwide and 3.6% in the USA.

The comparison of the three measurements of import tariffs applicable to Russia's exports in Figure 18 is perhaps more informative. Whereas the incremental measurement may be more or less than the cumulative measurement of import tariffs applied worldwide on Russia's goods, it is consistently lower in case of Germany. This signifies that Germany receives more embodied import tariffs payable on Russia's goods along the preceding value chain than it incrementally charges on embodied inputs of Russian origin. The difference between the cumulative and incremental tariffs arises because of a combination of factors, including the structure of import tariffs and imports, hence the weighting and aggregation scheme in the IO framework, and, not least, the relative position of exporter and partner within global value chains. Consider chemicals and chemical products: the direct import tariff in Germany is 2.3%, the cumulative tariff is 5.7%, but the incremental tariff is 3.8%. Interpret this is follows: 5.7% of the bilateral gross imports of chemicals from Russia is the cumulative value of import tariffs paid in Germany and elsewhere on made-in-Russia chemicals, embodied in any products entering Germany; 3.8% is the total value of import tariffs charged in Germany on made-in-Russia chemicals that enter Germany directly or as intermediate inputs embodied in the products of any exporting country. So the incremental term treats Germany as the country that is indeed responsible for charging import tariffs whereas the cumulative term does not. The latter treats Germany as only the recipient of embodied tariffs that have been charged anywhere along the upstream value chain.

The above consideration – consistency with the notion of partner responsibility to charge import tariffs – is in favour of prioritising the incremental tariff/resistance term for quantifying trade costs within global value chains. Another consideration is that the incremental measure is the one properly handling tariffs on services. The direct and cumulative import tariff measurements cannot account for tariffs incurred in the supply of services.[17] The Rouzet and Miroudot (2013) version of cumulative tariffs accounts for all import tariffs incurred in trading all intermediate inputs along the preceding service value chain. It is capable of counting tariffs on services because goods are embodied in those services, but is indifferent to the sector of origin of or the country responsible for taxing the intermediate flows of those goods. The advantage of the incremental resistance term is that it captures import tariffs on services that apply because services are embodied in goods. It is also capable of counting the tariffs incrementally paid at the border of each partner country.

[17] Unless the output of some service sectors includes goods subject to transport margins and tariffs, as a result of the application of the Eurostat Model D for the SUT-IOT transformation.

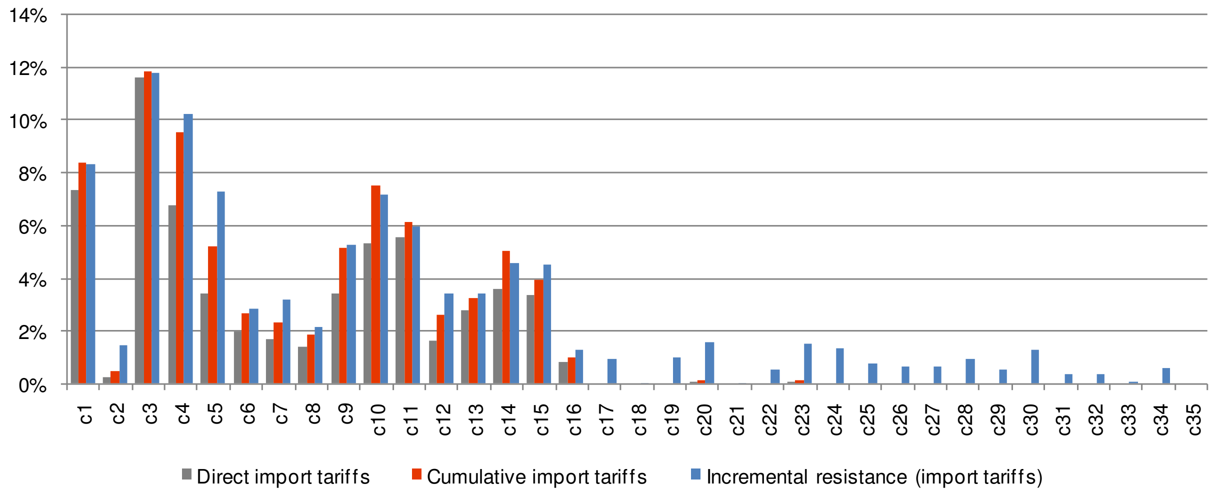

Figure 18.1: Import tariffs faced by Russia's exports worldwide.

