Comparison of Mathematical Methods for SUT Construction Using WIOD Database

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Abstract

We investigate the relative effectiveness of the projection methods of Supply and Use tables. The empirical basis of the study is founded on Use and Supply tables for 28 countries from the period 19952010 provided by the WIOD project. We conducted a comparative study of three mathematical methods that have proven the most effective in constructing projection of Use tables for Spain and the Netherlands from the empirical study by Temurshoev, Webb, and Yamano (2011). In these methods, Input-Output, Supply, and Use tables are constructed based on the benchmark table and the column and row totals of the table under construction. Whereas the results of *Op. Cit.* show that the GRAS method is the most effective for the IOTs and for the Supply tables for Netherlands and Spain, in the case of Use tables some quadratic methods show similar results as GRAS.

Essentially, our results confirm the conclusions of *Op. Cit.* The most effective of the considered methods is GRAS, a version of the classical RAS algorithm. The results of applying this method under the number of criteria are closer to the published tables than the results of the INSD method and Kuroda method, which are based on quadratic programming. At the same time we have shown that in some cases the table cannot be balanced by GRAS method because of significant changes in the structure of the table. In 80% of these cases the tables were successfully balanced by the two quadratic methods. In these cases the Kuroda method is the most effective.

Our work is motivated by a recent project of retrospective construction of SUT for Russia. We conclude that GRAS method is a priority in the extrapolation of Use tables of Russia. At the same time, if the structure of the table under construction is expected to be essentially different from the structure of the benchmark one then a version of Kuroda method is more appropriate.

Keywords: WIOD project, RAS, Kuroda method, Supply table, Use table
Classification JEL: D57, C53, C67

1. Introduction

Input-output tables are included into Systems of National Accounts (SNA) in many countries [Miller, Blair, 2009]. Supply and Use tables are paramount since they reflect the amount of use and output of particular categories of commodities in different industries of the economy. These tables are also used to build symmetrical commodity-by-commodity and industry-by-industry tables. In this paper we discuss both Supply and Use tables.

Use tables are usually built either in basic prices or in purchasers' prices. The latter are calculated as a sum of Use tables in basic prices, tables of trade and transportation margins and tables of taxes on commodities. Both Supply and Use tables are divided into four quadrants as follows. In Use tables, quadrant I contains the information about each industry's consumption of all commodities produced in the economy, in quadrant II a disaggregated final demand is represented, quadrant III provides information about the value added in each industry. Quadrant IV usually is not filled, but sometimes it could (partially) represent information about redistribution of the gross domestic product. In Supply tables, we are mainly interested in quadrant I (the production matrix, or the transposed make matrix) which contains the information about each industry's supply of all commodities produced in the economy (in basic prices). Quadrant II contains the column vector of import/export by products and valuation matrix (which contains product totals from the tables of trade and transportation margins and the table of taxes on commodities). The row totals of the whole quadrants I and II of Supply table are the supply totals of each product in purchasers' prices. The quadrants III and IV may contain additional rows with the information about CIF/FOB adjustments on imports, direct purchases abroad by residents, and the rows of division of output of each industry by market output, output for own final use, and other nonmarket output. Input-output tables are constructed by statistical services of different countries and could be united in tables of a group of countries (e.g. for the European Union). In terminology of Eurostat Manual [Eurostat, 2008], the quadrants I and II of Use tables are called tables of intermediate uses and table of final uses respectively.

Building the input-output tables system requires a large-scale survey of enterprises of all relevant industries. Such a survey is commonly provided by national statistical services not more often than once in several years (generally, once in five years). We refer to the resultant Supply and Use tables as benchmark tables. Since applied analysis and forecast problems require a time series of annual tables, for years when no such survey was conducted SUTs are constructed on the basis of the benchmark tables and the annual SNA data. This procedure is called projection (or extrapolation) of the tables.

