

Vertex centralities in input-output networks: A comparative analysis of energy and telecommunications industries between Mexico and OECD countries¹.

Topic: Input-Output and the Network Theory

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Abstract

The aim of this paper is to apply network theory in input-output analysis through vertex centrality measures based on random walks, among which are: random walk centrality and counting betweenness. Due to the large size of input-output models, an advantage of applying network theory is that it allows simplifying and describing in detail the scheme of intersectoral relationships of an economy. It also has a relational character that allows separability analysis, segmentation of internal causality or structural interdependence. These measures are applied to 45 OECD and non-OECD members using STAN IO (mid 2000) database for a comparative analysis of the energy and telecommunications sectors between Mexico and other countries. In this analysis we estimate the current status of these two sectors; the degree of domestic component in the energy sector and the degree of competitiveness and coverage in the telecommunications sector. This is important in Mexico for the design of public policies based on the implementation of structural reforms in these sectors in 2014 that intend to boost the national economy.

1. Introduction

This paper presents a comparative analysis of key sectors in the Mexican economy with respect to 45 countries (members and non-OECD) We use the STAN-IO database, which contains the input-output tables for these countries in 35 aggregated sectors, corresponding to the mid-2000s.

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Although other methods of network theory have been heavily used in recent years, in this work we use random walks (weighted networks) because they allow us a better analysis of highly aggregated input-output tables.

Input-output tables describe the flows of goods and services between sectors of an economic system that are required in sectoral production. These flows represent the interdependence between economic sectors. Thus a national economy is a complex system in which many different size agents interact selling and buying goods and services.

One way to understand the interactions in a systemic level is analyzing the relationships that underlie complex networks (Schweitzer *et al.* 2009). Therefore, we can interpret input-output table as a network, so that each sector is a vertex and economic flow from one activity to another is a directed and weighted edge.

In the complex networks theory, we can identify key sectors and their role in an economy by applying an appropriate measure of central node in an input-output graph.

Freeman (1977) proposes the concept of centrality of a graph; closeness centrality, is defined as the inverse of the average geodesic distance of all nodes to one.

Another measure is betweenness centrality, which is the average number of shortest paths between other pairs of nodes that pass through it. Intermediation flow is the maximum capacity of flows between nodes. Newman (2005) defines the betweenness of a random walk as the average of effective views of all possible random walks on a network.

These two measures of flows in the network allow us to find an ideal route or the shortest path from each node to his objective.

There are three advantages of weighted graphs in input-output analysis versus unweighted graphs. First, high levels of aggregation generate densely connected networks, so it is necessary insert the weights of the edges. Second, sectors bought and sold various quantities of inputs, so that if the graph is reflective, it is

not the same value. Third, self-loops play an important role in input-output tables because economic sectors often make a lot of purchases themselves, however there are few input-output approaches that incorporate self-loops in their analysis.

Thus, there are two measures that are suitable for the analysis of this type of network. Both are based on random walks and each has an economic interpretation.

In this paper we use the first measure; random walk centrality (RWC), which can reveal us some important aspects of the different national economies. Applying this measure to the data given by the STAN-IO database allows us to compare centrality of the nodes between different countries, in an intuitive way.

2. Theory of graphs and networks Input-Output

A graph $G = (V, E)$ consists of a set of vertices V and a set of edges E , where $E \subset V \times V$. Each edge $(i, j) \in E$ is directed and assigned a nonnegative real weight (a_{ij}) , the number of sectors is denoted by n . Our graph contain self-loops in each sector.

The graph can be represented by an adjacency matrix $a_{ij} n \times n$, where (i, j) elements are the weights a_{ij} of the egde of node i to node j .

Outdegree of node i is $k_i = \sum_{j=1}^n a_{ij}$. We denote the set of neighbors of i :

$$N(i) = \{j \mid (i, j) \in E\}.$$

In an input-output table, the adjacency matrix A can be defined as a network, here, the vertices are the sectors of an economy and the edges quantify the flows of goods and services between sectors.

We consider only the intermediate inputs matrix, that is to say, only the sales of goods and services of some firms, which are used as intermediate inputs by other firms. This matrix is not a closed system and the sum of the rows is not equal to the sum of the columns. Through the rows, sales not only include intermediate

demand from other sectors but also of final demand sectors that are not incorporated in the model; and through columns, purchases include not only those corresponding to intermediate consumption but payments to factors.

In network theory a random walker starts at a given position and repeatedly chooses an edge incident to another position.

But when considering weighted graph, the probability distribution is determined by the weight of each edge. The random walker continues till its final goal is reached.

