Counting borders in global value chains*

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Abstract

This paper introduces a new measure of the weighted average number of border crossings in value chain required for a product of one country to reach the final user in another country. It builds on the power series of a new multiplier matrix that models a "melting" part of the initial exports until it is entirely consumed (used) at an infinitely remote tier. Underlying the derivation of this new "global" inverse is a gross exports accounting framework that traces the destination of direct exports to their eventual users through multi-stage production. Data from the World Input-Output Database are employed for numerical tests, and the results are discussed and contrasted with other measures that quantify the number of production stages and accumulated protection in global value chains.

1 Introduction

It is widely recognised that the growing fragmentation of production across borders may have important implications for trade and investment policies. When value chains are global, intermediate inputs cross national borders multiple times as their value is carried forward from one production stage to the next. Multiple border crossings involve multiple trade barriers and associated costs.

Earlier investigations identified multiple border crossings as a key factor behind the rise of vertical specialization in trade. Hummels et al. (2001) point out that "the number of border-crossings matter because it gives the "multiplier" trade effect that results from a given change in trade barriers". They, however, note that while their measures of vertical specialization imply that a good-in-process crosses at least two borders, they cannot determine the average number of border-crossings.

Since then, the impact of global value chains on trade, the environment and jobs has been extensively studied with novel accounting techniques and newly available inter-country input-output tables. Yet the attempts to quantify the number of border crossings have been rare.

The notion of multiple border crossings is key to explaining the magnification effect of trade costs in global value chains. Yi (2003, 2010), Johnson and Moxnes (2013) develop theoretical trade models with embedded multi-stage production where goods produced at various stages in different countries cross national borders during the production process and thus incur trade costs multiple times. Tamamura (2010) and Koopman et al. (2010) are perhaps the first to provide numerical estimates of cumulative trade costs using inter-country input-output tables. Fally (2012) proposes a formula to compute cumulative transport costs

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and shows that the result has a linear relationship with his index of "embodied production stages". Rouzet and Miroudot (2013) present an elaborate exposition of the concept of the cumulative tariff and the relevant computational method. However, none of these papers provided a method to count the average number of border crossings.

Meanwhile, the measurement of the number of borders crossed is intimately related to the measurement of the number of production stages and the length of production chains that has attracted the interest of many input-output economists. Dietzenbacher et al. (2005) describe a method to compute the average number of steps it takes an exogenous change in one sector to affect the value of production in another sector, a measure they call the "average propagation length" (APL). First applications of the APL concept to measure the length of cross-border production chains appear in Dietzenbacher and Romero (2007) and Inomata (2008), though Oosterhaven and Bouwmeester (2013) warn that the APL should only be used to compare pure interindustry linkages and not to compare different economies or different industries.

Fally (2011, 2012) proposes the recursive definitions of two indices that quantify the "average number of embodied production stages" and the "distance to final demand". Miller and Temurshoev (2015), by analogy with Antràs et al. (2012), use the logic of the APL and derive the measures of "output upstreamness" and "input downstreamness" that indicate industry relative position with respect to the final users of outputs and initial producers of inputs. They show that their measures are mathematically equivalent to those of Fally and the well known indicators of, respectively, total forward linkages and total backward linkages. Fally (2012) indicates that the average number of embodied production stages may be split to account for the stages taking place within the domestic economy and abroad. This approach was implemented in OECD (2012), De Backer and Miroudot (2013) and elaborated in Miroudot and Nordstrom (2015).

Ye et al. (2015) generalize previous length and distance indices and propose a consistent accounting system to measure the distance in production networks between producers and consumers at the country, industry and product levels from different economic perspectives. Their "value added propagation length" may be shown to be equal to Fally's embodied production stages and Miller–Temurshoev's input downstreamness when aggregated across producing industries.

Various measures of distance and length from previous studies count production stages upstream or downstream the value chain irrespective of whether those stages link two industries within the same economy or different economies. Even if the APL indicator is split into domestic and international components as in OECD (2012) and De Backer and Miroudot (2013), the underlying computation method still treats foreign intermediate inputs in the same way as domestic inputs at each production stage.

This paper proposes a new APL-type measure of the weighted average number of border crossings. Importantly, it is capable of consistently delimiting domestic and international trade transactions and therefore only counts cross-border production tiers. At the core of this new measure is a new "global" inverse, derived from a gross exports accounting framework that traces the destination of direct exports to their eventual users through multi-stage production. The proposed indicator builds on the power series of the new "global" inverse where each term corresponds to a border crossing. The sum of the number of border crossings is weighted by the share of direct or indirect exports at each successive tier (border crossing) in the cumulative exports at all tiers. Its lowest value is 1 when a sector only exports final products. This is in line with the conventional wisdom: exported products cross borders at least once.

For a numerical test of the proposed indicator, the paper uses the inter-country input-output tables from the World Input-Output Database (WIOD) for 2001, 2005 and 2010.

The results show which countries and economic sectors export products with the longest cross-border production chain on the way to final users. The weighted average number of border crossings is found to slowly increase in 2001-2010 but this trend has not been uniform among the countries covered in WIOD. As expected, the number of border crossings is consistently lower than the total number of production stages a product has to undergo along the entire value chain, or the "output propagation length". It is explored whether the economic distance measured by the number of border crossings can be explained by the geographic distance. In an effort to capture a relationship between the number of border crossings and the cumulative protection along the value chain, the proposed indicator is paired with an input-output based measure of cumulative tariff.

2 Accounting framework

2.1 The input-output framework: notation and setup

Global value chain analysis requires a global input-output table where single-country tables are combined and linked via international trade matrices. Such inter-country or multi-regional input-output tables have been described by Isard (1951), Moses (1955), and Leontief and Strout (1963), among others, but have not been compiled at a global scale until late 2000s. The release of experimental global input-output datasets, including WIOD, Eora, Exiobase, OECD ICIO, GTAP-MRIO[1] and others,[2] has fuelled research into the implications of global value chains on trade, the economy and the environment.

If there are K countries and N economic sectors in each country, the key elements of the inter-country input-output system may be described by block matrices and vectors. The KN×KN matrix of intermediate demand $\mathbf{Z}$ is therefore as follows:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1k} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{k1} & \mathbf{Z}_{k2} & \cdots & \mathbf{Z}_{kk} \end{bmatrix} \quad \text{where a block element} \quad \mathbf{Z}_{rs} = \begin{bmatrix} z_{rs}^{11} & z_{rs}^{12} & \cdots & z_{rs}^{1n} \\ z_{rs}^{21} & z_{rs}^{22} & \cdots & z_{rs}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{rs}^{n1} & z_{rs}^{n2} & \cdots & z_{rs}^{nn} \end{bmatrix}$$

The lower index henceforth denotes a country with $r \in K$ corresponding to the exporting country and $s \in K$ to the partner country. The upper index denotes sector. $\mathbf{Z}_{rs}$ is therefore an N×N matrix where each element z_{rs}^{ij} is the monetary value of the intermediate inputs supplied by the producing sector $i \in N$ in country r to the purchasing (using) sector $j \in N$ in country s.

Similarly, the KN×K matrix of final demand is:

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}_{11} & \mathbf{f}_{12} & \cdots & \mathbf{f}_{1k} \\ \mathbf{f}_{21} & \mathbf{f}_{22} & \cdots & \mathbf{f}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{f}_{k1} & \mathbf{f}_{k2} & \cdots & \mathbf{f}_{kk} \end{bmatrix} \quad \text{where a block element} \quad \mathbf{f}_{rs} = \begin{bmatrix} f_{rs}^{1} \\ f_{rs}^{2} \\ \vdots \\ f_{rs}^{n} \end{bmatrix}$$

Each block $\mathbf{f}_{rs}$ is an N×1 vector with elements f_{rs}^{i} representing the value of the output of sector i in country r sold to final users in country s.

Total output of each sector is recorded in the KN×1 column vector $\mathbf{x}$:

[1] Multi-regional versions of GTAP input-output tables were compiled on an *ad hoc* basis in various research projects and were not publicly released.

[2] See the special issue of *Economic Systems Research*, 2013, vol. 25, no. 1 for an overview.

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_k \end{bmatrix} \quad \text{where a block element} \quad \mathbf{x}_r = \begin{bmatrix} x_r^1 \\ x_r^2 \\ \vdots \\ x_r^n \end{bmatrix}$$

And the value added by each sector is recorded in the 1×KN row vector $\mathbf{v}$:

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_k \end{bmatrix} \quad \text{where a block element} \quad \mathbf{v}_s = \begin{bmatrix} v_s^1 & v_s^2 & \cdots & v_s^n \end{bmatrix}$$

$\mathbf{v}_s$ is a 1×N vector where each element v_s^j describes the value added generated by sector j in country s throughout the production process.

To better reflect the results of production, net of any taxes, subsidies or margins related to sales, the transactions in $\mathbf{Z}$ and $\mathbf{F}$ should be valued at basic prices. Meanwhile, from the producer's perspective, intermediate inputs should enter the calculation at purchasers' prices, inclusive of all costs associated with their purchase. Accordingly, the taxes or margins payable on intermediate inputs should also be accounted for as inputs to production. These are usually recorded as 1×KN row vectors below $\mathbf{Z}$:

$$\mathbf{m}(g)_{(\mathbf{Z})} = \begin{bmatrix} \mathbf{m}(g)_{(\mathbf{Z})1} & \mathbf{m}(g)_{(\mathbf{Z})2} & \cdots & \mathbf{m}(g)_{(\mathbf{Z})k} \end{bmatrix}$$
$$\text{where a block element} \quad \mathbf{m}(g)_{(\mathbf{Z})s} = \begin{bmatrix} m(g)_{(\mathbf{Z})s}^1 & m(g)_{(\mathbf{Z})s}^2 & \cdots & m(g)_{(\mathbf{Z})s}^n \end{bmatrix}$$

$\mathbf{m}(g)_{(\mathbf{Z})s}$ is a 1×N row vector of the g^{th} margin where each element $m(g)_{(\mathbf{Z})s}^j$ is the amount of tax paid, subsidy received or trade/transport margin on all intermediate inputs purchased by sector j in country s. $\mathbf{m}(g)_{(\mathbf{Z})}$ is in fact a condensed form of the valuation layer that conforms to the dimension of $\mathbf{Z}$:

$$\mathbf{M}(g)_{(\mathbf{Z})} = \begin{bmatrix} \mathbf{M}(g)_{(\mathbf{Z})11} & \mathbf{M}(g)_{(\mathbf{Z})12} & \cdots & \mathbf{M}(g)_{(\mathbf{Z})1k} \\ \mathbf{M}(g)_{(\mathbf{Z})21} & \mathbf{M}(g)_{(\mathbf{Z})22} & \cdots & \mathbf{M}(g)_{(\mathbf{Z})2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}(g)_{(\mathbf{Z})k1} & \mathbf{M}(g)_{(\mathbf{Z})k2} & \cdots & \mathbf{M}(g)_{(\mathbf{Z})kk} \end{bmatrix}$$

$$\text{where a block element} \quad \mathbf{M}(g)_{(\mathbf{Z})rs} = \begin{bmatrix} \mathbf{M}(g)_{(\mathbf{Z})rs}^{11} & \mathbf{M}(g)_{(\mathbf{Z})rs}^{12} & \cdots & \mathbf{M}(g)_{(\mathbf{Z})rs}^{1n} \\ \mathbf{M}(g)_{(\mathbf{Z})rs}^{21} & \mathbf{M}(g)_{(\mathbf{Z})rs}^{22} & \cdots & \mathbf{M}(g)_{(\mathbf{Z})rs}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}(g)_{(\mathbf{Z})rs}^{n1} & \mathbf{M}(g)_{(\mathbf{Z})rs}^{n2} & \cdots & \mathbf{M}(g)_{(\mathbf{Z})rs}^{nn} \end{bmatrix}$$

In N×N matrices $\mathbf{M}(g)_{(\mathbf{Z})rs}$, each element $m(g)_{(\mathbf{Z})rs}^{ij}$ depicts the amount of g^{th} margin (tax paid, subsidy received or trade/transport cost) paid on intermediate inputs purchased by sector j in country s from sector i in country r. $\mathbf{M}(g)_{(\mathbf{Z})}$ is then a matrix of bilateral margins that changes the valuation of intermediate inputs. If the sector that produces the margins, e.g., domestic trade and transportation services, is modelled as endogenous to the inter-industry system (in other words, is inside $\mathbf{Z}$), the summation of $\mathbf{M}(g)_{(\mathbf{Z})}$ column-wise will result in a zero vector $\mathbf{m}(g)_{(\mathbf{Z})}$. Taxes and subsidies on products are usually recorded as exogenous to the system, so vector $\mathbf{m}(g)_{(\mathbf{Z})}$ contains non-zero values. International transport margins are also modelled as though they were provided from outside the system, which is the result of the "Panama assumption" (see Streicher and Stehrer, 2015 for an extensive discussion).

For a complete account of trade costs later in this section, valuation terms should also be compiled with respect to final products – 1×K row vector $\mathbf{m}(g)_{(\mathbf{F})}$ and KN×K matrix $\mathbf{M}(g)_{(\mathbf{F})}$.

The fundamental accounting identities in the monetary input-output system imply that total sales for intermediate and final use equal total output, $\mathbf{Zi}+\mathbf{Fi} = \mathbf{x}$, and the purchases of intermediate and primary inputs at basic prices plus margins and net taxes on intermediate inputs equal total input (outlays) that must also be equal to total output, $\mathbf{i'Z}+\sum_{g=1}^{G}\mathbf{m}(g)_{(\mathbf{Z})}+\mathbf{v} = \mathbf{x}'$, where $\mathbf{i}$ is an appropriately sized summation vector and G is the number of the valuation layers (margins).[3]

Gross bilateral exports in the inter-country input-output system may be obtained by summing the international sales of outputs for intermediate and final use:

$$\mathbf{E}_{bil} = \begin{bmatrix} 0 & \mathbf{e}_{12} & \cdots & \mathbf{e}_{1k} \\ \mathbf{e}_{21} & 0 & \cdots & \mathbf{e}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e}_{k1} & \mathbf{e}_{k2} & \cdots & 0 \end{bmatrix} \quad \text{where a block element} \quad \mathbf{e}_{rs} = \begin{bmatrix} e_{rs}^1 \\ e_{rs}^2 \\ \vdots \\ e_{rs}^n \end{bmatrix}$$

Block elements $\mathbf{e}_{rs}$ are N×1 vectors where each entry $e_{rs}^i = \sum_{j=1}^{N} z_{rs}^{ij} + f_{rs}^i$, $r \neq s$.

The key to the demand-driven input-output analysis is the Leontief inverse, which, in the case of the inter-country input-output table is defined as follows:

$$(\mathbf{I}-\mathbf{A})^{-1} = \begin{bmatrix} \mathbf{I}-\mathbf{A}_{11} & -\mathbf{A}_{12} & \cdots & -\mathbf{A}_{1k} \\ -\mathbf{A}_{21} & \mathbf{I}-\mathbf{A}_{22} & \cdots & -\mathbf{A}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{A}_{k1} & -\mathbf{A}_{k2} & \cdots & \mathbf{I}-\mathbf{A}_{kk} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} & \cdots & \mathbf{L}_{1k} \\ \mathbf{L}_{21} & \mathbf{L}_{22} & \cdots & \mathbf{L}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L}_{k1} & \mathbf{L}_{k2} & \cdots & \mathbf{L}_{kk} \end{bmatrix} = \mathbf{L}$$

$\mathbf{A}_{rs}$ blocks are N×N technical coefficient matrices where an element $a_{rs}^{ij} = \frac{z_{rs}^{ij}}{x_s^j}$ describes the amount of input by sector i in country r required per unit of output of sector j in country s. In block matrix form, $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$. Leontief inverse $\mathbf{L}$ is a KN×KN multiplier matrix that allows total output to be expressed as a function of final demand: $\mathbf{x} = \mathbf{Ax}+\mathbf{Fi} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{Fi} = \mathbf{LFi}$.

