STRUCTURAL DECOMPOSITION AND SHIFT-SHARE ANALYSES: LET THE PARALLELS CONVERGE

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ABSTRACT. Intuitively, structural decomposition analysis (SDA) demonstrates strong similarities to shift-share analysis (SSA). Both examine the effects of industry shifts due to growth (or decline) and some sort of difference in industry shares. But SSA works its shares across space while SDA works its shares again across industries via technology change (fabrication effects). Suffice it to say, SDA and SSA are related, and this chapter will formally combine the two disparate strands of literature. In particular, it will show how changes in regional growth differentials can be included into a structural decomposition analysis. Moreover, the present availability of a large number of input-output table panels appears to enable the detection of even more parallels between the two approaches. Between the formalization of the SSA-SDA relationship and the available I-O data, a wide range of new, policy-relevant empirical applications is possible. The chapter will conclude by suggesting a few avenues for future research.

1. Introduction

Both structural decomposition analysis (SDA) and shift-share analysis (SSA) have been widely applied in multi- and inter-regional input-output (I-O) studies. This paper shows how elements from SSA can be integrated in SDA. This adds a novel spatial perspective to decomposing the change over time in an endogenous variable into the changes in its constituent exogenous factors.

Rose and Casler (1996) forwarded the idea that the structural decomposition of input-output (I-O) tables was not unlike shift-share analysis (SSA). (Incidentally, they likened it to growth accounting and index number analysis as well.) Intuitively, structural decomposition analysis (SDA) demonstrates strong similarities to SSA. Both examine the effects of industry shifts due to growth (or decline) and some sort of difference in industry shares. But SSA works its shares across space while SDA works its shares again across industries via technology change (fabrication effects). Interestingly, using a set of multi-regional I-O tables from Spain over six years and without drawing parallels to either SSA or SDA, Oosterhaven and Escobedo-Cardeñoso (2011) demonstrated that regional I-O tables can be forecasted fairly well. One innovation they applied was lagging the “remainder” from the biproportional adjustment technique. This remainder looks remarkably like the “regional component” (also termed the “competitive effect”) in SSA. More recently, Arto and Dietzenbacher (2014) performed what might be termed a “dynamic” SDA to examine the effect of trade changes on the growth of global CO_2 emissions. This harkens parallels to dynamic SSA (Thirlwall, 1967; Barf and Prentice, 1988).

Suffice it to say, SDA and SSA are related and this chapter will formally combine the two disparate strands of literature. In particular, it will be shown how changes in regional growth differentials can be included into a structural decomposition analysis. Moreover, the present availability of a large number of I-O table panels appears to enable the detection of even more parallels between the two approaches. Between the formalization of the SSA-SDA relationship and the available I-O data, a wide range of new, policy-relevant empirical applications is possible. The method proposed in this chapter may be useful for several avenues of research.

2. Background

The notion of shift-share analysis (SSA) has been around since at least Creamer (1943).[1] SSA disaggregates regional change by industry (on a particular economic measure, generally employment) in order to identify the relative influence of components of that change. It is roughly predicated on the concept of regional comparative advantage. Consequently it is used to decompose growth into (a) general national trends, (b) nationwide industry deviations from that general trend, and (c) some remainder that is identified as the "regional component" of the industry's change. Occasionally, when the region of focus is a very small geographic unit, some interim political-geography growth trend differentials—both regional and industrial—are also applied. Key points of the continued popularity of the approach are its minimal data requirements and technical simplicity. Of course, it helps that despite these potential oversimplifications, SSA tends to do a fairly good job in identifying the relative importance of factors that influence industrywise change in a region's economy (Nazara and Hewings, 2004).