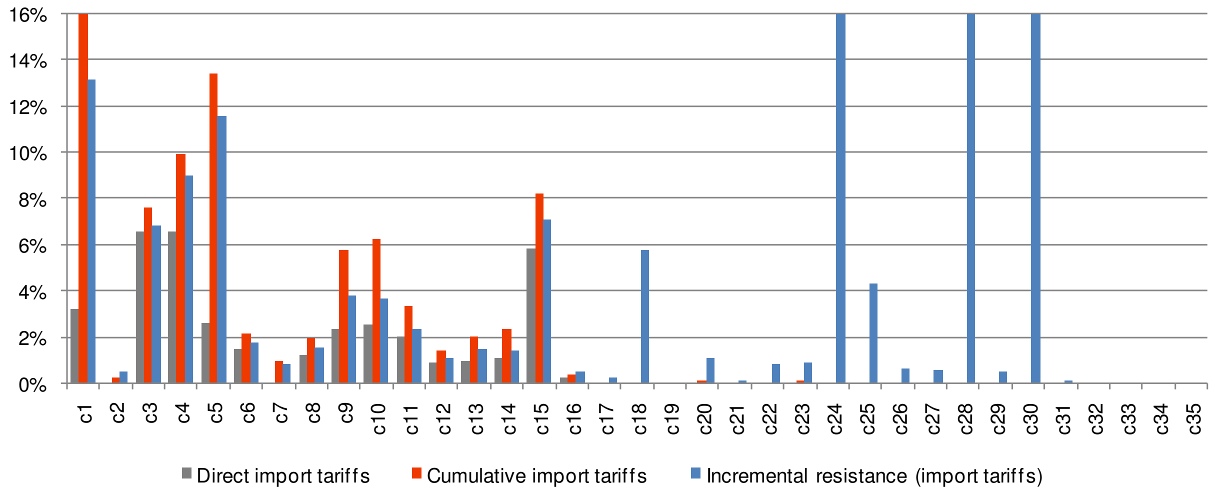

Figure 18.2: Import tariffs faced by Russia's exports to Germany.

Figure 18: Import tariffs faced by Russia's exports by three measurement concepts, 2010, per cent of gross bilateral exports.

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations.

Note: some results for services have been suppressed because of division by zero (no direct gross exports).

Finally, we map all three measurements of bilateral tariffs using a simple visualisation of data that appears in the aggregated form in Figure 16. The WIOD countries in Figures 19.1-19.3 are rearranged to provide an additional regional dimension of value chains. The maps are self-explanatory, and the transition from the direct to incremental measurement of import tariffs erodes the green cells that correspond to free trade in terms of import tariffs and highlights the red cells that correspond to higher tariffs. Noteworthy is that the intra-EU trade is not entirely free of import taxation with the incremental measurement. For example, the exports of Greece are indirectly subject to import tariffs in Spain, France, the Netherlands and other EU partners because these partners treat Greek inputs as non-European products. As a result, Greece indirectly faces resistance from its EU partners.