The supply-side counterpart to the Leontief inverse, or the matrix of output (demand) multipliers, is the Ghosh inverse, or the matrix of input (supply) multipliers:

$$(\mathbf{I}-\mathbf{B})^{-1} = \begin{bmatrix} \mathbf{I}-\mathbf{B}_{11} & -\mathbf{B}_{12} & \cdots & -\mathbf{B}_{1k} \\ -\mathbf{B}_{21} & \mathbf{I}-\mathbf{B}_{22} & \cdots & -\mathbf{B}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{B}_{k1} & -\mathbf{B}_{k2} & \cdots & \mathbf{I}-\mathbf{B}_{kk} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} & \cdots & \mathbf{G}_{1k} \\ \mathbf{G}_{21} & \mathbf{G}_{22} & \cdots & \mathbf{G}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{G}_{k1} & \mathbf{G}_{k2} & \cdots & \mathbf{G}_{kk} \end{bmatrix} = \mathbf{G}$$

where $\mathbf{B}_{rs}$ are N×N allocation coefficient matrices with elements $b_{rs}^{ij} = \frac{z_{rs}^{ij}}{x_r^i}$ that describe the proportion of output of sector i in country r sold as intermediate input to sector j in country s. In block matrix form, $\mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{Z}$. Ghosh inverse $\mathbf{G}$ is a KN×KN multiplier matrix that

[3] We assume here that the inter-country input-output table does not contain purchases abroad by residents or domestic purchases by non-residents or any statistical discrepancies. The sum of intermediate purchases at basic prices, net taxes, margins on intermediate inputs and value added at basic prices is therefore equal to the sector output at basic prices.

links total output and primary inputs in the following way: $\mathbf{x}' = \mathbf{x}'\mathbf{B} + \sum_{g=1}^{G} \mathbf{m}(g)_{(\mathbf{Z})} + \mathbf{v} = \left(\sum_{g=1}^{G} \mathbf{m}(g)_{(\mathbf{Z})} + \mathbf{v}\right)(\mathbf{I} - \mathbf{B})^{-1} = \left(\sum_{g=1}^{G} \mathbf{m}(g)_{(\mathbf{Z})}\right)\mathbf{G} + \mathbf{v}\mathbf{G}$.

2.2 Overview of available length indicators based on Leontief or Ghosh inverse

We will first briefly overview the indicators of the length of production chains from previous studies and examine their applicability for counting border crossings.

The sequence of production stages along the value chain can be approximated as a power series (see Miller and Blair, 2009):

$$\mathbf{x} = \mathbf{LFi} = \mathbf{Fi} + \mathbf{AFi} + \mathbf{AAFi} + \mathbf{AAAFi} + \ldots = \left(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots\right)\mathbf{Fi}$$

where $\mathbf{Fi}$ is the column vector of output for final use (row sum of matrix $\mathbf{F}$). In this backward decomposition, the production of final output $\mathbf{Fi}$ involves the use of intermediate inputs at each production stage (tier) t, equal to $\mathbf{A}^t\mathbf{Fi}$.[4] Each term in the brackets corresponds to the number of production stages between final output and intermediate inputs. For example, $\mathbf{I}$ signifies that delivering the output to the final user requires one stage. $\mathbf{A}$ signifies that direct intermediate inputs reach the final user in two stages. $\mathbf{A}^2$ signifies that indirect intermediate inputs embodied in direct intermediate inputs reach the final user in three stages, and the sequence continues to an infinitely remote production stage. The core idea behind the average propagation length (see Dietzenbacher et al., 2005) is to weigh the total number of production stages $1 + 2 + 3 + \cdots + t$ by decreasing shares of output at each successive production stage t. Virtually all length indicators build on this logic, and the difference is in the weighting/aggregation scheme.

Antràs et al. (2012) formulate a measure of industry upstreamness in value chain that indicates the position of industry i as a producer with respect to final users (lower indices for countries are dropped below for easier notation):

$$u^i = 1 \times \frac{f^i}{x^i} + 2 \times \frac{\sum_{j=1}^{N} a^{ij} f^j}{x^i} + 3 \times \frac{\sum_{j=1}^{N}\sum_{l=1}^{N} a^{il} a^{lj} f^j}{x^i} + 4 \times \frac{\sum_{j=1}^{N}\sum_{l=1}^{N}\sum_{m=1}^{N} a^{im} a^{ml} a^{lj} f^j}{x^i} + \ldots$$

Here the number of production stages is weighted by the share of total output generated because of the use of intermediate inputs at each successive stage. Antràs et al. (2012) observe that this measure is equal to Fally's distance to final demand and the widely used total forward linkages. Miller and Temurshoev (2015) explicitly show this in a compact matrix form:

$$\mathbf{u} = \hat{\mathbf{x}}^{-1}\left(\mathbf{I} + 2\mathbf{A} + 3\mathbf{A}^2 + 4\mathbf{A}^3 + \ldots\right)\mathbf{Fi} = \hat{\mathbf{x}}^{-1}\mathbf{LLFi} = \hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}}\mathbf{i} = \mathbf{Gi}$$

In the equation above, the weights are implicitly defined as the shares of output of sector i generated at stage t to satisfy total final demand:

$$(\mathbf{Lf}) \oslash (\mathbf{Lf}) = \mathbf{f} \oslash (\mathbf{Lf}) + (\mathbf{Af}) \oslash (\mathbf{Lf}) + (\mathbf{A}^2\mathbf{f}) \oslash (\mathbf{Lf}) + (\mathbf{A}^3\mathbf{f}) \oslash (\mathbf{Lf}) + \ldots$$

[4] The first tier is $t = 0$.

where $\oslash$ is the element-by-element division. The measure of industry upstreamness of Antràs et al. (2012) may be treated as a special case in a generalized accounting system that Ye et al. (2015) develop to measure the distance in production networks between producers and consumers at the country, industry and product levels. They define the value-added propagation length in a single-country N×N input-output system from sector i to final product of sector j (a scalar $\mathbf{U}^{ij}$, using the notation of this paper, that is equal to $\mathbf{D}^{ij}$, or value-added propagation length from a final product of sector j backwards to sector i):

$$\mathbf{U}^{ij} = \mathbf{D}^{ij} = \left(\mathbf{v}_c^i\left(1\mathbf{I} + 2\mathbf{A} + 3\mathbf{A}^2 + \dots\right)\mathbf{f}^j\right) / \left(\mathbf{v}_c^i\mathbf{L}\mathbf{f}^j\right) = \left(\mathbf{v}_c^i\mathbf{L}\mathbf{L}\mathbf{f}^j\right) / \left(\mathbf{v}_c^i\mathbf{L}\mathbf{f}^j\right)$$

where $\mathbf{v}_c^i$ is the row vector of value added coefficients $\mathbf{v}_c = \mathbf{v}\hat{\mathbf{x}}^{-1}$ with all entries except i set to zero and $\mathbf{f}^j$ is the column vector of final demand with all entries except j set to zero. $\mathbf{U}^{ij}$ counts the total number of stages, on average, through which the value added of a specific sector reaches final demand in the form of a specific product by the way of forward industrial linkages.

Re-writing the above in the block-matrix form, we will obtain a KN×KN matrix of value added propagation length between sector i in country r and final product of sector j in country s:

$$\mathbf{U} = \mathbf{D} = \left(\hat{\mathbf{v}}_c\left(1\mathbf{I} + 2\mathbf{A} + 3\mathbf{A}^2 + \dots\right)\hat{\mathbf{f}}\right) \oslash \left(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{f}}\right) = \left(\hat{\mathbf{v}}_c\mathbf{L}^2\hat{\mathbf{f}}\right) \oslash \left(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{f}}\right)$$

where $\hat{\mathbf{v}}_c$ and $\hat{\mathbf{f}}$ are, respectively, a diagonalized row vector of value added coefficients and a diagonalized column vector of final demand ($\mathbf{f} = \mathbf{F}\mathbf{i}$). In this case, the number of production stages is weighted by the share of value added (not total output as in the version of Antràs et al., 2012) generated at each successive stage and embodied in product of sector j:

$$(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{f}})\oslash(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{f}}) = (\hat{\mathbf{v}}_c\hat{\mathbf{f}})\oslash(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{f}}) + (\hat{\mathbf{v}}_c\mathbf{A}\hat{\mathbf{f}})\oslash(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{f}}) + (\hat{\mathbf{v}}_c\mathbf{A}^2\hat{\mathbf{f}})\oslash(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{f}}) + (\hat{\mathbf{v}}_c\mathbf{A}^3\hat{\mathbf{f}})\oslash(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{f}}) + \dots$$

Aggregating across the sectors that deliver final products will transform the value added propagation length into the measure of upstreamness or total forward linkages:

$$\mathbf{u} = \left(\hat{\mathbf{v}}\hat{\mathbf{x}}^{-1}\mathbf{L}\mathbf{L}\hat{\mathbf{f}}\mathbf{i}\right) \oslash \left(\hat{\mathbf{v}}\hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{f}}\mathbf{i}\right) = \left(\hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}}\mathbf{i}\right) \oslash \left(\hat{\mathbf{x}}^{-1}\hat{\mathbf{x}}\mathbf{i}\right) = \mathbf{G}\mathbf{i} \oslash \mathbf{i} = \mathbf{G}\mathbf{i}$$

Since each element in $\mathbf{U}$ is a ratio of the scalars:

$$u_{rs}^{ij} = \frac{v_r^i\left[\mathbf{L}^2\right]_{rs}^{ij} f_s^j}{v_r^i l_{rs}^{ij} f_s^j} = \frac{\left[\mathbf{L}^2\right]_{rs}^{ij}}{l_{rs}^{ij}}$$

then the value added propagation length $\mathbf{U}$ at the bilateral sector level may be simplified to:

$$\mathbf{U} = \left(\hat{\mathbf{v}}_c\mathbf{L}^2\hat{\mathbf{f}}\right) \oslash \left(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{f}}\right) = \mathbf{L}^2 \oslash \mathbf{L}$$

which is quite similar to the average propagation length described by Dietzenbacher et al. (2005). The only difference is that they subtract unity from the diagonal elements in $\mathbf{L}$ to neglect the initial effect when defining the weights.[5]

It is straightforward to show that the value added propagation length at the bilateral sector level may be equally defined in terms of the Ghosh inverse:

$$\begin{aligned}\mathbf{U} &= \left(\hat{\mathbf{v}}_c\mathbf{L}^2\hat{\mathbf{f}}\right) \oslash \left(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{f}}\right) = \left(\hat{\mathbf{v}}_c\hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{x}}^{-1}\hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{x}}^{-1}\hat{\mathbf{f}}\right) \oslash \left(\hat{\mathbf{v}}_c\hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{x}}^{-1}\hat{\mathbf{f}}\right) \\ &= \left(\hat{\mathbf{v}}\hat{\mathbf{x}}^{-1}\hat{\mathbf{x}}\mathbf{G}\mathbf{G}\hat{\mathbf{x}}^{-1}\hat{\mathbf{x}}\hat{\mathbf{f}}_c\right) \oslash \left(\hat{\mathbf{v}}\hat{\mathbf{x}}^{-1}\hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{x}}^{-1}\hat{\mathbf{x}}\hat{\mathbf{f}}_c\right) = \left(\hat{\mathbf{v}}\mathbf{G}\mathbf{G}\hat{\mathbf{f}}_c\right) \oslash \left(\hat{\mathbf{v}}\mathbf{G}\hat{\mathbf{f}}_c\right) = \mathbf{G}^2 \oslash \mathbf{G}\end{aligned}$$

[5]The APL in the original version of Dietzenbacher et al. (2005) may be written as $(\mathbf{L}(\mathbf{L}-\mathbf{I}))\oslash(\mathbf{L}-\mathbf{I})$.

where $\hat{\mathbf{f}}_c$ is the diagonalized column vector of the final demand coefficients $\mathbf{f}_c = \hat{\mathbf{x}}^{-1}\mathbf{f}$, describing the proportion of industry output allocated to final demand.

In the case of inter-country input-output tables, the bilateral measure of upstreamness may build on the following weights that describe the share of sector i's output in country r generated at stage t to satisfy total final demand in country s:

$$(\mathbf{LF}) \oslash (\mathbf{LF}) = \mathbf{F} \oslash (\mathbf{LF}) + (\mathbf{AF}) \oslash (\mathbf{LF}) + (\mathbf{A}^2\mathbf{F}) \oslash (\mathbf{LF}) + (\mathbf{A}^3\mathbf{F}) \oslash (\mathbf{LF}) + \ldots$$

The bilateral measure of upstreamness then equals:

$$\begin{aligned}\mathbf{U} &= 1\mathbf{F} \oslash (\mathbf{LF}) + 2(\mathbf{AF}) \oslash (\mathbf{LF}) + 3(\mathbf{A}^2\mathbf{F}) \oslash (\mathbf{LF}) + 4(\mathbf{A}^3\mathbf{F}) \oslash (\mathbf{LF}) + \ldots \\ &= \left(\left(\mathbf{I} + 2\mathbf{A} + 3\mathbf{A}^2 + 4\mathbf{A}^3 + \ldots\right)\mathbf{F}\right) \oslash (\mathbf{LF}) = (\mathbf{L}^2\mathbf{F}) \oslash (\mathbf{LF})\end{aligned} \tag{1}$$

$\mathbf{U}$ is a KN×K matrix where each element is an index of the average number of production stages that an output of sector i in country r needs to undergo before it ends up in final demand in country s. Aggregation across partner countries will again transform $\mathbf{U}$ into the familiar index of total forward linkages $\mathbf{Gi}$, but other aggregation options also exist (see examples in Appendix C). Equation (1) will be used further in this paper to compare the total number of production stages with the number of only cross-border stages.

It is clear that none of the measures overviewed here is designed to count only cross-border production stages. The underlying reason is that the "global" Leontief or Ghosh inverses inevitably mix inter-country and domestic effects. Each element l_{rs}^{ij} or g_{rs}^{ij} is the result of an infinite series of interactions among both domestic and foreign economic sectors. The next subsection proposes a method to draw an imaginary borderline between domestic and international value chains eventually leading to a consistent measure of the number of border crossings.

2.3 New "global" inverse, cumulative exports and the average number of border crossings

A gross exports accounting framework traces the destination of direct exports to their eventual users. This is a forward decomposition where the observed bilateral export flows are reallocated into the unobserved flows of embodied products as those pass through the downstream value chain. Koopman et al. (2010) and Wang et al. (2013) propose the accounting frameworks that may be classified under this type.[6]

An essential requirement for a gross exports accounting framework is the ability to account for sequential border crossings. The Leontief inverse $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ is not suitable because it is indifferent to the national origin of intermediate inputs. Another "global" inverse, described by Muradov (2015), addresses this issue:

$$\mathbf{H} = \left(\mathbf{I} - \check{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1}$$

where the modified "hat" and "check" operators extract, respectively, diagonal and off-diagonal block elements from block matrices but do not apply to the elements within those blocks. $\mathbf{H}$ is a KN×KN matrix of multipliers that is capable of sequentially identifying

[6] Muradov (2015) discusses the delimitation between the gross exports accounting framework and the value added accounting framework that is primarily intended for the reader's understanding of the underlying decomposition concept.

exports at tier t used to produce exports at the next tier $t+1$, or "exports embodied in exports" in a multi-country setting. Here, tiers denote production stages only when products cross national borders. An algebraic manipulation shows the relationship between the new "global" inverse and the standard Leontief inverse: $\mathbf{H} = \left(\mathbf{I} - \widehat{\mathbf{A}}\right)\mathbf{L}$. A detailed technical exposition may be found in the Appendix A.

