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TRACING VALUE-ADDED AND DOUBLE COUNTING IN SALES OF FOREIGN AFFILIATES AND DOMESTIC-OWNED COMPANIES

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ABSTRACT: The literature on trade in value-added has emphasised that gross trade flows do not adequately measure the income generated by trade when many intermediate inputs are imported. While this literature has deepened our understanding of global value chains, it is still missing an important element when analysing income generation along the value chain: the fact that domestic value-added often results from activities of foreign-owned companies. Studies that look at activities of multinational enterprises (MNEs) still rely on the concept of sales of foreign affiliates, which is a gross concept also subject to double counting when it comes to the use of intermediate inputs.

In this paper, we propose a new accounting framework for the decomposition of value-added into domestic, foreign and double counting terms in domestic sales. In this framework, we show where the value-added double counting is derived from and give an explicit expression of domestic and foreign double counting terms based on the Inter-Country Input-Output (ICIO) tables' Ghosh insight. We can distinguish domestic sales from exports and trace the value added and double counting in sales of foreign affiliates and domestic-owned enterprises. Based on this framework, we then calculate the value-added by foreign-owned and domestic-owned firms in exports and in domestic sales by using an Inter-Country Input-Output table split according to ownership (Cadestin et al., 2017). Preliminary results suggest that there is much more double counting in sales of foreign affiliates than in exports and that more value-added is created through exports than through sales of foreign affiliates in world GDP.

1. Introduction

The literature on trade in value-added (Johnson and Noguera, 2012; Koopman et al., 2014; Los et al., 2016; Nagengast and Stehrer, 2016) as well as empirical datasets, such as the TiVA indicators released by OECD and WTO in 2013, have emphasised that gross trade flows do not adequately measure the income generated by trade in a world characterised by global supply chains where intermediate products are traded across countries. New accounting frameworks have been developed to identify the domestic value-added in gross exports and in final demand and to remove the double counting of intermediate inputs that cross international borders more than once.

But trade is only one dimension in the activities of firms involved in global production. Some of these firms are multinational enterprises (MNEs) that rely on foreign affiliates to source inputs or produce abroad. According to UNCTAD (2013), 80% of global trade is co-ordinated by these MNEs (when including their arm's length trade transactions as well as trade flows related to franchising, contract manufacturing and strategic alliances).

The economic literature analysing activities of MNEs relies on the concept of sales of foreign affiliates, which is also a gross concept and includes some double counting with respect to foreign and domestic inputs. Somehow this concept has not yet been through the kind of 'value-added revolution' that has significantly changed the analysis of trade.

In this paper, we are interested in decomposing not only trade but also domestic sales in a consistent framework that can allow us to identify the activities of foreign-owned firms. Such a decomposition can shed light on the reasons why firms engage in FDI. The literature suggests that foreign affiliates can be involved: (1) in the production of final goods for domestic consumers (in the case of 'horizontal FDI'), (2) in the production of final goods for foreign consumers (in the case of 'export platform FDI') or in the production of inputs for other affiliates in the host economy or abroad (in the case of 'vertical FDI'). More recent work indicates that in many instances firms engage in 'complex FDI' combining horizontal and vertical motives (Alfaro and Charlton, 2009), or set up affiliates for other purposes than contributing to the production process such as 'conglomerate FDI' or FDI for financial purposes (Herger and McCorriston, 2016; Ray, 2016). There is therefore a need for more empirical work on value creation in relation to activities of foreign affiliates.

Moreover, by applying to sales of foreign affiliates the same kind of treatment applied to trade flows in the context of value-added analyses, we would like to provide a more accurate measurement of the importance of MNEs in global production. Figures such as the one proposed by UNCTAD seem to overstate the true importance of MNEs in trade and output as they are based on gross figures and not a value-added decomposition.

The paper is organised as follows. Section 2 details the methodology, first presenting an alternative mathematical framework to derive the domestic and foreign value-added multiplier coefficients in an inter-country input-output framework. Based on this, we provide a full decomposition of the value-added in domestic sales following the Ghosh insight and identifying double counting terms in addition to domestic and foreign value-added. In Section 3, we use this conceptual framework to decompose GDP in the context of ICIO tables that have an ownership dimension (splitting data for domestic-owned and foreign-owned firms). Section 4 concludes.

2. Methodology

This section introduces a new type of value-added decomposition in the inter-country input-output (ICIO) framework. The starting point is that gross output consists of domestic sales (i.e.. domestic shipments) and exports (i.e. shipments to foreign countries). Some important efforts have been devoted to tracing domestic and foreign value-added, as well as double counting, in gross exports (following Koopman et al., 2014), but not for domestic shipments. Domestic sales are also interesting in terms of their domestic and foreign value-added content, especially when these domestic sales result from activities of foreign-owned companies. Our objective is to provide a full decomposition of GDP in a given economy that would allow us to identify the domestic and foreign value-added in domestic sales and in exports, and ultimately to compare the foreign value-added coming from exports with the value added by foreign-owned firms in the domestic economy (which is part of the "domestic" value-added in current decompositions).

2.1 Value-added multiplier coefficients in domestic sales

Leontief (1936) established that the amount and type of intermediate inputs needed in the production of one unit of output can be estimated based on the input-output (IO) structure across industries. Using the linkages across industries, one can trace output in all stages of production needed to produce one unit of final goods. When the gross output flows associated with a specific level of final demand are known, value-added production and trade can be simply derived by multiplying these flows with the value added to gross output ratio in each industry.

In the IO framework, all gross output must be used either as an intermediate good or as a final good,

$$X = AX + Y \tag{1}$$

where, X is the $N \times 1$ gross output vector, Y is the $N \times 1$ final demand vector, and A is the $N \times N$ I-O coefficients matrix.

The accounting relationship between domestic sales H and final demand in an Inter-Country Input-Output (ICIO) model can be expressed as:

$$H = \bar{A}H + \bar{Y} \tag{2}$$

Here, $\bar{A} = A^D(I - A^F)^{-1}$, with A^D the domestic coefficients in the global ICIO table (i.e. the block-diagonal matrix of the A matrix in the ICIO table). A^F is the export matrix of A indicating the use of intermediate inputs from one country into another country. In addition, $\bar{Y} = Y^D + \bar{A}Y^F$, Y^D denoting domestic final demand and Y^F final demand in foreign countries. The full derivation is provided in the Appendix (Lemma 1).