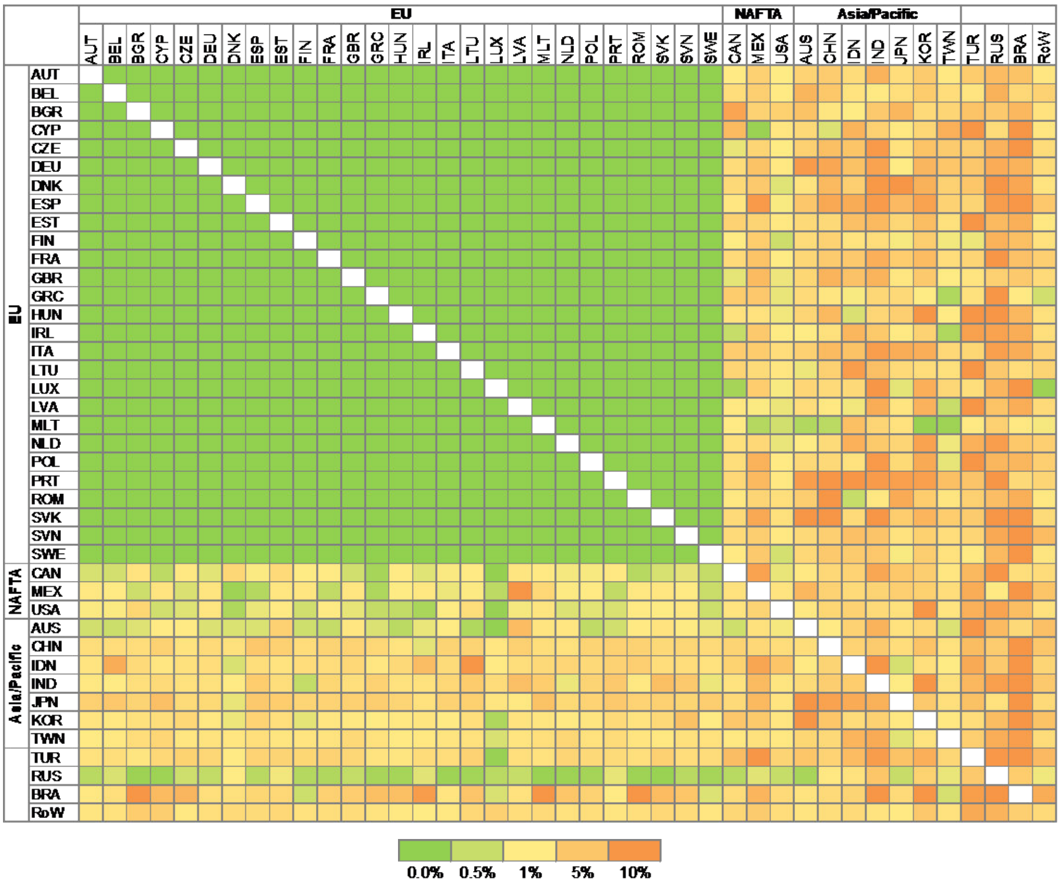

Figure 19.1: Direct bilateral import tariffs.

