Gravity in a world of global value chains

The international input-output structure as a determinant of bilateral trade

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Abstract: The gravity model has been the workhorse of trade analysis in the past 50 years. As an analogy to Newtonian physics, the model relates bilateral trade to the product of economic mass (the sales and expenditures in each country) and the inverse of the square of the distance separating them (a proxy for trade frictions). The model started has an empirical relationship but was then given solid theoretical micro-foundations. For many years, researchers have estimated the gravity equation with trade flows in gross terms and GDP figures as a proxy for "economic mass". This is only recently with the new literature on global value chains (GVCs) and trade in value added that it was pointed out that both the left side and the right side of the gravity equation should be in gross terms (or in value added terms).

In addition, the model itself may no longer provide a correct assessment of the determinants of bilateral trade. When trade is not limited to final goods but includes many intermediate products, bilateral trade is also a function of the economic mass of third countries and the trade frictions between these third countries and other countries through which inputs may travel before reaching final consumers. A first attempt at deriving a gravity equation fully incorporating the global value chain can be found in Noguera (2012). In addition to the traditional variables of the model, the estimation of a value-added gravity equation requires to know all the input-output relationships between the trading economies and their partners, as well as the partners of their partners. It can only be achieved with an international input-output table.

Against this backdrop, the paper compares estimates of gravity equations in gross terms and value-added terms and assesses to what extent the analysis of trade now requires a global input-output table. Using the WIOD dataset, it discusses the key parameters that have to be derived from input-output analysis and the bias introduced in trade analysis when omitting to take into account the input-output structure. The results differ across countries based on their size and their involvement in global value chains. But especially for small open economies, a significant share of bilateral trade is not explained by bilateral trade frictions or the economic mass of their partners. Third countries sometimes matter more to explain the volume of their trade. This result has important implications for trade policy.

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The gravity model has been described as the workhorse of the analysis of bilateral trade (Anderson, 2011; Head and Mayer, 2014). Since it was introduced by Tinbergen (1962), it has been very successful in explaining observed trade flows. More recently, the model received a solid micro-foundation (Anderson, 1979; Anderson and van Wincoop, 2003) and is now theoretically sound in what is called structural gravity (Anderson, 2011; Head and Mayer 2014). The basic intuition of the model, by analogy with Newtonian physics, is that bilateral trade can be approximated by the product of economic mass (the sales and expenditures in each country) and the inverse of the square of the distance separating the two trading economies (a proxy for trade frictions).

For many years, researchers have estimated the gravity equation with trade flows in gross terms and GDP figures as a proxy for "economic mass". This is only recently with the new literature on global value chains (GVCs) and trade in value added that it was pointed out that both the left side and the right side of the gravity equation should be in gross terms or in value added terms (Baldwin and Taglioni, 2011). When trade in intermediate inputs is pervasive, bilateral flows in value-added and gross terms can significantly differ (Johnson and Noguera, 2012; Koopman, Wang and Wei, 2014).

In addition, the model itself may no longer provide a correct assessment of the determinants of bilateral trade. When trade is not limited to final goods but includes many intermediate products, bilateral trade is also a function of the economic mass of third countries and the trade frictions between these third countries and other countries through which inputs may travel before reaching final consumers. A first attempt at deriving a gravity equation fully incorporating the global value chain can be found in Noguera (2012). In addition to the traditional variables of the model, the estimation of a value-added gravity equation requires to know all the input-output relationships between the trading economies and their partners, as well as the partners of their partners. It can only be achieved with an international input-output table.

1. The basic gravity equation and a description of the data used in the paper

The starting point of our analysis is the structural gravity equation derived by Anderson. The structural gravity model explains bilateral trade at user prices as a function of the expenditures in the importing country and sales in the exporting economy expressed as a share of world output (the frictionless value of trade) and a variable bilateral trade cost affected by trade costs with other partners (the distortion in trade induced by trade costs).