Each element of the $\bar{A}$ matrix describes how domestic intermediate goods are sent abroad (or transported domestically) to produce one unit of domestic sales in foreign countries (or in the domestic economy). For example, the element $\bar{A}_{ij}$ means that in order to produce one unit of domestic sales in country *j*, country *i* need to produce $\bar{A}_{ij}$ units of intermediate inputs that are then embodied in domestic sales in country *j*. $\bar{A}_{ij}H_j$ means that country *i* needs to produce $\bar{A}_{ij}H_j$ intermediate inputs for domestic sales H_j in country *j*, so we can call $\bar{A}$ as the 'direct domestic sales requirements matrix'. Re-arranging equation (2) above, we obtain $H = \bar{B}\bar{Y}$, and $\bar{B} = (I - \bar{A})^{-1}$, similar to $B = (I - A)^{-1}$ in the IO model. We can define matrix $\bar{B}$ as the 'total domestic sales requirements matrix'.

With respect to $\bar{B}$, we have

$$\begin{aligned}
&\bar{B} = (I - \bar{A})^{-1} = [I - A^D(I - A^F)^{-1}]^{-1} = [(I - A^F)(I - A^F)^{-1} - A^D(I - A^F)^{-1}]^{-1} \\
&= [(I - A^F - A^D)(I - A^F)^{-1}]^{-1} \\
&= (I - A^F)B = (I - A + A^D)B = I + A^D B
\end{aligned}$$

So, for any element in matrix $\bar{B}$, $\bar{B}_{ij} = \begin{cases} I + A_{ii}B_{ij} & i = j \\ A_{ii}B_{ij} & i \neq j \end{cases}$.

For H_i, the domestic sales in country *i*, all the intermediate inputs needed are $\sum_j^G \bar{A}_{ji}H_i$. We can thus calculate the value-added in domestic sales in country *i* as $VaH(i)^T = H_i - \sum_j^G \bar{A}_{ji}H_i$. This value-added does not only include country *i*'s value-added but also other countries' value-added. We can then express the value-added multiplier coefficients in domestic sales in the form of a matrix $\bar{V}$, defined as:

$$\bar{V} = u(I - \bar{A}) = u(I - A)(I - A^F)^{-1} = V(I - A^F)^{-1} \tag{3}$$

where V is a 1×N direct value-added coefficients vector. Each element of V_i gives the share of direct domestic value-added in total output. It is equal to one minus the intermediate input share from all countries (including domestically produced intermediates): $V_i = u[I - \sum_j^G A_{ji}]$, where u is a 1×N unit vector. If we use the notation $B^F = (I - A^F)^{-1}$, we obtain the expression for value-added coefficients in domestic sales in country i: $\overline{V}_i = V_i B_{ii}^F + \sum_{j \neq i}^G V_j B_{ji}^F$. They can be divided into two parts: the value-added from country i (domestic part) $V_i B_{ii}^F$ and the value-added from other countries (foreign part) $\sum_{j \neq i}^G V_j B_{ji}^F$.

Moreover, we can derive a consistent measure of the domestic and foreign value-added (or GDP) in domestic sales from the initial ICIO model. In the ICIO model, gross exports and gross output can be written as:

$$E = (I - A^F)^{-1} A^F H + (I - A^F)^{-1} Y^F \tag{4}$$

$$X = H + E = (I - A^F)^{-1} H + (I - A^F)^{-1} Y^F = B^F H + B^F Y^F \tag{5}$$

Based on the expression above, for country i's output X_i:

$$X_i = B_{ii}^F H_i + \sum_{j \neq i}^G B_{ij}^F H_j + \sum_j^G B_{ij}^F Y_j^F \tag{6}$$

Therefore, country i's GDP can be measured as:

$$GDP_i = V_i X_i = V_i B_{ii}^F H_i + V_i \sum_{j \neq i}^G B_{ij}^F H_j + V_i \sum_j^G B_{ij}^F Y_j^F \tag{7}$$

According to equation (7), country i's GDP can be divided into 3 parts. $V_i \sum_j^G B_{ij}^F Y_j^F$ is the share of GDP in exports of final products, $V_i B_{ii}^F H_i$ is GDP in country i's domestic sales and $V_i \sum_{j \neq i}^G B_{ij}^F H_j$ is GDP in foreign countries' domestic sales via exports of intermediates from country i to other countries and measured as foreign value-added for other countries. Similarly, we can also express domestic

value-added in domestic sales in a consistent way and regard coefficients $V_i B_{ii}^F$ as the domestic value-added multiplier coefficients for a country's domestic sales.

Symmetrically, we can obtain the expression of country *j*'s GDP as $GDP_j = V_j X_j = V_j B_{ji}^F H_i + V_j \sum_{k \neq i}^{G} B_{jk}^F H_k + V_j \sum_{k}^{G} B_{jk}^F Y_k^F$. From the point of view of country *j*'s GDP, the part $V_j B_{ji}^F H_i$ is included in country *i*'s domestic sales. If we sum up the value-added from all countries (except country *i*) in country *i*'s domestic sales, we also obtain an expression for the foreign value-added part of country *i*'s domestic sales, measured as $\sum_{j \neq i}^{G} V_j B_{ji}^F H_i$. Therefore, the value-added in country *i*'s domestic sales should be equal to the domestic part plus the foreign part: $V_i B_{ii}^F H_i + \sum_{j \neq i}^{G} V_j B_{ji}^F H_i = \bar{V}_i H_i$.

2.2 Tracing value-added in domestic sales: the Ghosh insight

The Ghosh model (Ghosh, 1958), in turn, is also known as the 'supply–driven' input-output model, since value-added is the exogenously specified driving force of the model. Although the Ghosh model is generally interpreted as a price model (Dietzenbacher, 1997), it can be applied to the analysis of the structure of value-added flows as an alternative to Leontief's 'demand-driven' model. The 'supply–driven' accounting identity states that country *i*'s total input (which should be equal to domestic sales here) is equal to the value of its initial inputs (domestic and foreign value-added) plus its intermediate input flows from all other countries (which can be interpreted as value-added double counting terms).

In the IO table, the output coefficient is defined as $l_{ij} = x_{ij} / x_i$. Output coefficients give the output percentage of industry *i* that is sold to industry *j*. The accounting equation can be rewritten as:

$$X^T = VA^T + X^T L = VA \cdot G \tag{8}$$

where $G = (I - L)^{-1}$ denotes the Ghosh inverse; Meanwhile, $G = \hat{X}^{-1} B \hat{X}$, where $\hat{X}$ is a $N \times N$ diagonal matrix with output on the diagonal.