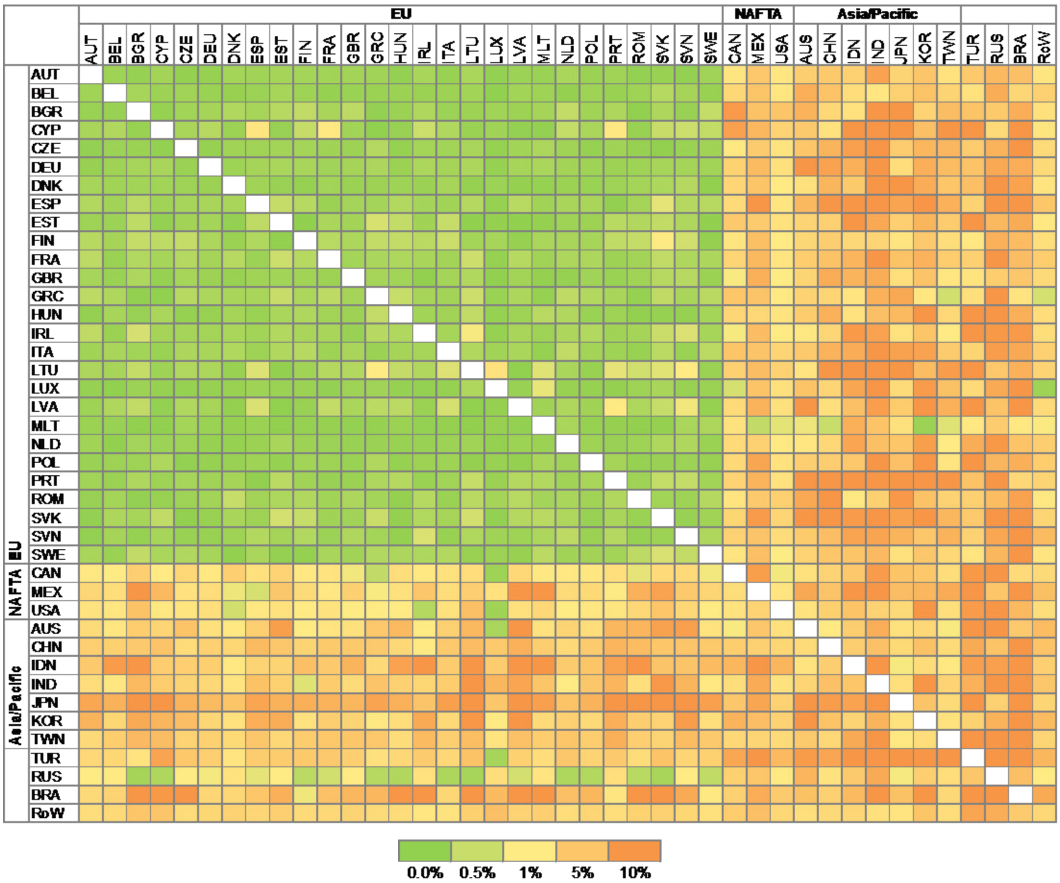

Figure 19.2: Cumulative (embodied) bilateral import tariffs.

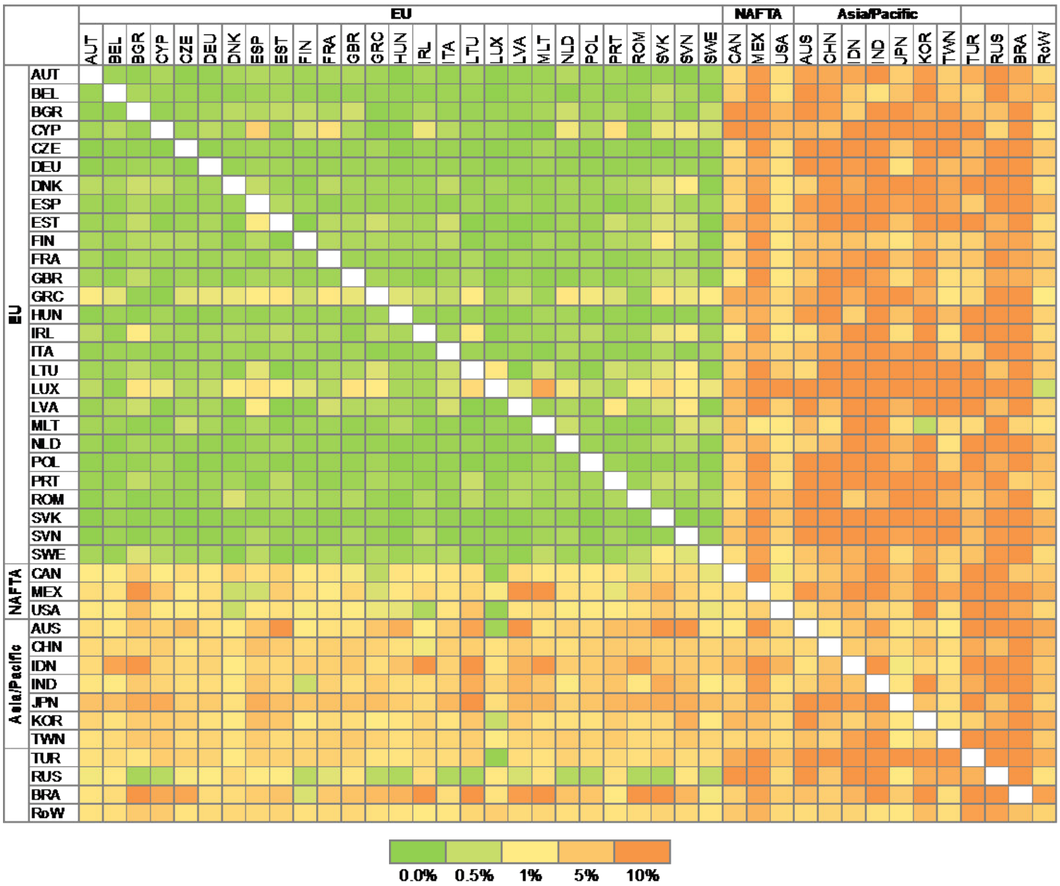

Figure 19.3: Incremental bilateral import tariffs (incremental resistance term).

Figure 19: Visualisation of bilateral import tariffs by regions, 2010, per cent of gross bilateral exports.

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations.

