Methods of computing the factor content of trade using the international input-output model[1]

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Abstract Computing the factor content of trade is important when testing Heckscher-Vanek-Ohlin theory. In existing studies, methods based on the international input-output model mainly refer to Deardorff's "actual" factor content of trade. These methods are plausible, but suffer from the problem of double counting. Thus, the first contribution of this paper is to propose method of computing the "actual" factor content of trade that resolves the problem of double counting. Deardorff also proposes a definition called "domestic" factor content of trade, but shows that this definition is implausible in terms of generalization and application. Therefore, as the second contribution, we revise the definition of Deardorff's "domestic" factor content of trade, and propose an appropriate computation method, then prove that the new definition and the method have ideal property for generalization and application. The differences between the actual and domestic factor contents of trade are also analyzed. For empirical analysis in this study, the methods of computing "actual" and "domestic" factor contents of trade are applied to analyzing value-added embodied in trade flows. Thus we derive the "actual" value added in trade and the "domestic" value added in trade. Then using World Input-Output Tables (WIOTs), the concepts related to value-added are computed and compared.

Key words: factor content of trade; international input-output model; intermediate trade

1.Introduction

Vanek's (1968) factor content of trade is used to test Heckscher-Vanek-Ohlin (HVO) theory of comparative advantage. Based on a series of assumptions, Deardorff (1982) gives several definitions for the factor content of trade, each with different properties, and generalizes the Heckscher-Ohlin Theorem (HO). His definitions include a "domestic" factor content of trade and an "actual" factor content of trade. When testing HVO in empirical analysis, the problem is how to compute the factor content of trade. One suitable method is to use input-output technique. For example, Leontief's early work in 1956 analyzes the factor proportions and structure of American trade by using the input-output model (Leontief, 1956). Today, in the context of production globalization, the international input-output model has becomes the primary method of computing the factor content of trade. Furthermore, using this model has become more feasible with the compilation of the international or world input-output tables by organizations around the world.

Many studies propose plausible methods of computing the factor content (or value added)

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embodied in trade based on the international input-output model, including Reimer (2006), Trefler & Zhu (2010), Johnson & Nogurea (2012), and Stehrer (2012) among others. However, it is still necessary to investigate how to measure factors (or value added) embodied in trade, because of the problem of double counting in the existing methods (Stehrer, 2012). Moreover, these methods mainly refer to Deardorff's "actual" factor content of trade rather than his "domestic" factor content of trade. This may be because the latter definition is implausible in terms of generalization and application, even though it reflects factor embodied in trade flows from a different and interesting angle.

The main contributions of this paper are as follows. First, it proposes a method of computing the "actual" factor content of trade that resolves the problem of double counting. Second, it revises the definition of Deardorff's "domestic" factor content of trade, improving its properties, and derives a proper method of computing it.

The remainder of this paper is structured as follows. The second section analyzes the current problem of double counting when computing the actual factor content of trade. Then, section 3 solves the double counting problem and proposes a method of computing the actual factor content of trade. In section 4, the domestic factor content of trade is redefined, and a method of calculating its value is proposed. Section 5 discusses the empirical analysis using value-added as an example. The final section concludes the paper.

2. The double counting problem when computing "actual" factor content of trade

Deardorff (1982) gives several definitions for the factor content of trade, including the actual and domestic factor contents of trade. The existing methods, represented by Trefler & Zhu (2010), refer to "actual" factor content of trade, which imputes to traded goods those factors actually used in their production wherever that took place.

Let T^j be the vector of country j's net exports, and G^{tj} be the actual factor reauirements matrix for country j's trade. The matrix traces backwards through the complete production history of each good that enters T^j, adding the factors actually used to produce intermediate inputs to its production, which may have occurred in a different country. Then, Deardorff's actual factor content of trade is

$$S^{tj} = G^{tj}T^j$$

Deardorff shows that in order to generalize HO theory, a necessary property is $\sum_{j=1}^{n} S^j \geq 0$, where S^j is the factor content of trade. Deardorff proves that actual factor content of trade, S^{tj}, has a stronger property, that is, $\sum_{j=1}^{n} S^{tj} = 0$. This is because every exported good of one country is an import of another, and the actual factor content is the same for both.

Trefer & Zhu (2010) (TZ) propose a widely used method of computing the actual factor content of trade based on the international input-output model. Other methods are similar to TZ. However, their method suffers from the problem of double counting (Stehrer, 2012), shown as follows. The main equation of their method is

$$f_i = D(I - A)^{-1}T_i \quad (1)$$

where D denotes the K*Nn matrix of direct factor requirement coefficients with each row representing the consumption of a particular factor per unit of output of the sectors in N countries. N is the number of countries and n the number of sectors. f_i is the net factor content embodied in country i's trade. $A = \begin{pmatrix} A^{11} & A^{12} & \cdots & A^{1N} \\ A^{21} & A^{22} & \cdots & A^{2N} \\ \cdots & \cdots & \cdots & \cdots \\ A^{N1} & A^{N2} & \cdots & A^{NN} \end{pmatrix}$, is the matrix of inter-country direct input coefficients, and A^{ij} represents the consumption of the intermediate exported goods from country i to country j for the production of country j. Therefore, the intermediate traded goods are generated endogenously in the international input-output model. Here, $(I\text{-}A)^{-1}$ is the inter-country Leontief inverse, where I is the Nn*Nn identity matrix. T_i denotes country i's net trade vector, $T_i = \begin{bmatrix} X_i \\ -M_{i2} \\ \vdots \\ -M_{iN} \end{bmatrix}$, where X_i is the vector of country i's exports including intermediate exports and final exports; M_{ij} denotes the imports of country i from country j, including intermediate imports and final imports. Then in equation (1), intermediate trade and final trade are treated in the same way. However, this is improper, because intermediate trade is endogenous, while final trade is exogenous. We can multiply $(I\text{-}A)^{-1}$ and final exports and imports, but we cannot do the same to intermediate exports and imports, because intermediate traded goods is used for the production of final trade and final domestic demand, and is already included in A.

In the international input-output model, the inter-country intermediate trade is endogenous. Therefore, part of the effects of intermediate trade is already included in the product of $D(I-A)^{-1}$ and final trade. This leads to double counting when employing equation (1) to measure the factor content of trade. For example, suppose that a car is manufactured in Japan and exported to China, but that some of the required parts and accessories are imported from China. At the same time, some of the inputs to the production of these parts and accessories made in China are steel products imported from Japan. That is, the car exported from Japan to China indirectly requires steel exported from Japan to China. Using equation (1), the manufacture of the car includes direct and indirect factor requirements. Thus, the factors used in the steel exported from Japan to China to produce the parts and accessories are already taken into account in the factors used to make the car. However, in equation (1), as intermediate trade, this part of steel is still included in the vector of exports X_i, which causes repeated computation of the factors used in this steel parts. For the intermediate traded goods used for the production of domestic final demand, the effect may also be double counted if we simply multiply this part of intermediate trade with $(I\text{-}A)^{-1}$.

3. A new method of computing "actual" factor content of trade

In this section, a new method of computing the actual factor content of trade is proposed. In Trefler & Zhu's method, intermediate trades are treated as final trades, which leads to the problem of double counting. Therefore, to solve this problem, we need to treat intermediate traded goods differently. Here, the way in this paper is to "transfer" or "shrink" the intermediate exports of each country into equivalent final exports. Then, we can treat them in the same way as final exports. The rule is that the output caused by the "transferred final" exports of the intermediate exports is equal to the output caused by the intermediate exports.

First we should find the method of computing the output induced by the countries' exports, especially the output induced by intermediate exports. Then we build the framework to transfer the intermediate exports into equivalent final exports.

3.1 The method of computing the output induced by exports

In this section, we first propose the method for two countries, and then extend it to the situation of N countries. There are three different paths to compute the output induced by exports (including final exports and intermediate exports), but each confirms the results of the others. For brevity, the first path is described here. The other two paths and the generalization from two to N countries are given in Appendix A.

Consider an international input-output model with two countries. Let Z denote the matrix of intermediate transactions, with superscript 1 and 2 denoting country 1 and country 2 respectively. For the columns in Z, Z^{11} and Z^{21} show the inputs in the production of Country 1, while for rows in Z, Z^{11} and Z^{12} show how the products of country 1 are used in the production of country 1 and country 2. Similarly, Z^{12}, Z^{22} and Z^{21} show the corresponding information for country 2. Final demand is also decomposed into that in country 1 and 2. Here, y^{11} and y^{22} denote the domestic final demand vectors of country 1 and country 2 respectively, y^{12} represents final exports from country 1 to country 2, and y^{21} represents final exports from country 2 to country 1.

Therefore, the exports of a country are divided into two parts, namely intermediate exports and final exports. For country 1, these are Z^{12} and y^{12} respectively, and they are Z^{21} and y^{21} for country 2. Recall that in international input-output model, intermediate exports are endogenous, and final exports are exogenous.

Then, the international input-output model is

$$q = (I - A)^{-1}(y^t + y^d) \tag{2}$$

where $A = \begin{pmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{pmatrix}$ and $A = Z(\hat{q})^{-1}$. $y^t = \begin{pmatrix} y^{12} \\ y^{21} \end{pmatrix}$ denotes the vector of final trade ,and $y^d = \begin{pmatrix} y^{11} \\ y^{22} \end{pmatrix}$ is the vector of domestic final demand.