Assuming identical preferences or technologies across countries (i.e. a globally common constant elasticity of substitution σ^k across varieties of products *k*), the structural gravity equation is:

$$X_{ij}^k = \frac{Y_i^k . Y_j^k}{Y^k} \left(\frac{t_{ij}^k}{P_j^k . \Pi_i^k} \right)^{1-\sigma^k} \tag{1}$$

where X_{ij}^k is the value of exports of product *k* from country *i* to country *j* at destination prices, Y_i^k the sales of product *k* (to all destinations, at destination prices) in country *i*, Y_j^k the expenditures on product *k* in country *j* (from all origins), and Y^k is world output of product *k* (the sum of all sales/expenditures in all countries) . $t_{ij}^k \geq 1$ is a variable bilateral trade cost between country *i* and country *j* for product *k* but bilateral trade is also affected by trade costs with other partners, summarised in two multilateral resistance terms:

$$\left(\Pi_i^k\right)^{1-\sigma^k} = \sum_j \left(\frac{t_{ij}^k}{P_j^k}\right)^{1-\sigma^k} \frac{Y_j^k}{Y^k} \quad (2)$$

$$\left(\mathrm{P}_j^k\right)^{1-\sigma^k} = \sum_i \left(\frac{t_{ij}^k}{\Pi_i^k}\right)^{1-\sigma^k} \frac{Y_i^k}{Y^k} \quad (3)$$

Π_i^k is the outward multilateral resistance and aggregates the incidence of all bilateral trade costs borne by the producers of product *k* in country *i*. P_j^k is the inward multilateral resistance and accounts for the incidence of all bilateral trade costs on buyers of product *k* in country *j*. These two multilateral resistance terms account for the fact that it is relative prices, and thus relative trade costs, that matter for the determination of the global pattern of trade and production. However, they are unfortunately not directly observable.

The usual trick in the literature is to rely on fixed effects to account for multilateral resistance without having to estimate the variables described in equations (2) and (3). The estimation of the gravity equation in its linear form then becomes:

$$lnX_{ij} = lnY_i + lnY_j - lnY^w - lnP_j^{1-\sigma} - ln\Pi_i^{1-\sigma} + \beta_1 lndist_{ij} + \beta_2 contig_{ij} + \beta_3 comlang_{ij} + \beta_4 comcol_{ij} + \varepsilon_{ij} \quad (4)$$

By introducing country fixed effects (for country *i* and country *j*), all the sales, expenditures and multilateral resistance terms are no longer needed and coefficients are obtained only for a set of variables that account for the bilateral trade costs. The traditional variables used as a proxy for trade costs are the log of distance ($lndist_{ij}$) and dummy variables indicating whether country *i* and country *j* share a common border ($lndist_{ij}$), a common language ($comlang_{ij}$) or had a colonial relationship in the past ($comcol_{ij}$).

With an estimation based on equation (4), there is no need to worry about the sales and expenditures variables and whether GDP or gross output should be used. But most authors are generally not satisfied with equation (4) and try to keep in the estimation Y_i and Y_j in order to estimate an elasticity between income and trade. To avoid the collinearity between these variables and the country fixed effects, panel regressions are generally run with time-invariant fixed effects (while Y_{it} and Y_{tj} become time-variant).

1.2 Trade, GDP and input-output data

In order to compare results across different versions of the gravity model, we rely on the WIOD dataset that has trade, GDP and input-output data for 41 countries over the period 1995-2011. A description of the dataset can be found in Timmer et al. (2014). It consists of a time-series of world input-output tables. The advantage is that these tables are consistent with national accounts, harmonised across countries and account for all inter-country and inter-industry transactions in 41 economies representing more than 85% of global production and trade. The World Input Output Tables have both the information on value-added and gross output, as well as the input-output structure that allows the calculation of trade flows in value-added terms.

To build the WIOD tables, a variety of assumptions are used and trade flows are adjusted to create a balanced world trade. On the one hand, this is an improvement over official trade statistics that are highly unbalanced and inconsistent across countries. On the other hand, balancing procedures are used to make the data consistent across countries and to solve the observed discrepancies between trade flows reported in national accounts and in trade statistics. These adjustments are based on assumptions that may introduce some bias in the estimation of the gravity equation. The consistent framework for trade and output should however be an advantage over a traditional gravity estimation mixing trade and output data from different inconsistent sources.

To estimate different versions of the gravity equation, we take advantage of the panel provided by the WIOD database. There are pros and cons in using panel data. Olivero and Yotov (2012) have criticised the use of panel regressions for estimating the gravity equation. But on the other hand, a dataset with several years gives us more observations and allows for a better treatment of fixed effects in order to account for multilateral resistance.