Similarly, in the domestic sales input-output table, domestic sales can be written as $H^T = VaH^T + H^T\overline{L} = VaH^T \cdot \overline{G}$. Here $\overline{G} = \hat{H}^{-1}\overline{B}\hat{H}$, $\overline{L} = \hat{H}^{-1}\overline{A}\hat{H}$ and $\overline{L}_{ij} = \hat{H}_i^{-1}\overline{A}_{ij}\hat{H}_j$. $\overline{L}_{ij}$ gives the share of country *i*'s goods in country *j*'s domestic sales.

To illustrate the relationship between domestic sales and value-added, we can refer to the Taylor expansion:

$$H^T = VaH^T(I + \overline{L} + \overline{L}^2 + \overline{L}^3 + \cdots) \tag{9}$$

In the value-added input VaH^T, the domestic sales value is H^T, which is decomposed into three value-added terms: an initial input VaH^T, a direct input $VaH^T \cdot \overline{L}$ in the first round and indirect input in subsequent rounds amounting to $VaH^T(\overline{L}^2 + \overline{L}^3 + \cdots)$.

Following the Ghosh insight, we can give the full decomposition for country *i*'s domestic sales:

$$\begin{aligned} H_i^T = {} & VaH(i)^T + VaH(i)^T\overline{L}_{ii} + \sum_{j\neq i}^{G} VaH(j)^T\overline{L}_{ji} \\ & + VaH(i)^T[\overline{L}]^2_{ii} + VaH(i)^T[\overline{L}]^3_{ii} + \cdots \\ & + \sum_{j\neq i}^{G} VaH(j)^T[\overline{L}]^2_{ji} + \sum_{j\neq i}^{G} VaH(j)^T[\overline{L}]^3_{ji} + \cdots \end{aligned} \tag{10}$$

The above expression provides an explicit interpretation of the decomposition of domestic sales according to the Ghosh insight. Every sub-term has an economic interpretation.

The initial effect is the value-added in country *i*'s domestic sales which is equal to $VaH(i)^T = \overline{V}_i\hat{H}_i = (V_iB_{ii}^F + \sum_{j\neq i}^{G} V_jB_{ji}^F)\hat{H}_i$. If we expand this term, it includes domestic value-added initial inputs $V_iB_{ii}^F\hat{H}_i$ and foreign value-added initial inputs $\sum_{j\neq i}^{G} V_jB_{ji}^F\hat{H}_i$ which are contained in goods imported from country *j*.

In the first round, it means that the value-added term which is already counted in the initial round propagates through the matrix $\overline{L}_{ii} = \hat{H}_i^{-1}\overline{A}_{ii}\hat{H}_i$ (having in mind that $\overline{A}_{ii} = A_{ii}B_{ii}^F = A_{ii} + A_{ii}[A^F]^2_{ii} + A_{ii}[A^F]^3_{ii} + \cdots$, this value-added propagation route includes not only what has gone across the country border but also across domestic sectors as intermediate inputs). Because this value-added was already measured in the initial round, it should be part of the value-added double counting terms in later rounds. The direct effect can be divided into two parts, the effect from

country *i*'s inputs (which is not domestic value-added here) and from other country *j*'s inputs (which is not foreign value-added here). Country *i*'s input is equal to:

$$VaH(i)^T \overline{L}_{ii} = \overline{V}_i \hat{H}_i \cdot \hat{H}_i^{-1} \overline{A}_{ii} \hat{H}_i = \overline{V}_i \overline{A}_{ii} \hat{H}_i \tag{11}$$

The other countries' value-added within intermediate inputs is imported from country *j*. These terms are equal to:

$$\sum_{j \neq i}^{G} VaH(j)^T \cdot \overline{L}_{ji} = \sum_{j \neq i}^{G} \overline{V}_j \hat{H}_j \cdot \hat{H}_j^{-1} \overline{A}_{ji} \hat{H}_i = \sum_{j \neq i}^{G} \overline{V}_j \overline{A}_{ji} \hat{H}_i \tag{12}$$

In the second round, the additional value-added has a similar interpretation and can also be divided into the domestic input and foreign input parts. It still accounts for value-added double counting terms passed from country *i*'s domestic propagation to the other countries and returned back home. This implies that for the domestic input part, the value-added coming from country *i* is $VaH(i)^T \sum_{k}^{G} \overline{L}_{ik} \overline{L}_{ki}$, reflecting value-added from country *i* $VaH(i)^T \overline{L}_{ik}$ propagated to country *k*. The $\overline{L}_{ki}$ part in country *k* returned back home. This part of value-added has already been measured in the initial round, so it should still be counted as value-added double counting term (domestic). We have

$$\begin{aligned} & VaH(i)^T [\overline{L}]^2_{ii} = VaH(i)^T \sum_{k}^{G} \overline{L}_{ik} \overline{L}_{ki} \\ & = \overline{V}_i \hat{H}_i \sum_{k}^{G} \hat{H}_i^{-1} \overline{A}_{ik} \hat{H}_k \cdot \hat{H}_k^{-1} \overline{A}_{ki} \hat{H}_i = \overline{V}_i \sum_{k}^{G} \overline{A}_{ik} \overline{A}_{ki} \hat{H}_i \end{aligned} \tag{13}$$

For the value-added contributed by country *j*, we have $VaH(j)^T \sum_{k}^{G} \overline{L}_{jk} \overline{L}_{ki}$, reflecting the value-added from country *j* $VaH(j)^T \overline{L}_{jk}$ propagated to country *k*. $\overline{L}_{ki}$ is the part in country *k* that has returned back to country *i*. This part has also already been measured in the initial round as foreign value-added input, so it should be counted as value-added double counting term (foreign). Also, we have

$$\begin{aligned} & \sum_{j \neq i}^{G} VaH(j)^T [\overline{L}]^2_{ji} = VaH(j)^T \sum_{k}^{G} \overline{L}_{jk} \overline{L}_{ki} \\ & = \overline{V}_j \hat{H}_j \sum_{k}^{G} \hat{H}_j^{-1} \overline{A}_{jk} \hat{H}_k \cdot \hat{H}_k^{-1} \overline{A}_{ki} \hat{H}_i = \overline{V}_j \sum_{k}^{G} \overline{A}_{jk} \overline{A}_{ki} \hat{H}_i \end{aligned} \tag{14}$$

Therefore, in round 2, the whole double counted value-added in the foreign part is $\sum_{j\neq i}^{G}\overline{V}_j\sum_{k}^{G}\overline{A}_{jk}\overline{A}_{ki}\hat{H}_i$.