A country's final exports are exogenous. Thus, the output for this part of export is as follows:

$$q^t = (I - A)^{-1} y^t \tag{3}$$

As endogenous variables, intermediate trade occurs in the production processes of final demand, including final trade and domestic final demand. Therefore, we can divide intermediate trade into the intermediate trade for the production of final trade, and intermediate trade for the production of domestic final demand. The effect of intermediate trade for production of final trade is already contained in equation (3). What we need to do is to compute the output induced by the intermediate trade for the production of domestic final demand. This means intermediate trade is required in order to fulfill the domestic final demand, which causes the output in the relevant countries to increase.

For a particular country i, we can divide the output caused by the intermediate trade for domestic final demands of all countries into two parts. The first is the effect of intermediate trade used for domestic final demands of all countries except for country i. The second is the effect of intermediate trade used for the production of domestic final demand of country i.

(1)The first effect:

For the two-country case, the production process of country 1's domestic final demand products will increase the outputs of the sectors in country 1 by using its domestic inputs. It will also increase country 2's outputs by using intermediate products imported from country 2. That is, country 2's output will increase, via intermediate trade, as a result of the production of country 1's domestic final demand products, and vice versa. Let q^{z1} be the output of country 1 resulting from country 2's domestic final demand, and q^{z2} be the output of country 2 resulting from country 1's domestic final demand. By considering all rounds, we can obtain q^{z1} & q^{z2} as follows.

$$\begin{pmatrix} q^{z1} \\ q^{d2} \end{pmatrix} = (I-A)^{-1} \begin{pmatrix} 0 \\ y^{22} \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} q^{d1} \\ q^{z2} \end{pmatrix} = (I-A)^{-1} \begin{pmatrix} y^{11} \\ 0 \end{pmatrix} \tag{5}$$

Let $\mathrm{L} = (I-A)^{-1} = \begin{pmatrix} L^{11} & L^{12} \\ L^{21} & L^{22} \end{pmatrix}$, and $q^{1z} = \begin{pmatrix} q^{z1} \\ q^{z2} \end{pmatrix}$. Then

$$q^{1z} = \begin{pmatrix} 0 & L^{12} \\ L^{21} & 0 \end{pmatrix} \begin{pmatrix} y^{11} \\ y^{22} \end{pmatrix} \tag{6}$$

(2)The second effect:

The production of country 1's domestic final products requires intermediate products imported from country 2. However, the production of these intermediate traded products in country 2 requires the intermediate inputs exported from country 1. Therefore, country 1's output increases further. This feedback loop can continue infinitely, as follows.

The first round: inputs imported from country 2 are required to produce one unit of final product of country 1. Then, to produce these exports, country 2 needs intermediate inputs imported from country 1. This will increase output in country 1: $B^1A^{12}B^2A^{21}B^1$, where $B^1 = (I-A^{11})^{-1}$,$B^2 = (I-A^{22})^{-1}$.

The second round: to produce the output of country 1, $\mathrm{B^1A^{12}B^2A^{21}B^1}$, requires imports from country 2. Then, the production of the imports from country 2 requires exports of country 1. Thus the output of country 1 increases further. That is $B^1A^{12}B^2A^{21}B^1A^{12}B^2A^{21}B^1$;

And so on.

In total, country 1' s vector of outputs from the intermediate trade used in the production of country 1's domestic final demand is

$$B^1A^{12}B^2A^{21}B^1 + B^1A^{12}B^2A^{21}B^1A^{12}B^2A^{21}B^1 + B^1A^{12}B^2A^{21}B^1A^{12}B^2A^{21}B^1A^{12}B^2A^{21}B^1 + \cdots$$
$$= B^1A^{12}B^2A^{21}B^1(I - A^{12}B^2A^{21}B^1)^{-1}y^{d1}$$

Similarly, we can obtain country 2's vector of outputs from the intermediate trade used in the production of country 2's domestic final demand: $B^2A^{21}B^1A^{12}B^2(I - A^{21}B^1A^{12}B^2)^{-1}y^{d2}$.

To sum these two effects, the vector of output induced by the intermediate trade used in the production of all countries' domestic final demands is

$$q^z = \begin{pmatrix} B^1A^{12}B^2A^{21}B^1(I - A^{12}B^2A^{21}B^1)^{-1} & L^{12} \\ L^{21} & B^2A^{21}B^1A^{12}B^2(I - A^{21}B^1A^{12}B^2)^{-1} \end{pmatrix} \begin{pmatrix} y^{d1} \\ y^{d2} \end{pmatrix} \tag{7}$$

Since $\begin{pmatrix} L^{11} & L^{12} \\ L^{21} & L^{22} \end{pmatrix} \begin{pmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$, we have

$$L^{11} = B^1 + B^1A^{12}B^2A^{21}B^1(I - A^{12}B^2A^{21}B^1)^{-1} \tag{8}$$

$$L^{12} = B^1A^{12}B^2(I - A^{21}B^1A^{12}B^2)^{-1} \tag{9}$$

$$L^{21} = B^2A^{21}B^1(I - A^{12}B^2A^{21}B^1)^{-1} \tag{10}$$

$$L^{22} = B^2 + B^2A^{21}B^1A^{12}B^2(I - A^{21}B^1A^{12}B^2)^{-1} \tag{11}$$

where, $B^1 = (I-A^{11})^{-1}$, and $B^2 = (I-A^{22})^{-1}$. By equation (8） and equation(11）, we know that

$$B^1A^{12}B^2A^{21}B^1(I - A^{12}B^2A^{21}B^1)^{-1} = L^{11} - B^1$$
$$B^2A^{21}B^1A^{12}B^2(I - A^{21}B^1A^{12}B^2)^{-1} = L^{22} - B^2$$

Let $B = \begin{pmatrix} B^1 & \\ & B^2 \end{pmatrix}$, then

$$q^z = \begin{pmatrix} L^{11} - B^1 & L^{12} \\ L^{21} & L^{22} - B^2 \end{pmatrix} \begin{pmatrix} y^{d1} \\ y^{d2} \end{pmatrix} = (L - B)y^d \quad (12)$$

Summing equation (3) and equation (12), we obtain the total output induced by exports, including final exports and intermediate exports:

$$q^t + q^z = Ly^t + (L - B)y^d \quad (13)$$

Equation (12) and (13) can be extended to N countries (see Appendix A).

3.2 Computing the "actual" factor content of trade

Trefler & Zhu's method of computing the actual factor content of trade is given by equation (1), with $T_i = \begin{bmatrix} X_i \\ -M_{i2} \\ \vdots \\ -M_{iN} \end{bmatrix}$, where X_i is the vector of country i's export, including intermediate exports and final product exports. Then, M_{ij} denotes the imports of country i from country j, including intermediate imports and final product imports. The effect is double counted if intermediate trade and final trade are treated in the same way, as explained in section 2. Furthermore, we have seen that the effects of intermediate trade for producing final trade products are already embodied in the factor content of final trade (see equation (3)). Therefore, what we need to do is to isolate the factor content embodied in intermediate trade occurring in producing domestic final products. The actual factor content of a country's exports includes the factor used domestically and abroad, while the actual factor content of a country's imports includes the factor used abroad and domestically. Therefore, it is difficult to use the previous method directly to compute the factor content of intermediate trade in producing domestic final products. Thus, we do so using indirect method.

The idea is to transfer or shrink this part of intermediate trade into equivalent final trade. That is, we derive the "net" equivalent final exports from intermediate exports in producing final domestic products that can be treated as final trade. Here, the rule is that the output caused by "net" equivalent final exports that can be treated in the same way as final exports is equal to the output induced by intermediate trade occurred in the production of domestic final products (see section 3.1, equation (12)). The conditions satisfied are as follows:

$$(L - B)y^d = L\bar{A}z^{dn} \quad (14)$$

$$y^{dn} = \bar{A}z^{dn} \quad (15)$$

$$Y^{dn} = \bar{A}\hat{z}^{dn} \quad (16)$$

where z^{dn} is vector of "gross" equivalent final trade transferred from intermediate trade for producing domestic final products, $\bar{A} = \begin{pmatrix} 0 & A^{12} & \cdots & A^{1N} \\ A^{21} & 0 & \cdots & A^{2N} \\ \cdots & \cdots & & \cdots \\ A^{N1} & A^{N2} & \cdots & 0 \end{pmatrix}$, y^{dn} is the vector of "net" equivalent final exports, and Y^{dn} is the matrix of net equivalent final trade. The left side of equation (14) represents the output induced by the intermediate trade occurred in the production of y^d, namely the vector of domestic final products. The right side of the equation implies the output caused by the net equivalent final trade flows, $\bar{A}z^{dn}$, transferred from the intermediate trade for the production of domestic final products y^d. According to equation (14), the transferred net equivalent final exports from intermediate exports serving the production of domestic final demand can be treated in the same way as final exports, but the output induced by these exports should not have any change. Using of $\bar{A}$ and z^{dn} makes it easy to obtain the matrix of net equivalent final exports showing the detailed inter-country intermediate trade flows, which is necessary for our computation.

