THE GENERALIZED DYNAMIC INPUT-OUTPUT

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1. Introduction

My paper, entitled as "The Generalized Dynamic Input-Output Principle ", has been advancing the Nonlinear Model of the Generalized Dynamic Input-Output System (NMGDIOS) and solving its optimal solution of Pontryagin maximum. The Generalized Dynamic Input-Output System(GDIOS), on the theoretical plane, which is going to be the result from the synthesis of the optimal control theory, the general reproduction, the productivity theory and the input-output analysis.

Facing a few of input-output tables, like the count of distinct digital map recorded around the world in a year or a certain period of the national economy. But, the input-output table of different years, however, can only indicate an isolated, static state at one time point of national economic situation, does not reveal which exists the inner and inevitable connection among different input-output tables.

In the current input and output system, time "t" is usually defined by the nature time. Such as the continuous dynamic input-output model by W.Leontief, But, as so long as limiting the dynamic input-output analysis on the basis of the nature time series, is to meet the inconsistency problem between the theoretical prediction and the actual event on the time step size or the time point etc., which does not have the inevitable reproducibility characteristics necessary to predict the future events.

The Production Function Structure Classification(PFSC) in my paper, as the result from my research for the evolution of labor instruments in human history, is the comparison study among laborers, his tools and its functions in historical evolution, abstracts have common characteristics and generalization. Therefore, carried out classifying sectors in accordance with the PFSC, to span of time step beyond nature and to get rid of the fixed time step and its sequence, thus makes the input-output analysis based on the qualitative change of productive forces in history, and is closely related to laborers, his tools and its functions, that come into being common rhythm or developed on the basis. So I use the PFSC to classify different structures for the input-output system and to approach the evolution of the varied typical input-output systems from lower type to higher one in accordance with different typical productive forces.

The GDIOS can be regard as making the choice in the varied time point, which connects with every type or each stage of productive forces and with the process of social reproductions, based upon the dynamic structure and balance conditions of the varied productive forces, so that the GDIOS has a few of characteristics, due to the PFSC, such as speeding up the economy development, shortening the production cycle and raising productivity etc.

In my paper, I try using the principle of Pontryagin maximum, by constructing the Hamiltonian, forming and solving the corresponding Jacobian matrix, calculating with in-homogeneous differential equations solution and general solution form, thus constitute the Pontryagin maximum solution for the Generalized Nonlinear Dynamic Input-output Model and deduce the coexistence of diverse types.

2.Building the first-order continuous dynamic input-output model

As the increment of a society total product, there is

$$\begin{aligned} X(t+1)-X(t) &= \Delta X(t+1) \\ &= A(t+1)\Delta x(t+1)+B(t+1)\Delta x(t+1)+\Delta c(t+1) \end{aligned}$$

Because of the goal, is the reproduction among different types on the basis of expanded reproduction, therefore there is

$$\Delta X(t+1)=X(t)-A(t)x(t)-B(t)x(t)-c(t)$$

3. As the continuity of the Generalized Dynamic Input-Output System (GDIOS)

GDIOS allows that the function or a control path are piecewise continuous, does not require that the control path is a un-interruption continuous.

The assumption is that: in case of piecewise continuous, uses the continuous time and the continuous function. There is

$$\lim_{\Delta t \to 0} \frac{\Delta X_j(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{X_j(t+\Delta t) - X_j(t)}{\Delta t} = \frac{dX_j(t)}{dt}$$

The investment demand of departments next year is

$$\dot{X}(t+1) = X(t) - A(t)x(t) - B(t)x(t) - c(t)$$

4. The discontinuity of GDIOS

Different subjects from different research Angle, can be definition and research for the discontinuity

The discontinuity of production process, may be due to a variety of reasons. Such as equipment overhaul, equipment upgrading, holidays, strike, war, and so on.

The discontinuity of GDIOS is mainly on the choice in each point in time, different types of productive forces and in the basis of the formation process of social reproduction for the dynamic input-output model .

The GDIOS can be regard as making the choice in the varied time point, which connects with every type or each stage of productive forces and with the process of social reproductions, based upon the dynamic structure and balance conditions of the varied productive forces.

5. The simple difference equation as the Linear Model of Dynamic Input-Output System (LMDIOS)

For the purpose of research, set a simple LMDIOS with the structure of productive forces.

Let the row vector x represent the n sectoral intermediate products,

$x_1, x_2, \ldots, x_n$, and h the corresponding row vector, $h_1, h_2, \ldots, h_n$, of deliveries to labor instruments, and y_j the corresponding row vector, of deliveries to the final net products.

Let the corresponding column vector is composed of the object of labor, labor instruments and Labor power L_i.

6. Building the specific I-O Table

<table>
<tr><td colspan="3" rowspan="2"></td><td>Intermediate product</td><td colspan="2">Final product</td><td rowspan="2">Total</td></tr>
<tr><td>Sector 1, 2,⋯,n</td><td>Sector 1, 2,⋯,n</td><td>Z_j</td></tr>
<tr><td rowspan="2">Material consumption</td><td>Object of labor</td><td>Sector 1, 2,⋯,n</td><td>$x_{11}, x_{12}, \cdots, x_{1n}$
$x_{21}, x_{22}, \cdots, x_{2n}$
⋮ ⋯ ⋮
$x_{n1}, x_{n2}, \cdots, x_{nn}$</td><td></td><td></td><td>$X_1, X_2, \cdots, X_n$</td></tr>
<tr><td>Labor instruments</td><td>Sector 1, 2,⋯,n</td><td></td><td>$h_{11}, h_{12}, \cdots, h_{1n}$
$h_{21}, h_{22}, \cdots, h_{2n}$
⋮ ⋯ ⋮
$h_{n1}, h_{n2}, \cdots, h_{nn}$</td><td>$Z_1, Z_2, \cdots, Z_n$</td><td>$H_1, H_2, \cdots, H_n$</td></tr>
<tr><td colspan="2">Labor</td><td>L_i</td><td></td><td></td><td></td><td>L_j</td></tr>
<tr><td colspan="3">Total</td><td>$X_1, X_2, \cdots, X_n$</td><td>$H_1, H_2, \cdots, H_n$</td><td>Z_i</td><td>X</td></tr>
</table>