When we consider weighted graphs, we define a matrix that shows the transition probabilities of the products of each sector. Regarding this, we must pay special attention to the transition probabilities of the outputs of each sector (accounted by M). These can be obtained by normalizing the input-output matrix given by the sum of their rows as:

$$(1) M = \hat{k}^{-1}A$$

where A is the incidence matrix, it is weighted by sales ratios relative to total production, \hat{k} is matrix of vector of weighted outdegree (sums through the rows of the weighted adjacency matrix) and M is transition probability matrix.

3. Methodology

The closeness centrality concept that has been used as standard in the input-output analysis is commonly defined as the inverse of the medium geodesic distance of all nodes to a given node. But a major limitation is that the shortest paths have little sense in dense networks, also they completely ignore the existence of self-loops.

One way to generalize the concept of closeness is using the concept of Medium First Passage Time (MFPT), Blöchl, *et al.* (2011). This distance solves the limitation of the standard measurement and is suitable for analysis of random walks. The MFPT is defined as the number of steps a random walker, starting at node s whose goal is to reach t in the first time, takes:

$$(2) H(s, t) = \sum_{r=1}^{\infty} r \cdot P(s \xrightarrow{r} t).$$

In (2) $P(s \xrightarrow{r} t)$ is the probability that the random walker that starts in s take r steps exactly before arriving to t for the first time.

MFPT focus in the first visit to the goal t . Blöchl, *et al.* (2011) propose to adopt an absorbent random walk, this is that once the objective is reached it is never abandoned. This requires modifying the transition matrix M in order to be able to remove the row and column t , leaving M_{-t} with a dimension $(n-1)(n-1)$.

The element (s, i) in the matrix $(M_{-t})^{r-1}$ gives us the probability to start in s and to reach i in $(r-1)$ steps before having gone through t . The modified transition probability as follows:

$$(3) P(s \xrightarrow{r} t) = \sum_{i \neq t} ((M_{-t})^{r-1})_{si} m_{it}$$

Substituting (3) in (2) we have:

$$(4) H(s, t) = \sum_{r=1}^{\infty} r \sum_{i \neq t} ((M_{-t})^{r-1})_{si} m_{it}$$

Where we get:

$$(5) H(s, t) = \sum_{i \neq t} (I - M_{-t})^{-2}_{si} m_{it}$$

To facilitate the calculation we can get a vectorized expression (5) $H(., t) = \sum_{i \neq t} (I - M_{-t})^{-2} m_{-t}$. Where $H(., t)$ is the average vector of the first passage time of random walker when ending in t , $m_{-t} = (m_{1t}, \dots, m_{t-1,t}, \dots, m_{t+1,t}, \dots, m_{nt})'$ is $(t-th)$ column with the deleted element m_{tt} .

We have $e =$ unit vector of dimension $(n-1)$, then $m_{-t} = (I - M_{-t})^{-1} e$, we can obtain the following expression: (5) $H(., t) = (I - M_{-t})^{-1} e$. Where we obtain the calculation of MFPT row by row through basic matrix operations.

Finally we define closeness centrality of a random walk as the inverse of the MFPT of a node as:

$$(6) C_{rw}(i) = \frac{n}{\sum_{j \in V} H(j,i)}$$

The interpretation of this measure is simple: it means the sensitivity of a node under a supply shock in the economy, when the shock occurs in a central sector and spreads proportionally to a remote sector (target).

4. Results

In this article, we analyze nine sectors for 45 OECD and non-OECD countries. Data were obtained from the input-output matrices of STAN IO OCDE database. The analyzed sectors are shown in Table 1 (To the right a short name is presented). The values of random walk centrality are presented in Appendix 1.