The idea behind computing cumulative exports is similar to that of output embodied in bilateral final demand **LF** (see Johnson and Noguera, 2012; Koopman et al., 2010). The KN×K matrix of cumulative exports $\mathbf{E}_{cum}$ may be computed in two alternate ways yielding the same result (see Appendix A for a detailed derivation procedure).

First, cumulative exports may be computed as a function of final demand in partner countries:

$$\mathbf{E}_{cum} = \mathbf{H}\breve{\mathbf{F}} + (\mathbf{H} - \mathbf{I})\widehat{\mathbf{F}} = \mathbf{HF} - \widehat{\mathbf{F}} \tag{2}$$

where the first term $\mathbf{H}\breve{\mathbf{F}}$ accumulates direct and indirect exports of final products after all border crossings, and the second term $(\mathbf{H} - \mathbf{I})\widehat{\mathbf{F}}$ accumulates direct and indirect exports of intermediates eventually transformed into final products for partner use. This formulation is required for the derivation of the weighted average number of border crossings, while a rearrangement into $\mathbf{HF} - \widehat{\mathbf{F}}$ is useful for the implementation of equation (2).

Second, cumulative exports may be computed as a function of bilateral and total gross exports:

$$\mathbf{E}_{cum} = \mathbf{H}\mathbf{E}_{bil} - (\mathbf{H} - \mathbf{I})\mathbf{E}_{tot} = \mathbf{H}(\mathbf{E}_{bil} - \mathbf{E}_{tot}) + \mathbf{E}_{tot} \tag{3}$$

In $\mathbf{E}_{cum}$, each element describes the amount of product of sector i in country r that is eventually used for final demand in country s, delivered as direct or indirect exports. Total cumulative exports to all destinations are equal to total direct gross exports:

$$\mathbf{E}_{cum}\mathbf{i} = \mathbf{E}_{bil}\mathbf{i}$$

The above is parallel to the summation of output embodied in final demand $\mathbf{LFi} = \mathbf{x}$.

An essential property of the multiplier matrix **H** is the ability to trace a "melting" portion of the initial exports until it is entirely consumed (used) at an infinitely remote t^{th} tier. Each t^{th} term in the power series of **H** therefore corresponds to a t^{th} border crossing.[7] The logic of the average propagation length suggests that the total number of border crossings $1 + 2 + 3 + \cdots + t$ be weighted by the share of direct and indirect exports at each successive tier in the cumulative exports at all tiers $\mathbf{E}_{cum}$:

$$c = 1 \times \frac{\substack{\textit{direct exports}\\ \textit{of final products}} + \substack{\textit{direct exports}\\ \textit{of intermediates}}}{\substack{\textit{cumulative}\\ \textit{exports}}} + 2 \times \frac{\substack{\textit{indirect exports}\\ \textit{of final products}\\ \textit{to } 2^{nd} \textit{ tier partner}} + \substack{\textit{indirect exports}\\ \textit{of intermediates}\\ \textit{to } 2^{nd} \textit{ tier partner}}}{\substack{\textit{cumulative}\\ \textit{exports}}} +$$
$$+ \cdots + t \times \frac{\substack{\textit{indirect exports}\\ \textit{of final products}\\ \textit{to } t^{th} \textit{ tier partner}} + \substack{\textit{indirect exports}\\ \textit{of intermediates}\\ \textit{to } t^{th} \textit{ tier partner}}}{\substack{\textit{cumulative}\\ \textit{exports}}}$$

where c is the weighted average number of border crossings and intermediates are those transformed into final products without leaving the territory of the t^{th} tier partner. For the derivation of this measure in block-matrix form, we will first define weights separately for

[7]The input-output model treats the border between exporter and partner as a single border.

each of the two terms in equation (2). The count of the number of borders crossed by final products $\mathbf{H}\breve{\mathbf{F}}$ starts from 1:

$$1\breve{\mathbf{F}} \oslash \mathbf{E}_{cum} + 2\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} + 3\left(\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{2}\breve{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} +$$
$$+ \cdots + t\left(\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t-1}\breve{\mathbf{F}}\right) \oslash \mathbf{E}_{cum}$$

And the count of the number of borders crossed by intermediates for final use in partner countries $(\mathbf{H}-\mathbf{I})\widehat{\mathbf{F}}$ starts from 0 because the first domestic delivery of final products does not involve border crossings:

$$0\widehat{\mathbf{F}} \oslash \mathbf{E}_{cum} + 1\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} + 2\left(\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{2}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} +$$
$$+ \cdots + t\left(\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum}$$

Adding up the two expressions above yields the bilateral weighted average number of border crossings:

$$\mathbf{C} = 1\left(\breve{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} + 2\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{F}} + \left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{2}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} +$$
$$+ \cdots + t\left(\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t-1}\breve{\mathbf{F}} + \left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum}$$

We may easily verify that the sum of all weights implicitly applied to $\mathbf{F}$ is a KN×K matrix where all elements are equal to 1. Pre-multiplying the numerator (the expressions in brackets) by $\left(\mathbf{I}-\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)$ and then by $\left(\mathbf{I}-\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{-1}$ shows that:

$$1\mathbf{I} + 2\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1} + 3\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{2} + \cdots + t\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t-1} = \mathbf{H}^{2}$$

$$0\mathbf{I} + 1\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1} + 2\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{2} + \cdots + t\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t} = \mathbf{H}(\mathbf{H}-\mathbf{I})$$

Then the equation of the weighted average number of border crossings can be simplified to:

$$\mathbf{C} = \left(\mathbf{H}^{2}\breve{\mathbf{F}} + (\mathbf{H}-\mathbf{I})\mathbf{H}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} = \left(\mathbf{H}^{2}\mathbf{F} - \mathbf{H}\widehat{\mathbf{F}}\right) \oslash \left(\mathbf{H}\mathbf{F} - \widehat{\mathbf{F}}\right) \tag{4}$$

The "hat" operator in equation (4) applies to the blocks of $\mathbf{F}$, not to the elements therein. $\mathbf{C}$ is a KN×K matrix where each element c_{rs}^{i} may be interpreted as the weighted average number of border crossings along the path of a product of sector i from country r to its final user in country s. The lowest value of the element c_{rs}^{i} is 1 when sector i in country r only exports final products. This is in line with the conventional wisdom confirming that exported products cross borders at least once.

3 Empirical application

3.1 The rise of the average number of border crossings is modest

For an empirical application of the proposed measures, this paper uses data from the World Input-Output Database (WIOD). The WIOD contains a series of national and inter-country supply-use tables and input-output tables supplemented by sets of socio-economic and environmental indicators for 1995-2011. It covers 27 European Union member states, 13 other major non-European economies, plus estimates for the rest of the world (RoW). The classification used in the WIOD discerns 35 industries and 59 products, based on NACE rev.1 (ISIC rev. 3) and CPA, respectively. The WIOD project is recognized for its benchmarking of inter-country input-output data against updated national account aggregates, ensuring accuracy in handling international merchandise and services trade statistics. It has been widely used for quantitative research into the various implications of global value chains (Timmer et al., 2015).[8] The world input-output tables in WIOD, used for the computations here, are compatible with the matrix setup in subsection 2.1 except the matrices of trade and transport margins and net taxes.[9]

The computation of the weighted average number of border crossings between each country/sector of origin and the country of final destination, as in equation (4), yields matrices in the dimension KN×K, which in the case of the WIOD is 1435×41. For sensible visualization, these data are reorganized by exporting country and exporting sector (see the aggregation options in Appendix C). The aggregated indicators are necessarily trade-weighted.

It is worth noting that the numbers reported below correspond to the downstream perspective, i.e. count the number of production stages or borders from the sector of origin to the country of final demand. This is distinct to other studies (e.g., Miroudot and Nordstrom, 2015) that adopt the upstream perspective and quantify the number of embodied production stages from final products or exports backwards to the sources of value added.

From the perspective of an exporting country, the weighted average number of border crossings across all partners ranged in 2010 from 1.23 for Mexico to 1.59 for Russia. As Fig. 1 reveals, the change in the number of border crossings has not been uniform. For 26 exporting countries in WIOD, this number increased in 2001-2005 but descended in 2005-2010. For 12 countries, it increased both in 2001-2005 and 2005-2010. 2 countries experienced a decline of this measure in both periods. The simple average number of border crossings for all exporters rose from 1.30 in 2001 to 1.35 in 2005 and stood at 1.34 in 2010.

In 2010, the products of the"Basic Metals and Fabricated Metal" industry (c12) had to cross more borders than any other product on the way to their eventual users – 1.59, while the products of "Food, Beverages and Tobacco" industry (c3) only crossed borders 1.08 times which was the lowest number (apart from the "Private Households with Employed Persons"). Exporting sectors with the longest cross-border value chains included auxiliary transport activities (c26), mining and quarrying (c2), inland transport (c23), electricity, gas and water supply (c17), rubber and plastics (c10). The cross-border value chains of health and social work (c33), leather products (c5) and textile products (c4) were the shortest and, for the latter, involved declining number of border crossings in 2001-2005-2010 as seen in Fig. 2.

[8] The database and related information are available at `http://www.wiod.org`.

[9] The WIOD records the information on valuation that is needed to change the national supply-use tables from purchasers' prices to basic prices, but does not utilize it to produce consistent valuation layers for the symmetric world table.

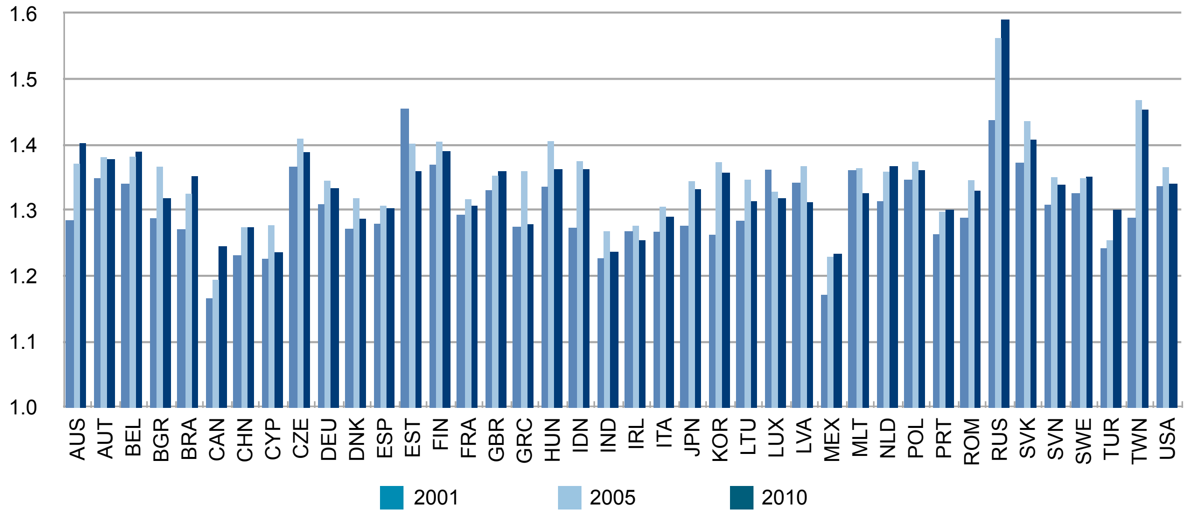


Figure 1: Weighted average number of border crossings, by exporting country
Source: WIOD database, author's calculations
Note: the full list of countries in WIOD is in Table D.1, Appendix D.

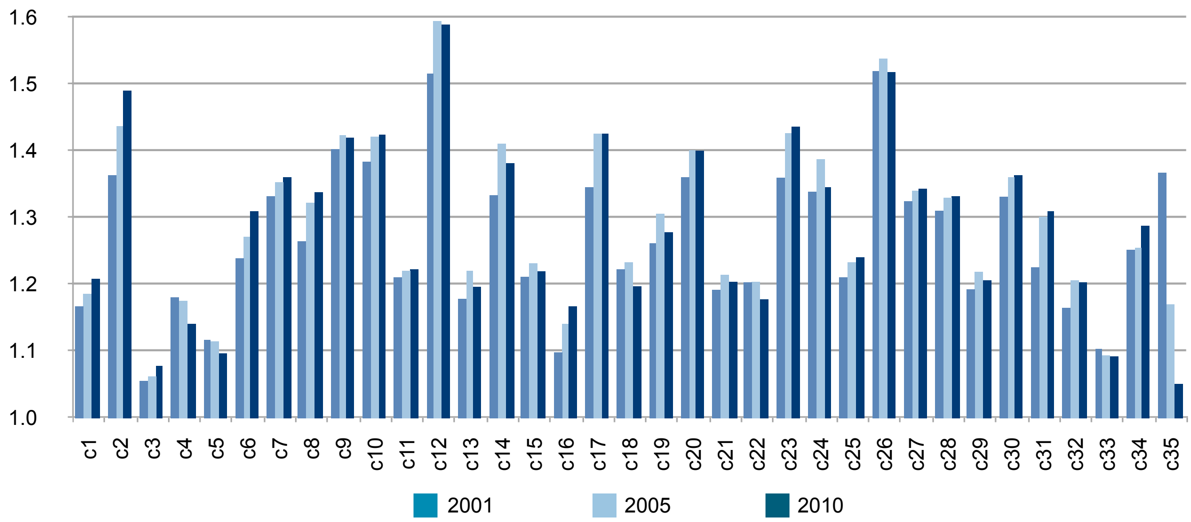


Figure 2: Weighted average number of border crossings, by exporting sector
Source: WIOD database, author's calculations
Note: the full list of sectors in the WIOD is in Table D.2, Appendix D.

3.2 The average number of border crossings is proportional to the average number of production stages

In the entire value chain, production stages may be classified into those that take place within economies and those between economies. The latter involve border crossings while the former do not. By intuition, the number of cross-border production stages should be smaller then the total number of production stages linking the producing (exporting) sector and the final user. It is also natural to suppose that there is a relatively stable relationship between the two numbers: the more production stages an average product has to undergo, the more border crossings it has to cross. The measures described in subsections 2.1 and 2.2 allow us to empirically test these assumptions and gauge the relative importance of the cross-border production chain.

The total length of the downstream value chain, or the total number of production stages from producer to the final user, may be quantified with the measure of upstreamness **U** from equation (1), aggregated across partner countries. For the ease of presentation, it is simultaneously aggregated across producing sectors or countries. Unaggregated results indicate that the domestic part of value chain involves less production stages, accounts for larger transactions and therefore heavily influences the weighted average numbers. That is the reason why two versions of the upstreamness measure are computed: (1) the standard measure that corresponds to the total length of value chain, or the total number of production stages and (2) the same measure disregarding the domestic blocks in **U** which corresponds to the length of exports value chain, or the number of production stages between a country/sector and all its partner countries. Obviously, the modified measure counts both cross-border and domestic production stages, but only those ending abroad.