And for the domestic part, the domestic double counted value-added is:

$$\begin{aligned}&VaH(i)^T\overline{L}_{ii}+VaH(i)^T[\overline{L}]^2{}_{ii}+VaH(i)^T[\overline{L}]^3{}_{ii}+\cdots=\\&\overline{V}_i(\overline{A}_{ii}+\sum_{j}^{G}\overline{A}_{ij}\overline{A}_{ji}+\sum_{k}^{G}\sum_{j}^{G}\overline{A}_{ij}\overline{A}_{jk}\overline{A}_{ki}+\cdots)\hat{H}_i=\overline{V}_i(\overline{B}_{ii}-I)\hat{H}_i=\overline{V}_iA_{ii}B_{ii}\hat{H}_i\end{aligned}\tag{15}$$

While the foreign double counted value-added is:

$$\begin{aligned}&\sum_{j\neq i}^{G}VaH(j)^T\cdot\overline{L}_{ji}+\sum_{j\neq i}^{G}VaH(j)^T[\overline{L}]^2{}_{ji}+\sum_{j\neq i}^{G}VaH(j)^T[\overline{L}]^3{}_{ji}+\cdots=\\&\sum_{j\neq i}^{G}\overline{V}_j(\overline{A}_{ji}+\sum_{k}^{G}\overline{A}_{jk}\overline{A}_{ki}+\sum_{t}^{G}\sum_{k}^{G}\overline{A}_{jk}\overline{A}_{kt}\overline{A}_{ti}+\cdots)\hat{H}_i=\sum_{j\neq i}^{G}\overline{V}_j\overline{B}_{ji}\hat{H}_i\end{aligned}\tag{16}$$

Merging the expression of value-added coefficients $\overline{V}_i=V_iB_{ii}^F+\sum_{j\neq i}^{G}V_jB_{ji}^F$ in the domestic sales ICIO framework, the domestic value-added in country *i*'s domestic sales should be equal to country *i*'s value-added portion in the initial round: $V_iB_{ii}^F\hat{H}_i$. Moreover, country *i*'s value-added double counting term in domestic sales should be equal to the sum of the country *i*'s value-added portion in the double counting content (include the domestic input term and foreign input term): $V_iB_{ii}^FA_{ii}B_{ii}\hat{H}_i+\sum_{j\neq i}^{G}V_iB_{ij}^F\overline{B}_{ji}\hat{H}_i$.

Theorem 1: In the value-added decomposition of domestic sales, the sum of the domestic value-added and the double counting term is equal to the domestic content in domestic sales.

$$V_iB_{ii}^F\hat{H}_i+V_iB_{ii}^FA_{ii}B_{ii}\hat{H}_i+\sum_{j\neq i}^{G}V_iB_{ij}^F\overline{B}_{ji}\hat{H}_i=V_iB_{ii}\hat{H}_i$$

Similarly, the foreign value-added for country *i*'s domestic sales should be equal to the sum of foreign countries' value-added in the initial round's foreign input: $\sum_{j\neq i}^{G}V_jB_{ji}^F\hat{H}_i$. Foreign value-added double counting term in country *i*'s domestic sales should be equal to the foreign value-added portion in the double counting content: $\sum_{j\neq i}^{G}V_jB_{ji}^FA_{ii}B_{ii}\hat{H}_i+\sum_{s\neq i}^{G}\sum_{j\neq i}^{G}V_sB_{sj}^F\overline{B}_{ji}\hat{H}_i$.

Theorem 2: In the value-added decomposition of domestic sales, the sum of the foreign value-added and the double counting term is equal to the foreign content in domestic sales.

$$\sum_{j\neq i}^{G} V_j B_{ji}^F \hat{H}_i + \sum_{j\neq i}^{G} V_j B_{ji}^F A_{ii} B_{ii} \hat{H}_i + \sum_{s\neq i}^{G}\sum_{j\neq i}^{G} V_s B_{sj}^F \overline{B}_{ji} \hat{H}_i = \sum_{j\neq i}^{G} V_j B_{ji} \hat{H}_i$$

See the appendix for the proof of these theorems.

2.3 GDP decomposition into exports and domestic sales (with an overlap)

As previously highlighted, the accounting relationship between domestic sales H and final demand in destination in the ICIO model can be written as $H = \overline{A}H + \overline{Y}$. In a similar way, we can also obtain the accounting relationship between gross exports E and final demand in different destinations in the Inter-Country Input-Output (ICIO) model (see the Appendix):

$$E = \tilde{A}E + \tilde{Y} \tag{17}$$

with $\tilde{A} = A^F (I - A^D)^{-1}$ and $\tilde{Y} = Y^F + \tilde{A} Y^D$.

Re-arranging equations (2) and (17), we can express gross exports and domestic sales as:

$$E = [I - A^F (I - A^D)^{-1}]^{-1} [A^F (I - A^D)^{-1} Y^D + Y^F] \tag{18}$$

$$H = [I - A^D (I - A^F)^{-1}]^{-1} [A^D (I - A^F)^{-1} Y^F + Y^D] \tag{19}$$

Therefore, in the ICIO model, gross output can be written as:

$$X = A^D X + Y + A^F X + Y^F = A^D X + Y + E \tag{20}$$

$$\text{Or } X = A^D X + Y + A^F X + Y^F = H + A^F X + Y^F \tag{21}$$

Rearranging equations (20) and (21), we get:

$$X = (I - A^D)^{-1} Y^D + (I - A^D)^{-1} E \tag{22}$$

$$\text{And } X = (I - A^F)^{-1} Y^F + (I - A^F)^{-1} H \tag{23}$$

The expression $(I - A^D)^{-1}$ is sometimes described as the local Leontief inverse in the ICIO.

GDP can then be calculated as follows:

$$GDP = VX = V(I - A^D)^{-1} Y^D + V(I - A^D)^{-1} E \tag{24}$$

$$\text{Or } GDP = VX = V(I - A^F)^{-1}Y^F + V(I - A^F)^{-1}H \tag{25}$$

According to equation (24), GDP can be divided into two parts. The first part is the share of GDP that does not participate in international trade and is just for domestic final demand. The second part, $V(I - A^D)^{-1}E$, is the share of GDP in exports. GDP in exports includes some value-added that can return home. This is why the split is not based on whether final consumption takes place in the domestic economy or abroad. Exports include both intermediate and final products.

From equation (25), GDP can also be decomposed into two parts along another dimension: $V(I - A^F)^{-1}H$ reflects the value-added in domestic sales while $V(I - A^F)^{-1}Y^F$ corresponds to value-added for the foreign final demand. Again, it does not indicate where value-added is ultimately going as the concept of domestic sales is still a mix of intermediate and final products.