1.3 Bilateral gravity variables

The other variables needed for the gravity equation come from the CEPII dataset described in Mayer and Zignano (2011). They are the traditional distance, common border, common language and past colonial relationship variables. Distance in this dataset is a weighted bilateral distance measured using city-level data to account for the geographic distribution of population inside each nation.

In addition, we use a RTA variable based on the information provided by the WTO on trade agreements.[2] The WTO database makes a distinction between economic integration agreements (EIAs), custom unions (CUs), free trade agreements (FTAs) and preferential trade agreements (PTAs). We create a RTA_{ijt} dummy variable that takes the value of one when one of these agreements is in force (for a given year) between a pair of countries.

1.4 Trade in value-added

The trade literature has recently put the emphasis on trade in value added. One way to measure trade in value added is to decompose gross exports and identify the domestic contribution in exports (Koopman et al., 2014). Another approach followed by Johnson and Noguera (2012) consists in measuring bilateral value added trade flows. Starting from final consumption in country j, they look at the origin of value added in country i, not only through the direct exports between country i and country j but also through inputs produced in i and then exported to country k to be further processed and shipped to country j. The same input can also transit through other countries l before reaching j. At the end, “exports of value added” account for all the value added generated in country *i* and ending up in final consumption in country *j*.

We rely on the Johnson and Noguera (2012) framework to calculate trade flows in value-added terms, i.e. the value-added of country *i* embodied in a good (or a service) for which final consumption takes place in country *j*. The starting point is the Leontief model in an international setting. We formulate the ICIO-model in block matrix notation in order to distinguish as clearly as possible between domestic and international transactions. The data are organized in three matrices:

[2] http://rtais.wto.org/UI/PublicMaintainRTAHome.aspx

$$Y = \underbrace{\begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mm} \end{bmatrix}}_{mn \times m}, \quad A = \underbrace{\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mm} \end{bmatrix}}_{mn \times mn},$$

$$F = \underbrace{\begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1m} \\ f_{21} & f_{22} & \cdots & f_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ f_{m1} & f_{m2} & \cdots & f_{mm} \end{bmatrix}}_{mn \times m},$$

where Y is a block matrix that defines production by origin, sector and destination markets, A is the intermediate consumption matrix with domestic IO-links on the diagonal blocs and international IO-links on the off-diagonal blocks, and where F is the final demand matrix by destination markets. In general equilibrium supply must equal demand in all sectors and countries, taking into account the intermediate consumption used in all production activities:

$$\begin{aligned} Y &= AY + F \\ &= [I - A]^{-1}F \end{aligned} \qquad (5)$$

The value-added trade flows are the off-diagonal elements of a *vbf* matrix calculated as:

$$vbf = v(I - A)^{-1}F \qquad (6)$$

where *v* is a diagonal matrix of the value-added shares in each country and industry, i.e. value-added divided by gross output. To simplify the analysis, no industry dimension is used in the rest of the paper and value-added variables are calculated for each country (aggregating the results obtained at the country/industry level).

2. Structural gravity in gross and value added terms

We start with the estimation of equation (4) as a benchmark for the coefficients observed on the bilateral gravity variables and before discussing differences between the gross and value-added gravity equations. As we use panel data, there is a time dimension for all the variables, except distance, common border, common language and past colonial relationship that are time-invariant. Time-variant country fixed effects are introduced. As previously mentioned, panel estimations of the gravity equation are generally not regarded as robust because of the potential correlation between trade and sales/expenditures data. To mitigate the issue, we rely on an estimation in difference. We also provide results for an OLS estimation with clustered standard errors (for pairs of countries) and a PPML estimation. Since we work with country-level trade flows and with 41 countries that are among the main trading economies, there are not too many zeroes in the dataset. But still Poisson estimates are regarded as particularly suited for gravity regressions (Santos Silva and Tenreyro, 2006).

Table 1 reports the estimation results. All the variables have the expected sign and magnitude in line with gravity model literature. In the first two columns, the dependent variable is gross trade while in the third and fourth columns we use the value added trade flow, as defined in Section 1.4.