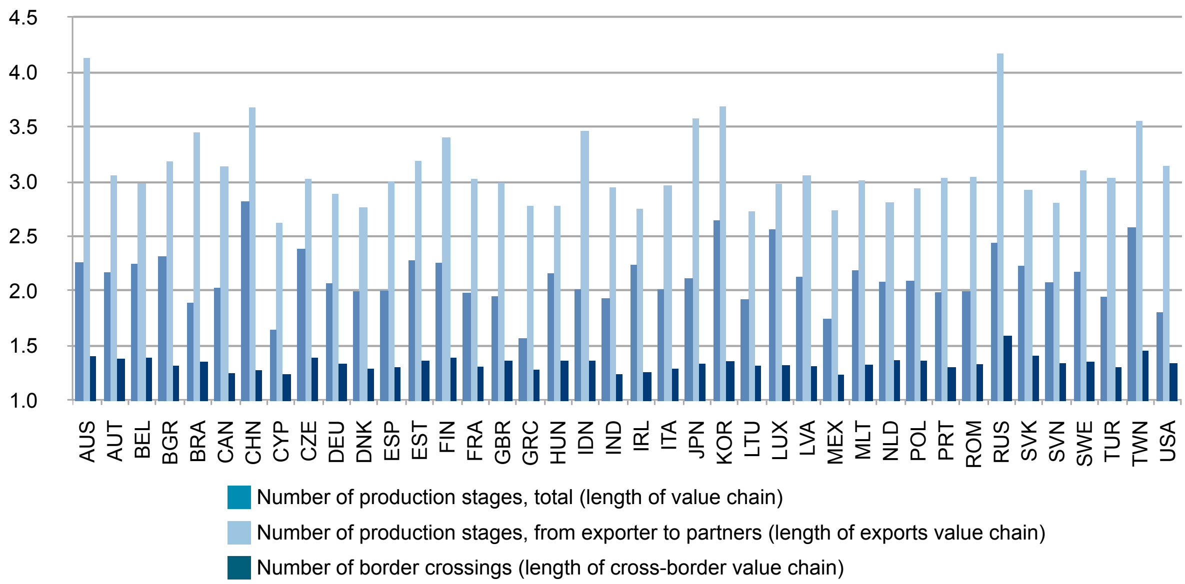


Figure 3: The number of production stages and border crossings, by exporting country, 2010
Source: WIOD, author's calculations
Note: the full list of countries in WIOD is in Table D.1, Appendix D.

In Fig. 3, these two measures are contrasted with the number of border crossings in 2010 from the producing/exporting country perspective. The chart confirms that the total length of an average product path towards its final user at all destinations is consistently greater than the length of its cross-border movements to those destinations. The ratio of the number of border crossings to the total number of production stages, computed as a simple average

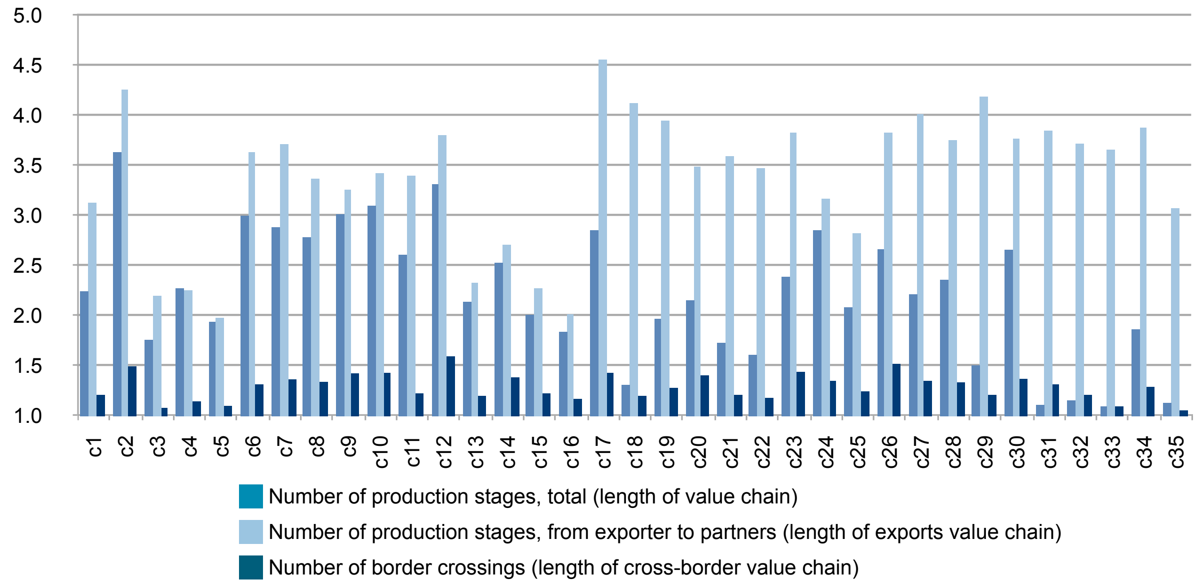


Figure 4: The number of production stages and border crossings, by exporting sector, 2010
Source: WIOD, author's calculations
Note: the full list of sectors in the WIOD is in Table D.2, Appendix D.

across all 40 WIOD countries, remained rather stable: 0.64 in 2001, 0.65 in 2005 and 0.64 in 2010.

When the domestic production chains (or more precisely, those originating and ending in the same country) are disregarded, the average number of production stages increases by a factor of 1.3-1.8. This modified measure of upstreamness is referred to here as the length of exports value chain. It is necessarily greater than the length of cross-border production chain. The ratio of the number of border crossings to the number of production stages ending abroad for all 40 WIOD countries stood at 0.44 in 2001, 0.45 in 2005 and 0.43 in 2010.

The ratio described above approximates the relative importance of border crossings in downstream value chains and changes thereof. In 2001-2005, this ratio increased for 26 countries in WIOD meaning that their products at average had to cross more borders per one production stage. In 2005-2010, the same may be observed for only 4 countries while the rest experienced a reverse trend. The average number of border crossings per one production stage was relatively low and declining in large Asian economies: from 0.38-0.35 in 2005-2010 in China, from 0.39-0.37 in Japan and from 0.40-0.37 in Korea.

From the perspective of the exporting sector (see Fig. 4), the weighted average number of border crossings is somewhat higher than the weighted average number of total production stages in 2010 for three service industries: "Public Administration and Defence, Compulsory Social Security", "Education" and "Health and Social Work". This is an unusual result that may be explained by the weighting and aggregation scheme with the prevalence of short domestic transactions in those services directly to final consumers. Moreover, at the most disaggregate, country-sector by country level, the ratio of the average number of border crossings to the average number of total production stages does not exceed 1. In 2001-2005, the modified ratio of border crossings to production stages ending abroad (i.e., excluding domestic transactions) increased for 23 out of 35 WIOD sectors, but it increased for only 1 sector in 2005-2010.

By and large, the average number of border crossings evolved in a proportion to the number of production stages that remained relatively stable over the period considered. The findings also suggest that the cross-border fragmentation of production was more pronounced in the first half of 2010s than in the second half of that decade.

3.3 The length of cross-border value chains does not depend on geographic distance but regional pattern may be discerned

One variable that may be expected to explain the variation in the number of border crossings is the geographic distance between countries. In the world of global value chains, direct bilateral trade between two distant countries, e.g. Russia and Australia or Brazil and Korea, may be modest, but it is non-negligible when indirect trade flows and embodied intermediates are accounted for. Value chains between two distant countries may be longer than those between two close or neighbouring countries.

In a scatter plot in Fig. 5 we examine a relationship between the number of border crossings and the natural logarithm of geographic distance[10] for all exporter-partner pairs in WIOD in 2010. The data points appear to be condensed in two "clouds", and this is not by chance. Shorter distances (left "cloud") correspond to intra-regional flows while longer distances (right "cloud") to the inter-regional flows. For this purpose, countries in the WIOD are grouped into three larger regions: EU, NAFTA and Asia Pacific (Brazil, Russia, Turkey and rest of the world are not included in any region). Fig. 5 indicates that the average number of border crossings is higher for inter-regional country pairs than for those intra-regional. Indeed, an additional calculation reveals that the variance of the number of border crossings between regions explains 31% of its total variance in 2010.

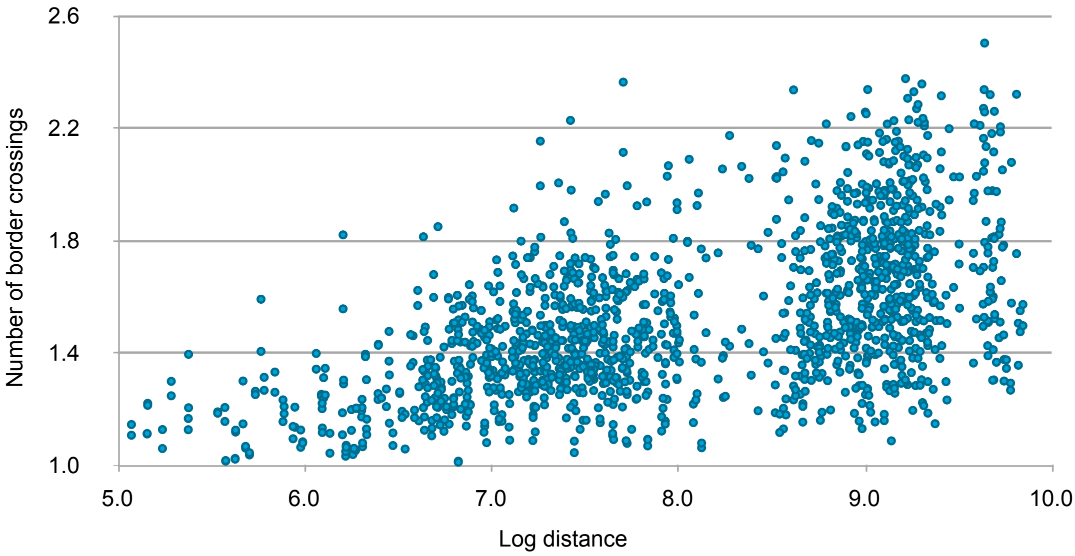
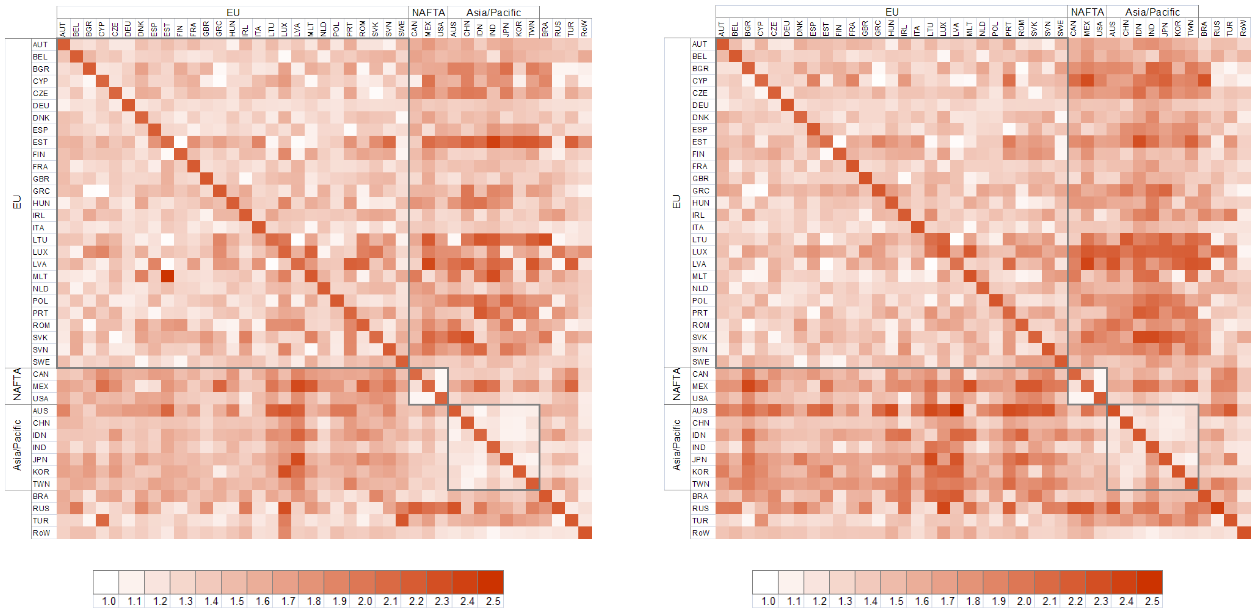


Figure 5: Relationship between bilateral number of border crossings and bilateral distance, by exporter-partner country pair, 2010

Source: WIOD, CEPII, author's calculations

The heatmaps in Fig. 6 confirm the regional pattern. The EU appears as an area where products have to cross relatively lesser number of borders if both producer and final user are in Europe. This is also true for NAFTA and, with some reservations, for Asia Pacific. Meanwhile, the economic distance between countries in different regions became longer. This conforms with the results of other studies, e.g. Miroudot and Nordstrom (2015) who report that the slicing of the value chain has primarily been extra-regional.

[10]Data on bilateral distances is drawn from the GeoDist dataset provided by CEPII.



(a) 2001 (b) 2010

Figure 6: Bilateral weighted average number of border crossings, reorganized by region, 2001 and 2010

Source: WIOD database, author's calculations

3.4 Multiple border crossings do not hinder trade liberalization

Previous studies, including Hummels et al. (2001) and Yi (2010), have identified multiple border crossings as a key factor behind the magnification of trade costs in global value chains.

Muradov (2015) explores the value added and gross exports accounting frameworks and derives two measure of accumulated bilateral trade costs in global value chains.[11] These methods allow the measurement of accumulated trade costs between the country of origin (exporter, producer) and the country of destination (partner, user), including direct and indirect costs.

The first method counts direct plus indirect trade costs when the latter apply to the transactions between the exporter and third countries. It treats trade costs as embodied inputs that eventually reach the destination. This measure, referred to as "cumulative trade costs", is derived from the value added accounting framework. The second method treats indirect trade costs when they apply to the transactions between third countries and the partner. It identifies trade costs that apply to the exporter inputs at the border of the partner where they are hidden in third country exports. The second measure, "incremental trade costs", takes root in the gross accounting framework (see Appendix B for the formulae and details on their derivation).

In the input-output framework, data on valuation layers are the most accessible for trade cost accounting. These data cover a significant share of trade costs, including distribution costs, i.e. trade and transport margins, and taxes less subsidies on traded products. The WIOD does not provide the valuation layers compatible with the global input-output matrices. However, two valuation layers may be readily compiled, creating only minor in-

[11]A value added accounting framework traces the origin of gross exports to the sectors that initially contribute value added to those exports. This is a backward decomposition that reallocates all observed bilateral export flows into the unobserved value added flows between origins and destinations. A gross exports accounting framework traces the destination of direct exports to their eventual users. This is a forward decomposition where the observed bilateral export flows are reallocated into the unobserved flows of embodied products as those pass through the downstream value chain.

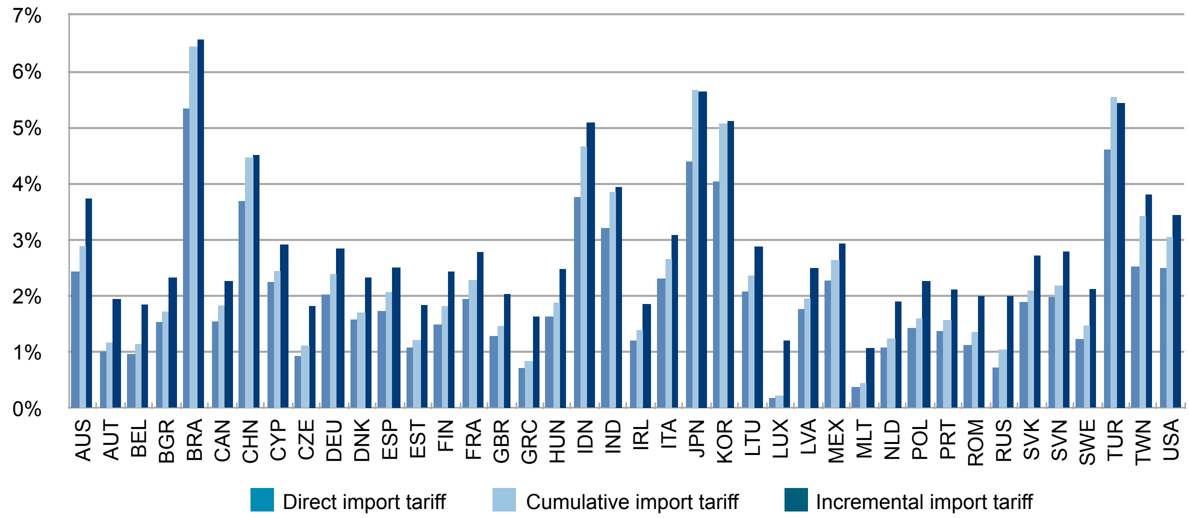


Figure 7: Direct and accumulated import tariffs faced by exporting country, 2010
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations
Note: the full list of countries in the WIOD is in Table D.1, Appendix D.

consistencies with the original world input-output tables in the WIOD – the matrices of international trade and transport margins and the matrices of import taxes at destination. These matrices were compiled for 2001, 2005 and 2010 in the product-by-industry format and were transformed into the symmetric industry-by-industry format. The underlying data were extracted from the UN Comtrade and UN TRAINS databases.[12]

Fig. 7 displays the so called magnification effect of import tariffs by comparing three versions of bilateral tariff measures, aggregated across partner countries. From the perspective of market access for exporters, the average direct tariffs across all partners are generally low and declining. Out of 40 countries in the WIOD (apart from the RoW), for only 7 countries did the average direct tariff exceed 3% in 2010, and for only one country was it higher than 5%. Brazil and Luxembourg faced, respectively, the highest (5.3%) and the lowest (0.2%) tariffs. The simple average import tariff for all 40 exporters declined from 3.2% in 2001 to 2.2% in 2005 and to 2.0% in 2010. The low average level of import tariffs is partly the result of accounting for bilateral and regional preferences arising from new free trade agreements. It also reflects the WIOD's focus on the European Union members that apply low MFN tariffs and zero tariffs with respect to their intra-regional imports.