6. Conclusion

Global inter-country IO tables are experimental datasets that extend the frontiers of the analysis of production and trade in a globalised economy. In largely the same way the national IO tables take full account of the inter-industry dependences within a single economy, global IO tables model the global production chain. As such, they provide essential information to estimate the inter-country flows of intermediate or primary inputs embodied in traded products that are invisible to the conventional gross trade statistics. To produce these estimates, trade and input-output economists proposed accounting frameworks that motivated the research efforts described above.

This paper argues and explains why it is unlikely to derive a unified accounting framework for all types of global value chain trade decompositions. It therefore suggests classifying the accounting frameworks into two types: value added accounting frameworks and gross exports accounting frameworks. The frameworks of the first type distill value added from gross trade flows, and the decomposition traces external demand backwards to the sector that contributes value added at the country of origin. The frameworks of the second type discern the eventual flows of exported products, and the decomposition traces sectoral gross exports forward to the country of ultimate destination.

Given this conceptual delineation, the paper has generalised the accounting frameworks of both types, simultaneously addressing the computational complexity. The result for the value added accounting framework includes a matrix of bilateral value added flows that can be decomposed into the sum of various matrices classified by the mode of delivery or the use at destination or both. This confirms the earlier results of Koopman et al. (2012) and Stehrer (2013) who handled bilateral or total exports individually at country level. The core product of the generalised gross exports accounting framework is a matrix of cumulative bilateral exports, i.e. gross exports that eventually reach their final destination in whatever form. There are at least two complementary ways to compute this matrix.

All computations need to be performed in the full block matrix environment and require simple matrix operations. This saves the computational resources, ensures the ease of aggregation and, not least, leads to the derivation of two matrices of the inter-sectoral flows of value added. These matrices explain the differences between the value added flows and the product flows for a particular sector at either origin or destination, i.e. account for the domestic value chain before or after exports take place. The single-flow decompositions as in the previous studies do not explicitly capture these terms.

Another contribution of this paper includes a generalised technique for the trade cost accounting in global value chain trade. Three accounting methods have been discussed and clear preference has been developed for the one based on the gross exports accounting framework. In case of import tariffs, the related incremental resistance term counts all import tariffs applied by direct and indirect partners to a sector exports on its way to the final destination along the downstream value chain.

The generalised value added and gross exports accounting frameworks have been exposed to numerical tests with the WIOD data for 2010.

This paper is thought to contribute to the ongoing research in global value chains by refining two accounting frameworks that allow to quickly translate the global inter-country IO tables into a wealth of indicators and visualisations. The value added accounting framework best suits the analysis which requires establishing a link between the use of a product at destination and the primary inputs (value added, employment, fixed capital) at origin, e.g. factor content, productivity, competitiveness. The gross exports accounting framework may be essential for the analysis which requires accounting for sequential border crossing, e.g. trade policy. However, it is perhaps the joint application of both frameworks that may best inform the user.

Finally, one must take note that any accounting framework is only the computational algorithm, and the results will bear the peculiarities inherent to the construction of the IO tables. The experience throughout writing this paper, in particular the trade cost accounting section, suggests the following room for improvements of the existing and upcoming IO datasets: (A) re-price international trade flows into the basic price of the exporting countries and compile the full set of at least six valuation layers as shown in Figure 6; (B) provide access to the underlying SUT with valuation layers so that the users may derive IO table in alternative product-by-product format which is thought to be more convenient for the trade cost analysis;[18] (C) increase sector resolution.

[18] Eurostat model D that is often use to obtain the industry-by-industry IO tables, is not entirely consistent with the product homogeneity requirement, because one industry may still produce more than one product (see Eurostat, 2008, p.317).

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Appendix A: Countries and industries in the WIOD database

Table A.1: List of countries in the WIOD database.

Country code	Country	Country code	Country
AUS	Australia	IRL	Ireland
AUT	Austria	ITA	Italy
BEL	Belgium	JPN	Japan
BGR	Bulgaria	KOR	Korea
BRA	Brazil	LTU	Lithuania
CAN	Canada	LUX	Luxembourg
CHN	China	LVA	Latvia
CYP	Cyprus	MEX	Mexico
CZE	Czech Republic	MLT	Malta
DEU	Germany	NLD	Netherlands
DNK	Denmark	POL	Poland
ESP	Spain	PRT	Portugal
EST	Estonia	ROM	Romania
FIN	Finland	RUS	Russian Federation
FRA	France	SVK	Slovak Republic
GBR	United Kingdom	SVN	Slovenia
GRC	Greece	SWE	Sweden
HUN	Hungary	TUR	Turkey
IDN	Indonesia	TWN	Chinese Taipei
IND	India	USA	United States
		RoW	Rest of the World

Source: Dietzenbacher et al., 2013; http://www.wiod.org.