Table 1. Structural gravity estimates in gross and value-added terms

	Gross trade		Value-added trade	
	OLS	Poisson	OLS	Poisson
	coef/se	coef/se	coef/se	coef/se
Distance	-1.156***	-0.730***	-0.931***	-0.633***
	(0.045)	(0.037)	(0.035)	(0.033)
Common border	0.405***	0.349***	0.417***	0.326***
	(0.114)	(0.066)	(0.091)	(0.060)
Common language	0.116	0.218***	0.133	0.189***
	(0.132)	(0.085)	(0.089)	(0.072)
Past colonial relationship	0.524*	0.045	0.422*	0.010
	(0.311)	(0.214)	(0.220)	(0.188)
RTA	0.123***	0.179**	0.075***	0.180***
	(0.038)	(0.071)	(0.028)	(0.064)
Number of observations	26,514	26,520	26,520	26,520
F	226.523		428.959	
Adjusted R2	0.884	.	0.934	.

note: *** p<0.01, ** p<0.05, * p<0.1

For the distance coefficient, Poisson estimates have lower values, in particular because zeroes are taken into account (absence of trade) and because of a better treatment of heteroskedasticity . The impact of RTAs is however stronger with the Poisson estimates. There is also a difference for the common language variable, which very significant with the Poisson estimation and not significant in the case of OLS with clustered standard errors. The opposite is observed for the past colonial relationship, although the variable is not strongly significant in the case of the OLS estimation. This can be explained by the sample of countries where not many "former colonies" are included.

Focusing now on the difference between the gross trade and the value-added estimates, there is clearly a difference, which is observed both with the OLS and with the Poisson estimation. The impact of distance is lower on the value-added trade dependent variable. This is not surprising as "trade in value-added" is not always a direct export between country *i* and country *j*. The fact that this simple structural gravity equation focuses on trade costs between country *i* and country *j* is not fully explaining the value-added flows. This is why we need to account for third country effects.

3. Accounting for trade costs in third countries and beyond

In this section, we move to more sophisticated versions of the gravity equation that introduce in the bilateral equation the gravity variables of third countries and their partners to fully account for global value chains.

3.1 Weighted trade cost terms

The intuition behind the value added gravity equation is that not only trade costs with the partner country *j* matter but also the subsequent trade costs in the value chain (or the previous trade costs is one thinks about imports of value added). The multilateral resistance terms from equations (1) and (2) account for differences in trade costs between country *j* and other potential trade partners. But they cannot account for the trade costs between country *j* and a third country *k* and between country *k* and another country *l*. One way to fit into the gravity equation these other trade costs is to add them in the linear equation with weights derived from the input-output structure.

[results to be added]

3.2 A full derivation of the value-added gravity equation

To derive a value-added gravity equation, Noguera (2012) starts from equations similar to (1), (2) and (3) but these equations have to be different for final products and intermediate inputs. In a GVC world, inputs are traded and incorporated in the value of exports. As a consequence, trade takes place between *i* and *j* also through inputs found in final exports of country *k* and through inputs from *k* further processed in *l* before reaching *j*.

The value-added gravity equation is then obtained through a first-order log-linear Taylor approximation. It expresses the change in bilateral value added trade flows, $\hat{V}_{ij}$, as a function of changes in economic mass variables ($\hat{Y}$), bilateral trade costs ($\hat{t}$), multilateral resistance terms ($\hat{\Pi}$ and $\hat{P}$), and the global input-output structure (with parameters s_{ikj} and ϕ_{iklj}):

$$\hat{V}_{ij} = \sum_k s_{ikj} \left[\hat{Y}_k + \hat{Y}_j + \hat{Y} + (1-\sigma)\left(\hat{t}_{kj} - \hat{\Pi}_k - \hat{P}_j\right) \right. \tag{7}$$