In the value, the I-O table can be expressed as follows

$$x_{i1} + x_{i2} + \cdots + x_{in} + h_{i1} + h_{i2} + \cdots + h_{in} + z_j = X_i$$

Due to the labor materials can be subdivided into production tools and production facilities, H=F+E, there is

$$x_{i1} + x_{i2} + \cdots + x_{in} + f_{i1} + f_{i2} + \cdots + f_{in} + e_j + z_j = X_i$$

Because the column vector of E and Z both belong to the final product,E+Z=Y,and therefore can be classified as a merger

$$x_{i1} + x_{i2} + \cdots + x_{in} + f_{i1} + f_{i2} + \cdots + f_{in} + y_j = X_i$$

So its matrix form is

$$AX + QX + Y = X$$

In which:

$$f_{ij} = \sum_{i=1}^{n} q_{ij} X_j = QX$$

7. A few hypotheses and Analytic for LMDIOS

(1)There is only one type of the difference dynamic input-output system based upon one type of productive forces in each stage. (2)Every transformation of the difference dynamic input-output system among varied types has been completed in one cycle of production. (3)Every sector has only one kind of technicality in its production in each stage, which means that there are different in the direct consumption coefficients and occupation coefficients of the difference dynamic input-output system among varied types, but they are fixed in every stage. (4)There is no combination type of production. In other words, each productive activity or sector produces only one kind of the products, so the number of the product kind equals the number of productive sectors. (5)All inputs are completely depleted in one production use. (6)All input-output models are linear.

Also, it is an essential prerequisite of my investigation, as extending on the theoretical plane, that I restore the value into the socially necessary labor, thus the extending category of the investment, fixed assets and liquid assets etc. in general, in order that my investigation is suitable for various economic formations including the formation of the non-commodity economy. Therefore the symbol expressed numeral unit can be the amount of the socially necessary labor for the formation of the non-commodity economy as well as the magnitude of value expressed price or monetary unit for the formation of the commodity economy in this way.

8. The function-structure classification

What is called the analytic of the function-structure is a classification, which separates the structure of the productive forces into several components, equipment and institutes of the interrelation and mutual differentiation in accordance with its productive function.

Due to the factors composing the productive forces are various, I can begin my research from different angles in accordance with different classifications. Here, I would like to use the function-structure classification to separately research into the productive forces, which can be used as the research basis for the transformation of the difference dynamic I-O model among varied types[1].

9. Various types of social product structure have been evolving

TYPE	Sel. target	Sel. route	Info. control	Power transmit	Labor trunk	Power source	Function transmit	Act on object
Type 1	labor	labor	labor	labor	labor	labor	labor	tool
Type 2	labor	labor	labor	labor	labor	labor	tool	tool
Type 3	labor	labor	labor	labor	labor	labor	tool	tool
Type 4	labor	labor	labor	tool	tool	livestock	tool	tool
Type 5	labor	labor	labor	tool	tool	natural energy	tool	tool
Type 6	labor	labor	labor	mach.	mach.	mach.	mach.	mach.
Type 7	labor	labor	mach.	mach.	mach.	mach.	mach.	mach.

Throughout the evolution of the social product among varied types, accompanying with the development of productive forces from low level to high level, is the process of transferring from laborers to tools within the function of production.

10. Changes in social productive forces and its structure of varied types

(Ⅰ) Laborer — single tool — object
(Ⅱ) Laborer — inlay tool — object
(Ⅲ) Laborer — transmissible tool — compositional tool — object
(Ⅳ) Laborer — original generator — transmissible tool — compositional tool — object
(Ⅴ) Laborer — natural energy — gearing appliances — compositional tool — object
(Ⅵ) Laborer — motor mechanism — transmission mech. — working mech. — object
(Ⅶ) Laborer — control mech. — motor mech. — transmission mech. — working mech. — object

Throughout the evolution of the social product among varied types, accompanying with the development of productive forces from low level to high level, is the process of transferring from laborers to tools within the function of production.

11. Essential types

The first type was the primitive gathering economy, in which the primitive man gathered natural edibles as his main source of livelihood means and resisted natural enemies, in the embryonic hunting, as his secondary source of subsistence means. This type of the economy, which came into being at the early Pleistocene and lasted for the early period and middle period of the Paleolithic Age, included the Oldowan culture of the Pebble tool, the Acheulean hand-axe culture and the Mousterian culture in archaeology. The trait of this typical economy are that man began making the single tool(*st*) without handle, such as the Oldowan pebble tool and the Acheulean hand-axe, and using them to labor. As fellowing:

$$
\begin{aligned}
&\dim(f_{\{nt\}}+f_{\{st\}}) \\
&= \dim f_{\{nt\}} + \dim f_{\{st\}} - (\dim f_{\{nt\}} \cap \dim f_{\{st\}}) \\
&= f_{\{st\}}
\end{aligned}
$$

The structure of its total product as I-O system in Type 1:

$$\overset{\{I\}\quad I}{X} = [x_{i1} + x_{i2} + \cdots + x_{in}] + f_{\{st\}} + y_j$$

The second type was the primitive gathering-hunting economy, in which the primitive man gathered wild eatable plants and hunted wild animals and caught fishes as his sources of subsistence means. It came into being at the latter period of the Paleolithic Age and lasted throughout the Mesolithic Age. It has a few obvious features that the single tool was replaced by the inlay tool (*it*) with its handle (*ha*), such as spears, bows and arrows.

$$\begin{aligned}&\dim(f_{\{st\}}+f_{\{ha\}})\\&=\dim f_{\{st\}}+\dim f_{\{ha\}}-\dim(f_{\{st\}}\cap f_{\{ha\}})\\&=f_{\{it\}}\end{aligned}$$

The structure of its total product as I-O system in Type 2:

$$\overset{\{II\}}{X}=\overset{II}{[}\ x_{i1}+x_{i2}+\cdots+x_{in}]+f_{\{it\}}+y_j$$