From the above equations,

$$z^{dn} = (L\bar{A})^{-1}(L-B)y^d \tag{17}$$

Then we can obtain Y^{dn} and y^{dn}. Let

$$T_m = Y^t + Y^{dn} \tag{18}$$

where Y^t is the matrix of final trade flows, $Y^t = \begin{pmatrix} 0 & y^{12} & \cdots & y^{1N} \\ y^{21} & 0 & \cdots & y^{2N} \\ \cdots & \cdots & & \cdots \\ y^{N1} & y^{N2} & \cdots & 0 \end{pmatrix}$. The vector of the row summation of Y^t is the vector of final exports, y^t, that is $y^t = Y^t i$, $i = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$.

$$e_x = T_m i \tag{19}$$

$$E_x = diag(e_x) \tag{20}$$

Then let

$$T = E_x - T_m = \begin{pmatrix} E_{x1} & -T_{m12} & \cdots & -T_{m1N} \\ -T_{m21} & E_{x2} & \cdots & -T_{m2N} \\ \cdots & \cdots & \cdots & \cdots \\ -T_{mN1} & -T_{mN2} & \cdots & E_{xN} \end{pmatrix} \tag{21}$$

This is a similar matrix to that of T defined by Trefler and Zhu. The following computation is the same as Trefler and Zhu,

$$F_i = DLT_i \tag{22}$$

where F_i is the vector of factor content of trade of country i.

4. Redefining and computing the "domestic" factor content of trade

4.1 Definition

Deardorff 's Domestic factor content of trade (1982) is defined based on a country's domestic production techniques, as

$$S^{dj} = G^{dj}T^j \tag{23}$$

where G^{dj} is the direct-plus-indirect factor requirements matrix based on the production techniques actually used in country j for trade. From this definition, the factors embodied the country's exports and imports are both computed using the technique coefficients of country j, even if the imports of country j are produced in other countries with different production techniques. This is why the sum across all countries of various factor trade vector, for domestic factor content of trade, may be positive, negative, or zero (i.e. $\sum_{j=1}^{n} S^{dj} \overset{>}{\underset{<}{=}} 0$). This is because imports tend to be inefficient to produce domestically, which leads to difficulties in generalizing the H-O Theorem (Deardorff, 1982). Necessarily, it is ideal if $\sum_{j=1}^{n} S^{dj} \geq 0$.

In this section, we first propose a new definition for the domestic factor content of trade that satisfies $\sum_{j=1}^{n} S^{dj} = 0$, the stronger property, and then derive a proper method to compute it.

New Definition: *domestic factor content of trade*

Suppose a country's exports to others are produced using its domestic techniques and factors, while its imports from other countries are produced using other countries' domestic techniques of production and factors. Therefore, the domestic factor content of trade in country j is defined as

$$S^{dj} = G^{dj}X^j - G^{oj}M^j \tag{24}$$

where G^{oj} is the direct-plus-indirect factor requirements matrix based on production techniques used in the countries exporting to country j, X^j is the vector of country j's exports and M^j is the vector of country j's imports. This definition satisfies all the assumptions that Deardorf proposes, with a small change, described as follows.

Deardorf (1982) gives 11 assumptions before he defines the index of the factor content of trade. He suppose a country's production, consumption and trade be described by a pair, $\left(L^j, Q'^j\right) = (L^j, C^j + T^j)$, where $L^j = (L_1^j, L_2^j, \cdots, L_l^j)$ is a l-vector representing country j's employment of the l factors; Q'^j is a m-vector of net outputs of goods in country j; C^j is a m-vector of final demands for goods in country j; and T^j is a m-vector of country j's net exports. Elements of T^j are negative for goods which are imported. Technology is characterized by a set *H* of all feasible pairs, $\left(L \ \ , Q'\right)$, and is common to all countries.

In this paper, we change the pair that describes a country's production, consumption and trade by $\left(L^j, Q^j\right) = \left(L^j, C^j + X^j\right)$. Here, X^j represents country j's gross exports, based on the idea that gross exports and domestic final demands are produced in country j, while gross imports are produced in their original countries. Therefore, technology is characterized by a set *H* of all feasible pairs, *(L,Q)*. Primary factors of production are available in fixed supply in each country, by vectors $\bar{L}^j$. In trade equilibrium, $\left(\bar{L}^j, C^j + X^j\right) \in H.$ The remaining basic assumptions are the same as those in Deardorff (Deardorff, 1982, pp.684-685).

This definition supposes that domestic factors are used to produce domestic final demand and traded exports, and it requires

$$G^{dj}(C^j + X^j) \leq \bar{L}^j \tag{25}$$

Then

$$G^{dj}C^j \leq \bar{L}^j - G^{dj}X^j \leq \bar{L}^j - S^{dj} \tag{26}$$

Therefore, the definition satisfies Deardorf's 11th assumption, namely that $(\bar{L}^j - S^{dj}, C^j) \in H.$

For $S^{dj} = G^{dj}X^j - G^{oj}M^j$, we have

$$\sum_{j=1}^{N} S^{dj} = \sum_{j=1}^{N} G^{dj}X^j - \sum_{j=1}^{N} G^{oj}M^j \tag{27}$$

Because one county's exports are other countries' imports, the sum of the factors used to produce all countries' exports should always equal to the sum of factors used to produce imports. Then

$$\sum_{j=1}^{N} G^{dj}X^j = \sum_{j=1}^{N} G^{oj}M^j \tag{28}$$

That is

$$\sum_{j=1}^{N} S^{dj} = 0 \tag{29}$$

Therefore, the new definition of domestic factor content of trade satisfies the even stronger property that Deardorff's assumption 12 requires.

Compared with the actual factor content of trade, we can see that the domestic factor content of trade shows the "pure" factor content embodied in a country's international trade. The domestic factor content of exports gives the domestic factor used in a country's production of its exports, while actual factor content of exports includes the domestic and foreign factor used in the production of a certain country's exports. That is, actual factor content of exports is a mix of domestic and foreign factor used in the production of a county's exports. This is similar for the factor content of imports. The domestic factor content of imports contains only the factor used for the production of a country's imports in the original countries where the imports of this country are produced. However, the actual factor content of a country's imports includes not only the factor used in the original countries, but the factor used in this country, for the production of its imports. Therefore, the domestic factor content of trade is pure, and the actual factor content of trade is mixed. They analyze the factor content of trade from different angles, and can complement each other.

Next, we derive a method of computing the domestic factor content of trade, as per the new definition, based on the international input-output model.

4.2 Computing the domestic factor content of trade

According to the above definition of domestic factor content of trade, we need to calculate the domestic factor contents of exports and imports respectively. We already give the method of computing the output caused by exports (see section 3.1, equation (13)). Thus, we can compute the domestic factor content of exports in country i by

$$F_i^x = \overline{D}^i[Ly^t + (L - B)y^d] \tag{30}$$

where $\overline{D}^i = (0 \quad \cdots \quad 0 \quad \overline{D}^i \quad 0 \quad \cdots \quad 0)$, and $\overline{D}^i$ is the vector of direct coefficients per output of a certain factor in country i.

Next, we determine the domestic factor content of imports. Then the difference between the domestic factor contents of exports and imports is the domestic factor content of trade of a country.

First, a certain factor embodied in a country's final imports should be given. Let

$$\mathrm{M^f} = \begin{pmatrix} \widehat{\overline{D}}^1 & & & \\ & \widehat{\overline{D}}^2 & & \\ & & \ddots & \\ & & & \widehat{\overline{D}}^N \end{pmatrix} \begin{pmatrix} L^{11} & L^{12} & \cdots & L^{1N} \\ L^{21} & L^{22} & \cdots & L^{2N} \\ \cdots & \cdots & \ddots & \cdots \\ L^{N1} & L^{N2} & \cdots & L^{NN} \end{pmatrix} \begin{pmatrix} 0 & y^{12} & \cdots & y^{1N} \\ y^{21} & 0 & \cdots & y^{2N} \\ \cdots & \cdots & \ddots & \cdots \\ y^{N1} & y^{N2} & \cdots & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \widehat{\overline{D}}^1 \sum_{j\neq 1} L^{1j}y^{j1} & \widehat{\overline{D}}^1 \sum_{j\neq 2} L^{1j}y^{j2} & \cdots & \widehat{\overline{D}}^1 \sum_{j\neq N} L^{1j}y^{jN} \\ \widehat{\overline{D}}^2 \sum_{j\neq 1} L^{2j}y^{j1} & \widehat{\overline{D}}^2 \sum_{j\neq 2} L^{2j}y^{j2} & \cdots & \widehat{\overline{D}}^2 \sum_{j\neq N}^{N} L^{2j}y^{jN} \\ \cdots & \cdots & \ddots & \cdots \\ \widehat{\overline{D}}^N \sum_{j\neq 1} L^{Nj}y^{j1} & \widehat{\overline{D}}^N \sum_{j\neq 2} L^{Nj}y^{j2} & \cdots & \widehat{\overline{D}}^N \sum_{j\neq N}^{N} L^{Nj}y^{jN} \end{pmatrix}$$

Here, the sum of the jth column of M^f without diagonal entry is the embodied factor in country jth final imports. The jth diagonal entry of M^f is the factor used in country j that occurred in the production of country j's final imports from other countries. This is because the production of country j's final imports in other countries requires country j's exports to other countries. However, this also means that one country's final exports will induce another country's production, thus require other country's factor use. For example, in $\widehat{\overline{D}}^1 \sum_{j\neq 1} L^{1j}y^{j1} = \widehat{\overline{D}}^1 L^{12}y^{21} + \widehat{\overline{D}}^1 L^{13}y^{31} + \cdots + \widehat{\overline{D}}^1 L^{1N}y^{N1}$, $\widehat{\overline{D}}^1 L^{1j}y^{j1}$ shows the amount of country 1's factor needed by country j's exports to country 1, because of the imports from country 1 needed to produce country j's exports to country 1. This is exactly country j's factor imports for its production of final exports y^{j1}. These parts of factor embodied in imports should not be ignored. We can include them by reorganizing the diagonal entries of M^f.