Cumulative and incremental tariffs in Fig. 7 indicate that the average resistance to exports does not significantly increase when the multi-stage production is taken into account. For all 40 exporters, the simple average cumulative tariff went down from 3.9% in 2001 to 2.7% in 2005 and to 2.4% in 2010. The incremental method produces consistently higher estimates: 4.4% in 2001, 3.2% in 2005 and 2.9% in 2010.

By definition, cumulative and incremental tariffs may be split into direct tariffs on exports plus indirect tariffs on embodied inputs identified in two different ways. The indirect portion provides a good indication of the accumulated resistance effect. The largest indirect tariffs in 2010 are revealed by the incremental approach for Indonesia (3.76% direct tariff + 1.33% indirect tariff), Australia (2.44%+1.30%), and Taiwan (2.52%+1.28%), in addition to the cumulative approach for Japan (1.39%+1.28%). In none of these cases does the average tariff for exporters double as a result of value chain accounting. An indirect tariff in 2010 is

[12]UN Comtrade and UN TRAINS were accessed via the World Integrated Trade Solution (WITS). Detailed description of the compilation procedures may be found in Muradov (2015).

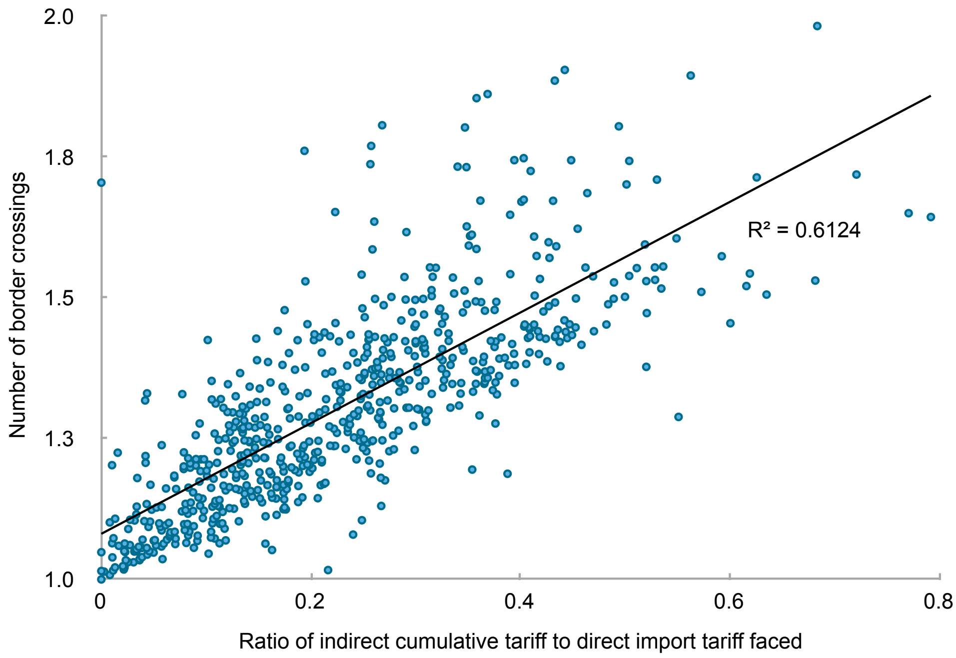


Figure 8: Relationship between indirect tariffs and number of border crossings, by exporting country-sector (goods only), 2010

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations

higher than a direct tariff when counted by the incremental approach only for Luxembourg (0.18%+1.02%), Malta (0.38%+0.69%), Russia (0.73%+1.27%) and Greece (0.72%+0.92%). These are also the countries that face some of the lowest direct import tariffs. Appendix B.3 includes a more detailed account of the relative importance of indirect tariff.

In Fig. 8, for each exporting country-sector, the tariff accumulation effect across all partners (horizontal axis) is related to the average number of border crossings across all destinations (vertical axis). The accumulation effect is defined as the ratio of indirect cumulative tariff between an exporting country-sector and all its partners to direct import tariff.[13] The incremental tariff is less relevant for this exercise because of its excessive focus on the tariffs applicable at the partner border. The scatter plot only shows the results for goods-producing sectors (c1 – c16 in the WIOD, see Appendix D) in 2010, as the results for service sectors may be biased because they face zero or minimal direct tariffs.

Fig. 8 confirms that, by and large, a higher accumulation effect is associated with more border crossings. However, the growing number of border crossings (see subsection 3.1) did not bring about an increase in cumulative tariffs in the five-year periods explored here. The cumulative tariff faced by total exports declined from 2001-2005 and from 2005-2010 for 29 countries in the WIOD. It first decreased but later increased for 10 countries and rose in both periods for only one country.

In sum, the number of border crossings rose slowly over 2001-2005-2010 while cumulative tariffs declined quickly. The continuous reduction in direct import tariffs neutralized the indirect tariff accumulation effect.

At the country-sector level, there is no clear unidirectional link between the change in the cumulative tariff (in percentage points) and the respective change in the number of border crossings (in dimensionless units). In Fig. 9, these changes are contrasted and differentiated between two periods. It is clear that from 2001-2005, a reduction in the cumulative tariff

[13]This measure is derived from equation B.3 in Appendix B.1: $\mathbf{T}_{cum} \oslash \mathbf{T} - 1$.

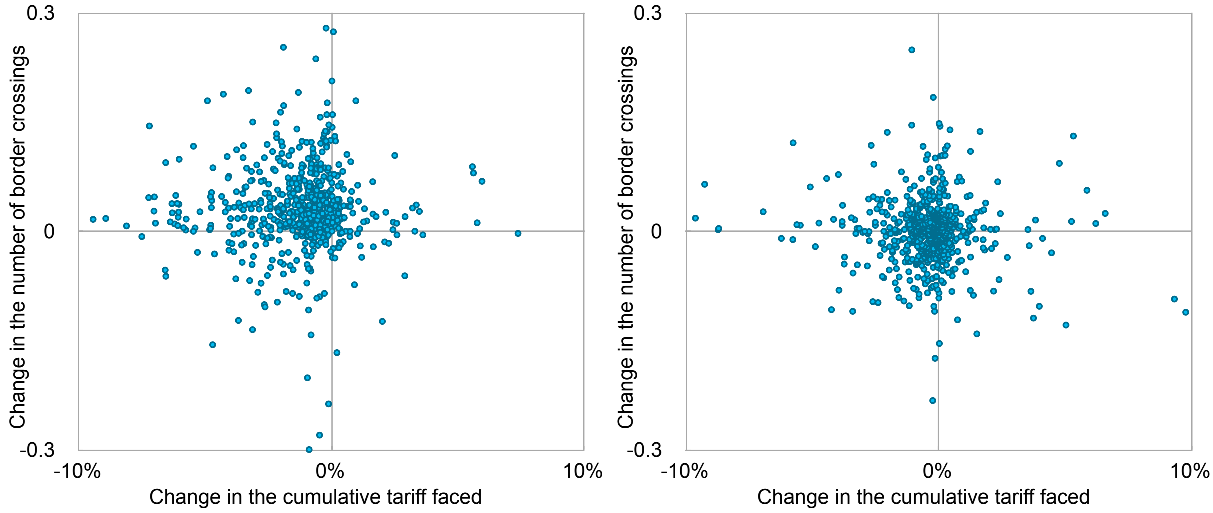


Figure 9: Relationship between the change in the number of border crossings and the change in the cumulative tariff faced, by exporting country-sector (goods only)

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations

among goods-producing sectors was, in the vast majority of cases, associated with an increase in the number of borders to be crossed. In 2005-2010, such a pattern is barely discernable. We may observe that from 2001-2005, the international fragmentation of production increased the average number of borders a product was required to cross before consumption, but trade liberalization ensured that exporters benefited from this and did not face greater protection along the downstream value chain. Over the next 5 years, both fragmentation of production and liberalization of trade slowed down with a mixed but mostly neutral effect on exporters. The global economic and trade collapse of the late 2000s might at least partially explain this result.

4 Conclusion

Multiple border crossings are known as an essential feature of global value chains. As such, they are responsible for the international fragmentation of production or vertical specialization in trade, but also for supposedly higher indirect trade costs. A number of reports discussed the theoretical underpinnings and implications of multiple border crossings. However, to the best of the author's knowledge, none of those proposed a method to consistently count the number of border crossings, or cross-border production stages, distilling those from purely domestic production lines.

This paper addressed that problem by applying the average propagation length principle to a measure of cumulative exports. The latter reallocate direct exports to the final users after multi-stage production. The result is an average number of border crossings weighted by a "melting" part of initial direct exports travelling along the downstream value chain in the form of embodied intermediates.

Although global value chains quickly evolved into a dominant feature of the world economy, we do not observe a dramatic increase in the average number of border crossings in 2000s. Moreover, the upward trend in the first half of the decade reversed in the second half.

By and large, the behaviour of the proposed measure is within the expected range. In most cases, unless the weighting and aggregation affects the result, it is lower than the total number of production stages extending from the producer to the final user. And it is consistently lower than the number of production stages between producer in one country and final users in all partner countries. The number of borders crossed per one production stage has been relatively stable, though slowly declining for such exporting economies as China, Japan and Korea.

There is no clear link between the length of cross-border value chains and the geographic distance. But adding the regional dimension to the analysis reveals that the number of borders a typical product has to cross increased between regions while remaining relatively stable within regions. Lastly, more border crossings do not bring about higher accumulated resistance to exports (in terms of import tariffs) as it is neutralized by the ongoing trade liberalization.

The measure of the average number of border crossings alone may have limited analytical value. It counts cross-border production stages without evaluating the associated border barriers. As follows from the experimental application in this paper, for policy-relevant results, the proposed measure should be combined with other indicators that show how protection, uncertainty and disruptions at the borders propagate along global value chains. Some of these indicators, e.g. cumulative tariffs, have been computed as part of this paper, while other, such as multi-stage trade facilitation, may be a topic for follow-up research.

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A Gross exports accounting framework and derivation of the new "global" inverse

A.1 The new "global" inverse

A gross exports accounting framework traces the destination of direct exports to their eventual users. This is a forward decomposition where the observed bilateral export flows are reallocated into the unobserved flows of embodied products as those pass through the downstream value chain.

By definition, bilateral gross exports comprise cross-border flows of intermediate and final products:

$$\mathbf{E}_{bil} = \breve{\mathbf{Z}}_{(KN\times K)} + \breve{\mathbf{F}}$$

Exports of intermediates can be expressed as a function of the partner country total output:

$$\breve{\mathbf{Z}}_{(KN\times K)} = \breve{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)}$$

where $\hat{\mathbf{x}}_{(KN\times K)}$ is the block-diagonalized vector of total output:

$$\hat{\mathbf{x}}_{(KN\times K)} = \begin{bmatrix} \mathbf{x}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{x}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{x}_k \end{bmatrix}$$

Total output $\hat{\mathbf{x}}_{(KN\times K)}$ is the sum of intermediates for domestic use, final products for domestic use and total exports, which in the KN×K block-diagonalized form can be written as:

$$\hat{\mathbf{x}}_{(KN\times K)} = \widehat{\mathbf{Z}}_{(KN\times K)} + \widehat{\mathbf{F}} + \mathbf{E}_{tot}$$

$\mathbf{E}_{tot}$ is the block-diagonalized matrix of total gross exports:

$$\mathbf{E}_{tot} = \begin{bmatrix} \mathbf{e}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{e}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{e}_k \end{bmatrix} \quad \text{where a block element} \quad \mathbf{e}_r = \begin{bmatrix} e_r^1 \\ e_r^2 \\ \vdots \\ e_r^n \end{bmatrix}$$

Block elements $\mathbf{e}_r$ are N×1 vectors where each entry $e_r^i = \sum_{s\neq r}^{K} \left(\sum_{j=1}^{N} z_{rs}^{ij} + f_{rs}^{i} \right)$.

Insert the decomposed $\hat{\mathbf{x}}_{(KN\times K)}$ into $\breve{\mathbf{Z}}_{(KN\times K)} = \breve{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)}$ and then into $\mathbf{E}_{bil} = \breve{\mathbf{Z}}_{(KN\times K)} + \breve{\mathbf{F}}$ to obtain:

$$\mathbf{E}_{bil} = \breve{\mathbf{A}}\widehat{\mathbf{Z}}_{(KN\times K)} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}}$$

Now, gross bilateral exports are a sum of (a) direct exports of intermediates for domestic intermediate use by partner, (b) direct exports of intermediates for domestic final use by partner, (c) direct exports of intermediates for exports by partner and (d) direct exports of final products. The eventual use of exported intermediates described by the first term $\breve{\mathbf{A}}\widehat{\mathbf{Z}}_{(KN\times K)}$ remains undetermined, i.e., these can either be embodied in domestic final use by partner or in partner exports. Accordingly, subsequent manipulations decompose this term until it is completely allocated between domestic final use and exports.

Using that $\widehat{\mathbf{Z}}_{(KN\times K)} = \widehat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} = \widehat{\mathbf{A}}\left(\widehat{\mathbf{Z}}_{(KN\times K)} + \widehat{\mathbf{F}} + \mathbf{E}_{tot}\right)$ leads to an infinite series of inter-industry interactions:

$$
\begin{aligned}
\mathbf{E}_{bil} &= \breve{\mathbf{A}}\widehat{\mathbf{Z}}_{(KN\times K)} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\widehat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\widehat{\mathbf{A}}\left(\widehat{\mathbf{Z}}_{(KN\times K)} + \widehat{\mathbf{F}} + \mathbf{E}_{tot}\right) + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\left(\widehat{\mathbf{Z}}_{(KN\times K)} + \widehat{\mathbf{F}} + \mathbf{E}_{tot}\right) + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \\
&+ \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \ldots
\end{aligned}
$$

Compiling and rearranging all terms after $t \to \infty$ rounds of interactions results in:

$$
\begin{aligned}
\underset{t\to\infty}{\mathbf{E}_{bil}} &= \breve{\mathbf{A}}\left[\widehat{\mathbf{A}}\right]^t \hat{\mathbf{x}}_{(KN\times K)} + \left(\breve{\mathbf{A}}\left[\widehat{\mathbf{A}}\right]^t + \cdots + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}} + \breve{\mathbf{A}}\widehat{\mathbf{A}} + \breve{\mathbf{A}}\right)\widehat{\mathbf{F}} + \\
&+ \left(\breve{\mathbf{A}}\left[\widehat{\mathbf{A}}\right]^t + \cdots + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}} + \breve{\mathbf{A}}\widehat{\mathbf{A}} + \breve{\mathbf{A}}\right)\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\left[\widehat{\mathbf{A}}\right]^t \hat{\mathbf{x}}_{(KN\times K)} + \breve{\mathbf{A}}\left(\left[\widehat{\mathbf{A}}\right]^t + \cdots + \widehat{\mathbf{A}}\widehat{\mathbf{A}} + \widehat{\mathbf{A}} + \mathbf{I}\right)\widehat{\mathbf{F}} + \\
&+ \breve{\mathbf{A}}\left(\left[\widehat{\mathbf{A}}\right]^t + \cdots + \widehat{\mathbf{A}}\widehat{\mathbf{A}} + \widehat{\mathbf{A}} + \mathbf{I}\right)\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= 0 + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \breve{\mathbf{F}}
\end{aligned}
$$

The elements in $\breve{\mathbf{A}}\left[\widehat{\mathbf{A}}\right]^t \hat{\mathbf{x}}_{(KN\times K)}$ are approaching zero with $t \to \infty$ because the column sums of $\mathbf{A}$ and $\widehat{\mathbf{A}}$ are less then 1 in a monetary IO table.