For this SUT projection a number of mathematical methods is designed, the most renowned among the methods is the classical RAS method. Every so often these methods are improved, new ones are designed, and comparative empirical studies are conducted in order to identify the most effective methods. One of the latest such studies considering the input-output tables of Spain and the Netherlands is undertaken in [Temurshoev, Webb, Yamano, 2011]. The task of choosing the most effective projection method is especially relevant now in Russia, since a number of similar tables for next and previous years should be built on the basis of the benchmark table system for 2011 published in March, 2017 by Russian Federal State Statistics Service.

The general purpose of this paper is to provide a more complete comparative empirical study of methods of SUT projection that will complement the results of [Temurshoev, Webb, Yamano, 2011]. Starting from [Temurshoev, Webb, Yamano, 2011], we assume known that the most effective existing methods could be GRAS, INSD and variations of Kuroda method (see Section 2), so we consider only these methods. In addition, since we concentrate on just several methods, this allows us to expand the empirical base of the study: we used data from the WIOD international project, in which Use tables for 40 countries for years 1995-2012 were collected. Efforts have been made to separate the benchmark tables, compiled on the basis of aggregation of statistical data, from those constructed by projection. Based on the analysis of the available information 52 benchmark pairs of Use and Supply tables from 29 countries were selected, which were compared with the tables we constructed based on various mathematical projection methods.

The results of the calculations showed that the modified bi-proportional GRAS is the most efficient of the three considered projection methods. The advantage of this method is insurmountable in the projection of the first quadrant of Use table, on which the method reduces to the method of RAS, and also in the results of the projection of the first and second quadrant with known totals by columns and rows. However, in a fairly large number of cases the table cannot be balanced by this method (for example, if the locations of the zero elements have changed significantly between the benchmark year and the forecasted one). Our computational experiments show that in 80% of such cases the quadratic projection methods Kuroda 1 and INSD give a satisfactory result.

The paper has the following structure. In Section 2 mathematical descriptions of some modern methods of SUT projection are given. Section 3 describes the initial data of our study. The essential part here is information about the years for which the countries included in the WIOD have produced benchmark SUTs. Based on this information we select SUTs for 28 countries from the WIOD database for computational experiments. Section 4 describes several traditional "metrics" that allow us to compare the tables constructed by our methods with real data. Both here and in Section 2 our presentation follows mainly [Temurshoev, Webb, Yamano, 2011]. The results of the calculations are presented in Section 5. At the first step of the calculation, a series of tables are selected for which all the methods considered produce a balanced result: for such tables, then the statistical results of the calculations are presented separately. Finally, Section 6 is devoted to a brief analysis of the outcomes.

This paper is an extended version of the Russian language paper [Kuznetsov et al., 2015].

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2. Selected methods of SUT projection

Firstly let us consider a bunch of so called proportional methods. All of them represent the modifications of the classical RAS method described below. Let $A_0 = (a^0{}_{ij})$ be the matrix that correspondes to the benchmark table, and let A be the unknown matrix that we need to estimate. Here it is also assumed that the marginal row and column totals of the matrix $A = (a_{ij})$ are given; notation for the vectors of row totals and column totals is u and v respectively.

RAS. The RAS method (a classical algorithm for biproportional method's realisation, proposed by R. Stone [Stone, 1961]) minimizes the objective function

$$(1) \qquad f(A, A_0) = \sum_{i,j} a_{ij} \ln \frac{a_{ij}}{e a_{ij}^0},$$

subject to the linear constraints fixing all row and column totals:

$$(2) \qquad \sum_j a_{ij} = u_i$$

$$(3) \qquad \sum_i a_{ij} = v_j .$$

One can solve this minimization problem by the formulae $A_{n+1} = RA_n$ (for even *n*) and $A_{n+1} = A_n S$ (for odd *n*), where

$$R=\begin{pmatrix} \frac{u_1}{\sum_j a_{1j}^n} & 0 & \cdots & 0 \\ 0 & \frac{u_2}{\sum_j a_{2j}^n} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{u_n}{\sum_j a_{nj}^n} \end{pmatrix}$$

and

$$R=\begin{pmatrix} \frac{u_1}{\sum_j a_{1j}^n} & 0 & \cdots & 0 \\ 0 & \frac{u_2}{\sum_j a_{2j}^n} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{u_n}{\sum_j a_{nj}^n} \end{pmatrix}.$$