For country 1, the embodied factor of imports hidden in diagonal entries of M^f is $\sum_{j\neq 1}\widehat{\widehat{D}}^j L^{j1} y^{1j}$. Similarly, for any country i, the embodied factor of imports hidden in the diagonal entries should be $\sum_{j\neq i}\widehat{\widehat{D}}^j L^{ji} y^{ij}$. If we substitute the ith diagonal entry of matrix M^f with $\sum_{j\neq i}\widehat{\widehat{D}}^j L^{ji} y^{ij}$, we have a new matrix $\bar{M}$ that gives complete information about the factors embodied in final imports:

$$\bar{M} = \begin{pmatrix} \sum_{j\neq 1}\widehat{D}^j L^{j1} y^{1j} & \widehat{D}^1 \sum_{j\neq 2} L^{1j} y^{j2} & \cdots & \widehat{D}^1 \sum_{j\neq N} L^{1j} y^{jN} \\ \widehat{D}^2 \sum_{j\neq 1} L^{2j} y^{j1} & \sum_{j\neq 2}\widehat{D}^j L^{j2} y^{2j} & \cdots & \widehat{D}^2 \sum_{j\neq N}^{N} L^{2j} y^{jN} \\ \cdots & \cdots & \ddots & \cdots \\ \widehat{D}^N \sum_{j\neq 1} L^{Nj} y^{j1} & \widehat{D}^N \sum_{j\neq 2} L^{Nj} y^{j2} & \cdots & \sum_{j\neq N}\widehat{D}^j L^{jN} y^{Nj} \end{pmatrix} \quad (31)$$

Therefore, the sum of the jth column of $\bar{M}$ is the "total" factor embodied in country j's final imports and the imports for final exports. Let $\bar{M}_{\cdot j} = \sum_{i=1}^{N} \bar{M}_{ij}$.

Second, the factors embodied in a country's intermediate imports for the production of all domestic final demands that are y^{11}, y^{22}, …, y^{NN}. Following the idea given in the second path in Appendix A, we compute the factor embodied in the intermediate imports of country 1 to produce domestic final demands, y^{11}, y^{22}, …, y^{NN}, as

$$m_1^d = (0 \quad \bar{D}^2 \quad \cdots \quad \bar{D}^N) \begin{pmatrix} B^1 & & & \\ & B^2 & & \\ & & \ddots & \\ & & & B^N \end{pmatrix} \begin{pmatrix} 0 & 0 & \cdots & 0 \\ A^{21} & 0 & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ A^{N1} & 0 & \cdots & 0 \end{pmatrix}$$

$$\begin{pmatrix} L^{11} & L^{12} & \cdots & L^{1N} \\ L^{21} & L^{22} & \cdots & L^{2N} \\ \cdots & \cdots & \ddots & \cdots \\ L^{N1} & L^{N2} & \cdots & L^{NN} \end{pmatrix} \begin{pmatrix} y^{11} & & & \\ & y^{22} & & \\ & & \ddots & \\ & & & y^{NN} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$= \bar{D} \begin{pmatrix} 0 & 0 & \cdots & 0 \\ B^2 A^{21} L^{11} y^{11} & B^2 A^{21} L^{12} y^{22} & \cdots & B^2 A^{21} L^{1N} y^{NN} \\ \cdots & \cdots & \ddots & \cdots \\ B^N A^{N1} L^{11} y^{11} & B^N A^{N1} L^{12} y^{22} & \cdots & B^N A^{N1} L^{1N} y^{NN} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

where $\bar{D} = (\bar{D}^1 \quad \bar{D}^2 \quad \ldots \quad \bar{D}^N)$ is the vector of direct factor coefficients of all N countries, for a certain factor. Similarly, for any country i, we have

$$m_i^d = (\bar{D}^1 \quad \cdots \quad \bar{D}^{i-1} \quad 0 \quad \bar{D}^{i+1} \quad \ldots \quad \bar{D}^N) \begin{pmatrix} B^1 & & & \\ & B^2 & & \\ & & \ddots & \\ & & & B^N \end{pmatrix} \begin{pmatrix} 0 & \cdots & A^{1i} & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & A^{i-1,i} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & A^{i+1,i} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & A^{Ni} & 0 & & 0 \end{pmatrix}$$

$$\begin{pmatrix} L^{11} & L^{12} & \cdots & L^{1N} \\ L^{21} & L^{22} & \cdots & L^{2N} \\ \cdots & \cdots & \ddots & \cdots \\ L^{N1} & L^{N2} & \cdots & L^{NN} \end{pmatrix} \begin{pmatrix} y^{11} & & & \\ & y^{22} & & \\ & & \ddots & \\ & & & y^{NN} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$= \bar{D} \begin{pmatrix} B^1A^{1i}L^{i1}y^{11} & B^1A^{1i}L^{i2}y^{22} & \cdots & B^1A^{1i}L^{ii}y^{ii} & \cdots & B^1A^{1i}L^{iN}y^{NN} \\ \vdots & \cdots & & \vdots \quad \vdots & & \cdots \quad \vdots \\ B^{i-1}A^{i-1,i}L^{i1}y^{11} & B^{i-1}A^{i-1,i}L^{i2}y^{22} & \cdots & B^{i-1}A^{i-1,i}L^{ii}y^{ii} & \cdots & B^{i-1}A^{i-1,i}L^{iN}y^{NN} \\ 0 & 0 & \cdots & 0 & \cdots & 0 \\ B^{i+1}A^{i+1,i}L^{i1}y^{11} & B^{i+1}A^{i+1,i}L^{i2}y^{22} & \cdots & B^{i+1}A^{i+1,i}L^{ii}y^{ii} & \cdots & B^{i+1}A^{i+1,i}L^{iN}y^{NN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B^NA^{Ni}L^{i1}y^{11} & B^NA^{Ni}L^{i2}y^{22} & \cdots & B^NA^{Ni}L^{ii}y^{ii} & \cdots & B^NA^{Ni}L^{iN}y^{NN} \end{pmatrix} i \tag{32}$$

Then the total factor content of country j's imports should be

$$F_j^m = \bar{M}_{\cdot j} + m_j^d \tag{33}$$

Therefore, the domestic factor content of trade for country j is

$$F_j^D = F_j^x - F_j^m \tag{34}$$

Next, we prove that the sum of all countries' domestic factor content of trade is zero, that is

$$\sum_{j=1}^N F_j^D = \sum_{j=1}^N F_j^x - \sum_{j=1}^N F_j^m = 0 \tag{35}$$

or

$$\sum_{j=1}^N F_j^x = \sum_{j=1}^N F_j^m \tag{36}$$

For $\sum_{j=1}^N F_j^x$, we have

$$\sum_{j=1}^N F_j^x = \bar{D}(LY^t + (L-B)\hat{Y})i \tag{37}$$

where $Y^t = \begin{pmatrix} 0 & y^{12} & \cdots & y^{1N} \\ y^{21} & 0 & \cdots & y^{2N} \\ \cdots & \cdots & \ddots & \cdots \\ y^{N1} & y^{N2} & \cdots & 0 \end{pmatrix}$, $\hat{Y} = \begin{pmatrix} y^{11} & & & \\ & y^{22} & & \\ & & \ddots & \\ & & & y^{NN} \end{pmatrix}$, $B = \begin{pmatrix} B^1 & & \\ & \ddots & \\ & & B^N \end{pmatrix}$,

and $i = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$.

Because the diagonal entries of $\bar{M}$ is the reorganized diagonal entries of M^f, the sum of each set of entries is equal. Therefore,

$$\sum_{j=1}^N F_j^m = \sum_{j=1}^N (\bar{M}_{\cdot j} + m_j^d) = \sum_{j=1}^N \bar{M}_{\cdot j} + \sum_{j=1}^N m_j^d \tag{38}$$

$$\sum_{j=1}^N \bar{M}_{\cdot j} = \bar{D}LY^t i \tag{39}$$

$$\sum_{i=1}^N \begin{pmatrix} 0 & \cdots & A^{1i} & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & A^{i-1,i} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & A^{i+1,i} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & A^{Ni} & 0 & 0 & 0 \end{pmatrix} L\hat{Y}i = \bar{A}L\hat{Y}i \tag{40}$$

$$\sum_{j=1}^N m_j^d = \bar{D}B\bar{A}L\hat{Y}i \tag{41}$$

where $\bar{A} = \begin{pmatrix} 0 & A^{12} & \cdots & A^{1N} \\ A^{21} & 0 & \cdots & A^{2N} \\ \cdots & \cdots & & \cdots \\ A^{N1} & A^{N2} & \cdots & 0 \end{pmatrix}$. Let $\hat{A} = \begin{pmatrix} A^{11} & & \\ & \ddots & \\ & & A^{NN} \end{pmatrix}$, then $B = \left(I - \hat{A}\right)^{-1}$, and

$\bar{A} = A - \hat{A}$, and we have

$$B\bar{A}L = \left(I - \hat{A}\right)^{-1}\left(I - \hat{A} - (I - A)\right)(I - A)^{-1}$$

$$= (I - A)^{-1} - \left(I - \hat{A}\right)^{-1} = L - B \quad (42)$$

Thus

$$\sum_{j=1}^{N} F_j^m = \sum_{j=1}^{N} \bar{M}_{\cdot j} + \sum_{j=1}^{N} m_j^d = \bar{D}LY^t i + \bar{D}(L - B)\hat{Y} i = \sum_{j=1}^{N} F_j^x$$

$$\sum_{j=1}^{N} F_j^D = \sum_{j=1}^{N} F_j^x - \sum_{j=1}^{N} F_j^m = 0 \quad (43)$$

Therefore, this method satisfies the plausible property that Deardorff illustrated.