It is worth noting that, due to the known property of the block-diagonal matrices, $\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}$ is equal to a block-diagonal matrix of local Leontief inverses:

$$
\begin{aligned}
(\mathbf{I} - \mathbf{A})^{-1} &= \begin{bmatrix} \mathbf{I} - \mathbf{A}_{11} & 0 & \cdots & 0 \\ 0 & \mathbf{I} - \mathbf{A}_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{I} - \mathbf{A}_{kk} \end{bmatrix}^{-1} = \\
&= \begin{bmatrix} (\mathbf{I} - \mathbf{A}_{11})^{-1} & 0 & \cdots & 0 \\ 0 & (\mathbf{I} - \mathbf{A}_{22})^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (\mathbf{I} - \mathbf{A}_{kk})^{-1} \end{bmatrix}
\end{aligned}
$$

The equation obtained above reallocates direct exports of sector i from the exporting country r according to their eventual use by the direct partner s:

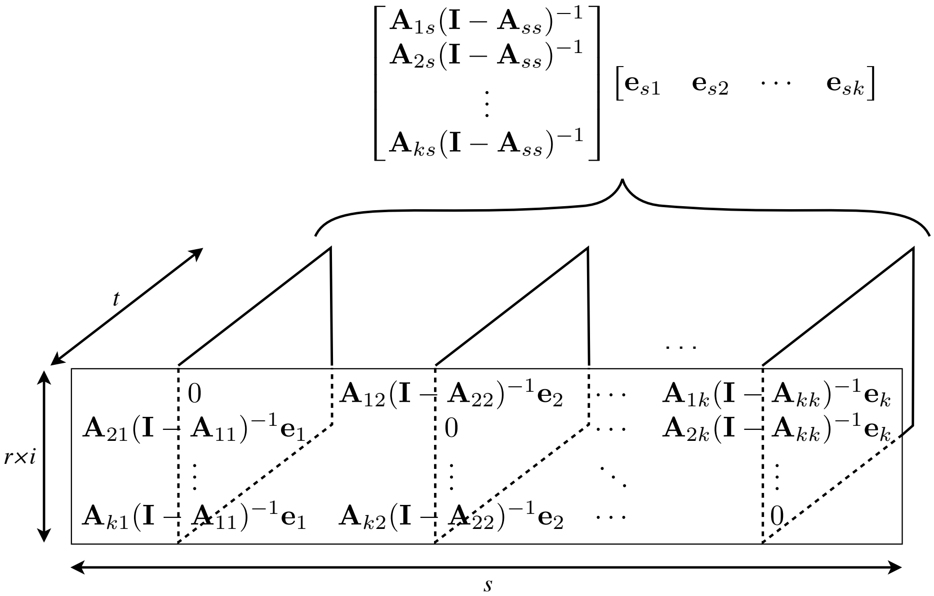

Figure A.1: Transformation of the $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ matrix into a 3$^{\text{rd}}$-order tensor

$$\underbrace{\mathbf{E}_{bil}}_{\substack{\text{bilateral}\\ \text{gross}\\ \text{exports}}} = \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}}_{\substack{\text{intermediates}\\ \text{eventually transformed}\\ \text{by partner into final products}\\ \text{for domestic use}}} + \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}}_{\substack{\text{intermediates}\\ \text{eventually used}\\ \text{by partner for exports}}} + \underbrace{\breve{\mathbf{F}}}_{\substack{\text{final products}\\ \text{directly exported}\\ \text{to partner}\\ \text{for domestic use}}} \tag{A.1}$$

Note that exports in this type of decomposition embody value added from all sectors and all countries of origin. The component matrices represent flows of products (not value added) and are necessarily confined to direct gross exports. In other words, value chains are confined to the national borders. Each component flow can be expressed as a share of direct gross exports and will not exceed 100%. This decomposition is conceptually close to those in Koopman et al. (2010) and Wang et al. (2013), though differs in the way of identifying the eventual use of direct exports.

In the decomposition above, it is still unknown where the re-exported term $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ is destined for. The next exercise will trace this flow to the next tiers of the value chain and allocate it according to its eventual use. A tier henceforth will correspond to cross-border flows of intermediate products.

The term $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ needs disaggregating according to the next country of destination, or second-tier partner. Given that $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ is a KN×K matrix that shows the flows among the exporting countries r and the first-tier partners s, our exercise requires extending the matrix to the third dimension KN×K×K. Then it will show the flows from the exporter r through the first-tier partner s to the second-tier partner t. This is visualized in Fig. A.1.

The result is a thee-dimensional matrix, or a 3$^{\text{rd}}$-order tensor where the third dimension is constructed by computing the outer product of the s^{th} column in $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}$ and s^{th} row in $\mathbf{E}_{bil}$:

$$
\begin{bmatrix}
\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1} \\
\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1} \\
\vdots \\
\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}
\end{bmatrix}
\begin{bmatrix} \mathbf{e}_{s1} & \mathbf{e}_{s2} & \cdots & \mathbf{e}_{sk} \end{bmatrix} =
$$

$$
= \begin{bmatrix}
\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \\
\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk}
\end{bmatrix}
$$

These KN×K matrices are perpendicular to $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ and their row sums are equal to the s^{th} column of $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$. So the tensor contraction along the third dimension results in reverting to the KN×K matrix $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$.

In principle, the s^{th} row in $\mathbf{E}_{bil}$ may be replaced with the sum of the rows in the component matrices from $\mathbf{E}_{bil} = \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \breve{\mathbf{F}}$. Then the re-exported term may be disaggregated again into the fourth dimension (KN×K×K×K) and so on, which may lead to a series of high-dimensional tensors.

In order to keep data in a manageable form for the decomposition to the next tiers, we opt for the tensor contraction along the second dimension, that is first-tier partners s:

$$
\sum_{s=1}^{K}
\begin{bmatrix}
\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1} \\
\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1} \\
\vdots \\
\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}
\end{bmatrix}
\begin{bmatrix} \mathbf{e}_{s1} & \mathbf{e}_{s2} & \cdots & \mathbf{e}_{sk} \end{bmatrix} =
$$

$$
= \begin{bmatrix}
\sum_{s=1}^{K}\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \sum_{s=1}^{K}\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \sum_{s=1}^{K}\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \\
\sum_{s=1}^{K}\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \sum_{s=1}^{K}\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \sum_{s=1}^{K}\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{s=1}^{K}\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \sum_{s=1}^{K}\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \sum_{s=1}^{K}\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk}
\end{bmatrix} =
$$

$$
= \begin{bmatrix}
0 & \mathbf{A}_{12}(\mathbf{I}-\mathbf{A}_{22})^{-1} & \cdots & \mathbf{A}_{1k}(\mathbf{I}-\mathbf{A}_{kk})^{-1} \\
\mathbf{A}_{21}(\mathbf{I}-\mathbf{A}_{11})^{-1} & 0 & \cdots & \mathbf{A}_{2k}(\mathbf{I}-\mathbf{A}_{kk})^{-1} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{A}_{k1}(\mathbf{I}-\mathbf{A}_{11})^{-1} & \mathbf{A}_{k2}(\mathbf{I}-\mathbf{A}_{22})^{-1} & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
0 & \mathbf{e}_{12} & \cdots & \mathbf{e}_{1k} \\
\mathbf{e}_{21} & 0 & \cdots & \mathbf{e}_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{e}_{k1} & \mathbf{e}_{k2} & \cdots & 0
\end{bmatrix} =
$$

$$
= \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}
$$

This operation results in a KN×K matrix $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$ where the country of origin is still r while the country of destination is t, or the second-tier partner. Replace $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ in equation (A.1) with $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$:

$$\underbrace{\mathbf{E}_{bil}}_{\text{1st + 2nd tier}} = \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}}_{\text{1st tier from } r \text{ to } s} + \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}}_{\text{2nd tier from } r \text{ to } t=s} + \underbrace{\breve{\mathbf{F}}}_{\text{1st tier from } r \text{ to } s} \tag{A.2}$$

The second term on the right side now captures intermediate exports from sector i of country r that are embodied in all exports to country s (which also appears as t at the next tier) via third countries. As a result, we disaggregate the second-tier partners at the expense of aggregating the first-tier partners. Importantly, the term on the left side in (A.2) no longer represents direct bilateral exports. Instead, it accounts for cumulative exports to the first- and second-tier partners.

Insert equation (A.1) into equation (A.2) to decompose bilateral exports to the second-tier partners:

$$\begin{aligned}
\underbrace{\mathbf{E}_{bil}}_{\text{1st + 2nd tier}} &= \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \\
&+ \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \breve{\mathbf{F}}\right) + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \\
&+ \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{F}} + \breve{\mathbf{F}}
\end{aligned}$$

Replace again $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ with $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$ and allocate the second-tier total exports to the third-tier bilateral exports:

$$\begin{aligned}
\underbrace{\mathbf{E}_{bil}}_{\text{1st + 2nd + 3d tier}} &= \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}}_{\text{1st tier from } r \text{ to } s} + \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}}_{\text{2nd tier from } r \text{ to } s} + \\
&+ \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}}_{\text{3rd tier from } r \text{ to } s} + \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{F}}}_{\text{2nd tier from } r \text{ to } s} + \underbrace{\breve{\mathbf{F}}}_{\text{1st tier from } r \text{ to } s}
\end{aligned}$$

In this way, further decomposing and reallocating exports along the value chain to the t^{th} tier results in:

$$\begin{aligned}
\underbrace{\mathbf{E}_{bil}}_{\text{1st + \ldots + } t\text{th tier}} &= \sum_{1}^{t}\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\widehat{\mathbf{F}} + \left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\mathbf{E}_{tot} + \\
&+ \sum_{1}^{t}\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t-1}\breve{\mathbf{F}} = \\
&= \sum_{0}^{t}\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\widehat{\mathbf{F}} - \widehat{\mathbf{F}} + \left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\mathbf{E}_{tot} + \\
&+ \sum_{0}^{t}\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\breve{\mathbf{F}} - \left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\breve{\mathbf{F}}
\end{aligned}$$

As the decomposition proceeds to an infinitely remote $t^{\text{th}} \to \infty$ tier, the re-exported term approaches zero and is eventually reallocated between intermediates and final products for domestic use:

$$\underset{\text{all tiers}}{\mathbf{E}_{bil}} = \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \widehat{\mathbf{F}} - \widehat{\mathbf{F}} + 0 + \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \breve{\mathbf{F}} - 0 =$$
$$= \left(\left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} - \mathbf{I}\right) \widehat{\mathbf{F}} + \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \breve{\mathbf{F}}$$

This is a way to trace bilateral exports throughout the whole value chain to the ultimate destination where they end up in partner final demand. The term on the left side can be treated as cumulative bilateral exports $\mathbf{E}_{cum}$ where the elements are smaller or larger than direct bilateral exports, subject to the mode of partner integration into the value chain:

$$\underbrace{\mathbf{E}_{cum}}_{\substack{\text{cumulative}\\ \text{exports}}} = \underbrace{\left(\left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} - \mathbf{I}\right) \widehat{\mathbf{F}}}_{\substack{\text{direct and indirect exports of intermediates}\\ \text{eventually transformed into final products}\\ \text{for domestic use}}} + \underbrace{\left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \breve{\mathbf{F}}}_{\substack{\text{direct and indirect exports}\\ \text{of final products}}} =$$
$$= \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \mathbf{F} - \widehat{\mathbf{F}} \tag{A.3}$$

Equation (A.3) is not a decomposition of actual trade flows. Rather, it should be understood as a way to compute cumulative bilateral exports $\mathbf{E}_{cum}$ where each element describes the amount of product by sector i of country r that is eventually used for final demand in country s, delivered as direct or indirect exports.

$\left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1}$ is a new "global" multiplier matrix that will be denoted by $\mathbf{H}$ for brevity.

The derivation of the equation of cumulative bilateral exports is also possible with the use of an alternative transformation at each tier:

$$\underbrace{\mathbf{E}_{bil}}_{\text{1st + 2nd tier}} = \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{bil} + \breve{\mathbf{F}} =$$
$$= \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{tot} + \breve{\mathbf{F}} -$$
$$- \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{tot} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{bil} =$$
$$= \underbrace{\mathbf{E}_{bil}}_{\text{1st tier}} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{tot} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{bil}$$

The continuous substitution of $\mathbf{E}_{bil}$ to an infinitely remote $t^{\text{th}} \to \infty$ tier will yield:

$$\mathbf{E}_{cum} = \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \mathbf{E}_{bil} - \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \mathbf{E}_{tot} + \mathbf{E}_{tot} =$$
$$= \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \mathbf{E}_{bil} - \left(\left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} - \mathbf{I}\right) \mathbf{E}_{tot} =$$
$$= \mathbf{H}\mathbf{E}_{bil} - (\mathbf{H} - \mathbf{I})\mathbf{E}_{tot} \tag{A.4}$$

Cumulative bilateral exports can therefore be expressed as a function of either final demand or bilateral and total gross exports.

A.2 The relationship of new "global" inverse to the standard Leontief "global" inverse

The following manipulations show the relationship of $\mathbf{H}$ to the standard Leontief "global" inverse $\mathbf{L}$:

$$
\begin{aligned}
\mathbf{L} &= (\mathbf{I}-\mathbf{A})^{-1} \\
\mathbf{L}(\mathbf{I}-\mathbf{A}) &= (\mathbf{I}-\mathbf{A})^{-1}(\mathbf{I}-\mathbf{A}) \\
\mathbf{L}(\mathbf{I}-\widehat{\mathbf{A}}-\check{\mathbf{A}}) &= \mathbf{I} \\
\mathbf{L}(\mathbf{I}-\widehat{\mathbf{A}})-\mathbf{L}\check{\mathbf{A}} &= \mathbf{I} \\
\mathbf{L}(\mathbf{I}-\widehat{\mathbf{A}})(\mathbf{I}-\widehat{\mathbf{A}})^{-1}-\mathbf{L}\check{\mathbf{A}}(\mathbf{I}-\widehat{\mathbf{A}})^{-1} &= \mathbf{I}(\mathbf{I}-\widehat{\mathbf{A}})^{-1} \\
\mathbf{L}-\mathbf{L}\check{\mathbf{A}}(\mathbf{I}-\widehat{\mathbf{A}})^{-1} &= (\mathbf{I}-\widehat{\mathbf{A}})^{-1} \\
\mathbf{L}\left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right) &= (\mathbf{I}-\widehat{\mathbf{A}})^{-1} \\
(\mathbf{I}-\widehat{\mathbf{A}})\mathbf{L} &= \left(\mathbf{I}-\check{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{-1} = \mathbf{H}
\end{aligned}
$$

The above also shows that $\mathbf{H}$ exists as long as does $\mathbf{L}$.