The third type was the cultivation and domestication economy, which was called the Neolithic Age by archaeology. The structure of its total product as:

$$\overset{\{III\}}{X}=\overset{III}{[}\ x_{i1}+x_{i2}+\cdots+x_{in}]+f_{\{ct\}}+f_{\{tt\}}+y_j$$

The difference between the previous two types and this one was that the simple compositional tool (*ct*), such as hoes, substituted for the inlay tool, and the transmissible tool, as belts and drilling bows, came into being and the power was transmitted throughout the transmissible tool from man to the compositional tool in very simple style.

$$\begin{aligned}&\dim(f_{\{ct\}}+f_{\{tt\}})\\&=\dim f_{\{ct\}}+\dim f_{\{tt\}}-\dim(f_{\{ct\}}\cap f_{\{tt\}})\\&=f_{\{ct\}}+f_{\{tt\}}\end{aligned}$$

The fourth type was the slavery economy, in which slaves were used on a large scale as the motive power of production at the early stage of the civilization. The structure of its total product as input-output system in Type 4:

$$\overset{\{IV\}\quad IV}{X} = [\ x_{i1} + x_{i2} + \cdots + x_{in}\ ] + f_{\{ct\}} + f_{\{tm\}} + y_j$$

It had a clear feature that the tool was not used directly by laborers' hand, but the tool was given by the impetus with gearing appliances, such as wheels, levers connecting rods or transmission shafts etc., because people invented the prime mover or the original generator (*og*) of forces which can be promoted by impetus and was linked up various transmission tools or gearing appliances. Some miracles of magnificent structures were made, such as the famous pyramids, the imperial palace in the ancient Assyrian and the Athenian Parthenon temple, etc.

$$\begin{aligned}
&\dim[(f_{\{ct\}} + f_{\{tt\}}) + f_{\{og\}}] \\
&= \dim[f_{\{ct\}} + f_{\{tt\}}] + \dim f_{\{og\}} - \dim[(f_{\{ct\}} + f_{\{tt\}}) \cap f_{\{og\}}] \\
&= f_{\{ct\}} + f_{\{tt\}} + f_{\{og\}} \\
&= f_{\{ct\}} + f_{\{tm\}}
\end{aligned}$$

The fifth type was the era, in which the natural forces such as animal draws, water-power and wind power were put to use widespread. It had distinct feature that the various natural forces and human power were mutually complementary and combinative, thus making up the basic motive power of production.

$$\overset{\{V\}\quad V}{X} = [\ x_{i1} + x_{i2} + \cdots + x_{in}\ ] + f_{\{ct\}} + f_{\{tm\}} + f_{\{ne\}} + y_j$$

The sixth type was the machine industry in modern time.

$$\overset{\{VI\}\quad VI}{X} = [\ x_{i1} + x_{i2} + \cdots + x_{in}\ ] + f_{\{mm\}} + f_{\{tm\}} + f_{\{wm\}} + y_j$$

The main feature of this type was that the classical integrated machinery consisted of the motor mechanism, the transmission mechanism and the working mechanism.

$$\begin{aligned}&\dim[(f_{\{ct\}}+f_{\{tm\}}+f_{\{ne\}})+f_{\{tc\}}]\\&=\dim^{V}[f_{\{ct\}}+f_{\{tm\}}+f_{\{ne\}}]+\dim f_{\{tc\}}-\dim[(f_{\{ct\}}+f_{\{tm\}}+f_{\{ne\}})\cap f_{\{tc\}}]\\&=f_{\{wm\}}+f_{\{tm\}}+f_{\{mm\}}\end{aligned}$$

The seventh type is now called the age of new industry revolution or the information network economy.

$$\overset{\{VII\}}{X}=\overset{VII}{[}\ x_{i1}+x_{i2}+\cdots+x_{in}]+f_{\{mm\}}+f_{\{tm\}}+f_{\{wm\}}+f_{\{cm\}}+y_j$$

It has a feature that the control mechanism is coming into being and the contemporary automatic machinery consists of the control mechanism, motor mechanism, the transmission mechanism and the working mechanism.

$$\begin{aligned}&\dim^{VII}[(f_{\{wm\}}+f_{\{tm\}}+f_{\{mm\}})+(f_{\{cm\}})]\\&=\dim^{VI}[f_{\{wm\}}+f_{\{tm\}}+f_{\{mm\}}]+\dim f_{\{cm\}}-\dim[(f_{\{wm\}}+f_{\{tm\}}+f_{\{mm\}})\cap f_{\{cm\}}]\\&=f_{\{wm\}}+f_{\{tm\}}+f_{\{mm\}}+f_{\{cm\}}\end{aligned}$$

12. COMPARATIVE RESEARCH

First, in the field of the structure, the production tool, as the main body of labor instruments in every stage, can always be divided into two basic parts:

One part is *Fo*, which can be generalized the result that the Former labor function have been objectified by the property of some substances, as the basis or prerequisite of the transformation of the productive forces among different types. If let the natural tool take as the starting point for my investigation, the natural tool, single tool, inlay tool, primitive production means, classic production means, traditional production means and classical integrated machinery consist of the basis or prerequisite of the transformation of the productive forces among varied stages.

Another part is the new quality, *Nt*, which can be generalized the outcome of the New transformation of the labor function, leads to the replacement of the productive forces from a lower type to a higher one. Such as, the single tool, handle, transmissible tool, original generator, natural resource, working institute and controlling institute, have ever been the new quality and consists of the sufficient and necessary condition of the transformation of the productive forces among varied types.