5. **Empirical analysis**

Value added can be considered as the factors employed in productions. Therefore, we can apply the definitions and methods for factor contents described in this paper to value added. In this section, we conduct an empirical analysis using value-added as an example based on its relative importance in recent research on international trade. First, several concepts about value added embodied in trade are explained. Then using World Input-Output Tables (WIOT) (Timmer (ed), 2012; Dietzenbacher, Los, Stehrer, Timmer and Vries, 2013), the different value added flows embodied in trade are computed.

5.1 Concepts of value added embodied in trade flows

There are two types of concepts frequently used in the analysis of value added in trade flows in literature, namely trade in value added, and value added in trade (Stehrer, 2012). In a two-country case, the former "accounts for the value added of one country directly and indirectly contained in final consumption of another country." The latter, value added in trade, "calculates the value added contained in gross trade flows between two countries". Clearly, this is the same as the definition of "actual" factor content of trade.

For the concept of "value added in trade", Stehrer (2012) follows the "actual factor content of trade" method of Trefler & Zhu's (2010), and the problem of double accounting still exists, as discussed in section 2. To solve the problem, we calculate value-added in trade using the method of actual factor content of trade described in section 3 of this paper, namely the "actual" value added in trade. Following equation (22), we have

$$VAiT_i^a = VLT_i \quad (44)$$

where V is the vector of value added rates of all countries, and V^i is vector of value added rates in country i, $V = (V^1 \quad V^2 \quad \cdots \quad V^N)$.

Using the method of computing the domestic factor content of trade in this paper, we obtain another value added flow that can be called "domestic" value added in trade. Next we substitute direct factor coefficients per output in equations (30)-(34) with value added rates, thus obtaining the domestic value added in exports and imports. The difference between the two is the domestic value added in trade. Domestic value-added in imports of country i is

$$VA_i^m = \bar{T}_{\cdot i} + t_i^d \quad (45)$$

where $\bar{T}_{\cdot i} = \sum_{j=1}^{N} \bar{T}_{ji}$, $\bar{T} = \begin{pmatrix} \sum_{j\neq 1} \hat{V}^j L^{j1} y^{1j} & \hat{V}^1 \sum_{j\neq 2} L^{1j} y^{j2} & \cdots & \hat{V}^1 \sum_{j\neq N} L^{1j} y^{jN} \\ \hat{V}^2 \sum_{j\neq 1} L^{2j} y^{j1} & \sum_{j\neq 2} \hat{V}^j L^{j2} y^{2j} & \cdots & \hat{V}^2 \sum_{j\neq N}^{N} L^{2j} y^{jN} \\ \cdots & \cdots & \ddots & \cdots \\ \hat{V}^N \sum_{j\neq 1} L^{Nj} y^{j1} & \hat{V}^N \sum_{j\neq 2} L^{Nj} y^{j2} & \cdots & \sum_{j\neq N} \hat{V}^j L^{jN} y^{Nj} \end{pmatrix}$;

$$t_i^d = V \begin{pmatrix} B^1 A^{1i} L^{i1} y^{11} & B^1 A^{1i} L^{i2} y^{22} & \cdots & B^1 A^{1i} L^{ii} y^{ii} & \cdots & B^1 A^{1i} L^{iN} y^{NN} \\ \vdots & \cdots & & \vdots \quad \vdots & & \cdots \quad \vdots \\ B^{i-1} A^{i-1,i} L^{i1} y^{11} & B^{i-1} A^{i-1,i} L^{i2} y^{22} & \cdots & B^{i-1} A^{i-1,i} L^{ii} y^{ii} & \cdots & B^{i-1} A^{i-1,i} L^{iN} y^{NN} \\ 0 & 0 & \cdots & 0 & \cdots & 0 \\ B^{i+1} A^{i+1,i} L^{i1} y^{11} & B^{i+1} A^{i+1,i} L^{i2} y^{22} & \cdots & B^{i+1} A^{i+1,i} L^{ii} y^{ii} & \cdots & B^{i+1} A^{i+1,i} L^{iN} y^{NN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B^N A^{Ni} L^{i1} y^{11} & B^N A^{Ni} L^{i2} y^{22} & \cdots & B^N A^{Ni} L^{ii} y^{ii} & \cdots & B^N A^{Ni} L^{iN} y^{NN} \end{pmatrix} i$$

The domestic value-added in exports is

$$VA_i^x = \bar{V}^i [L y^t + (L - B) y^d] \tag{46}$$

where $\bar{V}^i = (0 \quad \cdots \quad 0 \quad V^i \quad 0 \quad \cdots \quad 0)$. Then the domestic value-added in trade is

$$VAiT_i^D = VA_i^x - VA_i^m \tag{47}$$

Next, for the popular used concept, trade in value-added, we give the method employed in existing studies. For the gross trade level, "Value added exports of country *r* to all other countries include value added created in country *r* to satisfy final demand in countries *s* and *t*" (Stehrer (2012)). Therefore, the formulae of value added exports and imports of country i are

$$T_{TiVA}^e = \hat{V}^i \sum_{m\neq i} \sum_{j=1}^{N} L^{ij} y^{jm} \tag{48}$$

$$T_{TiVA}^i = \sum_{m\neq i} \hat{V}^m \sum_{j=1}^{N} L^{mj} y^{ji} \tag{49}$$

Trade in value added is the difference between value added exports and imports, that is

$$T_{TiVA} = \hat{V}^i \sum_{m\neq i} \sum_{j=1}^{N} L^{ij} y^{jm} - \sum_{m\neq i} \hat{V}^m \sum_{j=1}^{N} L^{mj} y^{ji} \tag{50}$$

These are exactly the methods used in literature (Stehrer, 2012).

5.2 Results of the empirical analysis

Using value-added as example and WIOT 2007 (Timmer (ed), 2012; Dietzenbacher, Los, Stehrer, Timmer and Vries, 2013), we compute the actual and domestic value added in trade defined in this paper, value added in trade by Trefler and Zhu's method, and trade in value added that is popular used. As a part of the World Input-Output Database (WIOD), the WIOT data include 35 industries and cover 40 countries (EU-27 countries, Turkey, Canada, USA and Mexico, Japan, Korea, Taiwan, Australia, Brazil, Russia, India, Indonesia and China). For simplicity, first we aggregate the EU-27 countries to one category, and then do the computation. The results are shown as follows in table 1 to table 3.

Table 1 Gross trade, TZ's value added in trade, and trade in value added in million US_$

	Gross trade			TZ's Value added in trade			Trade in value added		
	gross exports	gross imports	NET	VA in exports	VA in imports	NET VAiT	Exports	Imports	net TiVA
AUS	195359	203614	-8255	188270	198589	-10319	166384	176703	-10319

EU-27	2550024	2247994	302031	2403622	2201453	202169	2083012	1880842	202169
BRA	182673	153432	29241	168193	149257	18935	154127	135192	18935
CAN	479761	429994	49766	463999	421619	42381	389676	347296	42381
CHN	1342004	973631	368373	1319803	946362	373441	1063184	689743	373441
IDN	125466	102583	22883	124093	100217	23877	105843	81966	23877
IND	242725	268350	-25625	230611	262454	-31843	194870	226713	-31843
JPN	772058	639124	132935	766638	624384	142255	687298	545044	142255
KOR	436153	385383	50771	410327	377502	32825	310739	277913	32825
MEX	275760	277634	-1874	268698	271863	-3165	225704	228870	-3165
RUS	326696	241530	85165	308870	232332	76538	282632	206094	76538
TUR	119530	164455	-44925	111413	158197	-46784	93081	139865	-46784
TWN	277126	235846	41280	268464	231038	37426	181629	144203	37426
USA	1530925	2161403	-630477	1520734	2098352	-577618	1324238	1901856	-577618
ROW	2656330	3027619	-371289	2632824	2912943	-280119	1939322	2219441	-280119
SUM	11512592	11512592	0	11186561	11186561	0	9201740	9201740	0

As proved by Stehrer (2012), the net VAiT of a country is equal to its net export in gross terms, and is also equal to its net TiVA. Table 1 illustrates that for each country, the net VAiT is equal to its net TiVA. For example, China's net TiVA is 373441, which is also the amount of its net VAiT. Note that in gross terms, net exports of China equal to gross exports minus gross imports, 368373, and is very close to its net TiVA and net VAiT. Theoretically, they should equal each other. The reason for the slight difference is international transport margins and net taxes on products. In WIOD data, value added is not equal to gross output minus intermediate inputs because international transport margins and net taxes on products are taking into account. This is also mentioned in Stehrer (2012).