It is well known (see [Miller, Blair, 2009]), that for non-negative benchmark matrix A_0 the convergence of this method could be guaranteed (with exception of the some sparse A_0, see [Miller, Blair, 2009, 7.4.9]). In this case the matrix A occurs non-negative as well.

GRAS. The classical RAS method is unapplicable to matrices which contain negative elements. For such matrices a Generalize RAS (GRAS) is used [Günlük-Şenesen, Bates, 1988], [Temurshoev, Miller, Bouwmeester, 2013]. We will minimize the same objective function (1) subject to the same linear constraints and the additional condition that all elements of the matrix preserves the signs. Let

$$A = P - N,$$

where P and N are matrices of the same size as A such that P contains all positive elements of A and zeroes on all other places. Then N contains the absolute values of all negative elements of A and zeroes on all other places.

In each iteration, the matrix elements are computed by the following formulae. In the case of even n, $a_{ij}{}^{n+1} = r_i^{(n)} a_{ij}{}^{n}$ if $a_{ij}{}^{n} > 0$ and $a_{ij}{}^{n+1} = (r_i^{(n)})^{-1} a_{ij}{}^{n}$ if $a_{ij}{}^{n} < 0$, and in the case of odd n $a_{ij}{}^{n+1} = a_{ij}{}^{n} s_j{}^{(n)}$ if $a_{ij}{}^{n} > 0$ and $a_{ij}{}^{n+1} = a_{ij}{}^{n} (s_j{}^{(n)})^{-1}$ if $a_{ij}{}^{n} < 0$.

Here the multipliers r_i and s_i following are defined in the following recurrent way. Using the current values of the multipliers $r_i = r_i^{(n)}$ и $s_j = s_j{}^{(n)}$ and the current values of the matrices $P = (p_{ij})$, $N = (n_{ij})$, let us introduce the notation $p_i(s) = \sum_j p_{ij} \cdot s_j$, $p_j(r) = \sum_i r_i \cdot p_{ij}$, $n_i(s) = \sum_j n_{ij} \cdot s_j^{-1}$ and $n_j(r) = \sum_j n_{ij} r_i^{-1}$. Then the recurrent formulas for evaluating the next values of $r_i = r_i^{(n+1)}$ and $s_j = s_j{}^{(n+1)}$ has the form

$$r_i = \begin{cases} \dfrac{u_i + \sqrt{u_i^2 + 4p_i(s)n_i(s)}}{2p_i(s)}, p_i(s) > 0 \\ -\dfrac{n_i(s)}{u_i}, p_i(s) = 0 \end{cases}$$

(4)

and

$$s_j = \begin{cases} \dfrac{v_j + \sqrt{v_j^2 + 4p_j(r)n_j(r)}}{2p_j(r)}, p_j(r) > 0 \\ -\dfrac{n_j(r)}{u_j}, p_j(r) = 0 \end{cases}$$

INSD. The widely used method [Friedlander, 1961] (*improved normalized squared difference)* has the objective function

$$f = \sum_i \sum_j \frac{(a_{ij}^0 - a_{ij})^2}{a_{ij}^0}. \quad (5)$$

Here summation is performed over all *i* and *j* such that $a_{ij} \neq 0$. In contrast to the least squares method (LSM), this projection method is more careful with respect to small flows, while LSM almost ignores them.