As mentioned earlier, Rose and Casler (1996) draw parallels between structural decomposition analysis (SDA) of input-output (I-O) tables and SSA. SDA has been used to disaggregate economic change, more generally, into its proximate change components. The larger count of economic indicators available in I-O tables, as opposed to the SSA convention of using just employment or wage data, enables more variation in the analyses. But the lower frequency and time delay of I-O table production for a fixed geographic space has made available fewer data points of analysis. At its outset, SDA controlled for the three components

[1] Certainly Victor Fuchs (1959), Edgar Dunn (1960), Lowell Ashby (1964), and Anthony Thirlwall (1967) were major players in the technique's early development, and the prominence of these authors in the field of regional science and planning certainly induced SSA's popular appeal.

of change—activity level and industry mix (as in SSA), plus technology change. But as many as 14 different components of change have been analyzed simultaneously using the approach (Rose and Chen, 1991). And while regional and multiregional SDAs have been performed, both have only used pairs or multiple pairs of regional or multiregional tables to perform the analysis.

In summary, while SSA and SDA have similar roots, it is clear that, as yet, no SDA analysis has examined how a regional economy differs from its nation parent over time, which is the point of SSA. The purpose of the present paper is to lay out an SDA approach for performing such an analysis.

Of course, this then begs the question of why it might be desirable to perform SSA in an SDA context. SSA reveals how well a region's industries are performing relative to the nation, or other economy that contains the salient region, along a dimension of change. Thus, the focal quantities of SSA are the "regional components," which show the distance of actual regional performance from expectations. The expected values are derived by assuming regional industries grow at the national average rates. In this vein, the actual and relative distances from expectations for industries can help reveal a region's competitive strengths and weaknesses relative to national performance. This feature can be important in developing strategic regional development initiatives. As presently formulated, SDA does not offer this sort of result.

While the above explains why SSA-type findings are of value, it does not explain why performing them in an SDA context could be worthwhile. As mentioned earlier, input-output accounts, upon which SDA is formulated, offer a myriad of different economic indicators. Sophisticated sets of indicators have generally been applied in a much more limited fashion within SSA. Nonetheless, theoretical underpinnings of SSA, as articulated by Casler (1989), have been extended by Graham and Spence (1998) to unfold employment-based SSA's "regional component" further into partials related to input-price- and technology-related trends by using regression analysis to develop a productivity-growth decomposition within SSA. But their approach requires a panel of regional data on wage rates and output as well as employment, albeit a shallow one. And most countries do not release such panels of data by region. Meanwhile, an SDA equivalent would demand similar data for any region that is analyzed, but for only two points in time. Moreover, only data for the focal region and the nation of the analysis are needed. That is, given that I-O tables pre-exist, the data needs of SDA-based SSA should be far less demanding than that of the standard, regression-based SSA with equivalent complexity insofar as the array of applied indicators is concerned. Recall that, along with its intuitive implications, SSA's low data requirements have been key to its popularity. It would seem that SDA could minimize data requirements in certain shift-share settings and yet enable sophistication in the approach's theoretical underpinnings.

Yet another feature of SDA over conventional SSA is that it is able to measure the contribution of indirect (spillover and feedback) effects across regions. This is not to say that such effects are immeasurable by SSA. Indeed, Nazara and Hewings (2004) account for three components of change rather than the conventional two: the first is the usual national average growth component, and the second accounts for national sectoral growth differentials, and the third accounts for differential between the national sectoral growth and the weighted average sectoral growth rate for neighboring regions. Some spatial statistical approaches also have been applied to examine interregional spillover effects of shift-share components (Le Gallo and Kamarianakis, 2011; Li and Haynes, 2011).

A limitation of conventional shift–share that parallels the interregional aspect mentioned above is its omission of intersectoral relationships. Using an approach parallel to that of Nazara and Hewings (2004), Romanjo and Màrquez (2008) and Màrquez, Romanjo, and Hewings (2009) have suggested an extension that account for such interindustry shift-share contributions to change. That is, they define and use as the interindustry structure component for a particular referent industry the difference between the weighted-average growth rate of all other industries within the referent region and the weighted-average national growth rate of those same industries.

But like most extensions of SSA, the data demands for each additional component can be quite challenging to meet. Moreover as more variables are added, more degrees of freedom are consumed by the analysis, which in turn require more data observations (years and regions). This is not so much the case on SDA.