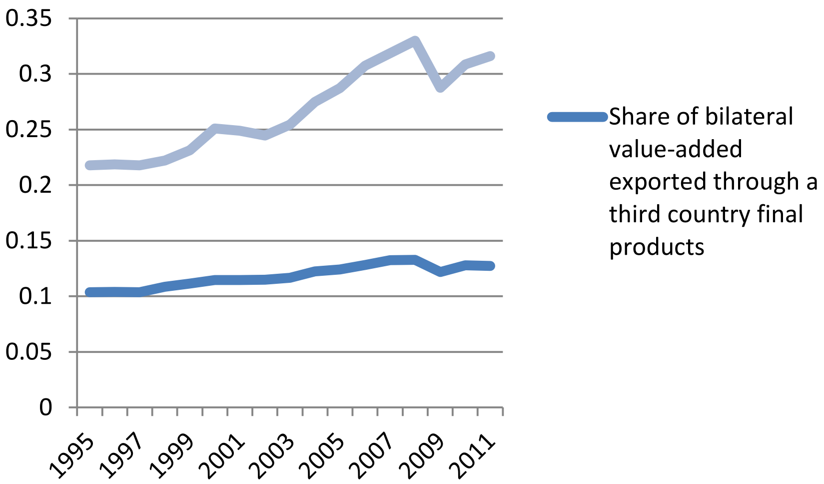
$$\left. + \sum_k \sum_l \phi_{iklj}\left[\hat{Y}_k - \hat{Y} + (1-\sigma)\left(\hat{t}_{kl} - \hat{\Pi}_k - \hat{P}_l\right)\right]\right.$$

In the absence of intermediate products, the above equation simplifies into a linear version of equation (1), in particular because there is no country *k* different from *i* (and no country *l* either). What is different in equation (7) is that bilateral trade is a function of the economic mass of all countries *k* (including country *i*) and the trade costs between these countries *k* and other countries *l* through which the inputs can transit as part of the global value chain.

The incidence of the economic mass and trade costs (and multilateral resistance terms) from these *k* and *l* countries depends on the input-output structure of the world economy. The first parameter, s_{ikj}, indicates the share of value added from country *i* to country *j* embodied in a country *k*'s final product to country *j*. The second parameter, ϕ_{iklj}, is the value added from country *i* embodied in intermediate inputs produced in country *k* which, after travelling through possibly many countries *l*, are ultimately absorbed as final demand in country *j*, relative to the value-added exports from *i* to *j*.

Using the WIOD dataset, Figure 1 shows the evolution of s_{ikj} and ϕ_{iklj} over time (an average across reporters and partners) when countries *k* and *l* are different from *i* and *j*. Over time, the contribution of economic mass variables and trade frictions from third countries has increased as determinants of bilateral trade flows.

Figure 1. Value-added gravity parameters, 1995-2011



Note: Simple average of s_{ikj} and ϕ_{iklj} across 40 countries for *k* and *l* different from *i* and *j*.
Source: Author's calculations based on the WIOD dataset.

The estimation results are presented in Table 2 where the same estimation in difference is applied to gross trade flows as well.

Table 2. Estimation in difference: gross trade equation and value-added gravity equation

	Gross trade coef/se	Value-added trade coef/se
dRTA_ij	0.168***	0.337**
	(0.016)	(0.005)
Distance x 1996	-0.006***	0.018***
	(0.002)	(0.001)
Distance x 1997	(dropped)	0.019***
		(0.001)
Distance x 1998	-0.017***	0.008***
	(0.002)	(0.001)
Distance x 1999	-0.016***	0.010***
	(0.002)	(0.001)
Distance x 2000	0.002	0.021***
	(0.002)	(0.001)
Distance x 2001	-0.013***	0.012***
	(0.002)	(0.001)
Distance x 2002	-0.009***	0.017***
	(0.002)	(0.001)
Distance x 2003	0.009***	0.032***
	(0.002)	(0.001)
Distance x 2004	0.009***	0.034***
	(0.002)	(0.002)
Distance x 2005	0.002	0.025***
	(0.002)	(0.001)
Distance x 2006	0.006***	0.027***
	(0.002)	(0.001)
Distance x 2007	0.009***	0.032***
	(0.002)	(0.001)
Distance x 2008	0.004**	0.025***
	(0.002)	(0.001)
Distance x 2009	-0.048***	-0.019***
	(0.002)	(0.001)
Distance x 2010	-0.010***	0.015***
	(0.002)	(0.001)
Distance x 2011	0.004**	0.024***
	(0.002)	(0.001)
Number of observations	24,952	24,960
F	31.440	73.340
R2	0.107	0.219
Adjusted R2	0.104	0.216