A.3 The equivalence between total cumulative exports and total direct gross exports

An important property is that total cumulative exports to all destinations are equal to total direct gross exports:

$$\mathbf{E}_{cum}\mathbf{i} = (\mathbf{H}\mathbf{E}_{bil} - (\mathbf{H}-\mathbf{I})\mathbf{E}_{tot})\,\mathbf{i} = \mathbf{H}\mathbf{E}_{bil}\mathbf{i} - \mathbf{H}\mathbf{E}_{tot}\mathbf{i} + \mathbf{E}_{tot}\mathbf{i} = \mathbf{E}_{bil}\mathbf{i}$$

The formulation above utilizes that, by definition, the sum of bilateral exports across all partners equals total exports.

B Measures of accumulated trade costs in global value chains

B.1 Cumulative trade costs based on the value added accounting framework

A value added accounting framework traces the origin of gross exports to the sectors that initially contribute value added to those exports. This is a backward decomposition that reallocates all observed bilateral export flows into the unobserved value added flows between origins and destinations. The key element in a value added accounting framework is the "global" Leontief inverse $\mathbf{L}$. Koopman et al. (2012) and Stehrer (2013) are well known examples of such decomposition. Replacing the value added coefficients $\mathbf{v}_c$ with the margin or tax coefficients $\mathbf{m}(g)_{c(\mathbf{Z})}$, i.e., the amount of margin or tax payable per unit of output, enables the analyses of trade costs as embodied valuation terms.

For an illustrative purpose, split bilateral gross exports into exports of intermediate and final products:

$$\mathbf{E}_{bil} = \breve{\mathbf{Z}}_{(KN\times K)} + \breve{\mathbf{F}}$$

where the modified "check" operators extract off-diagonal block elements from block matrices but do not apply to the elements within those blocks. $\breve{\mathbf{Z}}_{(KN\times K)}$ is the matrix of intermediate demand condensed to the KN×K dimension (i.e., aggregated across partner country sectors) with the diagonal blocks set to zero:

$$\breve{\mathbf{Z}}_{(KN\times K)} = \begin{bmatrix} 0 & \mathbf{Z}_{12}\mathbf{i} & \cdots & \mathbf{Z}_{1k}\mathbf{i} \\ \mathbf{Z}_{21}\mathbf{i} & 0 & \cdots & \mathbf{Z}_{2k}\mathbf{i} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{k1}\mathbf{i} & \mathbf{Z}_{k2}\mathbf{i} & \cdots & 0 \end{bmatrix} \quad \text{where a block element} \quad \breve{\mathbf{Z}}_{(KN\times K)rs} = \begin{bmatrix} z_{rs}^{1\bullet} \\ z_{rs}^{2\bullet} \\ \vdots \\ z_{rs}^{n\bullet} \end{bmatrix}$$

In the formulation above, $\mathbf{i}$ is an N×1 summation vector and the upper index $n\bullet$ signifies that the intermediate inputs of the producing sector n are aggregated across purchasing sectors.

A respective direct bilateral g^{th} valuation layer is given by:

$$\underset{t=0}{\mathbf{M}(g)_{(\mathbf{E})}} = \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{(\mathbf{F})}$$

The above margins/taxes change the valuation of direct exports, or exports at tier 0.

Following the logic of sequential production stages, exports of intermediate and final products require intermediate inputs at the previous stage: $\mathbf{A}\breve{\mathbf{Z}}_{(KN\times K)} + \mathbf{A}\breve{\mathbf{F}}$. This involves the corresponding valuation at tier 1, counting tiers backwards:

$$\underset{t=1}{\mathbf{M}(g)_{(\mathbf{E})}} = \mathbf{M}(g)_{c(\mathbf{Z})}\breve{\mathbf{Z}}_{(KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\breve{\mathbf{F}}$$

The above changes the valuation of intermediate inputs involved in the production of direct exports $\breve{\mathbf{Z}}_{(KN\times K)}$ and $\breve{\mathbf{F}}$. To show this explicitly, we will zoom in a typical block element in $\mathbf{M}(g)_{c(\mathbf{Z})}\breve{\mathbf{Z}}_{(KN\times K)}$:

$$\left[\mathbf{M}(g)_{c(\mathbf{Z})}\breve{\mathbf{Z}}_{(KN\times K)}\right]_{rs} = \sum_{t\neq s}^{K} \begin{bmatrix} \sum_{u=1}^{N} m(g)_{c(\mathbf{Z})rt}^{1u} z_{ts}^{u\bullet} \\ \sum_{u=1}^{N} m(g)_{c(\mathbf{Z})rt}^{2u} z_{ts}^{u\bullet} \\ \vdots \\ \sum_{u=1}^{N} m(g)_{c(\mathbf{Z})rt}^{nu} z_{ts}^{u\bullet} \end{bmatrix}$$

For a pair of exporter r and partner s, each element in the matrix above extracts the margin or tax incurred in the production of intermediate input z of sector u exported to country s at tier 0 and allocates that margin or tax to country r because it supplied the products subject to those margins or taxes at tier 1. Similarly, a typical block element in $\mathbf{M}(g)_{c(\mathbf{Z})}\breve{\mathbf{F}}$ is:

$$\left[\mathbf{M}(g)_{c(\mathbf{Z})}\check{\mathbf{F}}\right]_{rs} = \sum_{t\neq s}^{K} \begin{bmatrix} \sum_{u=1}^{N} m(g)^{1u}_{c(\mathbf{Z})rt} f^{u}_{ts} \\ \sum_{u=1}^{N} m(g)^{2u}_{c(\mathbf{Z})rt} f^{u}_{ts} \\ \vdots \\ \sum_{u=1}^{N} m(g)^{nu}_{c(\mathbf{Z})rt} f^{u}_{ts} \end{bmatrix}$$

In fact, the matrix of margin coefficients $\mathbf{M}(g)_{c(\mathbf{Z})}$ applies here in the same way that the matrix of technical coefficients $\mathbf{A}$ does, but counts embodied primary, not intermediate inputs.

Intermediate inputs two tiers back are equal to: $\mathbf{A}\mathbf{A}\check{\mathbf{Z}}_{(KN\times K)} + \mathbf{A}\mathbf{A}\check{\mathbf{F}}$. And the corresponding valuation at tier 2 is:

$$\underset{t=2}{\mathbf{M}(g)_{(\mathbf{E})}} = \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\check{\mathbf{Z}}_{(KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\check{\mathbf{F}}$$

The above changes the valuation of embodied intermediate inputs two tiers back. Each element in either matrix counts the amount of g^{th} margin/tax payable on inputs supplied at tier 2.

This decomposition can be continued backwards to an infinitely remote tier. Compiling the valuation of intermediate inputs at all tiers will result in:

$$\begin{aligned} \underset{t=1,2,\ldots,\infty}{\mathbf{M}(g)_{(\mathbf{Z},KN\times K)}} &= \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\check{\mathbf{Z}}_{(KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\check{\mathbf{Z}}_{(KN\times K)} + \\ &+ \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\mathbf{A}\check{\mathbf{Z}}_{(KN\times K)} + \cdots + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}^{t}\check{\mathbf{Z}}_{(KN\times K)} = \\ &= \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\left(\mathbf{I} + \mathbf{A} + \mathbf{A}\mathbf{A} + \cdots + \mathbf{A}^{t}\right)\check{\mathbf{Z}}_{(KN\times K)} = \\ &= \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\check{\mathbf{Z}}_{(KN\times K)} \end{aligned}$$

Similarly, the cumulative valuation of final products will yield:

$$\begin{aligned} \underset{t=1,2,\ldots,\infty}{\mathbf{M}(g)_{(\mathbf{F})}} &= \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{c(\mathbf{Z})}\check{\mathbf{F}} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\check{\mathbf{F}} + \\ &+ \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\mathbf{A}\check{\mathbf{F}} + \cdots + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}^{t}\check{\mathbf{F}} = \\ &= \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{c(\mathbf{Z})}\left(\mathbf{I} + \mathbf{A} + \mathbf{A}\mathbf{A} + \cdots + \mathbf{A}^{t}\right)\check{\mathbf{F}} = \\ &= \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\check{\mathbf{F}} \end{aligned}$$

Combining the multi-tiered valuation of intermediate and final products allows for the cumulative accounting of trade costs corresponding to the g^{th} valuation layer:

$$\begin{aligned} \mathbf{M}(g)_{(\mathbf{E})cum} &= \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\check{\mathbf{Z}}_{(KN\times K)} + \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\check{\mathbf{F}} = \\ &= \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\mathbf{E}_{bil} \end{aligned} \tag{B.1}$$

The $\mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\mathbf{E}_{bil}$ term involves the double-counting of embodied valuation in the same way that $\hat{\mathbf{v}}_c\mathbf{L}\mathbf{E}_{bil}$ involves the double-counting of value added. The core difference is that value added does not move internationally and $\hat{\mathbf{v}}_c$ is therefore a KN×KN diagonal matrix, unlike $\mathbf{M}(g)_{c(\mathbf{Z})}$.

If g corresponds to import tariffs τ, $\mathbf{M}(\tau)_{(\mathbf{Z},KN\times K)}$ can be written as $\check{\mathbf{Z}}_{(KN\times K)} \circ \mathbf{T}$ and $\mathbf{M}(g)_{(\mathbf{F})}$ can be written as $\check{\mathbf{F}} \circ \mathbf{T}$. The matrix of margin coefficients becomes equal to:

$$\mathbf{M}(\tau)_{c(\mathbf{Z})} = \mathbf{M}(\tau)_{(\mathbf{Z})}\hat{\mathbf{x}}^{-1} = \check{\mathbf{Z}} \circ \mathbf{T}_{(KN\times KN)}\hat{\mathbf{x}}^{-1} = \mathbf{A} \circ \mathbf{T}_{(KN\times KN)}$$

where $\circ$ signifies the element-by-element multiplication. Then the cumulative import tariff is:

$$\begin{aligned}\mathbf{M}(\tau)_{(\mathbf{E})cum} &= \check{\mathbf{Z}}_{(KN\times K)} \circ \mathbf{T} + \check{\mathbf{F}} \circ \mathbf{T} + \left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right) \mathbf{L}\mathbf{E}_{bil} = \\ &= \mathbf{E}_{bil} \circ \mathbf{T} + \left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right) \mathbf{L}\mathbf{E}_{bil} \end{aligned} \tag{B.2}$$

where $\mathbf{M}(\tau)_{(\mathbf{E})cum}$ is the KN×K matrix of cumulative import tariffs in monetary terms and $\mathbf{T}$ is the matrix of bilateral import tariff rates in the country-sector by country (KN×K) dimension. Read this equation as follows: cumulative tariffs (in monetary terms) are equal to the direct tariffs on bilateral exports plus the tariffs embodied in bilateral exports throughout the entire value chain. An important distinction as compared to the formula of Rouzet and Miroudot (2013) is that the embodied valuation term $\left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right) \mathbf{L}\mathbf{E}_{bil} = \mathbf{M}(\tau)_{c(\mathbf{Z})}\mathbf{L}\mathbf{E}_{bil}$ is not uniform across producing countries. It accounts for tariffs as the embodied primary inputs payable on the products of sector i in country r regardless of whether r is a direct or t^{th} tier supplier. Thus, it traces cumulative tariffs backwards to the origin of the products subject to those tariffs. To put it more explicitly, it captures the tariffs payable on inputs at their origin and records these as embodied inputs at their destination. Therefore, one important drawback of this measure is that it cannot capture the indirect valuation of services.[14]

Finally, the element-by-element ratios of cumulative tariffs (or margins and net taxes, in general) to gross bilateral exports translate the estimates in monetary terms into percentages that are more convenient for trade policy analysis, e.g., in comparison with direct tariff rates:[15]

$$\mathbf{T}_{cum} = \mathbf{M}(\tau)_{(\mathbf{E})cum} \oslash \mathbf{E}_{bil} = \mathbf{T} + \left(\left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right) \mathbf{L}\mathbf{E}_{bil}\right) \oslash \mathbf{E}_{bil} \tag{B.3}$$

where $\oslash$ is the element-by-element division. For brevity, $\mathbf{T}_{cum}$ will be referred to as "cumulative tariffs".

B.2 Incremental trade costs based on the gross exports accounting framework

A gross exports accounting framework traces the destination of direct exports to their eventual users. This is a forward decomposition where the observed bilateral export flows are reallocated into the unobserved flows of embodied products as those pass through the downstream value chain. Koopman et al. (2010) and Wang et al. (2013) propose the accounting frameworks that may be classified under this type.[16]

[14] Since equation (B.2) captures the tariffs at origin, and the direct tariffs on services are zero, the indirect (embodied) tariffs on services will also be zero.

[15] It is impossible to obtain the tariff rate in percentage terms if the respective bilateral exports are zero. This also applies to the implicit tariff rates suggested in subsection B.2.

[16] The delimitation between the gross exports accounting framework and the value added accounting framework is primarily intended for the reader's understanding of the underlying decomposition concept. In the existing literature, the elements of the backward and forward decompositions may be combined in a single formulation. For example, Wang et al. (2013) employ value added multipliers while tracing the use of direct exports. This helps in discerning the country of origin of added value contained therein, but not in discerning its sectoral origin.

An essential requirement for a gross exports accounting framework is the ability to account for sequential border crossings. The Leontief inverse $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ is not suitable because it is indifferent to the national origin of intermediate inputs. The new "global" inverse addresses this issue:

$$\mathbf{H} = \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1}$$

where the modified "hat" and "check" operators extract, respectively, diagonal and off-diagonal block elements from block matrices but do not apply to the elements within those blocks. $\mathbf{H}$ is a KN×KN matrix of multipliers that is capable of sequentially identifying exports at tier t used to produce exports at the next tier $t+1$, or "exports embodied in exports" in a multi-country setting. Here, tiers denote production stages only when products cross national borders.

The power series of $\mathbf{H}$ model the path of a "melting" portion of the initial exports until it is entirely consumed (used) at an infinitely remote t^{th} tier:

$$\mathbf{HE}_{bil} = \mathbf{E}_{bil} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil} + \left(\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{2}\mathbf{E}_{bil} + \cdots + \left(\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{t}\mathbf{E}_{bil}$$

Each term in this decomposition describes a portion of the initial exports that reaches partner after t tiers or border crossings. Replacing $\mathbf{E}_{bil}$ with a matrix of bilateral margins or taxes (subsidies) $\mathbf{M}(g)_{(\mathbf{E})}$ leads to the incremental valuation of those initial exports at the partner side:

$$\begin{aligned}\mathbf{HM}(g)_{(\mathbf{E})} = \mathbf{M}(g)_{(\mathbf{E})} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\mathbf{M}(g)_{(\mathbf{E})} + \left(\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{2}\mathbf{M}(g)_{(\mathbf{E})} + \\ + \cdots + \left(\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{t}\mathbf{M}(g)_{(\mathbf{E})}\end{aligned}$$

Obviously, $\mathbf{M}(g)_{(\mathbf{E})}$ is the margin or tax paid on direct exports. The second term $\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\mathbf{M}(g)_{(\mathbf{E})}$ records the margin or tax paid on partner bilateral exports (2nd tier) which are in fact a part of the initial exports from the country of origin (1st tier). The remaining terms record margins or taxes in the same way at each successive tier, or after each border crossing. In other words, at t^{th} tier from the origin, the respective term in the power series above reallocates direct margins at destination in proportion to indirect exports at origin.