Second, in the field of the function, from the primitive single tool, inlay tool and compositional tool to the classical integrated machinery and contemporary automatic machinery, they not only go through a changing process from simply to comprehensive in their structure, but also undertake more productive functions. As the coming and developing of the production tool with the new transformation of the labor function in every stage, the former labor functions taken by labors, such as his acting directly upon the labor object, transforming energy and function, acting as the limbs and truck of the laboring process, controlling and regulating the information of the production process, have successively be replaced by production tools. Thus, the series of above expressions of the dynamic function-structure of labor instruments can be generalized by the following: $F = Fo + Nt$

Final, with the productive function transforming from laborers to tools, the relation between laborers and labor instruments is changing, the division of productive functions between laborers and tools is resolved and combined to form on the new basis again, the division of works is shared and cooperated in the new economic and technical basis. Furthermore, with the replacement of the social productive sector and social division, the vast amount of population has transferred again and again, to form new work structure for labor powers.

$$\overset{\Omega+\tau}{F} = \overset{\Omega+\tau}{\{} Fo+Nt\}=\overset{\Omega}{\{} Fo\}+\overset{\tau}{\{}Nt\}$$

13. The Linear Model of Dynamic Input-Output System (LMDIOS)

Comprehensive above, LMDIOS can be expressed as follows:

$$\overset{\{\Omega+\tau\}}{X} = x_{1n}+x_{2n}+\cdots+x_{in}+\overset{\Omega}{F}_{\{Fo\}}+\overset{\tau}{f}_{\{Nt\}}+y_j \quad (i,j=1,2,\cdots,n)$$

Its matrix form is

$$\overset{\{\Omega+\tau\}}{X} = AX + QX_{\{Fo+Nt\}} + Y_j \qquad (i,j=1,2,\cdots,n)$$

In equation, Ω is a cardinal number, representing the existing type of the input-output system before the transformation, τ is an alternative number of zero or of one only, representing the transformation of the difference dynamic input-output system among varied types and whether *Nt* occurs or not.

As τ=0 and Ω is a certain natural number from one to seven, which means new productive sector does not occur and represents the transformation of the difference dynamic input-output system among varied types not exist, so the subscript is *Fo*.

Therefore, new accumulation or investment is constantly invested into the original productive structure, forming the reproduction just in the extension. Under this circumstance, the subscript can be eliminated from the above equation.

As τ=1, Ω is a certain natural number form one to seven, which means the transformation of the difference dynamic input-output system has been occurring from a lower type to a higher one.

The subscript is *Fo*+*Nt*, which represents the new structure of the social production is coming into being and making the accumulation or investment transform in accordance with a new productive structure. Now the subscript, as the time lag or production circle, represents a lot of circles needed in the process of completing this transformation.

The composition and dimension of the dynamic input-output system are varying with the new productive sector coming into being. As the relationship between laborers and labor instruments alters, the labor division is developing and the new labor structure ($L_1, L_2, \ldots L_n$) is coming into being.

Furthermore, the transformation of the social productive structure and of the product structure changes the social consumptive structure (Y_1, Y_2， $\ldots\ldots Y_n$). So, the difference dynamic input-output model has finished a transformation from a lower type to a higher one.

14. The general solution of LMDIOS

If that is a singular matrix, its general solution for generalized inverse is

$$\overset{\{\Omega+\tau\}}{X} = G^{+}Y + (I - G^{+}G)c$$

In which, c is an arbitrary column vector within n-dimensions.

15. Introduce the concept of the discount rate

We assume that the population would grow in "g", but the population size of the moment must be discounted, before the social utility would be aggregated at any point in the future. If the population size of the moment would be the discount rate, the objective function could be represented with the following form[2]:

$$\int_0^\infty U(c)L(t)e^{-rt}dt = \int_0^\infty U(c)L_0 e^{gt}e^{-rt}dt$$
$$= L_0\int_0^\infty U(c)e^{-(r-g)t}dt$$

In order to ensure convergence, we can assume: $\rho \equiv r - g$, this is equivalent to a positive discount rate $r - g > 0$. If selecting the unit as

$L_0 = 1$, this functional can be simplified to

$$\int_0^{\infty} U(c)e^{-pt}dt \quad (\rho \equiv r - g > 0)$$

16. The Nonlinear Model of the Generalized Dynamic Input-Output System (NMGDIOS)

Sum up the above, combining with the type sequence of the above total product and the structure of investment in labor instruments, NMGDIOS can be formulated as follow

$$\max U = \int_0^{\infty} U[c(t)]e^{-pt}dt$$

$$s.t. \begin{cases} \dot{X}(t+1) = \overset{\{\Omega+\tau\}}{X(t)} - A(t)x(t) - B(t)\overset{\{F+N\}}{x(t)} - c(t) \\ c(t) = e^{-pt}\eta_k \quad (\eta_k \in R^n, \quad k = 1, \cdots, n) \\ X(t) \geq 0, \quad X(t_0) = X_0, \quad (t = 1, 2, \ldots, n-1) \\ 0 \leq c(t) \leq X(t) \end{cases}$$

In which:

A--the coefficient matrix of the direct consumption;

B--the coefficient matrix of the labor instrument or fixed asset;

c--the coefficient matrix of the final net product;

U--the total output of the final net product in t year;

ρ--the discount rate.

Its objective function is the maximum output in the plan stage. Also, its constraint is that the total output,from the base to plan period, should be greater than the middle demand, investment demand and the final net product demand.

17. A few hypotheses for NMGDIOS

With a view to facilitating research, making a few hypothesis: ⑴There is only one type of the difference dynamic input-output system based upon one type of productive forces in each stage. ⑵Every transformation of the difference dynamic input-output system among varied types has been completed in one cycle of production. ⑶Every sector has only one kind of technicality in its production in each stage, which means that there are different in the direct consumption coefficients and occupation coefficients of the difference dynamic input-output system among varied types, but they are fixed in every stage. ⑷There is no combination type of production. In other words, each productive activity or sector produces only one kind of the products, so the number of the product kind equals the number of productive sectors. ⑸All inputs are completely depleted in one production use.

Also, making a hypothesis: The new productive sector does not occur and the transformation of the difference dynamic input-output system among varied types not exist, so the superscript of is Ω only. Under this circumstance, the superscript of NMGDIOS can be eliminated from the above equation. So it is return as follow

$$\dot{X}(t+1) = X(t) - A(t)x(t) - B(t)x(t) - c(t)$$

18. Constructing the current-value Hamiltonian Function

$$H = U[c(t)]e^{-pt} + \varpi\{X(t) - [A(t) + B(t)]x(t) - c(t)\}$$

In which: ϖ --Lagrange multiplier, which is used as the displacement variable of the current-value.