In summary then, in this paper we undertake a sort of technical reconnaissance into the potential of SDA for performing SSA. SDA can simultaneously account for interregional and interindustry effects while also accounting for nationwide and industrywide trends. In the original vein of SSA, SDA also has the potential to provide solid insight using few data points. But SDA has not examined regional trends in light of national trends in the manner that SSA does. We hope we sufficiently demonstrate how such an approach might be formulated. We conclude by pointing out the myriad types of analyses that might follow based on the SDA-based SSA that we formulate.

3. The input-output framework

SDA works on I-O accounts. So let us start with an interregional I-O table. For our purposes, we use the accounts shown in Table 1, which are for a country with three regions (*R*, *S*, and *T*).[2]

Table 1. A Interregional Input-Output Table

	Intermediate deliveries			*Final demands*				*Total*
	R	*S*	*T*	*R*	*S*	*T*	*Exp*	
R	$\mathbf{Z}^{RR}$	$\mathbf{Z}^{RS}$	$\mathbf{Z}^{RT}$	$\mathbf{f}^{RR}$	$\mathbf{f}^{RS}$	$\mathbf{f}^{RT}$	$\mathbf{e}^{R}$	$\mathbf{x}^{R}$
S	$\mathbf{Z}^{SR}$	$\mathbf{Z}^{SS}$	$\mathbf{Z}^{ST}$	$\mathbf{f}^{SR}$	$\mathbf{f}^{SS}$	$\mathbf{f}^{ST}$	$\mathbf{e}^{S}$	$\mathbf{x}^{S}$
T	$\mathbf{Z}^{TR}$	$\mathbf{Z}^{TS}$	$\mathbf{Z}^{TT}$	$\mathbf{f}^{TR}$	$\mathbf{f}^{TS}$	$\mathbf{f}^{TT}$	$\mathbf{e}^{T}$	$\mathbf{x}^{T}$
VA	$(\mathbf{v}^{R})'$	$(\mathbf{v}^{S})'$	$(\mathbf{v}^{T})'$					
Imp	$(\mathbf{m}^{R})'$	$(\mathbf{m}^{S})'$	$(\mathbf{m}^{T})'$					
Total	$(\mathbf{x}^{R})'$	$(\mathbf{x}^{S})'$	$(\mathbf{x}^{T})'$					
Labor	$(\mathbf{c}^{R})'$	$(\mathbf{c}^{S})'$	$(\mathbf{c}^{T})'$					

Here, $\mathbf{Z}^{RS}$ is an $n \times n$ matrix and its element z_{ij}^{RS} gives the intermediate deliveries from industry *i* in region *R* to industry *j* in region *S*; $\mathbf{f}^{RS}$ is an *n*-element (column) vector with typical element f_i^{RS} indicating the final demand (including household consumption, private investments, and government expenditures) by region *S* for the produce of industry *i* in region *R*; $\mathbf{e}^{R}$ is is an *n*-element (column) vector with typical element e_i^{R} indicating the exports by industry *i* in region *R*; $\mathbf{x}^{R}$ is an *n*-element (column) vector with typical element x_i^{R} indicating the output of (or total amount of production by) industry *i* in region *R*; $(\mathbf{v}^{R})'$ is an *n*-element (row) vector with typical element v_j^{R} indicating the value added generated in industry *j* in region *R*; and $(\mathbf{m}^{R})'$ is an *n*-element (row) vector with typical element m_j^{R} indicating the imports of industry *j* in region *R*. In addition, information from satellite accounts is often available. For example, the use of labor (say in hours worked). In that case, $(\mathbf{c}^{R})'$ is an *n*-element (row) vector with typical element c_j^{R} indicating the use of labor in industry *j* in region *R*.