note: *** p<0.01, ** p<0.05, * p<0.1

4. Concluding remarks

Accounting for the role of third countries in bilateral trade has important and obvious policy implications. Policymakers tend to focus on their direct trade partners (i.e. the countries from which they source their inputs and the countries to which they supply intermediate and final products). For some countries and in particular large economies, focusing on gross trade flows might be a good approximation of how important each partner is, but for other economies –and particularly small countries located next to large partners-, trade costs between third countries might be more important than the trade costs with direct trade partners. For example, small economies in Central America might be more impacted by a trade agreement between the EU and the US (or between Canada and the EU) than by the trade agreements they have themselves negotiated with the EU and the US. The geography of trade costs (and potential gains from trade liberalization) is different in a GVC world.

Moreover, a value added gravity model is important to understand the role of global value chains in changing the relationship between trade and income. In gross terms, there is potentially an overestimation of trade (exports or imports) because of the double counting of inputs. This double counting does not exist in GDP where only the contribution of net trade is measured (X-M). When estimating the gravity equation in value added terms, the measure of value-added trade is consistent with the definition of GDP as the sum of sectoral value-added. What we can learn from the value added gravity equation is not that trade frictions are systematically higher in a GVC world, nor that the impact of economic mass is higher or lower, but that the determinants of trade go beyond bilateral variables. Third countries providing inputs, and the relationship between these third countries and other countries through which inputs transit, influence bilateral trade flows. The overall impact depends on the coefficients of the global input-output structure but there are no reasons to assume that trade frictions are systematically higher or lower.

References

Anderson, J. (2011). “The Gravity Model”, *Annual Review of Economics* 3(1), 133-160.

Anderson, J. (1979). “A theoretical foundation for the gravity equation”, *American Economic Review* 69(1), 106-116.

Anderson, J. and E. van Wincoop (2003). “Gravity with Gravitas: A Solution to the Border Puzzle”, *American Economic Review* 93(1), 170-192.

Anderson, J. E. and E. Van Wincoop (2004). “Trade Costs”, *Journal of Economic Literature* 42(3), 691-751.

Baldwin, R. and D. Taglioni (2011). “Gravity chains: Estimating bilateral trade flows when parts and components trade is important”, *NBER Working Paper* No. 16672, January.

Head, K. and T. Mayer (2014). “Gravity Equations: Workhorse, Toolkit, and Cookbook”. In: G. Gopinath, E. Helpman and K. Rogoff (eds), *Handbook of International Economics Vol. 4*, Elsevier.

Head, K. and J. Ries (2001). “Increasing returns versus national product differentiation as an explanation for the pattern of US-Canada trade”, *American Economic Review* 91(4), 858-876.

Johnson, R. and G. Noguera (2012). “Accounting for intermediates: Production sharing and trade in value added”, *Journal of International Economics* 82(2), 224-236.

Koopman, R., Z. Wang and S.-J. Wei (2014). “Tracing Value Added and Double Counting in Gross Exports”, *American Economic Review* 104(2), 459-494.

Noguera, G. (2012). “Trade costs and gravity for gross and value added trade”, job market paper, UC Berkely and Columbia University.

Novy, D. (2013). “Gravity redux: Measuring international trade costs with panel data”, *Economic Inquiry* 51(1), 101-121.

Novy, D., N. Chen (2011). “Gravity, trade integration, and heterogeneity across industries”, *Journal of International Economics* 85(2), 206-221.

Olivero and Yotov (2012). “Dynamic gravity: endogenous country size and asset accumulation”, *Canadian Journal of Economics* 45(1), 64-92.

Santos Silva, J., S. Tenreyro (2006). “The log of gravity”, *Review of Economics and Statistics* 88(4), 641-658.

Timmer, M., E. Dietzenbacher, B. Los, R. Stehrer, G. de Vries (2014). “The World Input-Output Database : Content, concepts and applications”, *Groningen Growth and Development Centre Research Memorandum* 144. Groningen: University of Groningen.

Tinbergen, J. (1962). *Shaping the World Economy: Suggestions for an International Economic Policy*. New York: Twentieth-Century Fund.