Let ϖ join the above Hamiltonian equation, so as to have

$$H = e^{-pt}\{U[c(t)] + \lambda[X(t) - A(t)x(t) - B(t)x(t) - c(t)]\}$$

Solving the above Hamiltonian equation, we can get

$$\begin{cases} \dot{X}(t+1) = X(t) - A(t)x(t) - B(t)x(t) - c(t) \\ \dot{\lambda}(t) = \lambda(t)\left[\rho - X'(t) + A(t) + B(t)\right] \end{cases}$$

In its optimal trajectory, which shall meet the above system of differential equations[2][3].

19. Constructing related Jacobian matrix

At the E equilibrium values, it can be composed of the Jacobian matrix as follows

$$J_E = \begin{bmatrix} \dfrac{\partial \dot{X}}{\partial x} & \dfrac{\partial \dot{X}}{\partial \lambda} \\ \dfrac{\partial \dot{\lambda}}{\partial x} & \dfrac{\partial \dot{\lambda}}{\partial \lambda} \end{bmatrix}_{(\bar{x},\bar{\lambda})}$$

In which: $\rho = X'(t) - A(t) - B(t)$. Pay attention to $\lambda = U'(c)$, so as to have

$$\begin{cases} \left.\dfrac{\partial \dot{X}}{\partial x}\right|_{E(\bar{x},\bar{\lambda})} = X'(t) - A(t) - B(t) = \rho \\ \left.\dfrac{\partial \dot{X}}{\partial \lambda}\right|_{E(\bar{x},\bar{\lambda})} = 0 \\ \left.\dfrac{\partial \dot{\lambda}}{\partial x}\right|_{E(\bar{x},\bar{\lambda})} = -U'(c)X''(t) \\ \left.\dfrac{\partial \dot{\lambda}}{\partial \lambda}\right|_{E(\bar{x},\bar{\lambda})} = \rho - X'(t) + A(t) + B(t) = 0 \end{cases}$$

As a result, which can get its Jacobian matrix, according to the Cayley - Hamilton law, which can know that its characteristic equation is

$$
\begin{aligned}
|J_E| &= \begin{vmatrix} \rho & 0 \\ -U'(c)X''(t) & 0 \end{vmatrix} \\
&= \begin{vmatrix} s-\rho & 0 \\ U'(c)X''(t) & s \end{vmatrix} \\
&= s^2 - \rho s = 0
\end{aligned}
$$

Here, we need to pay attention to the value range of NMGDIOS, which is

$$1 \geq [\rho = X'(t) - A(t) - B(t)] \geq 0$$

In which, $A(t)$ and $B(t)$ are the non-singular matrix.

Assuming that $A(t)$ and $B(t)$ are non-negative matrixes and can be differential, reversible and observable, the solution solved from the determinant of the Jacobian matrix, must be the optimal linear approximation value of the extreme surface equation in its extreme value point, while NMGDIOS as a nonlinear model.

If the above Jacobian matrix is the non-singular matrix, it can be solved and get two groups of the characteristic root:

$$\begin{cases} s_1 = X'(t) - A(t) - B(t) \\ s_2 = 0 \end{cases}$$

As the optimal linear approximation value endlessly approximate to its extreme value point. In which, $s_2 = 0$, its state of being degenerated and given up.

Let $G(t) = A(t) + B(t)$ into s1, there is $X'(t) = G(t)$. This is a first-order homogeneous differential equations. Its solution is the equation with the base solution matrix as following

$$X = e^{\lambda_i t}$$

It need to meet the initial condition $\varphi(0) = x_0$ and to has the solution

$$\varphi(t) = e^{Gt} x_0 \quad (t \in R)$$

20. To prove

According to the law of decomposition pedigrees, there is

$$e^{Gt} = \sum_{k=1}^{s} \sum_{j=0}^{m_k - 1} t^j e^{G_k} E_{kj}$$

So the only solution of its initial value is

$$\begin{aligned} x(t) &= \sum_{k=1}^{s} \sum_{j=0}^{m_k - 1} t^j e^{G_k} E_{kj} x_0 \\ &= \sum_{k=1}^{s} \left(z_{k0} + z_{k1} + \cdots + z_{k,m_k - 1} t^{m_k - 1} \right) e^{\lambda_k t} \end{aligned}$$

In which: $z_{kj} = E_{kj} x_0; \quad j = 1, \ldots, m_k - 1; \quad k = 1, 2, \ldots, s$

While G is a simple matrix, the above equation degenerate as

$$x(t) = \sum_{k=1}^{s} z_{k0} e^{\lambda_k t}$$

(Its proving complete.)

21. As a special solution of the in-homogeneous differential equations with general solution

In NMGDIOS,the final net product of departments can be shown in the time index polynomial as following

$$C_i(t)=e^{-\rho_1 t}\eta_{i1}+e^{-\rho_2 t}\eta_{i2}+\cdots+e^{-\rho_k t}\eta_{ik}\quad (i=1,2,\cdots,n)$$

Its special solution can be expressed as

$$X_i(t)=\varsigma_{i1}e^{-\rho_1 t}+\varsigma_{i2}e^{-\rho_2 t}+\cdots+\varsigma_{ik}e^{-\rho_k t}\quad (i=1,2,\cdots,n)$$

According to the method of constant variation, NMGDIOS can be expressed as the general solution of non-homogeneous differential equations

$$y=\sum_{k=1}^{\infty}e^{\lambda_k t}\left\{\varsigma_k+\int_{t_0}^{t}\left(\sum_{k=1}^{\infty}e^{\lambda_k t}\right)^{-1}\left[e^{-pt}\eta_k(t)\right]dt\right\}$$

After simplifying, In the value of $\lambda_k(t)+\rho(t)$ is within the scope of positive discount rates; $\eta_k(t)$ is a bound matrix of constant coefficients, the general solution of NMGDIOS can be expressed as

$$y=e^{\lambda_k t}\varsigma_k+e^{\lambda_k t}\frac{\eta_k(t)}{\lambda_k(t)+\rho(t)}$$