Merging equations (18), (19), (24) and (25), we obtain the following GDP decomposition:

$$\begin{aligned} GDP = {} & VBA^F(I - A^D)^{-1}Y^D + V(I - A^D)^{-1}Y^D \\ & + VBA^D(I - A^F)^{-1}Y^F + V(I - A^F)^{-1}Y^F \end{aligned} \tag{26}$$

For a specific country *i,* the equation can be written as follows:

$$\begin{aligned} GDP_i = {} & V_i \sum_j^G \sum_{s \neq j}^G B_{is} A_{sj} (I - A_{jj})^{-1} Y_j^D + V_i (I - A_{ii})^{-1} Y_i^D \\ & + V_i \sum_j^G \sum_s^G B_{is} A_{ss} B_{sj}^F Y_j^F + V_i \sum_j^G B_{ij}^F Y_j^F \end{aligned} \tag{27}$$

Equation (27) is the decomposition that we will use in the empirical part of the paper and that allows us to divide GDP into 4 terms that are interesting for the analysis of global production. It highlights that domestic value-added contributes to four different types of "value chains". The first term, $V_i \sum_j^G \sum_{s \neq j}^G B_{is} A_{sj} (I - A_{jj})^{-1} Y_j^D$, measures the value-added which is propagating in the domestic economy and going into exports. The second term, $V_i (I - A_{ii})^{-1} Y_i^D$, measures the purely domestic value added which has not been part of GVCs or international trade and ends up in domestic final demand. The third term, $V_i \sum_j^G \sum_s^G B_{is} A_{ss} B_{sj}^F Y_j^F$, measures the value-added that has participated in the domestic propagation and is ultimately absorbed by foreign countries. The

last term $V_i \sum_j^G B_{ij}^F Y_j^F$ measures the value-added that has not participated in the domestic propagation and is absorbed by foreign countries.

Terms 1, 3 and 4 are equal to the domestic value-added in exports, as measured by Koopman, Wang and Wei (2014) or by Los et al. (2016), which includes the value-added in exports coming back to the domestic economy. The second term corresponds to value-added going into domestic final demand without having transited through other countries.

Theorem 3: The GDP decomposition in equation (27) is consistent with GDP decomposition according to final demand. We have:

$$VBA^F(I-A^D)^{-1}Y^D + V(I-A^D)^{-1}Y^D = VBY^D$$
$$VBA^D(I-A^F)^{-1}Y^F + V(I-A^F)^{-1}Y^F = VBY^F$$

From the above decomposition, we can also provide expressions for the value-added in exports and in domestic sales as follows:

$$\begin{aligned} V(I-A^D)^{-1}E &= VBA^F(I-A^D)^{-1}Y^D + VBA^D(I-A^F)^{-1}Y^F + V(I-A^F)^{-1}Y^F \\ V(I-A^F)^{-1}H &= VBA^F(I-A^D)^{-1}Y^D + VBA^D(I-A^F)^{-1}Y^F + V(I-A^D)^{-1}Y^D \end{aligned} \quad (28)$$

These equations highlight an important feature of this value-added decomposition. There is an overlap between the value-added in exports and in domestic sales (as some domestic sales are intermediates that are then incorporated into exports). The overlap can be seen in portions of $VBA^F(I-A^D)^{-1}Y^D$ and $VBA^D(I-A^F)^{-1}Y^F$, as these two terms not only participate in the domestic propagation but also in international trade.

Based on the discussions around Koopman et al. (2014) and the comments made by Nagengast and Stehrer (2016), we are not sure at this stage whether further decomposition and additional terms could help to disentangle the value-added in domestic sales and exports and our choice is just to accept this overlap which is needed to provide some analysis on the basis of domestic sales versus exports.

3. Empirical results

In this section, we present two types of results. First, we implement the decomposition proposed in equation (27) in the World Input-Output Database ICIO. Some information on WIOD tables can be found in Timmer et al. (2015). We use the 2016 update of the database (Timmer et al., 2016). We then further decompose our results according to ownership to analyse the value-added in sales and exports of foreign-owned firms. For this empirical work, we rely on split WIOD tables that have been developed in the context of an OECD project (Cadestin et al., 2017) by merging the WIOD data with statistics on the output, value-added, exports and imports of domestic-owned and foreign-owned firms in each country and industry. These split tables are on-going work and only a preliminary version is available. Therefore, the results below are just presented to illustrate the kind of analysis that can be performed with such data and with the decomposition proposed in this paper. These results are likely to change and should be interpreted with caution at this stage.

3.1 Four terms GDP decomposition

From equation (27) we can analyse how important is for each economy the four types of "value chains" or domestic value-added propagation. Table 1 below reports results from calculations done with the original World Input-Output Table (WIOT) for 2014, not yet the table split by ownership. It provides some kind of benchmark before analysing the same calculations for domestic-owned and foreign-owned firms.

Not surprisingly, in almost all economies the second term (T2, the value-added in GDP which is not part of global value chains or international trade) has the highest share. It is very high in the United States (90.4%). Luxembourg is the only country where T2 is not the highest term. However, one should keep in mind that the value-added in exports corresponds to T1+T3+T4. Therefore, Ireland is also a country where more value-added goes into exports than domestic final demand as its share of T2 is also below 50%.

T3 and T4 generally represent a smaller share of GDP as compared to T1. In most countries, value-added 'transits' through several industries before ending up in exports. Only T4 measures value-added that has not been part of the domestic propagation and is directly in exports. It is generally small, except for countries such as Ireland or Luxembourg exporting mostly services (in addition to being generally speaking stronger exporters).