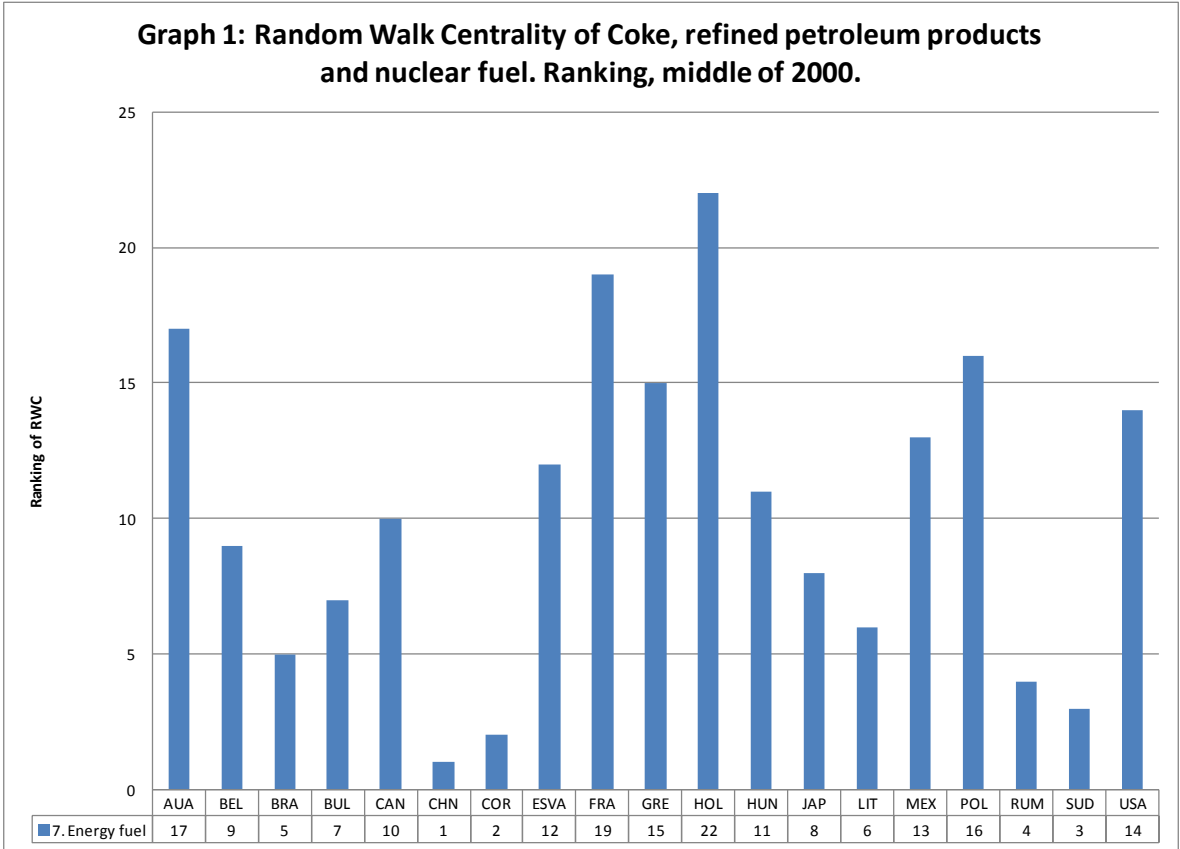
Table 1: Industry groups included in the study	
SECTORS	SHORT NAME
Energy fuels	
7: Coke, refined petroleum products and nuclear fuel	Energy fuels
Automotive Industry	
18: Motor vehicles, trailers and semi-trailers	Cars & trucks
19: Other transport equipment	Other automotive
Electronic & Electric industries	
14: Office, accounting and computing machinery	Computers
15: Electrical machinery and apparatus	Electrical industries
16: Radio, television and communication equipment	Radio, TV & Comm. Eq.
17: Medical, precision and optical instruments	Med., Prec. & Opt.
ICT industries	
26: Post and telecommunications	Telecommunications
30: Computing and related activities	Computing
Source: OECD, STAN I-O, Mid-2000.	

We calculate the centrality of random walks for 45 countries according to equation (6) of the preceding section. The results for 9 major industries, according to the described sector groups, are reported in Table 1.

We show the results in 4 graphs, where we include the top 15 countries according to their relative measures of centrality by random walks. Finally, Table 2 reports the results for all countries according to the ranking of the random walk centrality of the 9 industries considered in each country. Moreover, in all graphs we include Mexico in order to contrast the relative importance of their industries on a comparative analysis.

It is noteworthy pointing out that China, South Korea, Romania and Japan are among the first places for the 9 selected sectors (see all graphs), i.e. their RWC measure is important in all industries.