22. Coexisting two types of NMGDIOS

There are three kinds of situations:

(1) the transformation of the difference dynamic I-O system among varied types not exist, so its begin and end state are in the same type, so that we can set its head status as

$$X(t_0)=x_0$$

and set its end state as

$$\Psi[x(t_f),t_f]=0$$

(2) The transformation of the difference dynamic input-output system has been occurring, so its begin and end state are not in the same type, so that we can set its head status as

$$X\big|_{t_0=\{\Omega+0\}} = \overset{\{\Omega+0\}}{X_0} = 1$$

and set its end state as

$$\overset{\{\Omega+1\}}{\Psi} [x(t_f),t_f] = \overset{\{\Omega+1\}}{X_{t_f}} = \gamma$$

(3) There are two type or multiple types in the same period. We can set its head status as

$$\overset{\{\Omega\&\Omega+1\}}{X(t_0)} = 0$$

and set its end state as

$$\overset{\{\Omega+1\}}{\Psi} [x(t_f),t_f] = \gamma$$

23. There are two types in the same period

With a view to facilitating research, assuming that there are only two departments and use two types of labor instruments (or fixed assets):

$$\begin{cases} \dot{X}_1(t+1) = \overset{\{V\}}{X_1(t)} - A_1(t)x(t) - B_1(t)\overset{\{F+0\}}{x(t)} - c_1(t) \\ \dot{X}_2(t+1) = \overset{\{VI\}}{X_2(t)} - A_2(t)x(t) - B_2(t)\overset{\{F+N\}}{x(t)} - c_2(t) \end{cases}$$

So we can construct their Hamiltonian Function as follows

$$\begin{cases} H_1 = U_1[c(t)]e^{-pt} + \varpi\left[\overset{\{V\}}{X}(t) - A(t)x(t) - B_1(t)\overset{\{\Omega+0\}}{x(t)} - c_1(t)\right] \\ H_2 = U_2[c(t)]e^{-pt} + \varpi\left[\overset{\{VI\}}{X}(t) - A(t)x(t) - B_2(t)\overset{\{\Omega+\tau\}}{x(t)} - c_2(t)\right] \end{cases}$$

and

$$\begin{cases} H_1 = e^{-pt}\left\{U_1[c(t)] + \lambda\left[\overset{\{V\}}{X}(t) - A(t)x(t) - B_1(t)\overset{\{\Omega+0\}}{x(t)} - c(t)\right]\right\} \\ H_2 = e^{-pt}\left\{U_2[c(t)] + \lambda\left[\overset{\{VI\}}{X}(t) - A(t)x(t) - B_2(t)\overset{\{\Omega+\tau\}}{x(t)} - c(t)\right]\right\} \end{cases}$$

Solving the above Hamiltonian equations, we can get

$$\begin{cases} \dot{\lambda}_1(t) = \lambda_1(t)\left[\rho_1 - \overset{\{V\}}{X_1'}(t) + A_1(t) + B_1(t)\right] \\ \dot{X}_1(t+1) = \overset{\{V\}}{X_1}(t) - A_1(t)x(t) - B_1(t)\overset{\{V\}}{x(t)} - c_1(t) \end{cases}$$

and

$$\begin{cases} \dot{\lambda}_2(t) = \lambda_2(t)\left[\rho_2 - \overset{\{VI\}}{X_2'}(t) + A_2(t) + B_2(t)\right] \\ \dot{X}_2(t+1) = \overset{\{VI\}}{X_2}(t) - A_2(t)x(t) - B_2(t)\overset{\{VI\}}{x(t)} - c_2(t) \end{cases}$$

In its optimal trajectory, which shall meet the above system of differential equations. While two subsystems separately tend to its extreme value point so that there are

$$\begin{cases} \dot{\lambda}_1 = 0 & \rho_1 = \overset{\{V\}}{X_1'}(t) - A_1(t) - B_1(t) \\ \dot{X}_1(t+1) = 0 & \overset{\{V\}}{X_1}(t) = A_1(t)x(t) + B_1(t)\overset{\{V\}}{x(t)} + c_1(t) \end{cases}$$

and

$$\begin{cases} \dot{\lambda}_2 = 0 & \rho_2 = \overset{\{VI\}}{X'_2}(t) - A_2(t) - B_2(t) \\ \dot{X}_2(t+1) = 0 & \overset{\{VI\}}{X_2}(t) = A_2(t)x(t) + B_2(t)\overset{\{VI\}}{x(t)} + c_2(t) \end{cases}$$

24. The total output appears a constant value as the threshold of its type

While two sub-systems separately reach its extreme value point, the total outputs have reached its extreme value point and the consumption level of average person also have peaked.