Table 1 – Four terms GDP decomposition (Thousands USD and %), 2014

Country	GDP in 2014	T1	T2	T3	T4	Share T1	Share T2	Share T3	Share T4
Australia	1,384,375	178,015	1,139,888	42,360	24,112	12.9%	82.3%	3.1%	1.7%
Austria	405,334	63,534	270,394	32,181	39,225	15.7%	66.7%	7.9%	9.7%
Belgium	502,125	97,462	309,379	41,894	53,391	19.4%	61.6%	8.3%	10.6%
Bulgaria	52,644	10,436	33,350	4,476	4,382	19.8%	63.3%	8.5%	8.3%
Brazil	2,263,703	145,973	2,030,729	54,654	32,347	6.4%	89.7%	2.4%	1.4%
Canada	1,698,021	269,035	1,276,187	75,548	77,252	15.8%	75.2%	4.4%	4.5%
Switzerland	698,662	122,983	438,954	62,550	74,175	17.6%	62.8%	9.0%	10.6%
China	10,398,721	812,549	8,388,524	875,075	322,572	7.8%	80.7%	8.4%	3.1%
Czech Republic	198,330	38,124	111,076	23,660	25,470	19.2%	56.0%	11.9%	12.8%
Germany	3,620,310	517,943	2,415,282	339,828	347,257	14.3%	66.7%	9.4%	9.6%
Denmark	316,640	46,443	211,562	25,107	33,527	14.7%	66.8%	7.9%	10.6%
Spain	1,304,250	111,798	1,037,440	83,640	71,371	8.6%	79.5%	6.4%	5.5%
Estonia	24,777	5,896	14,507	2,035	2,339	23.8%	58.6%	8.2%	9.4%
Finland	248,360	35,166	183,391	15,197	14,607	14.2%	73.8%	6.1%	5.9%
France	2,649,132	243,996	2,106,476	158,966	139,695	9.2%	79.5%	6.0%	5.3%
United Kingdom	2,806,453	308,042	2,205,333	142,122	150,956	11.0%	78.6%	5.1%	5.4%
Greece	216,622	21,563	177,789	8,593	8,677	10.0%	82.1%	4.0%	4.0%
Hungary	126,627	24,374	70,924	12,074	19,254	19.2%	56.0%	9.5%	15.2%
Indonesia	880,103	110,324	707,565	35,561	26,654	12.5%	80.4%	4.0%	3.0%
India	2,106,492	134,967	1,814,309	76,518	80,698	6.4%	86.1%	3.6%	3.8%
Ireland	236,524	65,591	102,509	19,064	49,360	27.7%	43.3%	8.1%	20.9%
Italy	1,997,966	173,786	1,566,620	154,864	102,697	8.7%	78.4%	7.8%	5.1%
Japan	4,489,205	294,248	3,860,031	193,662	141,264	6.6%	86.0%	4.3%	3.1%
Korea	1,366,597	216,833	914,137	142,263	93,363	15.9%	66.9%	10.4%	6.8%
Lithuania	46,134	10,408	25,167	3,872	6,688	22.6%	54.6%	8.4%	14.5%
Luxembourg	61,088	22,382	21,529	7,419	9,757	36.6%	35.2%	12.1%	16.0%
Latvia	28,886	5,534	18,854	2,211	2,287	19.2%	65.3%	7.7%	7.9%
Mexico	1,226,364	116,809	986,788	57,507	65,260	9.5%	80.5%	4.7%	5.3%
Netherlands	829,488	202,260	474,744	70,561	81,922	24.4%	57.2%	8.5%	9.9%
Norway	476,410	108,070	321,857	25,840	20,642	22.7%	67.6%	5.4%	4.3%
Poland	512,028	74,401	340,095	49,875	47,657	14.5%	66.4%	9.7%	9.3%
Portugal	215,514	26,714	163,243	12,416	13,141	12.4%	75.7%	5.8%	6.1%
Romania	186,867	28,629	130,379	14,045	13,814	15.3%	69.8%	7.5%	7.4%
Russian Federation	1,724,354	347,187	1,268,859	72,644	35,664	20.1%	73.6%	4.2%	2.1%
Slovak Republic	97,465	18,269	55,395	10,745	13,058	18.7%	56.8%	11.0%	13.4%
Slovenia	45,221	8,874	26,187	4,452	5,709	19.6%	57.9%	9.8%	12.6%
Sweden	533,215	82,071	366,668	37,843	46,632	15.4%	68.8%	7.1%	8.7%
Turkey	750,556	73,562	573,737	56,874	46,383	9.8%	76.4%	7.6%	6.2%
United States	17,416,846	862,714	15,749,924	415,635	388,573	5.0%	90.4%	2.4%	2.2%

3.2 Value added in exports and domestic sales by foreign-owned and domestic-owned firms

To further analyse the role of domestic-owned and foreign-owned firms in exports and domestic sales, we do similar calculations to what was presented in the previous sub-section but with a different ICIO. In addition to countries and industries, this ICIO is fully split according to ownership (domestic-owned versus foreign owned firms).

Figure 1 first provides a full decomposition of world GDP indicating whether value-added is derived from domestic sales or from exports and then whether this value-added is generated by domestic-owned firms or foreign-owned firms. It shows that more value-added is created through trade than through the sales of foreign affiliates. The value-added in exports by domestic-owned firms and foreign-owned firms

(16%+4%=20%) is considerably larger than the value-added in domestic sales by foreign-owned firms (6%). The overlap between trade and investment is relatively small – i.e. 4% of world GDP - which corresponds to the value added in exports by foreign-owned firms. This 4% seem to suggest that at the world level foreign affiliates account for a rather small share of trade in value added terms, an indication that GVCs operate with many arm's length trade transactions and maybe less within pure MNE networks (e.g. inputs transferred between affiliates).

Figure 1 also highlights that 74% of world GDP is value-added created by domestic-owned firms in domestic sales. For most products and particularly for services, a high share of value-added is domestic. And even when products are imported, trade margins create domestic value added if the wholesaler and retailer are domestic firms (and possibly domestic-owned). But MNEs can still play a role in this domestic value that was added by domestic-owned firms. First, there are domestic MNEs and their operations also create domestic value-added. Second, with the analysis in value-added terms, the origin of value-added is identified, independently of how it has further transited through foreign firms. For example, domestic inputs can be used by foreign firms, exported and then come back embodied in intermediate imports that are then incorporated in domestic sales by domestic-owned companies. These inputs are also contributing in this case to the domestic value-added in domestic sales. Figure 1 should therefore not be interpreted as the share of GDP under the control of MNEs.