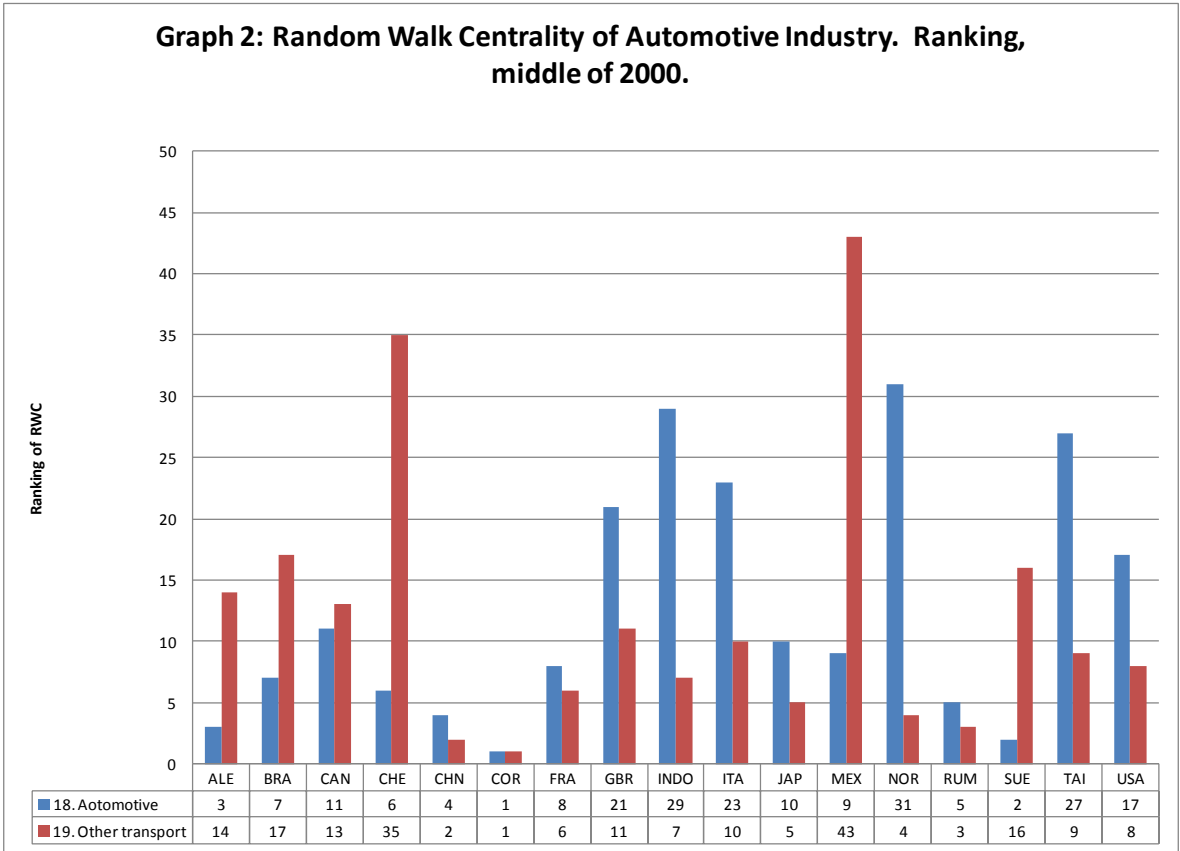
Graph 1 shows the results for the industry No. 7, concerning to coke, refined petroleum products and nuclear fuel.



Graph 1 highlights the importance of the industry Coke, refined petroleum products and nuclear fuel for the RWC values in countries like South Africa, Lithuania, Bulgaria, Belgium and Slovakia. Brazil ranks 5 and Canada ranks 10, while Mexico ranks 13th followed by the US which ranks 14.

Countries that appear in the more distant ranking are: Holland, Greece, France, Australia and others.

Graph 3 shows the results for the automotive industry that consists of two sectors: Motor vehicles, trailers and semi-trailers and Other transport equipment.