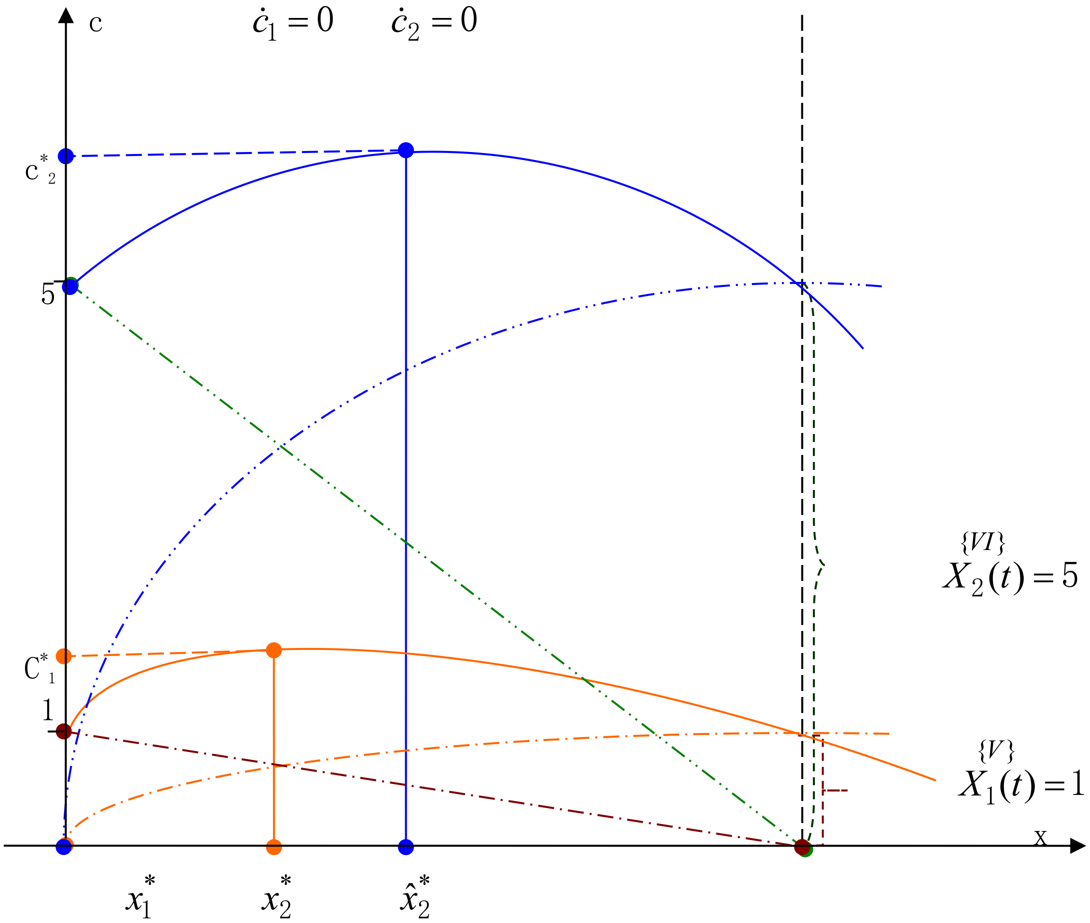
$$\begin{cases} \dot{X}_1(t+1) = 0 & \overset{\{\Omega+0\}}{X_1}(t) = A(t)x(t) - B(t)\overset{\{\Omega+0\}}{x(t)} - c(t) = 1, \quad (\tau = 0) \\ \dot{X}_2(t+1) = 0 & \overset{\{\Omega+1\}}{X_2}(t) = A(t)x(t) - B(t)\overset{\{\Omega+1\}}{x(t)} - c(t) = \gamma, \quad (\gamma > 1) \end{cases}$$

For example, as is known to all, the Janny Machine which Hagerives invented in 1760s.The efficiency of Jenny machine was five or six times as old spinning wheel.

In material structure, if subsystem 1 of its total output was 1 unit, therefore subsystem 2 of its total output could be 5 unit. Only 5 times to simulate its growth (such as following)

In value, there are

$$\begin{cases} \dot{X}_1(t+1) = 0 & \overset{\{V\}}{X_1}(t) = A_1(t)x(t) + B_1(t)\overset{\{V\}}{X}(t) + c_1(t) = 1/6 \\ \dot{X}_2(t+1) = 0 & \overset{\{VI\}}{X_2}(t) = A_2(t)x(t) + B_2(t)\overset{\{VI\}}{X}(t) + c_2(t) = 5/6 \end{cases}$$



In which:

$$\overset{\{VI\}}{X_2}(t) = A_2(t)x(t) + B_2(t)\overset{\{\Omega+1\}}{x(t)} + c_2 \quad \dot{X}_2 = 0$$

$$\overset{\{V\}}{X_1}(t) = A_1(t)x(t) + B_1(t)\overset{\{\Omega+0\}}{x(t)} + c_1 \quad \dot{X}_1 = 0$$