Table A.2: List of industries in the WIOD database.

WIOD code	NACE Rev.1 / ISIC Rev.3	Industry
c1	A – B	Agriculture, Hunting, Forestry and Fishing
c2	C	Mining and Quarrying
c3	15 – 16	Food, Beverages and Tobacco
c4	17 – 18	Textiles and Textile Products
c5	19	Leather, Leather and Footwear
c6	20	Wood and Products of Wood and Cork
c7	21 – 22	Pulp, Paper, Paper , Printing and Publishing
c8	23	Coke, Refined Petroleum and Nuclear Fuel
c9	24	Chemicals and Chemical Products
c10	25	Rubber and Plastics
c11	26	Other Non-Metallic Mineral
c12	27 – 28	Basic Metals and Fabricated Metal
c13	29	Machinery, Nec
c14	30 – 33	Electrical and Optical Equipment
c15	34 – 35	Transport Equipment
c16	36 – 37	Manufacturing, Nec; Recycling
c17	E	Electricity, Gas and Water Supply

c18	F	Construction
c19	50	Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel
c20	51	Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles
c21	52	Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods
c22	H	Hotels and Restaurants
c23	60	Inland Transport
c24	61	Water Transport
c25	62	Air Transport
c26	63	Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies
c27	64	Post and Telecommunications
c28	J	Financial Intermediation
c29	70	Real Estate Activities
c30	71 – 74	Renting of M&Eq and Other Business Activities
c31	L	Public Admin and Defence; Compulsory Social Security
c32	M	Education
c33	N	Health and Social Work
c34	O	Other Community, Social and Personal Services
c35	P	Private Households with Employed Persons

Source: Dietzenbacher et al., 2013; http://www.wiod.org.

Appendix B: "Zoom in" view on block matrices

For illustrative example, take the matrix of domestic value added that is re-directed to partner via third countries (indirect bilateral "domestic value added in trade" matrix) $\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]$. This matrix appears in equation (9), a decomposition of value added flows benchmarked against gross bilateral exports.

Zoom in the matrix, with the resolution at the block level:

$$\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]=\begin{bmatrix} \mathbf{0} & \mathbf{V_{c,1}}\sum_{\mathbf{t\neq 1}}^{\mathbf{K}}\mathbf{L_{1t}e_{t2}} & \cdots & \mathbf{V_{c,1}}\sum_{\mathbf{t\neq 1}}^{\mathbf{K}}\mathbf{L_{1t}e_{tk}} \\ \mathbf{V_{c,2}}\sum_{\mathbf{t\neq 2}}^{\mathbf{K}}\mathbf{L_{2t}e_{t1}} & \mathbf{0} & \cdots & \mathbf{V_{c,2}}\sum_{\mathbf{t\neq 2}}^{\mathbf{K}}\mathbf{L_{2t}e_{tk}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V_{c,k}}\sum_{\mathbf{t\neq k}}^{\mathbf{K}}\mathbf{L_{kt}e_{t1}} & \mathbf{V_{c,k}}\sum_{\mathbf{t\neq k}}^{\mathbf{K}}\mathbf{L_{kt}e_{t2}} & \cdots & \mathbf{0} \end{bmatrix}$$

Zoom in a single (off-diagonal) block, with the resolution at the individual element level:

$$\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]_{rs}=\begin{bmatrix} v_{c,r}^{1} & 0 & \cdots & 0 \\ 0 & v_{c,r}^{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_{c,r}^{n} \end{bmatrix}\sum_{t\neq r}^{K}\begin{bmatrix} l_{rt}^{11} & l_{rt}^{12} & \cdots & l_{rt}^{1n} \\ l_{rt}^{21} & l_{rt}^{22} & \cdots & l_{rt}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ l_{rt}^{n1} & l_{rt}^{n2} & \cdots & l_{rt}^{nn} \end{bmatrix}\begin{bmatrix} e_{ts}^{1} \\ e_{ts}^{2} \\ \vdots \\ e_{ts}^{n} \end{bmatrix}=\sum_{t\neq r}^{K}\begin{bmatrix} \sum_{u=1}^{N} v_{c,r}^{1} l_{rt}^{1u} e_{ts}^{u} \\ \sum_{u=1}^{N} v_{c,r}^{2} l_{rt}^{2u} e_{ts}^{u} \\ \vdots \\ \sum_{u=1}^{N} v_{c,r}^{n} l_{rt}^{nu} e_{ts}^{u} \end{bmatrix}$$