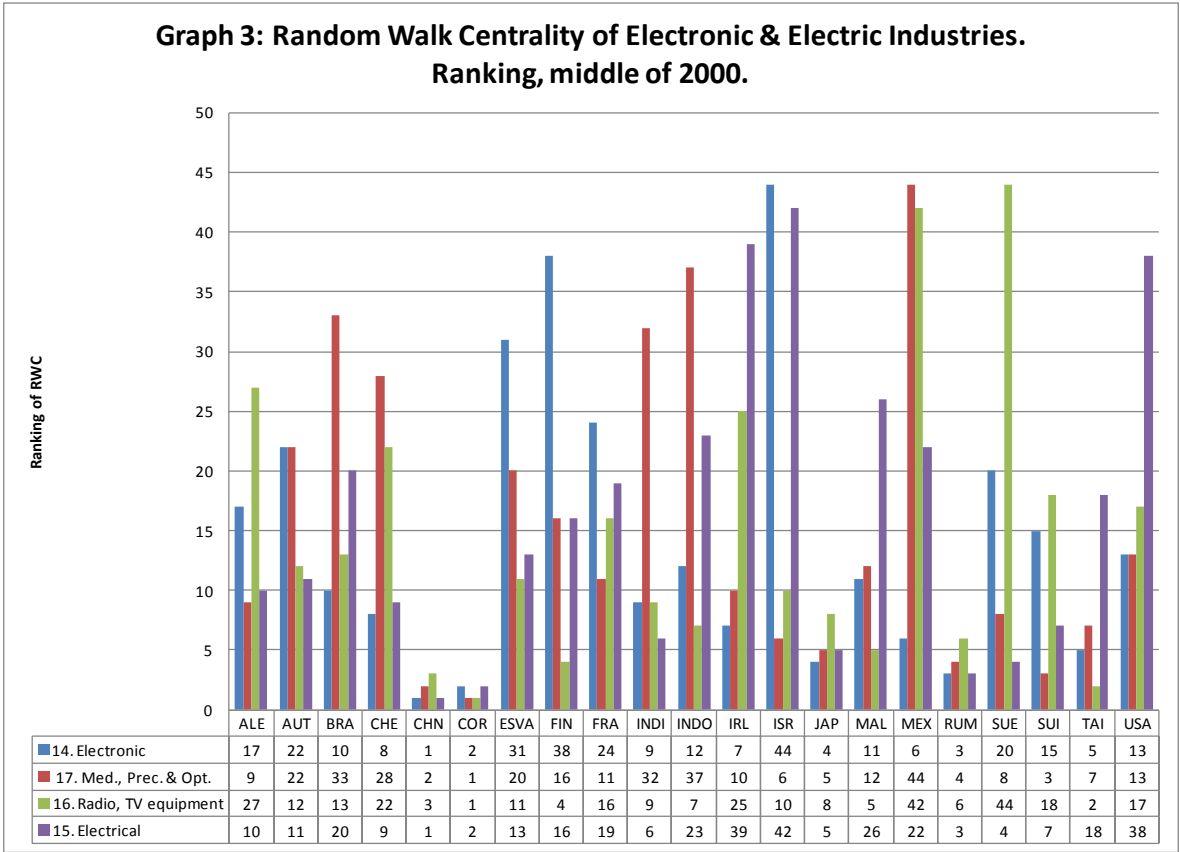
The positions of China, South Korea, Romania, Japan, France and Canada (slightly behind) stand out at the top of the ranking for the two industries considered simultaneously.

A number of countries (e.g. Sweden, Germany, Brazil, Czech Republic and Mexico) show a high value in the RWC in the sector of Motor vehicles, trailers and

semi-trailers and less important values in Other transport equipment. The graph emphasizes both values of RWC of Brazil, which exceeds those of Mexico especially in Other transport equipment (RWC = 43rd).

Moreover there is another group of countries (e.g. United States, Britain, Italy, Thailand, Indonesia and Norway) that show an important value in the ranking in Other transport equipment, but they have more lagged values in the ranking of Motor vehicles, trailers and semi-trailers.

Graph 3 shows the ranking of countries that occupy the first places in the group of Electronic & Electric industries, composed by Office, accounting and computing machinery; Medical, precision and optical instruments; Radio, television and communication equipment; and electrical machinery and apparatus.



In the corresponding group Electronic & Electric industries, China, South Korea, Romania and Japan stand out in the four sectors, followed by Switzerland and Thailand.

Mexico stands out in the Office, accounting and computing machinery sector. Followed by Thailand, Ireland, India and Malaysia and the United States.