Figure 1. Treemap of world GDP, 2014

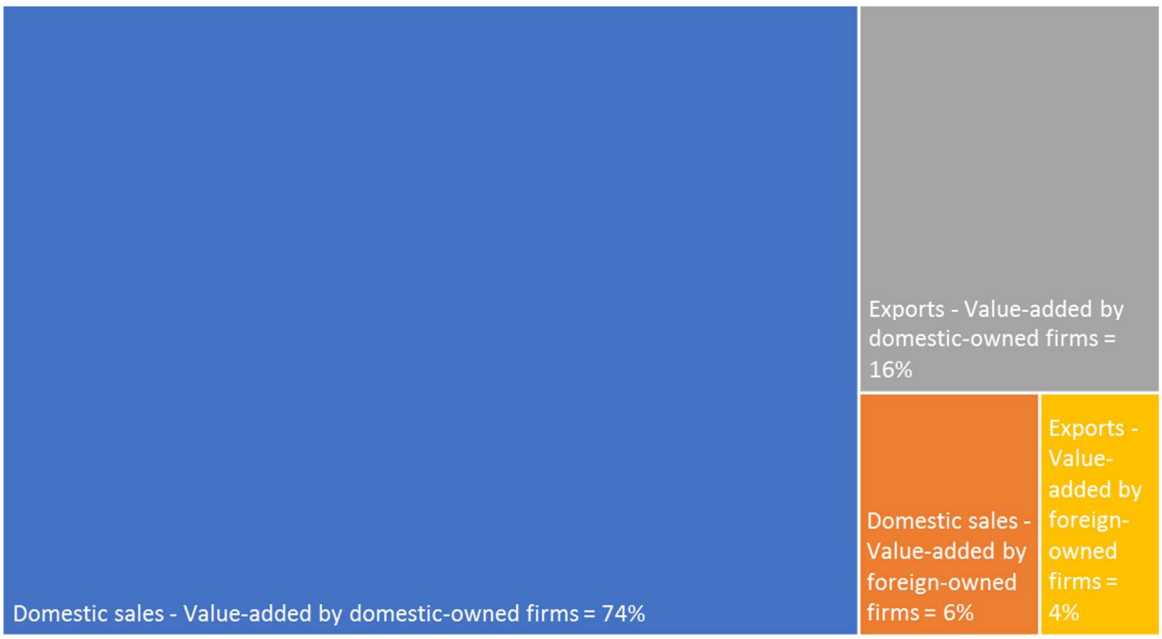

Table 2 below provides a decomposition of domestic sales by country and by type of ownership, showing this time the double counting terms. It compares sales by domestic-owned firms with

sales by foreign-owned firms (the respective importance of the two is only given through the value in USD of domestic sales, the percentages expressed in other columns applying to these values).

Table 2 – Decomposition of domestic sales by domestic-owned and foreign-owned firms (Thousands USD and %), 2014

Country	Domestic-owned firms					Foreign-owned firms				
	Domestic sales in Thousands USD	Domestic VA	Domestic double counting	Foreign VA	Foreign double counting	Domestic sales in Thousands USD	Domestic VA	Domestic double counting	Foreign VA	Foreign double counting
Australia	2,122,783	52.0%	38.7%	3.0%	6.3%	313,793	46%	38%	8%	8%
Austria	490,565	55.5%	26.9%	7.8%	9.8%	108,071	46%	30%	12%	11%
Belgium	651,324	52.8%	28.2%	8.3%	10.8%	76,419	43%	26%	19%	12%
Brazil	3,408,188	57.2%	34.7%	3.0%	5.1%	425,051	45%	39%	8%	8%
Bulgaria	72,285	45.1%	30.0%	10.0%	14.8%	18,889	48%	22%	17%	14%
Canada	2,318,879	55.3%	32.4%	4.7%	7.7%	369,785	42%	32%	15%	11%
China	27,192,850	34.3%	54.5%	2.6%	8.6%	2,126,789	23%	59%	6%	12%
Czech Republic	225,035	46.4%	30.4%	9.7%	13.5%	106,167	41%	30%	14%	16%
Denmark	398,786	57.2%	26.4%	7.4%	9.0%	45,503	51%	21%	18%	11%
Estonia	27,530	52.8%	26.8%	8.9%	11.5%	8,687	47%	23%	17%	13%
Finland	361,650	52.9%	29.9%	7.0%	10.2%	51,554	45%	27%	15%	12%
France	3,750,263	57.4%	30.3%	5.0%	7.2%	510,217	42%	36%	13%	10%
Germany	4,383,928	57.5%	30.9%	4.8%	6.8%	1,000,560	44%	33%	13%	10%
Greece	296,776	61.4%	27.6%	5.4%	5.7%	22,208	56%	27%	10%	6%
Hungary	124,153	53.7%	23.1%	11.0%	12.1%	43,832	49%	20%	18%	13%
India	3,498,684	54.5%	32.9%	4.6%	8.0%	115,387	33%	41%	13%	14%
Indonesia	1,337,864	52.3%	35.8%	4.8%	7.2%	165,880	51%	31%	9%	10%
Ireland	171,750	54.6%	17.7%	15.6%	12.1%	74,976	52%	16%	20%	13%
Italy	3,069,126	53.5%	34.6%	4.1%	7.8%	417,691	40%	36%	13%	11%
Japan	7,563,495	53.7%	33.2%	4.7%	8.5%	287,727	48%	37%	6%	9%
Korea	2,601,126	43.0%	33.1%	7.7%	16.3%	104,794	41%	31%	13%	16%
Latvia	40,930	46.1%	33.0%	7.9%	13.0%	9,077	49%	27%	13%	11%
Lithuania	43,201	58.5%	21.1%	11.1%	9.3%	9,744	62%	22%	8%	8%
Luxembourg	75,825	38.5%	13.3%	25.1%	23.1%	17,704	34%	10%	36%	20%
Mexico	1,566,138	62.1%	26.1%	5.5%	6.3%	196,166	56%	28%	9%	7%
Netherlands	954,218	55.0%	27.7%	7.5%	9.7%	141,892	51%	25%	14%	11%
Norway	528,228	58.3%	29.6%	4.9%	7.2%	118,721	49%	29%	13%	9%
Poland	630,598	50.5%	30.7%	7.5%	11.3%	223,205	45%	32%	11%	13%
Portugal	286,828	55.4%	28.8%	6.7%	9.1%	50,820	53%	28%	10%	10%
Romania	214,677	49.5%	32.1%	7.2%	11.1%	105,956	46%	29%	12%	12%
Russian Federation	2,778,295	50.7%	41.4%	2.9%	5.0%	108,996	44%	39%	9%	8%
Slovak Republic	112,695	51.5%	28.2%	8.5%	11.7%	34,475	39%	26%	18%	17%
Slovenia	55,658	53.2%	26.9%	8.7%	11.2%	7,767	48%	29%	12%	11%
Spain	1,843,363	55.4%	31.8%	4.8%	8.0%	335,537	44%	34%	10%	11%
Sweden	629,345	56.7%	29.5%	5.8%	8.0%	153,491	50%	28%	12%	10%
Switzerland	822,605	54.1%	31.4%	5.5%	9.0%	223,491	44%	32%	12%	11%
Turkey	1,140,564	53.3%	33.7%	5.4%	7.6%	104,081	53%	31%	9%	8%
United Kingdom	3,642,384	56.3%	32.9%	4.3%	6.6%	889,480	46%	34%	10%	9%
United States	26,677,432	58.1%	35.6%	2.4%	4.0%	2,366,500	44%	40%	9%	7%

A first comment is that double counting is very high as compared to what is measured in the trade in value-added literature for exports, both for domestic-owned and foreign-owned firms. It is however expected as industries in the domestic economy are much more integrated than industries across borders. The double counting coming from value-added going back and forth

is to be more expected within the same border in the domestic economy. It explains while the domestic double counting is higher than the foreign double counting in Table 2.