Kuroda's Method. This method was proposed in [Kuroda, 1988]. The objective function to be minimized is

$$f(A, A_0) = \frac{1}{2}\sum_i \sum_j \left(\frac{a_{ij}}{u_i} - \frac{a_{ij}^0}{u_i^0}\right)^2 w_{ij} + \frac{1}{2}\sum_i \sum_j \left(\frac{a_{ij}}{v_i} - \frac{a_{ij}^0}{v_i^0}\right)^2 v_{ij},$$

where the weights w_{ij} and v_{ij} differ for different variations of the method (the enumeration is taken from [Temurshoev, Webb, Yamano, 2011]):

- Kuroda 1: $w_{ij} = \dfrac{(u_i^0)^2}{a_{ij}^2}, v_{ij} = \dfrac{(v_i^0)^2}{(a_{ij}^0)^2}$ [Kuroda, 1988, Case (2)]
- Kuroda 2: $w_{ij} = \dfrac{u_i^2}{2}, v_{ij} = \dfrac{v_i^2}{2}$ [Wilcoxen, 1989]
- Kuroda 3: $w_{ij}=v_{ij}=1$

In all cases, only terms with non-zero denominators are summarized.

Our study of the projection methods has been motivated by the recent situation with the SUTs in Russia. The benchmark SUTs for 2011 were published in March, 2017 by the Russian Federal State Statistics Service. Then the perspective and retrospective sequences of SUTs need to be constructed (for all future years before the release of next benchmark table and for all previous ones till the closest official table). Some issues concerning construction of the retrospective SUT series are discussed in [Baranov, Kim, Starytsina, 2011], [Baranov, Kim, Piontkovski, Starytsina, 2014].

One of the last comparative empirical studies of SUT and IOT projection methods was conducted in [Temurshoev, Webb, Yamano, 2011]. In this study, the projection results of 10 methods were compared to the real SUTs and IOTs of Spain and the Netherlands. The study shows that GRAS method ended to be the most effective. However, in the special case of Use tables the results are not convincing. In this case, the quadratic INSD method and Kuroda's method performed as effective as RAS and GRAS. In particular, Kuroda's method performed better on the projecting Netherland's Use table in purchasers' prices from 1995 to 2000, while GRAS is more effective for projecting the same tables from 2000 to 2005 [Temurshoev, Webb, Yamano, 2011, Table 3]. Analogously, for projecting Use tables for Spain in basic prices from 2000 to 2005 years Kuroda's method showed better results for the first quadrant while GRAS method looks better for the second quadrant (see the last two plots on Fig.1 and Table 4 in Op.Cit).).

3. Sources of the data for computational experiments

For selecting tables out of WIOD for the computational experiments we had to take into account that, for constructing SUTs on the basis of the benchmark tables national statistical services also use mathematical methods (see [Eurostat, 2008, 8.6.1]). Thereby, a simple comparison of projecting methods using raw empirical data can lead to distortions: for instance, if an official table for 2001 was projected from one for 2000 by the RAS method, then, obviously, this method will be chosen as the best for the mentioned projection, but will give us no relevant information concerning forecasting of the true economic conditions. Analogously, if there were years with no corresponding official SUTs, then missing ones were artificially projected within the WIOD project [Erumban et al., 2012], in particular, using a special variation of RAS proposed in [Timurshoev, Timmer, 2011]. For instance, SUTs for Russia were constructed using this method on the basis of 1995 benchmark SUTs, so, as our computational experiments confirmed, under the WIOD data the RAS method appears to be more precise for projecting use tables for Russia (from 1995 to 2000s years) then for other countries.

To avoid such statistical distortions, we have decided to use only those WIOD tables that were obtained by aggregation of the national benchmark tables. However, despite the fact that the sources of data for the WIOD project are completely described by the authors [Erumban et al., 2012], unfortunately, for some countries we failed to identify which SUTs were benchmark ones among all SUTs constructed by national statistical services. Excluding these countries and also countries likewise Russia, for which only one benchmark table was published during the period under our interest (or even none), there are 28 countries left. The empirical base of our research consists of the data corresponding to these left countries, see Table 1.