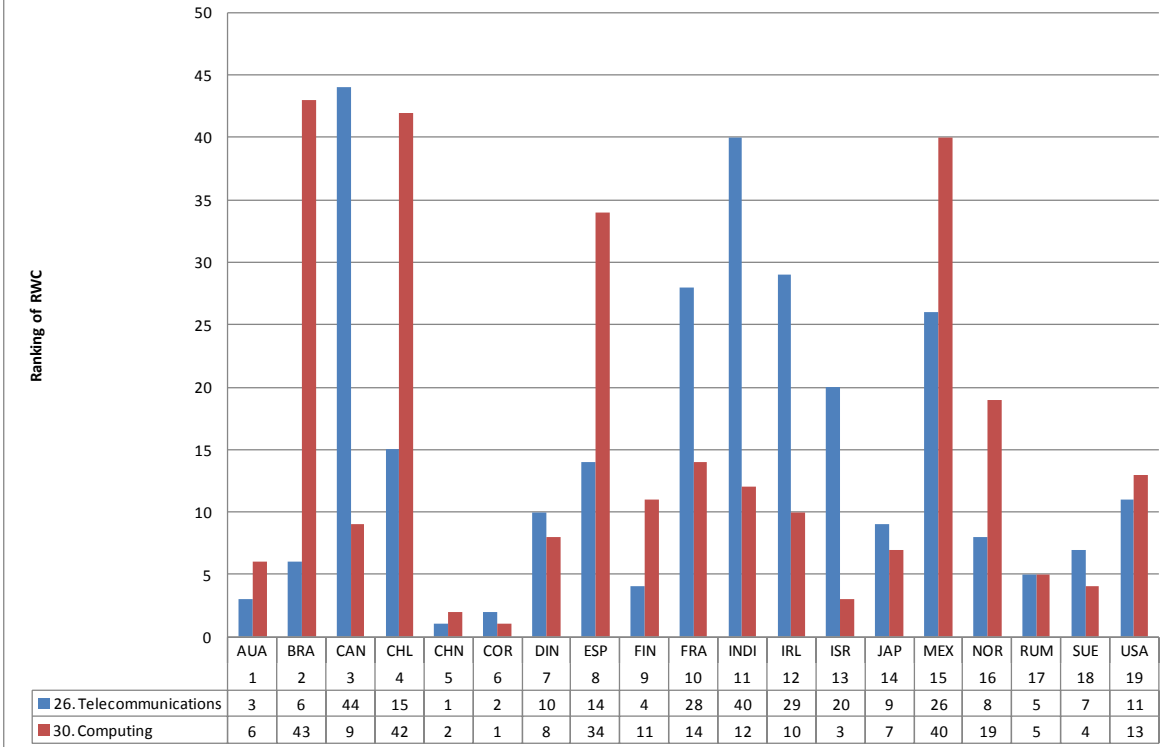
In the Medical, precision and optical instruments sector, Israel, Sweden, Thailand, Germany, Ireland and France are important. Mexico is still far behind, ranking in the 44th place.

In the industry of radio, television and communication equipment, the second most important group of countries are: Thailand, Malaysia, Finland, India, Indonesia and Israel. The Mexican economy is far behind in the 42nd place.

Finally, in the Electrical machinery and apparatus, Sweden, Switzerland, India, Czech Republic, Germany and Austria are position after the countries with the best ranking of RWC (mentioned above). Mexico is lagging behind, located at number 22.

Graph 4 shows the results of the ranking of countries with the most important values of RWC on ICT Industries that consist of the Post and Telecommunications and Computing and related activities sectors.

Graph 4: Random Walk Centrality of ICT Industries. Ranking, middle of 2000.



China, South Korea, Romania and Japan are again the countries with the best ranking in both sectors within the ICT. These countries are followed by Australia, Sweden, Finland, Denmark and Norway.

Brazil stands out in the telecommunications sector with a ranking of 6, while in Mexico this industry is far behind in the place 26th. In the United States, Spain and Chile, this industry is between the 11th and 15th places.

Regarding the information industry (Computing), the second group ranked between 6th and 11th places includes the following countries: Israel, Sweden, Denmark, Finland, Ireland and Canada. The United States and France are located in a third group of importance of RWC with the rankings 13 and 14. In addition, there is a large backlog in the Mexican economy with the number 40 ranking.

5. Conclusions

The countries showing the best rankings over the nine sectors analyzed are China, South Korea, Romania and Japan. These are generally located in the first 5 places, except that Japan is among the top 10 in the 18) Motor vehicles, trailers and semi-trailers, and Post and Telecommunications sectors.

United States is placed in the group with rankings between 11 and 17, except in Other transport equipment where it ranks 8.

The cases of Germany and France are very heterogeneous; Germany is located in the 1-5 places only in the sector 18) Motor vehicles, trailers and semi-trailers; in the 6-10 places in the sectors 15), 17), 19) and 14); in the 16-20 places in the sectors 16) and 7); and in the 41-45 places in the 26) and 30) sectors. On the other hand, France is located in the 1-5 places in the sectors 18) Motor vehicles, trailers and semitrailers and 19) Other transport equipment; in the 6-10 places in the sectors 16), 17) and 30); in the 11-15 places in the 7), 15) and 14) sectors; and in the 16-20 places only in the sector 26).

The centrality of the sectors in Brazil and India does not have any pattern. In the case of Brazil, we find that the centrality of the sectors 7), 18) and 26) had a higher ranking (among 1-5 places); sectors 14) 16) and 19 are located between the 6-10 places; only the 15) sector in the 11-15 places; the 17) sector in the 16-20 places; and the 30) sector in the 41-45 places. India has a ranking among the 1-5 places in the sector 15); among the 6-10 places in the sectors 14), 16), 18) and 30); among the 11-15 places in the sectors 7) and 19); among the 16-20 places in the sector 17); and among the 21-25 places in the 26).