A second comment is that while domestic value-added is higher within the sales of domestic-owned firms, the difference is not as high as we could have expected. It seems that most affiliates of foreign firms are relying on a large extent on domestic inputs and not on inputs coming from the parent economy. There are however also higher percentages for the foreign double counting in the case of foreign-owned firms (and at the end a higher overall double-counting as compared to domestic-owned firms). It is also consistent with more value-added going back and forth with the parent country of the foreign affiliates.

4. Concluding remarks

This paper has introduced a new type of GDP decomposition that allows us to trace value-added and double counting not only in exports but also in domestic sales. The motivation is that traditional I-O analysis looking at value-added in final demand is not sufficient to discuss trade and investment in global value chains. Looking at trade in value-added terms and decomposing gross exports has brought many interesting analytical results and led to new policy implications. We believe the same will happen when looking at activities of multinational enterprises in value-added terms and decomposing sales of foreign affiliates. This is why we need for domestic sales tools similar to what was developed for gross exports.

By using our methodology to compare the value-added in exports and in sales of domestic-owned and foreign-owned firms, there are already interesting findings. In particular, it seems that the double counting in sales of foreign affiliates is much more pronounced than in exports, as affiliates of foreign firms rely even more on inputs from the host economy. The whole literature on the benefits of FDI and the impact of activities of MNEs will have to be revisited in light of this value-added analysis which can provide a better indication of how income is generated and who really benefits from production by foreign-owned firms.

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Appendix

Lemma 1 : The accounting relationship between domestic shipments H and final demand in destination in an Inter-Country Input-Output (ICIO) model can be expressed as:

$$H = \overline{A}H + \overline{Y}$$

Proof: Gross output X is the sum of gross exports E and gross domestic shipments H. From the accounting identity in equation (1), $X = AX + Y$, we can express E the vector of exports and H the vector of gross domestic shipments as:

$$E = A^F(E+H) + Y^F$$
$$H = A^D(E+H) + Y^D$$

Solving for E, we obtain:

$$E = (I - A^F)^{-1}A^F H + (I - A^F)^{-1}Y^F$$

Merging the expression for H and for E, we obtain:

$$\begin{aligned} H &= A^D(E+H) + Y^D \\ &= A^D[H + (I - A^F)^{-1}A^F H + (I - A^F)^{-1}Y^F] + Y^D \\ &= A^D[I + (I - A^F)^{-1}A^F]H + A^D(I - A^F)^{-1}Y^F + Y^D \\ &= A^D(I - A^F)^{-1}H + A^D(I - A^F)^{-1}Y^F + Y^D \\ &= \overline{A}H + \overline{Y} \end{aligned}$$

with $\overline{A} = A^D(I - A^F)^{-1}$ and $\overline{Y} = \overline{A}Y^F + Y^D$.

Lemma 2 : The accounting relationship between gross exports E and final demand in destination in an Inter-Country Input-Output (ICIO) model can be expressed as:

$$E = \tilde{A}E + \tilde{Y}$$

Proof: Similar to Lemma1.

Lemma 3：In the domestic sales accounting framework, we have

$$B^F \overline{B} = B$$

Here, $B^F = (I - A^F)^{-1}$ and $\overline{B} = (I - \overline{A})^{-1}$, B is the 'total requirements matrix' in the ICIO table which is $B = (I - A)^{-1}$.

Proof: Expanding the expression of B^F and $\overline{B}$, we obtain:

$$\begin{aligned}
&B^F \overline{B} = (I - A^F)^{-1}(I - \overline{A})^{-1} = [(I - \overline{A})(I - A^F)]^{-1} = \{[I - A^D(I - A^F)^{-1}](I - A^F)\}^{-1} \\
&= \{[(I - A^F)(I - A^F)^{-1} - A^D(I - A^F)^{-1}](I - A^F)\}^{-1} \\
&= [(I - A^F - A^D)(I - A^F)^{-1}(I - A^F)]^{-1} \\
&= (I - A)^{-1} = B
\end{aligned}$$

Theorem 1: In the value-added decomposition of domestic sales, the sum of the domestic value-added and the double counting term is equal to the domestic content in domestic sales.

$$V_i B_{ii}^F \hat{H}_i + V_i B_{ii}^F A_{ii} B_{ii} \hat{H}_i + \sum_{j \neq i}^{G} V_i B_{ij}^F \overline{B}_{ji} \hat{H}_i = V_i B_{ii} \hat{H}_i$$

Proof: According to lemma 3, we can obtain the submatrix *i*'s expression as

$$\sum_{j}^{G} B_{ij}^F \overline{B}_{ji} = B_{ii}^F \overline{B}_{ii} + \sum_{j \neq i}^{G} B_{ij}^F \overline{B}_{ji} = B_{ii}$$

merging the expression of matrix $\overline{B}_{ii} = I + A_{ii} B_{ii}$, we have

$$B_{ii}^F + B_{ii}^F A_{ii} B_{ii} + \sum_{j \neq i}^{G} B_{ij}^F \overline{B}_{ji} = B_{ii}$$

Theorem 2: In the value-added decomposition of domestic sales, the sum of the foreign value-added and the double counting term is equal to the foreign content in domestic sales.

$$\sum_{j\neq i}^{G} V_j B_{ji}^F \hat{H}_i + \sum_{j\neq i}^{G} V_j B_{ji}^F A_{ii} B_{ii} \hat{H}_i + \sum_{s\neq i}^{G}\sum_{j\neq i}^{G} V_s B_{sj}^F \overline{B}_{ji} \hat{H}_i = \sum_{j\neq i}^{G} V_j B_{ji} \hat{H}_i$$

Proof: Similar to theorem 1.

Theorem 3: The four terms GDP decomposition is consistent with GDP decomposition in the final demand. We have:

$$VBA^F(I-A^D)^{-1}Y^D + V(I-A^D)^{-1}Y^D = VBY^D$$
$$VBA^D(I-A^F)^{-1}Y^F + V(I-A^F)^{-1}Y^F = VBY^F$$

Proof: For the first equation, we have

$$BA^F(I-A^D)^{-1} + (I-A^D)^{-1} = (BA^F + I)(I-A^D)^{-1}$$
$$= [BA^F + B(I-A)](I-A^D)^{-1} = [B(I-A+A^F)](I-A^D)^{-1}$$
$$= B$$

The same can be done with the second equation.