The summation of terms in this forward decomposition may therefore be treated as an incremental resistance term $\mathbf{M}(g)_{(\mathbf{E})inc}$ because trade costs arise incrementally in the exporter–partner relationship:

$$\mathbf{M}(g)_{(\mathbf{E})inc} = \mathbf{HM}(g)_{(\mathbf{E})} \tag{B.4}$$

where $\mathbf{M}(g)_{(\mathbf{E})} = \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{(\mathbf{F})}$.

For an intuitive interpretation of equation (B.4), consider the specific case of import tariffs:

$$\mathbf{M}(\tau)_{(\mathbf{E})inc} = \mathbf{H}(\mathbf{E}_{bil} \circ \mathbf{T}) \tag{B.5}$$

Each element in the KN×K matrix $\mathbf{M}(\tau)_{(\mathbf{E})inc}$ counts all tariffs (in monetary terms) payable on the products of sector i in country r at the border of country s regardless of whether s is a direct or t^{th} tier partner. Like the cumulative measure of tariffs $\mathbf{M}(\tau)_{(\mathbf{E})cum}$ derived from the value added accounting framework above, the $\mathbf{M}(\tau)_{(\mathbf{E})inc}$ term involves double counting of the import tariffs paid. However, it does so in a different way: it incrementally captures the tariffs payable at (the border of) destination and records these as exports at origin. Equation (B.5) is therefore capable of quantifying the indirect tariffs on services because it keeps track of services embodied in goods that are subject to tariffs.

The implicit tariff rates in this case are as follows:

$$\mathbf{T}_{inc} = \mathbf{M}(\tau)_{(\mathbf{E})inc} \oslash \mathbf{E}_{bil} = (\mathbf{H}(\mathbf{E}_{bil} \circ \mathbf{T})) \oslash \mathbf{E}_{bil} = \mathbf{T} + ((\mathbf{H} - \mathbf{I})(\mathbf{E}_{bil} \circ \mathbf{T})) \oslash \mathbf{E}_{bil} \qquad \text{(B.6)}$$

where $\oslash$ is the element-by-element division. For brevity, $\mathbf{T}_{inc}$ will be referred to as "incremental tariffs".[17]

B.3 Importance of indirect tariffs in global value chains

While direct import tariffs tend to decline, the change in the relative importance of indirect tariff exhibits a complex pattern. In terms of cumulative tariff, the ratio of indirect tariff to direct import tariff across all export markets decreased both from 2001-2005 and from 2005-2010 for 14 countries in the WIOD. For 21 countries, this ratio first increased but later decreased, and it was lower in 2010 than in 2001 for 14 countries of those 21 (see Fig. B.1). The cumulative accounting therefore indicates that the accumulated resistance effect has become somewhat less significant.

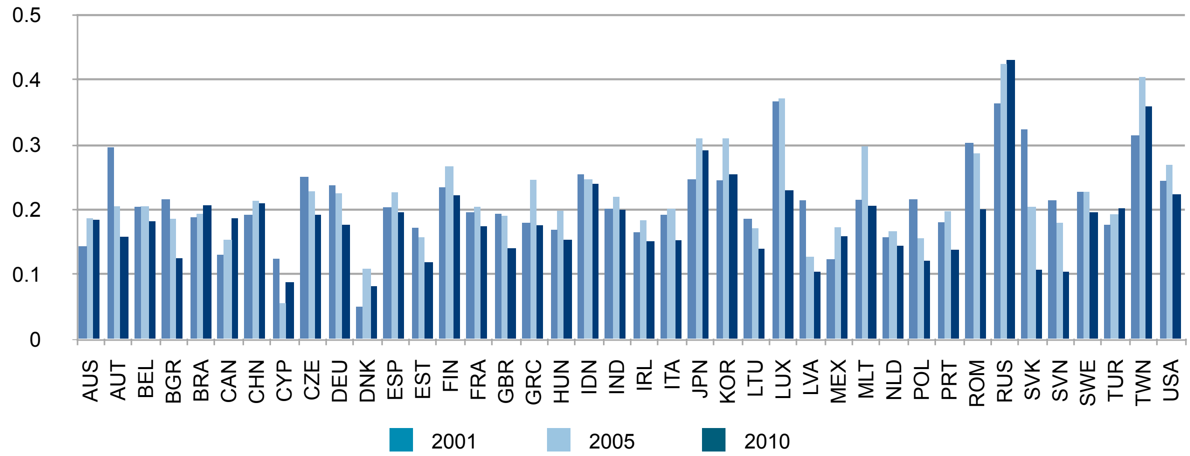

Figure B.1: Ratio of indirect tariff to direct import tariff faced by exporting country: the cumulative approach

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations

Note: the full list of countries in the WIOD is in Table D.1, Appendix D.

In terms of incremental tariff, the ratio of indirect tariff to direct import tariff that exporters face in foreign markets increased both from 2001-2005 and from 2005-2010 for 14 countries. This ratio first increased but then decreased for 18 countries, and only for 4 countries in the WIOD did it decrease in both periods (see Fig. B.2).

Interpret this as follows. For example, Indonesia faced indirect tariff because third countries levied tariffs on its intermediate exports (i.e. cumulative indirect tariff) equal to 0.25 of

[17]The terms "cumulative" and "incremental" are introduced here for easier reference to the two different accounting techniques.

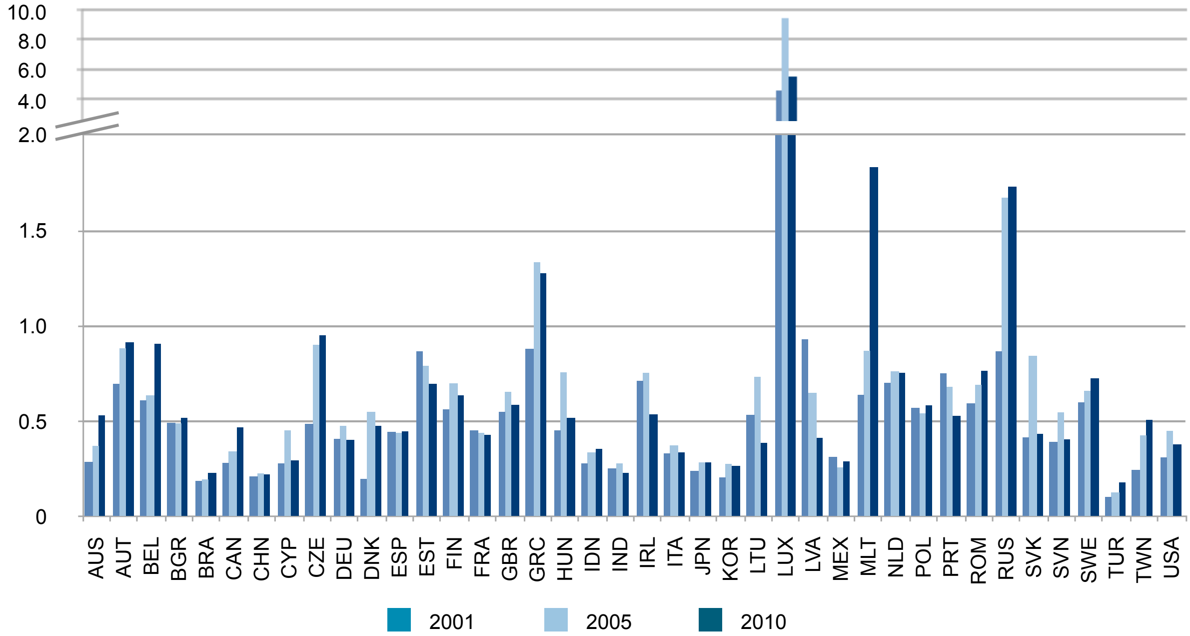

Figure B.2: Ratio of indirect tariff to direct import tariff faced by exporting country: the incremental approach
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations
Note: the full list of countries in the WIOD is in Table D.1, Appendix D.

the direct tariff faced in 2001 and 0.24 of the direct tariff in 2010. However, Indonesia faced indirect tariff because partners levied tariffs on third country exports (i.e. incremental indirect tariff) equal to 0.28 of the direct tariff faced in 2001 and 0.35 of the direct tariff faced in 2010. The accumulation effect of protection becomes more pronounced further downstream in the value chain.

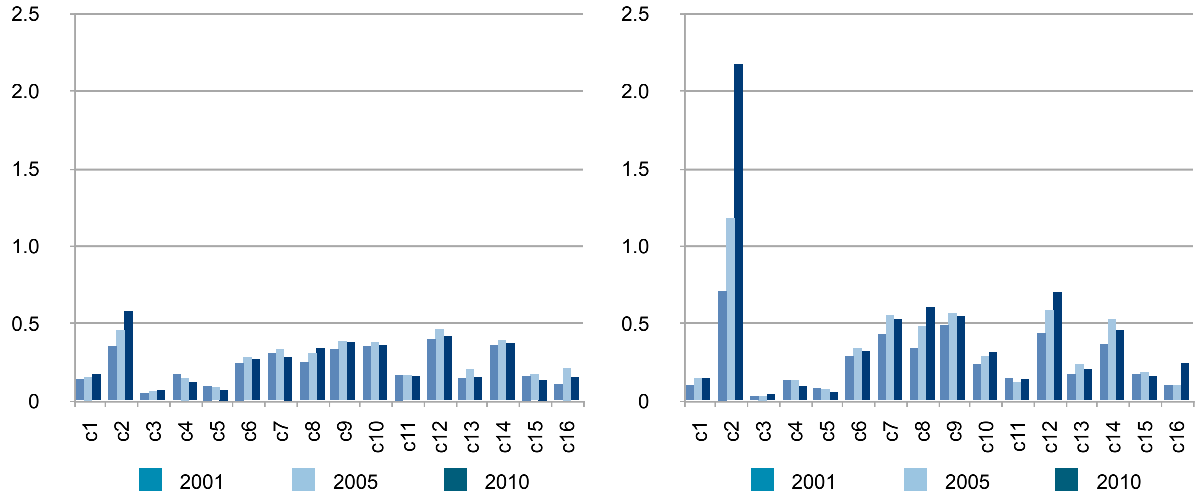

Figure B.3: Ratio of indirect tariff to direct import tariff faced by exporting sector (goods only): the cumulative approach (left) and incremental approach (right), 2010
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations
Note: the full list of sectors in the WIOD is in Table D.2, Appendix D.

The accumulation effect, measured by the ratio of indirect tariff to direct import tariff faced in Fig. B.3,[18] remained relatively stable in the cumulative valuation with the exception

[18]Service sectors are not shown in Fig. B.3 because the ratio involves division by direct tariffs that are

of the products of mining and quarrying sector for which the ratio increased from 0.36 in 2001 to 0.46 in 2005 and 0.58 in 2010. The incremental valuation reveals a more significant accumulation of resistance to exports. For the mining and quarrying sector, it increased from 0.71 in 2001 to 1.18 in 2005 and 2.17 in 2010. Indirect protection also accumulates at the partner border with respect to other sectors that produce inputs such as coke, petroleum products, basic metals and fabricated metal products. But the accumulation effect is less significant for sectors that export primarily final products: food, textiles, leather and footwear.

C Aggregation options

To reduce the dimension of the results at the bilateral country-sector level for the ease of analysis and visualization, this paper employs two aggregation matrices and a summation vector.

The sector-wise aggregation matrix $\mathbf{S}_N$ is constructed from the N×1 summation vectors $\mathbf{i}$:

$$\mathbf{S}_N = \begin{bmatrix} \mathbf{i} & 0 & \cdots & 0 \\ 0 & \mathbf{i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{i} \end{bmatrix}$$

The dimension of $\mathbf{S}_N$ is KN×K. Pre-multiplying a KN×K matrix by $\mathbf{S}'_N$ compresses it to the K×K (country by country) dimension.

The country-wise aggregation matrix $\mathbf{S}_K$ requires N×N identity matrices $\mathbf{I}$:

$$\mathbf{S}_K = \begin{bmatrix} \mathbf{I} & \mathbf{I} & \cdots & \mathbf{I} \end{bmatrix}$$

The dimension of $\mathbf{S}_K$ is N×KN. Pre-multiplying a KN×K matrix by $\mathbf{S}_K$ compresses it to the N×K (sector by country) dimension.

Lastly, post-multiplication of any matrix by an appropriately sized summation vector transforms it to the KN×1, K×1, or N×1 dimensions where all partners are added up.

Consider the aggregation of the weighted average number of border crossings across exported products and partner countries (equation 4):

$$\mathbf{c}_{K\times 1} = \left(\mathbf{S}'_N(\mathbf{H}^2\mathbf{F} - \mathbf{H}\widehat{\mathbf{F}})\mathbf{i}\right) \oslash \left(\mathbf{S}'_N(\mathbf{H}\mathbf{F} - \widehat{\mathbf{F}})\mathbf{i}\right)$$

The above formulation was used to draw Fig. 1.

The following is an example of the aggregation of the bilateral measure of upstreamness across exporting countries (equation 1):

$$\mathbf{U}_{N\times 1} = (\mathbf{S}_K\mathbf{L}^2\mathbf{F}\mathbf{i}) \oslash (\mathbf{S}_K\mathbf{L}\mathbf{F}\mathbf{i})$$

This informed drawing of Fig. 3.

close to zero.

D Countries and industries in the WIOD database

Table D.1: List of countries in the WIOD database

Country code	Country	Country code	Country
AUS	Australia	IRL	Ireland
AUT	Austria	ITA	Italy
BEL	Belgium	JPN	Japan
BGR	Bulgaria	KOR	Korea
BRA	Brazil	LTU	Lithuania
CAN	Canada	LUX	Luxembourg
CHN	China	LVA	Latvia
CYP	Cyprus	MEX	Mexico
CZE	Czech Republic	MLT	Malta
DEU	Germany	NLD	Netherlands
DNK	Denmark	POL	Poland
ESP	Spain	PRT	Portugal
EST	Estonia	ROM	Romania
FIN	Finland	RUS	Russian Federation
FRA	France	SVK	Slovak Republic
GBR	United Kingdom	SVN	Slovenia
GRC	Greece	SWE	Sweden
HUN	Hungary	TUR	Turkey
IDN	Indonesia	TWN	Chinese Taipei
IND	India	USA	United States
		RoW	Rest of the World

Source: Dietzenbacher et al., 2013; `http://www.wiod.org`

Table D.2: List of industries in the WIOD database

WIOD code	NACE Rev.1/ ISIC Rev.3	Industry
c1	A – B	Agriculture, Hunting, Forestry and Fishing
c2	C	Mining and Quarrying
c3	15 – 16	Food, Beverages and Tobacco
c4	17 – 18	Textiles and Textile Products
c5	19	Leather, Leather and Footwear
c6	20	Wood and Products of Wood and Cork
c7	21 – 22	Pulp, Paper, Paper , Printing and Publishing
c8	23	Coke, Refined Petroleum and Nuclear Fuel
c9	24	Chemicals and Chemical Products
c10	25	Rubber and Plastics
c11	26	Other Non-Metallic Mineral
c12	27 – 28	Basic Metals and Fabricated Metal
c13	29	Machinery, Nec
c14	30 – 33	Electrical and Optical Equipment
c15	34 – 35	Transport Equipment
c16	36 – 37	Manufacturing, Nec; Recycling
c17	E	Electricity, Gas and Water Supply
c18	F	Construction
c19	50	Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel
c20	51	Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles
c21	52	Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods
c22	H	Hotels and Restaurants
c23	60	Inland Transport
c24	61	Water Transport
c25	62	Air Transport
c26	63	Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies
c27	64	Post and Telecommunications
c28	J	Financial Intermediation
c29	70	Real Estate Activities
c30	71 – 74	Renting of M and Eq and Other Business Activities
c31	L	Public Admin and Defence; Compulsory Social Security
c32	M	Education
c33	N	Health and Social Work
c34	O	Other Community, Social and Personal Services
c35	P	Private Households with Employed Persons

Source: Dietzenbacher et al., 2013; `http://www.wiod.org`