For the majority of these 28 countries, the sources of information about the benchmark tables were the official national statistical services' websites, where one can find the tables themselves and the corresponding information. Unfortunately, for some countries we have failed to find such data. For *Cyprus,* there is no benchmark SUT for the given period. For *Mexico,* only one benchmark SUT for the year 2003 was constructed, hence it was not used in this research. Same story appear for *Canada* (there was only one benchmark SUT system for the year 1997); and *Russia* (only benchmark SUT system for the year 1995, since the very recent official SUTs for 2011 are not still incorporated in WIOD). As for *Netherlands* and *Luxemburg* we have the opposite case: as far as we know (see sources of data in [Piontkovski, Sokolov, Starchikova, 2015]), in these countries the benchmark tables are constructed annually. Since for the other countries under consideration the 5-year gap for benchmark tables was the most common, we decided to reduce examined Netherlands' and Luxemburg's tables to the ones for 2000 and 2005 yy. Also, we could not find an exact data concerning *Austria*, *Denmark*, *Latvia*, *Lithuania*, *Slovakia*, *France* and *Estonia*, that is why the corresponding tables were not used in the current research.

Table 1. Information about existence of benchmark SUTs for some of countries (relevant years are marked with the plus sign)

	Years (1995–2011)																
Countries\	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	10	11
Australia		+							+	+				+			
Belgium	+					+											
Brazil	+	+				+					+						
Bulgaria	+					+				+							
UK				+						+				+			
Hungary						+					+					+	
Germany						+					+						+
Greece											+					+	
India				+					+					+			
Indonesia	+					+					+						
Ireland	+					+					+						
Spain	+					+					+						
Italy	+					+					+						
Canada			+														
China			+					+					+				
Cyprus																	
Korea	+					+					+			+			
Luxembourg						+					+					+	
Malta						+	+										
Mexico									+								
Netherlands						+					+					+	
Poland						+					+						
Portugal							+					+					
Russia	+																
Romania						+						+					
Slovenia						+								+			
USA			+					+					+				
Taiwan							+					+					
Turkey				+				+									
Finland	+					+											
Czech Republic	+					+					+					+	
Sweden	+					+					+						
Japan	+					+					+						+

For detailed references to each country's data sources see [Piontkovski, Sokolov, Starchikova, 2015]. Note that in some cases only a part of SUT is benchmark in our sense (e.g., in our data the Brasil Supply table in 1996 seems to be benchmark whereas the Use table in purchasers' prices in 1996 seems to be a projection of the 1995 table). We have not separated such cases.

4. SUT projecting: general methodology and metrics to compare the results

We will consider projections of the following possible parts of Use tables: quadrant I (intermediate use), quadrant II (final use) or the united quadrants I and II referred to as simply Use table. We consider also quadrant I (production matrix) of Supply tables .

Suppose that we have a matrix A_g^s is a corresponding part (quadrant I, or II, or both) of benchmark Use or Supply table constructed for the country s for year g. Suppose also that row and column totals are known (from national accounts or other sources) for the similar matrix $A_{(g+t)}^s$ with unknown elements for the year $(g+t)$, where $t \geq 1$. Let n be the number of rows and let m be the number of columns in each of the two matrices. Note that in WIOD format which is used in this paper, for Use tables we have $n=59$ and m equals to 36, 6 or 42 depending on the considered part of the table: quadrant I, quadrant II, or the united quadrants I & II; for the first quadrant of WIOD Supply tables, we have $n=59$ and m = 35. There are m+n linear constraints for the elements of the matrix $A_{(g+t)}^s$ meaning that the row and column totals are fixed. Consider a linearization of these linear constraints

$$G\ \boldsymbol{a}_{g+t}^s = \boldsymbol{c}_{g+t}^s, \qquad (6)$$

where G is a matrix of size $(n+m)\times(n\cdot m)$ consisting of zeros and ones, $\boldsymbol{a}_{g+t}^s$ is the vectorization of the unknown matrix A_{g+t}^s (of size $(n\cdot m)\times 1$), and $\boldsymbol{c}_{g+t}^s$ is a vector of size $(n+m)\times 1$ which consists of the row and column totals of the matrix A_{g+t}^s .