Following the recent discussion on global value chains and trade in value added (or trade in emissions), one of the questions at the regional level would be: "Who works (or

[2] There is of course, no reason this could not be four or even more regions. But three regions typically takes any analysis beyond a trivial case.

emits) for whom?" (Serrano and Dietzenbacher, 2010; Koopman, Wang and Wei, 2014). That is, how much labor is (directly and indirectly) necessary in region *R* for the final demand bundle of region *T*? Using an interregional I-O model, the answer would be given by the element π_{RT} of the 3×3 matrix **Π**, which is defined as

$$\mathbf{\Pi} = \begin{bmatrix} \pi_{RR} & \pi_{RS} & \pi_{RT} \\ \pi_{SR} & \pi_{SS} & \pi_{ST} \\ \pi_{TR} & \pi_{TS} & \pi_{TT} \end{bmatrix} = \mathbf{HF} = \begin{bmatrix} (\mathbf{h}^{RR})' & (\mathbf{h}^{RS})' & (\mathbf{h}^{RT})' \\ (\mathbf{h}^{SR})' & (\mathbf{h}^{SS})' & (\mathbf{h}^{ST})' \\ (\mathbf{h}^{TR})' & (\mathbf{h}^{TS})' & (\mathbf{h}^{TT})' \end{bmatrix} \begin{bmatrix} \mathbf{f}^{RR} & \mathbf{f}^{RS} & \mathbf{f}^{RT} \\ \mathbf{f}^{SR} & \mathbf{f}^{SS} & \mathbf{f}^{ST} \\ \mathbf{f}^{TR} & \mathbf{f}^{TS} & \mathbf{f}^{TT} \end{bmatrix} \quad (1)$$

Note that **H** is a 3× $3n$ matrix with labor multipliers and **F** is a $3n \times 3$ matrix with regional final demands. The elements of the matrix **H** are obtained as follows

$$\mathbf{H} = \begin{bmatrix} (\mathbf{d}^{R})' & 0 & 0 \\ 0 & (\mathbf{d}^{S})' & 0 \\ 0 & 0 & (\mathbf{d}^{T})' \end{bmatrix} \begin{bmatrix} \mathbf{L}^{RR} & \mathbf{L}^{RS} & \mathbf{L}^{RT} \\ \mathbf{L}^{SR} & \mathbf{L}^{SS} & \mathbf{L}^{ST} \\ \mathbf{L}^{TR} & \mathbf{L}^{TS} & \mathbf{L}^{TT} \end{bmatrix} \quad (2)$$

The vector $(\mathbf{d}^R)'$ contains the direct labor input coefficients and is defined as $(\mathbf{d}^R)' = (\mathbf{c}^R)'(\hat{\mathbf{x}}^R)^{-1}$ or $d_j^R = c_j^R / x_j^R$. The second matrix on the right-hand side of (2) gives the partitioned Leontief inverse, i.e. $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$. **A** is the $3n \times 3n$ matrix with input coefficients which in partitioned form is given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^{RR} & \mathbf{A}^{RS} & \mathbf{A}^{RT} \\ \mathbf{A}^{SR} & \mathbf{A}^{SS} & \mathbf{A}^{ST} \\ \mathbf{A}^{TR} & \mathbf{A}^{TS} & \mathbf{A}^{TT} \end{bmatrix}$$

where the input coefficients are defined as $\mathbf{A}^{RS} = \mathbf{Z}^{RS}(\hat{\mathbf{x}}^S)^{-1}$ or $a_{ij}^{RS} = z_{ij}^{RS} / x_j^S$.

Note that the *j*th element of the vector $(\mathbf{h}^{RS})'$, i.e. h_j^{RS}, gives the total amount of labor used in region *R* that is necessary for one dollar of final demand for product *j* from region *S*. The scalar $(\mathbf{h}^{RS})'\mathbf{f}^{ST}$ then gives the total amount of labor used in region *R* that is embodied in the final demand of region *T* for products from region *S*. The element $\pi_{RT} = (\mathbf{h}^{RR})'\mathbf{f}^{RT} + (\mathbf{h}^{RS})'\mathbf{f}^{ST} + (\mathbf{h}^{RT})'\mathbf{f}^{TT}$ then gives the total amount of labor used in region *R* that is necessary for all final demands by region *T*.

Observe that our calculations take indirect linkages and interregional feedback effects into account, as far as they are national. For example, final demands in *T* require inputs from *S* that require inputs from *R*. Indirectly, final demands in *T* require production and therefore labor use in *R*. What is not included in our analysis are feedback effects that run through foreign countries. Exactly the same example can be used with region *S* replaced by a foreign country.