Mexico has only one sector, in which it is located among the 1-5 places, i.e. sector 14) Office, accounting and computing machinery. Moreover, Mexico is placed in the 6-10 places in the sectors 7) and 18); in the 11-15 places in the sectors 15) and 26); and in the 41-45 places in the sectors 16), 17), 19) and 30). These results contrast with the results obtained by Bouchain, Velázquez and Conde (2014),

where the electronics industry appears as a key sector, and Telecommunications and Computing sector are lagging behind sectors.

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Appendix 1

RANDOM WALK CENTRALITY VALUES																	
Energy fuels	Automotive Industry				Electronic & Electric industries						ICT industries						
7. Coke, refined petroleum products and nuclear fuel	18. Motor vehicles, trailers and semi-trailers		19. Other transport equipment		14. Office, accounting and computing machinery		15. Electrical machinery and apparatus n.e.c		16. Radio, television and communication equipment		17. Medical, precision and optical instruments		26. Post and telecommunications		30. Computer and related activities		
CHN	0.049	COR	0.084	COR	0.050	CHN	0.057	CHN	0.061	COR	0.073	COR	0.047	CHN	0.053	COR	0.066
COR	0.044	SUE	0.054	CHN	0.046	COR	0.044	COR	0.054	TAI	0.055	CHN	0.036	COR	0.053	CHN	0.031
SUD	0.039	ALE	0.053	RUM	0.031	RUM	0.022	RUM	0.027	CHN	0.045	SUI	0.028	AUA	0.032	ISR	0.030
RUM	0.036	CHN	0.051	NOR	0.022	JAP	0.020	SUE	0.025	FIN	0.028	RUM	0.024	FIN	0.031	SUE	0.025
BRA	0.034	RUM	0.038	JAP	0.019	TAI	0.017	JAP	0.023	MAL	0.023	JAP	0.020	RUM	0.030	RUM	0.024
LIT	0.032	CHE	0.038	FRA	0.012	MEX	0.014	INDI	0.020	RUM	0.023	ISR	0.017	BRA	0.026	AUA	0.023
BUL	0.019	BRA	0.035	INDO	0.008	IRL	0.009	SUI	0.015	INDO	0.023	TAI	0.011	SUE	0.023	JAP	0.021
JAP	0.019	FRA	0.032	USA	0.007	CHE	0.007	ESVE	0.014	JAP	0.020	SUE	0.008	NOR	0.022	DIN	0.020
BEL	0.018	MEX	0.031	TAI	0.007	INDI	0.007	CHE	0.013	INDI	0.018	ALE	0.006	JAP	0.022	CAN	0.017
CAN	0.017	JAP	0.026	ITA	0.006	BRA	0.007	ALE	0.013	ISR	0.013	IRL	0.006	DIN	0.018	IRL	0.016
HUN	0.015	CAN	0.025	GBR	0.006	MAL	0.004	AUT	0.013	ESVA	0.013	FRA	0.006	USA	0.018	FIN	0.016
ESVA	0.013	ESVE	0.022	AUT	0.006	INDO	0.004	DIN	0.012	AUT	0.009	MAL	0.005	ITA	0.017	INDI	0.015
MEX	0.012	POL	0.021	CAN	0.006	USA	0.003	ESVA	0.011	BRA	0.009	USA	0.005	HUN	0.017	USA	0.012
USA	0.011	AUT	0.021	ALE	0.006	GBR	0.003	ITA	0.011	HUN	0.009	ESVE	0.005	ESP	0.016	FRA	0.010
GRE	0.011	SUD	0.020	FIN	0.006	SUI	0.002	POL	0.010	HOL	0.009	DIN	0.005	CHL	0.016	GBR	0.010
POL	0.008	INDI	0.019	SUE	0.006	HUN	0.002	FIN	0.009	FRA	0.007	FIN	0.004	POL	0.015	ITA	0.010