The results in table 1 illustrate that at the aggregate level, using TZ's method of actual factor content of trade, VAiT does not give us any new information on a country's trade, because for a country, value added in exports, value added in imports and net VAiT are equal to its gross exports, gross imports and net exports respectively. This is also illustrated by Stehrer (2012). With regard to trade in value added, a country's net TiVA is equal to its net exports in gross terms. The amount of its value added exports is smaller than its gross exports, and its value added imports is less than its gross imports, since there is no double counting in TiVA.

Table2 Actual value added in trade and domestic value-added in trade using the proposed methods in this paper in million US-$

	Actual value added in trade			Domestic value added in trade		
	Value added in exports	Value added in imports	net VAiT	Value added in exports	Value added in imports	Net VAiT
AUS	149040	177435	-28395	167116	175999	-8883
EU-27	2021797	1985212	36585	2187381	1850724	336658
BRA	143076	135576	7500	154511	129546	24965
CAN	417925	349803	68121	392184	378982	13203
CHN	1169672	701924	467748	1075365	717997	357368

IDN	100287	82202	18085	106079	83298	22781
IND	200858	227519	-26661	195677	215690	-20013
JPN	630836	554739	76097	696994	522221	174773
KOR	338051	279857	58195	312682	303070	9613
MEX	243623	229749	13874	226584	239919	-13335
RUS	254777	207618	47159	284156	214949	69207
TUR	99789	140070	-40282	93287	136079	-42793
TWN	215949	144775	71174	182201	182992	-791
USA	1253532	1977554	-724022	1399936	1831002	-431066
ROW	2257860	2303038	-45178	2022919	2514604	-491686
SUM	9497071	9497071	0	9497071	9497071	0

Using the proposed methods of computing actual and domestic factor contents of trade, we obtain the results of the actual and domestic value added in trade, shown in table 2. Clearly, the sum of actual value added in exports of all countries equals to the sum of domestic value added in exports, and the sum of actual value added in imports equals to that of domestic value-added in trade. The results also show that the sum of net actual value added in trade is zero, which is the good property Deardorff mentions (Deardorff, 1982). Finally, the results show that domestic value added in trade also has this property. For a certain country, the actual value added in trade does not equal to the domestic value added in trade, because they show the contained value added from different angles. For example, China's actual value added in exports is 1169672, and its domestic value added in exports is 1075365. The reason of the difference is that the actual value added in exports of China includes the value added created inside and outside China in order to produce these exports, and that its domestic value added in exports contains the value added created in China for the production of these exports and the value added created in China for the production of the new exports induced by these exports.

Table 3 Differences between TZ, TiVA and the proposed methods

	Dif between TZ and the proposed actual VAiT			Dif between TiVA and the proposed actual VAiT			DIF between TiVA and the proposed domestic VAiT		
	Exports	Imports	Net	Exports	Imports	Net	Exports	Imports	Net
AUS	26.32%	11.92%	-63.66%	11.64%	-0.41%	-63.66%	-0.44%	0.40%	16.17%
EU-27	18.89%	10.89%	452.60%	3.03%	-5.26%	452.60%	-4.77%	1.63%	-39.95%
BRA	17.55%	10.09%	152.47%	7.72%	-0.28%	152.47%	-0.25%	4.36%	-24.15%
CAN	11.02%	20.53%	-37.79%	-6.76%	-0.72%	-37.79%	-0.64%	-8.36%	221.01%
CHN	12.84%	34.82%	-20.16%	-9.10%	-1.74%	-20.16%	-1.13%	-3.94%	4.50%
IDN	23.74%	21.92%	32.03%	5.54%	-0.29%	32.03%	-0.22%	-1.60%	4.81%
IND	14.81%	15.35%	19.43%	-2.98%	-0.35%	19.43%	-0.41%	5.11%	59.11%
JPN	21.53%	12.55%	86.94%	8.95%	-1.75%	86.94%	-1.39%	4.37%	-18.61%
KOR	21.38%	34.89%	-43.59%	-8.08%	-0.69%	-43.59%	-0.62%	-8.30%	241.48%
MEX	10.29%	18.33%	-122.81%	-7.35%	-0.38%	-122.81%	-0.39%	-4.61%	-76.26%
RUS	21.23%	11.90%	62.30%	10.93%	-0.73%	62.30%	-0.54%	-4.12%	10.59%
TUR	11.65%	12.94%	16.14%	-6.72%	-0.15%	16.14%	-0.22%	2.78%	9.33%
TWN	24.32%	59.58%	-47.42%	-15.89%	-0.39%	-47.42%	-0.31%	-21.20%	-4830.01%

USA	21.32%	6.11%	−20.22%	5.64%	−3.83%	−20.22%	−5.41%	3.87%	34.00%
ROW	16.61%	26.48%	520.03%	−14.11%	−3.63%	520.03%	−4.13%	−11.74%	−43.03%
SUM	17.79%	17.79%		−3.11%	−3.11%		−3.11%	−3.11%	

Note: Dif denotes the difference between TZ and the proposed methods, or the difference between TiVA and the proposed methods. Take the results of proposed methods as the base.

The differences between TZ's actual value added content of trade and actual value added content of trade by the method of this paper show that for value added in exports and in imports, the results of TZ method are much larger than the results of the method in this paper because of the problem of double counting in TZ's method. Generally, the double counting part is about 17.8%, and cannot be ignored.

The difference between TiVA and the actual value added in trade using method in this paper shows that in average, TiVA is smaller than VAiT, about 3.11%. The reason is that TiVA does not consider the effects of intermediate exports occurred for the production of a country's own domestic final products, y^{ii}.

6. Conclusion

How to calculate the factor content of trade is a significant issue when testing the theory of comparative advantage, as well as in analyses of value-added in international trade. In literature, the method commonly used is Deardorff's actual factor content of trade, proposed by Trefer and Zhu. Although the method is plausible, it suffers from the problem of double counting. Thus, this study proposes an alternative method of measuring the actual factor content of trade that resolves this problem.

The second contribution of this study is with regard to the domestic factor content of trade. Deardorff proves that the domestic factor content of trade has unsatisfactory properties in terms of application. However, this concept supplies a valuable view of production if we focus on the exports and imports flows in a certain region. Therefore, we redefined the domestic factor content of trade, and proposed a method to compute its value. Furthermore, we proved that the redefined concept and the calculation method have promising property. The difference between the actual and domestic factor contents of trade is also analyzed.

In our empirical analysis, we applied the methods to value-added. First, several indexes are introduced. First is trade in value-added, which is popular in value-added chain analyses. Second is value-added in trade using TZ's method, also employed frequently, and is based on the concept of actual factor content of trade. Then, using the method of this paper, we obtained actual value added in trade that resolves the problem of double counting, and domestic value-added in trade, based on the redefined concept of domestic factor content of trade. Lastly, using WIOT 2007, we computed the indexes and compared the results of these different concepts.

The analyses in this study are all on the gross trade level of a country. Thus, the factor content of trade at the bilateral trade level is not investigated. We leave this as a topic for future research using the methods proposed here.

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Appendix A

A.1 Path 2 of deriving the method of computing the factors embodied in exports

The output caused by final trade is the same as that in path 1 (see section 3, equation (3)). Therefore, similarly to that in path 1, we need to measure the effect of intermediate trade used for all countries' domestic final demands. In this path, we compute the amount of intermediate trade used for domestic final demand first, and then find a way to compute the output induced by this amount of intermediate trade.

First, the output induced by domestic final demands is

$$q^d = (I-A)^{-1}y^d \qquad \text{(A.1)}$$

where y^d denotes the vector of domestic final demands, for two country case, $y^d = \begin{pmatrix} y^{11} \\ y^{22} \end{pmatrix}$.Then, the intermediate trade used for the above output q^d is

$$z^d = \begin{pmatrix} z^{d1} \\ z^{d2} \end{pmatrix} = \begin{pmatrix} 0 & A^{12} \\ A^{21} & 0 \end{pmatrix} q^d = \begin{pmatrix} 0 & A^{12} \\ A^{21} & 0 \end{pmatrix} (I-A)^{-1}y^d \qquad \text{(A.2)}$$

Next, we measure the outputs of the two countries caused by z^d. This cannot be treated in the same way as final trade. Here, we use the idea of the single country model, and consider z^{d1} & z^{d2} as exogenous variables of the single production systems of country 1 and country 2. Then, we have

$$q^{zd1} = (I-A^{11})^{-1}z^{d1} \qquad \text{(A.3a)}$$

$$q^{zd2} = (I-A^{22})^{-1}z^{d2} \qquad \text{(A.3b)}$$

Let $q^{zd}=\begin{pmatrix}q^{zd1}\\q^{zd2}\end{pmatrix}$. Then,

$$q^{zd}=\begin{pmatrix}(I-A^{11})^{-1} & 0\\ 0 & (I-A^{22})^{-1}\end{pmatrix}\begin{pmatrix}0 & A^{12}\\ A^{21} & 0\end{pmatrix}q^{d}$$
$$=\begin{pmatrix}(I-A^{11})^{-1} & 0\\ 0 & (I-A^{22})^{-1}\end{pmatrix}\begin{pmatrix}0 & A^{12}\\ A^{21} & 0\end{pmatrix}(I-A)^{-1}y^{d}$$

This result is consistent with that of the first path. We prove it as follows.