4. Adding shift-share elements

The next step is to introduce shift-share elements into the equation (1). To this end write the first n rows of the $3n \times n$ matrix **F** as follows.

$$[\mathbf{f}^{RR} \quad \mathbf{f}^{RS} \quad \mathbf{f}^{RT}] = \{\mathbf{T}^R \otimes \mathbf{\Sigma} \otimes \mathbf{S}\}\mathbf{R}f^{NAT} = \tag{3}$$

$$\{[\mathbf{t}^{RR} \quad \mathbf{t}^{RS} \quad \mathbf{t}^{RT}] \otimes [\boldsymbol{\sigma}^R \quad \boldsymbol{\sigma}^S \quad \boldsymbol{\sigma}^T] \otimes [\mathbf{s}^{NAT} \quad \mathbf{s}^{NAT} \quad \mathbf{s}^{NAT}]\}\begin{bmatrix} r^R & 0 & 0 \\ 0 & r^S & 0 \\ 0 & 0 & r^T \end{bmatrix} f^{NAT}$$

Going through the equation from right to left, the scalar f^{NAT} indicates the total amount of national final demand. That is, $f^{NAT} = \sum_{I=R,S,T}\sum_{J=R,S,T}\sum_{i=1}^{n} f_i^{IJ}$, the sum of all elements in the matrix **F**. The diagonal elements of the 3×3 matrix **R** give the share of the regional total final demand in the national final demand. For example, $r^R = \sum_{I=R,S,T}\sum_{i=1}^{n} f_i^{IR} / f^{NAT}$ and observe that $r^R + r^S + r^T = 1$. The $n \times 3$ matrix **S** consists of three times the vector $\mathbf{s}^{NAT}$ with the national final demand mix. Note that the final demand mix does not distinguish between the region of origin, it matters for example what households consume of (domestically produced) good *i*, not where the consumer goods come from. That is, $s_i^{NAT} = \sum_{I=R,S,T}\sum_{J=R,S,T} f_i^{IJ} / f^{NAT}$ and note that the shares add to one (i.e. $\sum_{i=1}^{n} s_i^{NAT} = 1$).

The regional final demand shares (for example for region *R*) are obtained as $\sum_{I=R,S,T} f_i^{IR} / \sum_{I=R,S,T}\sum_{i=1}^{n} f_i^{IR}$. The discrepancies between the regional and the national shares of final demands are given by the elements of the $n \times 3$ matrix $\mathbf{\Sigma}$. That is, $\sigma_i^R = \sum_{I=R,S,T} f_i^{IR} / (s_i^{NAT}\sum_{I=R,S,T}\sum_{i=1}^{n} f_i^{IR})$. The operator $\otimes$ stands for the Hadamard product of elementwise multiplication. The element in row *i* and column *R* of the matrix $\mathbf{\Sigma} \otimes \mathbf{S}$ thus equals $\sigma_i^R s_i^{NAT} = \sum_{I=R,S,T} f_i^{IR} / \sum_{I=R,S,T}\sum_{i=1}^{n} f_i^{IR}$, the share of good *i* in the total final demands of region *R*. Finally, the elements of the $n \times 3$ matrix $\mathbf{T}^R$ give the trade coefficients, indicating the share of a region's final demand for product *i* that originates from region *R*. For example, the *i*th element of the *n*-element vector $\mathbf{t}^{RS}$ yields $t_i^{RS} = f_i^{RS} / \sum_{I=R,S,T} f_i^{IS}$.

The expression for the full $3n \times n$ matrix **F** then becomes

$$\mathbf{F}=\begin{bmatrix}\mathbf{f}^{RR} & \mathbf{f}^{RS} & \mathbf{f}^{RT}\\ \mathbf{f}^{SR} & \mathbf{f}^{SS} & \mathbf{f}^{ST}\\ \mathbf{f}^{TR} & \mathbf{f}^{TS} & \mathbf{f}^{TT}\end{bmatrix}=\left\{\begin{bmatrix}\mathbf{T}^{R}\\ \mathbf{T}^{S}\\ \mathbf{T}^{T}\end{bmatrix}\otimes\begin{bmatrix}\mathbf{\Sigma}\\ \mathbf{\Sigma}\\ \mathbf{\Sigma}\end{bmatrix}\otimes\begin{bmatrix}\mathbf{S}\\ \mathbf{S}\\ \mathbf{S}\end{bmatrix}\right\}\mathbf{R}f^{NAT}=\{\mathbf{T}\otimes\bar{\mathbf{\Sigma}}\otimes\bar{\mathbf{S}}\}\mathbf{R}f^{NAT} \quad (4)$$