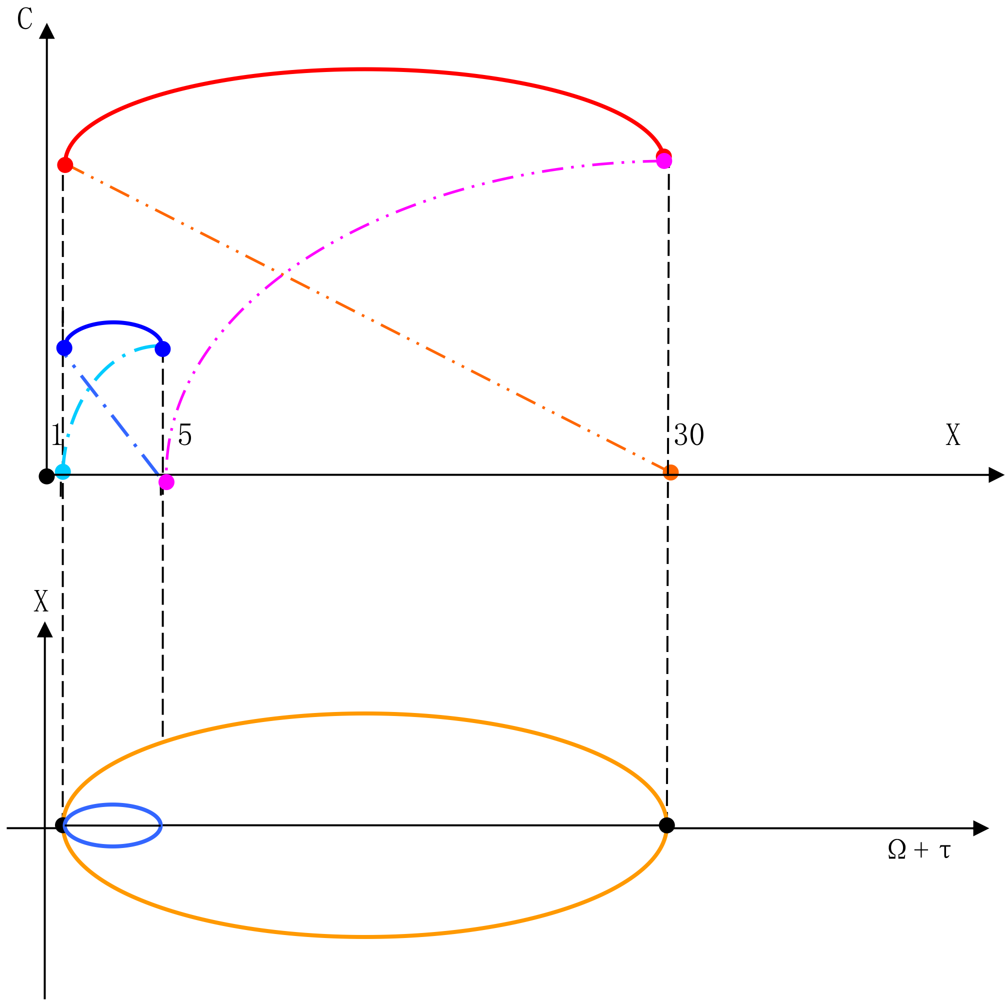
$U_2(c)e^{-pt}$ $U_1(c)e^{-pt}$

$$A_2(t)x(t) + B_2(t)\overset{\{VI\}}{x(t)} + c_2$$

$$A_1(t)x(t) + B_1(t)\overset{\{V\}}{x(t)} + c_1$$

Also known as “mule machine”, It combines the “Jenny machine” and the hydraulic characteristics of spinning machines.The efficiency of mule machine was thirty times more than old spinning wheel.

In material structure, if subsystem 1 of its total output was 1 unit, therefore subsystem 2 of its total output could be 30 unit (such as following).



In which:

$$\overset{\{VI\}}{X_2}(t) = A_2(t)x(t) + B_2(t)\overset{\{\Omega+1\}}{x(t)} + c_2 \quad \dot{X}_2 = 0$$

$$\overset{\{V\}}{X_1}(t) = A_1(t)x(t) + B_1(t)\overset{\{\Omega+0\}}{x(t)} + c_1 \quad \dot{X}_1 = 0$$

$U_2(c)e^{-pt}$ $U_1(c)e^{-pt}$

$A_2(t)x(t)+B_2(t)\overset{\{VI\}}{x}(t)+c_2$

$A_1(t)x(t)+B_1(t)\overset{\{V\}}{x}(t)+c_1$

In value, there are

$$\begin{cases} \dot{X}_1(t+1)=0 \quad \overset{\{V\}}{X}_1(t)=A_1(t)x(t)+B_1(t)\overset{\{V\}}{X}(t)+c_1(t)=1/31 \\ \dot{X}_2(t+1)=0 \quad \overset{\{VI\}}{X}_2(t)=A_2(t)x(t)+B_2(t)\overset{\{VI\}}{X}(t)+c_2(t)=30/31 \end{cases}$$

In the domestic trade in goods and the free movement of laborer under a single market, the discount rate between different departments can't be a big difference during the same period. But due to the different fixed assets for production,which lead to the difference of their total outputs between two sub-systems.There is

$$\overset{\{V\}}{X}_1(t) \le \overset{\{VI\}}{X}_2(t)$$

They can be composed of the Jacobian matrix as follows

$$\begin{cases} J_1=\begin{pmatrix} \dfrac{\partial \dot{X}_1}{\partial x} & \dfrac{\partial \dot{X}_1}{\partial \lambda} \\ \dfrac{\partial \dot{\lambda}_1}{\partial x} & \dfrac{\partial \dot{\lambda}_1}{\partial \lambda} \end{pmatrix}_{(\bar{x},\bar{\lambda})} = \begin{pmatrix} \overset{\{V\}}{X'_1}(t)-A_1(t)-B_1(t) & 0 \\ -U'_1(c)\overset{\{V\}}{X''_1}(t) & 0 \end{pmatrix} \\ J_2=\begin{pmatrix} \dfrac{\partial \dot{X}_2}{\partial x} & \dfrac{\partial \dot{X}_2}{\partial \lambda} \\ \dfrac{\partial \dot{\lambda}_2}{\partial x} & \dfrac{\partial \dot{\lambda}_2}{\partial \lambda} \end{pmatrix}_{(\bar{x},\bar{\lambda})} = \begin{pmatrix} \overset{\{VI\}}{X'_2}(t)-A_2(t)-B_2(t) & 0 \\ -U'_2(c)\overset{\{VI\}}{X''_2}(t) & 0 \end{pmatrix} \end{cases}$$

Which can know that their characteristic equation, according to the Cayley - Hamilton law, there are

$$
\begin{cases}
|J_1| = \begin{vmatrix} r - \left[\overset{\{V\}}{X_1'}(t) - A_1(t) - B_1(t) \right] & 0 \\ U_1'(c)\,\overset{\{V\}}{X_1''}(t) & r \end{vmatrix} \\
|J_2| = \begin{vmatrix} s - \left[\overset{\{VI\}}{X_2'}(t) - A_2(t) - B_2(t) \right] & 0 \\ U_2'(c)\,\overset{\{VI\}}{X_2''}(t) & s \end{vmatrix}
\end{cases}
$$

If the above Jacobian matrix are the non-singular matrix, they can be solved and got their characteristic root

$$
r\begin{cases} r_1 = \overset{\{V\}}{X_1'}(t) - A_1(t) - B_1(t) \\ r_2 = 0 \end{cases} \Rightarrow \overset{\{V\}}{x_1} = e^{\lambda_1 t}
$$

$$
s\begin{cases} s_1 = \overset{\{VI\}}{X_2'}(t) - A_2(t) - B_2(t) \\ s_2 = 0 \end{cases} \Rightarrow \overset{\{VI\}}{x_2} = e^{\lambda_2 t}
$$

Their linear combination is

$$X = \varsigma_1 e^{\lambda_1 t}\xi_1 + \varsigma_2 e^{\lambda_2 t}\xi_2$$

According to the Perron-Froubenius Law, if $(I - A)^{-1}B$ is the irreducible matrix, there is a non-negative real characteristic root λ_2 within the maximum modulus among its characteristic roots. λ_2 would be obviously on behalf of the department in the more advanced types

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