$$q^{zd}=\begin{pmatrix}(I-A^{11})^{-1} & 0\\ 0 & (I-A^{22})^{-1}\end{pmatrix}\begin{pmatrix}0 & A^{12}\\ A^{21} & 0\end{pmatrix}q^{d}$$
$$=\begin{pmatrix}(I-A^{11})^{-1} & 0\\ 0 & (I-A^{22})^{-1}\end{pmatrix}\begin{pmatrix}0 & A^{12}\\ A^{21} & 0\end{pmatrix}(I-A)^{-1}y^{d}$$
$$=\begin{pmatrix}0 & B^{1}A^{12}\\ B^{2}A^{21} & 0\end{pmatrix}\begin{pmatrix}L^{11} & L^{12}\\ L^{21} & L^{22}\end{pmatrix}\begin{pmatrix}y^{11}\\ y^{22}\end{pmatrix}$$
$$=\begin{pmatrix}B^{1}A^{12}L^{21} & B^{1}A^{12}L^{22}\\ B^{2}A^{21}L^{11} & B^{2}A^{21}L^{12}\end{pmatrix}\begin{pmatrix}y^{11}\\ y^{22}\end{pmatrix}$$

Using $\begin{pmatrix}A^{11} & A^{12}\\ A^{21} & A^{22}\end{pmatrix}\begin{pmatrix}L^{11} & L^{12}\\ L^{21} & L^{22}\end{pmatrix}=\begin{pmatrix}I & 0\\ 0 & I\end{pmatrix}$, we can deduce that

$$B^{1}A^{12}L^{21}=B^{1}A^{12}B^{2}A^{21}B^{1}(I-A^{12}B^{2}A^{21}B^{1})^{-1}$$
$$B^{1}A^{12}L^{22}=B^{1}A^{12}(B^{2}+B^{2}A^{21}B^{1}A^{12}B^{2}(I-A^{21}B^{1}A^{12}B^{2})^{-1})$$
$$=B^{1}A^{12}B^{2}(I-A^{21}B^{1}A^{12}B^{2})^{-1}$$
$$=L^{12}$$
$$B^{2}A^{21}L^{11}=B^{2}A^{21}(B^{1}+B^{1}A^{12}B^{2}A^{21}B^{1}(I-A^{12}B^{2}A^{21}B^{1})^{-1})$$
$$=B^{2}A^{21}B^{1}(I-A^{12}B^{2}A^{21}B^{1})^{-1}$$
$$=L^{21}$$
$$B^{2}A^{21}L^{12}=B^{2}A^{21}B^{1}A^{12}B^{2}(I-A^{21}B^{1}A^{12}B^{2})^{-1}$$

Then

$$q^{zd}=\begin{pmatrix}B^{1}A^{12}B^{2}A^{21}B^{1}(I-A^{12}B^{2}A^{21}B^{1})^{-1} & L^{12}\\ L^{21} & B^{2}A^{21}B^{1}A^{12}B^{2}(I-A^{21}B^{1}A^{12}B^{2})^{-1}\end{pmatrix}\begin{pmatrix}y^{11}\\ y^{22}\end{pmatrix}$$
$$=q^{z}$$
$$q^{zd}=q^{z}=\begin{pmatrix}L^{11}-B^{1} & L^{12}\\ L^{21} & L^{22}-B^{2}\end{pmatrix}\begin{pmatrix}y^{11}\\ y^{22}\end{pmatrix}\qquad\text{(A.4)}$$

After adding equation (3) and（A.4）, we have $q^{t}+q^{zd}=Ly^{t}+(L-B)y^{d}$, which is the same as that in path 1.

A.2 Path 3 of deriving the method of computing the factor embodied in exports

In path 3, we start with the single country model, and consider final trade and intermediate trade as exogenous for a particular country's production system. Then, we deduce the method based on the international model through the production connections of the countries.

Without loss of generality, we discuss country 1 first. Let y^{t1} be country 1's vector of final export, then in the simplified two country case, $y^{t1}=y^{12}$; let z^{1} denotes the intermediate export of country 1, and y^{d1} represents the vector of country 1's domestic final demand, $y^{d1}=y^{11}$.For country 1, the three kinds of final demands resulting in the following output

$$q^{ft1}=(I-A^{11})^{-1}y^{t1}\qquad\text{(A.5)}$$

$$q^{fz1} = (I - A^{11})^{-1} z^1 \quad (A.6)$$
$$q^{d1} = (I - A^{11})^{-1} y^{d1} \quad (A.7)$$

The total of these three parts is the vector of total output of country 1. Then, the sum of the former two equations is the vector of output induced by country 1's exports based on the single country input-output model. Furthermore, the production process of country 1 requires the imports from country 2, therefore we have the following production connections between the two countries.

The vector of import from country 2 used for the production of country 1's final export is

$$z_2^{t1} = A^{21} q^{ft1} = A^{21}(I - A^{11})^{-1} y^{t1} \quad (A.8)$$

The vector of import from country 2 used for the production of country 1's intermediate export is

$$z_2^{z1} = A^{21} q^{fz1} = A^{21}(I - A^{11})^{-1} z^1 \quad (A.9)$$

The vector of import from country 2 used for the production of country 1's domestic final demand is

$$z_2^{d1} = A^{21} q^{d1} = A^{21}(I - A^{11})^{-1} y^{d1} \quad (A.10)$$

Then, the sum of the above three parts is the intermediate exports of country 2 to country 1, that is

$$z^2 = z_2^{t1} + z_2^{z1} + z_2^{d1} \quad (A.11)$$

We have

$$z^2 = z_2^{t1} + z_2^{z1} + z_2^{d1} = A^{21}(I - A^{11})^{-1}(y^{t1} + z^1 + y^{d1}) \quad (A.12)$$

Similarly, for country 2, we have

$$z^1 = z_1^{t2} + z_1^{z2} + z_1^{d2} = A^{12}(I - A^{22})^{-1}(y^{t2} + z^2 + y^{d2}) \quad (A.13)$$

Next, we deduce the method based on the international model using the above production connections of the two countries.

The vectors of output incurred by final export and intermediate export by using the single country model are

$$q^{ft} = \begin{pmatrix} B^1 & \\ & B^2 \end{pmatrix} \begin{pmatrix} y^{t1} \\ y^{t2} \end{pmatrix} \quad (A.14)$$

$$q^{fz} = \begin{pmatrix} B^1 & \\ & B^2 \end{pmatrix} \begin{pmatrix} z^1 \\ z^2 \end{pmatrix} \quad (A.15)$$

According to the production connections of the two countries, and considering intermediate trade as unknown variables, we can calculate them by solving the following equations

$$z^1 = z_1^{t2} + z_1^{z2} + z_1^{d2} = A^{12} B^2 (y^{t2} + z^2 + y^{d2}) \quad (A.16a)$$
$$z^2 = z_2^{t1} + z_2^{z1} + z_2^{d1} = A^{21} B^1 (y^{t1} + z^1 + y^{d1}) \quad (A.16b)$$

Then, we have

$$z^1 = (I - A^{12}B^2A^{21}B^1)^{-1}(A^{12}B^2y^{t2} + A^{12}B^2A^{21}B^1y^{t1} + A^{12}B^2y^{d2} + A^{12}B^2A^{21}B^1y^{d1})$$
$$z^2 = (I - A^{21}B^1A^{12}B^2)^{-1}(A^{21}B^1y^{t1} + A^{21}B^1A^{12}B^2y^{t2} + A^{21}B^1y^{d1} + A^{21}B^1A^{12}B^2y^{d2})$$

Or

$$\begin{pmatrix} z^1 \\ z^2 \end{pmatrix} = \begin{pmatrix} (I - A^{12}B^2A^{21}B^1)^{-1}A^{12}B^2A^{21}B^1 & (I - A^{12}B^2A^{21}B^1)^{-1}A^{12}B^2 \\ (I - A^{21}B^1A^{12}B^2)^{-1}A^{21}B^1 & (I - A^{21}B^1A^{12}B^2)^{-1}A^{21}B^1A^{12}B^2 \end{pmatrix} \begin{pmatrix} y^{t1} + y^{d1} \\ y^{t2} + y^{d2} \end{pmatrix}$$

The output brought by intermediate trade will be

$$\begin{pmatrix} q^{fz1} \\ q^{fz2} \end{pmatrix} = \begin{pmatrix} B^1 & \\ & B^2 \end{pmatrix} \begin{pmatrix} z^1 \\ z^2 \end{pmatrix}$$

$$= \begin{pmatrix} B^1 & \\ & B^2 \end{pmatrix} \begin{pmatrix} (I - A^{12}B^2A^{21}B^1)^{-1}A^{12}B^2A^{21}B^1 & (I - A^{12}B^2A^{21}B^1)^{-1}A^{12}B^2 \\ (I - A^{21}B^1A^{12}B^2)^{-1}A^{21}B^1 & (I - A^{21}B^1A^{12}B^2)^{-1}A^{21}B^1A^{12}B^2 \end{pmatrix} \begin{pmatrix} y^{t1} + y^{d1} \\ y^{t2} + y^{d2} \end{pmatrix}$$

$$= \begin{pmatrix} B^1A^{12}B^2A^{21}B^1(I - A^{12}B^2A^{21}B^1)^{-1} & L^{12} \\ L^{21} & B^2A^{21}B^1A^{12}B^2(I - A^{21}B^1A^{12}B^2)^{-1} \end{pmatrix} \begin{pmatrix} y^{t1} + y^{d1} \\ y^{t2} + y^{d2} \end{pmatrix}$$

$$= \begin{pmatrix} B^1A^{12}B^2A^{21}B^1(I - A^{12}B^2A^{21}B^1)^{-1} & L^{12} \\ L^{21} & B^2A^{21}B^1A^{12}B^2(I - A^{21}B^1A^{12}B^2)^{-1} \end{pmatrix} \begin{pmatrix} y^{t1} \\ y^{t2} \end{pmatrix}$$

$$+ \begin{pmatrix} B^1A^{12}B^2A^{21}B^1(I - A^{12}B^2A^{21}B^1)^{-1} & L^{12} \\ L^{21} & B^2A^{21}B^1A^{12}B^2(I - A^{21}B^1A^{12}B^2)^{-1} \end{pmatrix} \begin{pmatrix} y^{d1} \\ y^{d2} \end{pmatrix} \quad \text{(A.17)}$$