A similar distinction can be made for the 3× $3n$ matrix **H** with labor multipliers. That is,

$$\mathbf{H}=\begin{bmatrix}(\mathbf{h}^{RR})' & (\mathbf{h}^{RS})' & (\mathbf{h}^{RT})'\\ (\mathbf{h}^{SR})' & (\mathbf{h}^{SS})' & (\mathbf{h}^{ST})'\\ (\mathbf{h}^{TR})' & (\mathbf{h}^{TS})' & (\mathbf{h}^{TT})'\end{bmatrix}=$$

$$\begin{bmatrix}(\boldsymbol{\gamma}^{RR})' & (\boldsymbol{\gamma}^{RS})' & (\boldsymbol{\gamma}^{RT})'\\ (\boldsymbol{\gamma}^{SR})' & (\boldsymbol{\gamma}^{SS})' & (\boldsymbol{\gamma}^{ST})'\\ (\boldsymbol{\gamma}^{TR})' & (\boldsymbol{\gamma}^{TS})' & (\boldsymbol{\gamma}^{TT})'\end{bmatrix}\otimes\begin{bmatrix}(\mathbf{h}^{NAT,R})' & (\mathbf{h}^{NAT,S})' & (\mathbf{h}^{NAT,T})'\\ (\mathbf{h}^{NAT,R})' & (\mathbf{h}^{NAT,S})' & (\mathbf{h}^{NAT,T})'\\ (\mathbf{h}^{NAT,R})' & (\mathbf{h}^{NAT,S})' & (\mathbf{h}^{NAT,T})'\end{bmatrix}=\mathbf{\Gamma}\otimes\mathbf{H}^{NAT} \quad (5)$$

The elements of the matrix $\mathbf{H}^{NAT}$ give the national labor multipliers. For example, $h_j^{NAT,S}$ gives the total amount of labor that is used nationally for the final demand of one dollar of good *j* produced by region *S*. This amount equals the sum of the labor use in each region, i.e. $h_j^{NAT,S}=\sum_{I=R,S,T}h_j^{IS}$. The elements of the matrix $\mathbf{\Gamma}$ then give the shares of the national labor use that take place in each of the regions. That is, $\gamma_j^{RS}=h_j^{RS}/h_j^{NAT,S}$ and note that the shares add to one ($\sum_{I=R,S,T}\gamma_j^{IJ}=1$, for $J=R,S,T$ and $j=1,\dots,n$).

Combining Equations (1), (4), and (5), yields

$$\mathbf{\Pi}=[\mathbf{\Gamma}\otimes\mathbf{H}^{NAT}][\mathbf{T}\otimes\bar{\mathbf{\Sigma}}\otimes\bar{\mathbf{S}}]\mathbf{R}f^{NAT} \quad (6)$$

5. The structural decomposition

Structural decomposition analysis splits the growth in some variable (here, the matrix **Π**) into the contributions of the growth in its components (here, the matrix **Γ** is one of these components). That is, one decomposes $\Delta\mathbf{\Pi}=\mathbf{\Pi}_1-\mathbf{\Pi}_0$, the change in **Π** between two points in time, indicated by 0 and 1. One possible decomposition is