In this "zoom in" view, the lower index designates countries and the upper index designates industries. It's apparent that each entry in this block describes the value added that originates in one industry of the exporting country and flows to the partner country as part of all products via any third countries and any intermediary industries.

In the extended value accounting framework, this matrix is as follows:

$$\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{E}}_{bil(KN\times KN)}\right]=\begin{bmatrix}\mathbf{0} & \mathbf{V_{c,1}}\sum_{t\neq 1}^{K}\mathbf{L_{1t}}\hat{\mathbf{e}}_{t2} & \cdots & \mathbf{V_{c,1}}\sum_{t\neq 1}^{K}\mathbf{L_{1t}}\hat{\mathbf{e}}_{tk}\\ \mathbf{V_{c,2}}\sum_{t\neq 2}^{K}\mathbf{L_{2t}}\hat{\mathbf{e}}_{t1} & \mathbf{0} & \cdots & \mathbf{V_{c,2}}\sum_{t\neq 2}^{K}\mathbf{L_{2t}}\hat{\mathbf{e}}_{tk}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{V_{c,k}}\sum_{t\neq k}^{K}\mathbf{L_{kt}}\hat{\mathbf{e}}_{t1} & \mathbf{V_{c,k}}\sum_{t\neq k}^{K}\mathbf{L_{kt}}\hat{\mathbf{e}}_{t2} & \cdots & \mathbf{0}\end{bmatrix}$$

Then zoom in a single block:

$$\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{E}}_{bil(KN\times KN)}\right]_{rs}=\begin{bmatrix}v_{c,r}^{1} & 0 & \cdots & 0\\ 0 & v_{c,r}^{2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & v_{c,r}^{n}\end{bmatrix}\sum_{t\neq r}^{K}\begin{bmatrix}l_{rt}^{11} & l_{rt}^{12} & \cdots & l_{rt}^{1n}\\ l_{rt}^{21} & l_{rt}^{22} & \cdots & l_{rt}^{2n}\\ \vdots & \vdots & \ddots & \vdots\\ l_{rt}^{n1} & l_{rt}^{n2} & \cdots & l_{rt}^{nn}\end{bmatrix}\begin{bmatrix}e_{ts}^{1} & 0 & \cdots & 0\\ 0 & e_{ts}^{2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & e_{ts}^{n}\end{bmatrix}=$$

$$\sum_{t\neq r}^{K}\begin{bmatrix}v_{c,r}^{1}l_{rt}^{11}e_{ts}^{1} & v_{c,r}^{1}l_{rt}^{12}e_{ts}^{2} & \cdots & v_{c,r}^{1}l_{rt}^{1n}e_{ts}^{n}\\ v_{c,r}^{2}l_{rt}^{21}e_{ts}^{1} & v_{c,r}^{2}l_{rt}^{22}e_{ts}^{2} & \cdots & v_{c,r}^{2}l_{rt}^{2n}e_{ts}^{n}\\ \vdots & \vdots & \ddots & \vdots\\ v_{c,r}^{n}l_{rt}^{n1}e_{ts}^{1} & v_{c,r}^{n}l_{rt}^{n2}e_{ts}^{2} & \cdots & v_{c,r}^{n}l_{rt}^{nn}e_{ts}^{n}\end{bmatrix}$$

This matrix in the country-sector by country-sector (KN×K) dimension does not sum the rows and is therefore capable to differentiate among the products received by partner. Now, each entry in this block describes the value added that originates in one industry of the exporting country and flows to the partner country as part of the product of the same or another industry via any third countries and any intermediary industries.

Similar "zoom in" views with the resolution at the level of blocks and elements can be generated for any matrix that appears in the value added or gross accounting frameworks, though the interpretation will not always be as intuitive as in the example above.