That is, the output caused by the intermediate trade is decomposed into two parts, attributed to the effects of final trade and domestic final demand respectively. The former is the effect of intermediate trade used for the production of final trade, and the latter is the effect of intermediate trade used for the production of domestic final demand. After adding the former to equation (A.14) , which is the output induced by final exports, we have

$$\begin{pmatrix} B^1 + B^1A^{12}B^2A^{21}B^1(I - A^{12}B^2A^{21}B^1)^{-1} & L^{12} \\ L^{21} & B^2 + B^2A^{21}B^1A^{12}B^2(I - A^{21}B^1A^{12}B^2)^{-1} \end{pmatrix} \begin{pmatrix} y^{t1} \\ y^{t2} \end{pmatrix}$$

$$= \begin{pmatrix} L^{11} & L^{12} \\ L^{21} & L^{22} \end{pmatrix} \begin{pmatrix} y^{t1} \\ y^{t2} \end{pmatrix} \quad \text{(A.18)}$$

This is the same result obtained from equation (3).

The second part of equation (A.17) is the output induced by the intermediate trade used for the domestic final demand products, which is

$$\begin{pmatrix} B^1A^{12}B^2A^{21}B^1(I - A^{12}B^2A^{21}B^1)^{-1} & L^{12} \\ L^{21} & B^2A^{21}B^1A^{12}B^2(I - A^{21}B^1A^{12}B^2)^{-1} \end{pmatrix} \begin{pmatrix} y^{d1} \\ y^{d2} \end{pmatrix}$$

$$= \begin{pmatrix} L^{11} - B^1 & L^{12} \\ L^{21} & L^{22} - B^2 \end{pmatrix} \begin{pmatrix} y^{d1} \\ y^{d2} \end{pmatrix} \quad \text{(A.19)}$$

The result is consistent with equation (12). Thus it is proved that the result of the third path is just the same as that of the former two paths. Therefore, they are equivalent, and can be verified by each other.

A.3 Generalizing the method: From two countries to N countries

In this section, we generalize the method from two countries to N countries. That is, the outputs induced by final trade and by intermediate trade are

$$q^t = Ly^t = \begin{pmatrix} L^{11} & L^{12} & \cdots & L^{1N} \\ L^{21} & L^{22} & \cdots & L^{2N} \\ \cdots & \cdots & \cdots & \cdots \\ L^{N1} & L^{N2} & \cdots & L^{NN} \end{pmatrix} \begin{pmatrix} y^{t1} \\ y^{t2} \\ \cdots \\ y^{tN} \end{pmatrix} \quad \text{(A.20)}$$

and

$$q^z = (L - B)y^d = \begin{pmatrix} L^{11} - B^1 & L^{12} & \cdots & L^{1N} \\ L^{21} & L^{22} - B^2 & \cdots & L^{2N} \\ \cdots & \cdots & \cdots & \cdots \\ L^{N1} & L^{N2} & \cdots & L^{NN} - B^N \end{pmatrix} \begin{pmatrix} y^{d1} \\ y^{d2} \\ \cdots \\ y^{dN} \end{pmatrix} \quad \text{(A.21)}$$

where N denotes the number of the countries, and $B^i = \left(I - A^{ii}\right)^{-1}$, $B = \begin{pmatrix} B^1 & & & \\ & B^2 & & \\ & & \ddots & \\ & & & B^N \end{pmatrix}$,

$y^{ti} = \sum_{j \neq i} y^{ij}$, and $y^{di} = y^{ii}$. Here, y^{ij} denotes the exports of country i to country j, and y^{ii} is country i's domestic final demand. The proof follows the idea of path 3. First, obtain the vector of intermediate trade by solving the following equations:

$$z^1 = A^{12}B^2z^2 + A^{13}B^3z^3 + \cdots + A^{1N}B^Nz^N + (A^{12}B^2(y^{t2} + y^{d2}) + A^{13}B^3(y^{t3} + y^{d3}) + \cdots + A^{1N}B^N(y^{tN} + y^{dN}))$$

$$z^2 = A^{21}B^1z^1 + A^{23}B^3z^3 + \cdots + A^{2N}B^Nz^N + (A^{21}B^1(y^{t1} + y^{d1}) + A^{23}B^3(y^{t3} + y^{d3}) + \cdots + A^{2N}B^N(y^{tN} + y^{dN}))$$

$$\cdots$$

$$z^N = A^{N1}B^1z^1 + A^{N2}B^2z^2 + \cdots + A^{N,N-1}B^Nz^N + (A^{N1}B^1(y^{t1} + y^{d1}) + A^{N3}B^3(y^{t3} + y^{d3}) + \cdots + A^{N,N-1}B^{N-1}(y^{t,N-1} + y^{d,N-1}))$$

Or in matrix

$$\begin{pmatrix} I & -A^{12}B^2 & \cdots & -A^{1N}B^N \\ -A^{21}B^1 & I & \cdots & -A^{2N}B^N \\ \vdots & \vdots & & \vdots \\ -A^{N1}B^1 & -A^{N2}B^2 & \cdots & I \end{pmatrix} \begin{pmatrix} z^1 \\ z^2 \\ \vdots \\ z^N \end{pmatrix}$$

$$= \begin{pmatrix} 0 & A^{12}B^2 & \cdots & A^{1N}B^N \\ A^{21}B^1 & 0 & \cdots & A^{2N}B^N \\ \vdots & \vdots & & \vdots \\ A^{N1}B^1 & A^{N2}B^2 & \cdots & 0 \end{pmatrix} \begin{pmatrix} y^{t1} + y^{d1} \\ y^{t2} + y^{d2} \\ \vdots \\ y^{tN} + y^{dN} \end{pmatrix}$$

Let

$$z = \begin{pmatrix} z^1 \\ z^2 \\ \vdots \\ z^N \end{pmatrix}, \bar{A} = \begin{pmatrix} 0 & A^{12} & \cdots & A^{1N} \\ A^{21} & 0 & \cdots & A^{2N} \\ \cdots & \cdots & & \cdots \\ A^{N1} & A^{N2} & \cdots & 0 \end{pmatrix}, \hat{A} = \begin{pmatrix} A^{11} & & & \\ & A^{22} & & \\ & & \ddots & \\ & & & A^{NN} \end{pmatrix}.$$

Then, $A = \hat{A} + \bar{A}$, and $B = (I - \hat{A})^{-1}$.

Therefore,

$$(I - \bar{A}B)z = \bar{A}B(y^t + y^d) \qquad \text{(A.22)}$$

Solving it, we have

$$z = (I - \bar{A}B)^{-1}\bar{A}B(y^t + y^d) \qquad \text{(A.23)}$$

The output induced by the intermediate trade is

$$q^{ze} = B(I - \bar{A}B)^{-1}\bar{A}B(y^t + y^d) = B\bar{A}B(I - \bar{A}B)^{-1}y^t + B\bar{A}B(I - \bar{A}B)^{-1}y^d \qquad \text{(A.24)}$$

From the above equation, the output induced by the intermediate trade used for the domestic final demand is

$$q^{zd} = B\bar{A}B(I - \bar{A}B)^{-1}y^d \qquad \text{(A.25)}$$

And the output brought by final exports is

$$q^t = By^t + B\bar{A}B(I - \bar{A}B)^{-1}y^t = (B + B\bar{A}B(I - \bar{A}B)^{-1}) = (I - B\bar{A})^{-1}By^t \qquad \text{(A.26)}$$

Since

$$L = \left(I - \hat{A} - \bar{A}\right)^{-1} = \left(I - \left(I - \hat{A}\right)^{-1}\bar{A}\right)^{-1}\left(I - \hat{A}\right)^{-1}$$

$$= (I - B\bar{A})^{-1}B$$

we have

$$q^{zd} = (L - B)y^d \qquad \text{(A.27)}$$

$$q^t = Ly^t \qquad \text{(A.28)}$$

By using the vector of direct factor coefficients, we can get the factor embodied in exports. Let

$\overline{D}$ be the vector of direct requirement coefficients of a certain factor, $\overline{D} = (\overline{D}^1 \quad \cdots \quad \overline{D}^N)$, where $\overline{D}^i$ is the vector of direct coefficients per output of the certain factor in country I, the factor embodied in exports is

$$F = \widehat{\widehat{D}}[Ly^t + (L - B)y^d] \tag{A.29}$$

where $\widehat{\widehat{D}} = \begin{pmatrix} \widehat{\overline{D}}^1 & & & \\ & \widehat{\overline{D}}^2 & & \\ & & \ddots & \\ & & & \widehat{\overline{D}}^N \end{pmatrix}$.