$$\begin{aligned}
&\Delta\mathbf{\Pi}=\mathbf{\Pi}_1-\mathbf{\Pi}_0\\
&=[\mathbf{\Gamma}_1\otimes\mathbf{H}_1^{NAT}][\mathbf{T}_1\otimes\bar{\mathbf{\Sigma}}_1\otimes\bar{\mathbf{S}}_1]\mathbf{R}_1f_1^{NAT}-[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_0\otimes\bar{\mathbf{\Sigma}}_0\otimes\bar{\mathbf{S}}_0]\mathbf{R}_0f_0^{NAT}\\
&=[\mathbf{\Gamma}_1\otimes\mathbf{H}_1^{NAT}][\mathbf{T}_1\otimes\bar{\mathbf{\Sigma}}_1\otimes\bar{\mathbf{S}}_1]\mathbf{R}_1f_1^{NAT}-[\mathbf{\Gamma}_0\otimes\mathbf{H}_1^{NAT}][\mathbf{T}_1\otimes\bar{\mathbf{\Sigma}}_1\otimes\bar{\mathbf{S}}_1]\mathbf{R}_1f_1^{NAT}\\
&+[\mathbf{\Gamma}_0\otimes\mathbf{H}_1^{NAT}][\mathbf{T}_1\otimes\bar{\mathbf{\Sigma}}_1\otimes\bar{\mathbf{S}}_1]\mathbf{R}_1f_1^{NAT}-[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_1\otimes\bar{\mathbf{\Sigma}}_1\otimes\bar{\mathbf{S}}_1]\mathbf{R}_1f_1^{NAT}\\
&+[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_1\otimes\bar{\mathbf{\Sigma}}_1\otimes\bar{\mathbf{S}}_1]\mathbf{R}_1f_1^{NAT}-[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_0\otimes\bar{\mathbf{\Sigma}}_1\otimes\bar{\mathbf{S}}_1]\mathbf{R}_1f_1^{NAT}\\
&+[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_0\otimes\bar{\mathbf{\Sigma}}_1\otimes\bar{\mathbf{S}}_1]\mathbf{R}_1f_1^{NAT}-[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_0\otimes\bar{\mathbf{\Sigma}}_0\otimes\bar{\mathbf{S}}_1]\mathbf{R}_1f_1^{NAT}\\
&+[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_0\otimes\bar{\mathbf{\Sigma}}_0\otimes\bar{\mathbf{S}}_1]\mathbf{R}_1f_1^{NAT}-[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_0\otimes\bar{\mathbf{\Sigma}}_0\otimes\bar{\mathbf{S}}_0]\mathbf{R}_1f_1^{NAT}\\
&+[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_0\otimes\bar{\mathbf{\Sigma}}_0\otimes\bar{\mathbf{S}}_0]\mathbf{R}_1f_1^{NAT}-[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_0\otimes\bar{\mathbf{\Sigma}}_0\otimes\bar{\mathbf{S}}_0]\mathbf{R}_0f_1^{NAT}\\
&+[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_0\otimes\bar{\mathbf{\Sigma}}_0\otimes\bar{\mathbf{S}}_0]\mathbf{R}_0f_1^{NAT}-[\mathbf{\Gamma}_0\otimes\mathbf{H}_0^{NAT}][\mathbf{T}_0\otimes\bar{\mathbf{\Sigma}}_0\otimes\bar{\mathbf{S}}_0]\mathbf{R}_0f_0^{NAT}
\end{aligned}$$

Or, more in more compact form:

$$\begin{aligned}
&\Delta\mathbf{\Pi} = \mathbf{\Pi}_1 - \mathbf{\Pi}_0 \\
&= [(\Delta\mathbf{\Gamma}) \otimes \mathbf{H}_1^{NAT}][\mathbf{T}_1 \otimes \bar{\mathbf{\Sigma}}_1 \otimes \bar{\mathbf{S}}_1]\mathbf{R}_1 f_1^{NAT} \\
&+[\mathbf{\Gamma}_0 \otimes (\Delta\mathbf{H}^{NAT})][\mathbf{T}_1 \otimes \bar{\mathbf{\Sigma}}_1 \otimes \bar{\mathbf{S}}_1]\mathbf{R}_1 f_1^{NAT} \\
&+[\mathbf{\Gamma}_0 \otimes \mathbf{H}_0^{NAT}][(\Delta\mathbf{T}) \otimes \bar{\mathbf{\Sigma}}_1 \otimes \bar{\mathbf{S}}_1]\mathbf{R}_1 f_1^{NAT} \\
&+[\mathbf{\Gamma}_0 \otimes \mathbf{H}_0^{NAT}][\mathbf{T}_0 \otimes (\Delta\bar{\mathbf{\Sigma}}) \otimes \bar{\mathbf{S}}_1]\mathbf{R}_1 f_1^{NAT} \\
&+[\mathbf{\Gamma}_0 \otimes \mathbf{H}_0^{NAT}]\big[\mathbf{T}_0 \otimes \bar{\mathbf{\Sigma}}_0 \otimes (\Delta\bar{\mathbf{S}})\big]\mathbf{R}_1 f_1^{NAT} \\
&+[\mathbf{\Gamma}_0 \otimes \mathbf{H}_0^{NAT}][\mathbf{T}_0 \otimes \bar{\mathbf{\Sigma}}_0 \otimes \bar{\mathbf{S}}_0](\Delta\mathbf{R}) f_1^{NAT} \\
&+[\mathbf{\Gamma}_0 \otimes \mathbf{H}_0^{NAT}][\mathbf{T}_0 \otimes \bar{\mathbf{\Sigma}}_0 \otimes \bar{\mathbf{S}}_0]\mathbf{R}_0(\Delta f^{NAT})
\end{aligned}$$

The change variable in parentheses (recognized by the Δ symbol) identifies how to interpret each of the seven terms or components.

1) The first shows the contribution from the change in regional shares in national labor use (**Γ**),
2) the second reveals the effects of change in the national labor multipliers (**H**),
3) the third identifies the effects due to changes in the supplying region's share of the regional final demand (**T**),
4) the fourth reveals the effects of due to changes in the differences between regional and national final demand mixes ($\bar{\mathbf{\Sigma}}$),
5) the fifth reports the effects due to changes in the national final demand mix ($\bar{\mathbf{S}}$),
6) the sixth shows the effects due to changes in the shares of regional total final demand in the national final demand (**R**), and
7) the seventh reports the effects of due to changes in total national final demand (f^{NAT}).

Clearly this is just one extreme decomposition of many we could have expressed to achieve the same analysis. Fortunately, Dietzenbacher and Los (1997) have noted that a simple average of the above decomposition and its polar opposite very reasonably represents the average of all possible decompositions. Moreover, while the above is an additive decomposition, multiplicative decompositions are also conceivable (see, e.g., Dietzenbacher, Hoen and Los, 2000; Dietzenbacher, Lahr and Los, 2004). Indeed, they can offer the added benefit of decomposing on supply- and demand-side factors simultaneously.

A key point to be made here is that the sort of analyses we suggest here would be hampered by roughly estimated final demand accounts. If performed using a multiplicative approach, the analyses could be even more compromised if value added components were also only roughly estimated. In this vein, analyses of national I-O tables, for which extraordinary care has been taken to formulate the input-output accounts, within a broader framework of nations—perhaps those sharing a trade agreement (e.g., EU, BRICs, NAFTA)—could be ideal targets for the sort of SSA-SDA analyses that we are suggesting. In such instances, regions *R* and *S* in the framework described above would be representative of countries in the trade group to be analyzed (effectively the "nation" in our framework), and *T* reflecting relationships with countries outside of it. Still, the basic form of the equations would remain the same, but the number of regions and, hence, partitions composing the matrices would generally be greater and requiring specific adaptations.

In a similar vein, note that our framework only identifies two periods. Clearly more periods could be analyzed using the framework we have outlined. They need only be studied serially, following the example of traditional dynamic SSA (Thirlwall, 1967; Barf and Prentice, 1988). Indeed, many authors have already applied SDA in such a fashion. Perhaps the best example is Arto and Dietzenbacher (2014), who performed what might be termed a "dynamic" SDA to examine the effect of trade changes on the growth of global CO_2 emissions. Indeed, armed with this SSA-SDA approach and a panel of interregional I-O tables, it might be interesting to revisit the aims of Oosterhaven and Escobedo-Cardeñoso (2011) who demonstrated that regional I-O tables can be forecasted fairly well, using a set of multi-regional I-O tables over a number of years.

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