AUA	0.008	USA	0.018	BRA	0.005	ALE	0.002	HUN	0.009	USA	0.007	NOR	0.004	SUI	0.015	HUN	0.009
INDI	0.007	AUA	0.015	INDI	0.005	HOL	0.002	TAI	0.009	SUI	0.006	HUN	0.004	MAL	0.015	GRE	0.009
FRA	0.006	ESP	0.015	HOL	0.005	ESVE	0.002	FRA	0.008	LIT	0.006	ITA	0.004	GBR	0.015	NOR	0.009
INDO	0.006	BEL	0.015	POL	0.005	SUE	0.001	BRA	0.008	TUR	0.005	ESVA	0.004	ISR	0.014	LUX	0.008
CHL	0.006	GBR	0.014	EST	0.005	POL	0.001	BUL	0.008	CAN	0.005	GBR	0.004	TUR	0.014	HOL	0.008
HOL	0.006	HUN	0.014	BUL	0.005	AUT	0.001	MEX	0.008	CHE	0.005	AUT	0.004	INDO	0.013	AUT	0.007
CHE	0.006	ITA	0.014	LET	0.005	ITA	0.001	INDO	0.007	ESVE	0.005	HOL	0.004	BUL	0.013	SUI	0.007
ITA	0.005	ESVA	0.013	GRE	0.005	FRA	0.001	POR	0.007	POR	0.005	POL	0.003	CHP	0.013	ESVA	0.007
ISR	0.005	TUR	0.011	ESP	0.004	ESP	0.001	AUA	0.006	IRL	0.004	EST	0.002	LET	0.013	CHE	0.007
TUR	0.005	POR	0.011	SUI	0.004	EST	0.001	MAL	0.006	EST	0.004	LIT	0.002	MEX	0.012	BEL	0.007
AUT	0.004	TAI	0.008	DIN	0.004	BUL	0.001	BEL	0.006	ALE	0.004	LUX	0.002	AUT	0.012	CHP	0.006
TAI	0.004	HOL	0.007	LIT	0.004	DIN	0.001	TUR	0.006	GBR	0.004	CHE	0.002	FRA	0.012	LET	0.006
SUI	0.004	INDO	0.007	ISR	0.004	CAN	0.001	NOR	0.006	POL	0.004	AUA	0.002	IRL	0.012	ESVE	0.005
GBR	0.004	EST	0.003	AUA	0.004	LET	0.001	LIT	0.005	ITA	0.003	BUL	0.002	HOL	0.011	TAI	0.005
ALE	0.003	NOR	0.003	VIN	0.004	ESVA	0.000	ESP	0.005	BEL	0.003	BEL	0.002	LIT	0.011	MAL	0.005
FIN	0.003	CHL	0.002	MAL	0.004	POR	0.000	SUD	0.005	NOR	0.002	INDI	0.002	SUD	0.011	POL	0.005
ESP	0.003	FIN	0.002	ESVA	0.004	BEL	0.000	LET	0.005	DIN	0.002	BRA	0.002	ESVE	0.010	EST	0.004
DIN	0.003	DIN	0.002	ESVE	0.004	LIT	0.000	EST	0.005	SUD	0.002	ESP	0.001	POR	0.010	ESP	0.004
EST	0.003	GRE	0.002	CHE	0.004	NOR	0.000	GBR	0.004	GRE	0.002	POR	0.001	ESVA	0.010	POR	0.003
SUE	0.002	SUI	0.002	HUN	0.003	TUR	0.000	GRE	0.003	ESP	0.002	LET	0.001	BEL	0.009	LIT	0.003
POR	0.002	LET	0.001	POR	0.002	GRE	0.000	CAN	0.003	BUL	0.001	INDO	0.001	EST	0.008	BUL	0.003
VIN	0.000	IRL	0.001	BEL	0.002	FIN	0.000	USA	0.003	LET	0.001	TUR	0.001	ALE	0.008	ALE	0.003
ESVE	0.000	LIT	0.001	TUR	0.001	CHP	0.000	IRL	0.003	VIN	0.001	CHP	0.001	TAI	0.008	TUR	0.001
LET	0.000	VIN	0.001	IRL	0.001	SUD	0.000	HOL	0.003	CHP	0.000	GRE	0.001	INDI	0.006	MEX	0.000
MAL	0.000	CHP	0.001	CHP	0.000	CHL	0.000	VIN	0.003	LUX	0.000	VIN	0.000	GRE	0.005	INDO	0.000
CHP	0.000	BUL	0.001	LUX	0.000	AUA	0.000	ISR	0.002	MEX	0.000	CAN	0.000	CHE	0.005	CHL	0.000
NOR	0.000	ISR	0.000	MEX	0.000	VIN	0.000	CHP	0.001	CHL	0.000	SUD	0.000	LUX	0.005	BRA	0.000
IRL	0.000	LUX	0.000	SUD	0.000	ISR	0.000	CHL	0.001	SUE	0.000	MEX	0.000	CAN	0.004	SUD	0.000
LUX	0.000	MAL	0.000	CHL	0.000	LUX	0.000	LUX	0.000	AUA	0.000	CHL	0.000	VIN	0.002	VIN	0.000