The target matrix should satisfy the system (6). For finding the most appropriate solution we need an initial approximation of the unknown vector it which should have a similar structure as the target one. Then the vectorization $\boldsymbol{a}_g^s$ of the given matrix A_g^s play the role of such an initial approximation. Finally, we use the obtained values of G, $\boldsymbol{a}_g^s$ and $\boldsymbol{c}_{g+t}^s$ as inputs for the forecasting procedure and as a result receive the forecast $\tilde{A}_{g+t}^s$ of the matrix A_{g+t}^s.

In each computational experiment, we will consider a certain projecting method as the most effective if it will yield the closest, in some sense, result to the true matrix A_{g+t}^s. Thus, to compare the projecting results for any two methods M_1 and M_2, we will need to know the benchmark table not only for the initial year g but also for a target year A_{g+t}^s.

The mentioned closeness of the two matrices can be formalized by various ways. Here we follow [Temurshoev, Webb, Yamano, 2011] and consider the following quasi-metrics. All of them are well-known, see [Miller, Blair, 2009, 7.4.8]. We follow the notation from [Temurshoev, Webb, Yamano, 2011, chapter 3]: in each of these five cases a particular "metric" is computed for a pair of matrices (x_{ij}) and (x_{ij}^{true}). In the next section, we will compute them for $(x_{ij}^{true}) = A_{g+t}$ (a benchmark table) and $(x_{ij}) = A_{(g+t),m}$ (the forecast matrix obtained by the *m*-th method).

1) Mean absolute percentage error:

$$MAPE = \frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left|x_{ij}-x_{ij}^{true}\right|}{\left|x_{ij}^{true}\right|}\times 100$$

2) Weighted absolute percentage error:

$$WAPE = \sum_{i=1}^{n}\sum_{j=1}^{m}\left(\frac{\left|x_{ij}^{true}\right|}{\sum_k\sum_l x_{kl}^{true}}\right)\frac{\left|x_{ij}-x_{ij}^{true}\right|}{\left|x_{ij}^{true}\right|}\times 100$$

3) Standardized weighted absolute difference:

$$SWAD = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left|x_{ij}^{true}\right|\times\left|x_{ij}-x_{ij}^{true}\right|}{\sum_k\sum_l (x_{kl}^{true})^2}$$

4) Psi-statistic (PsiStat):

$$\hat{\psi} = \frac{1}{\sum_k\sum_l x_{kl}^{true}}\sum_i\sum_j\left[\left|x_{ij}^{true}\right|\times\left|\ln\left(\frac{x_{ij}^{true}}{s_{ij}}\right)\right|+\left|x_{ij}\right|\times\left|\ln\left(\frac{x_{ij}}{s_{ij}}\right)\right|\right],$$

where $s_{ij}=\left(\left|x_{ij}^{true}\right|+\left|x_{ij}\right|\right)/2$

5) RSQ, the squared coefficient of correlation between obtained and target matrices.

As a result of our computations, we obtain three values for each metric (one for each of the three methods) for each matrix pair, A_g and A_{g+t}. Then we range these values for all metrics as follows: in descend order (the less, the better) for metrics 1), 2), 3), 4) – in descending order (the less, the better); and in ascending order for metric 5) (since correlation increases with the closeness). As a result we obtain three five-dimensional vectors of ranks R_m for each of the three methods ($m = 1,2,3$): the *i*-th component of the vector R_m equals to the rank of the *m*'s method among all three methods with respect to the metric *i*, where rank 1 corresponds to the most effective method, and rank 3 corresponds to the least effective one.

Finally, we compute the sums of all elements in each of the three vectors R_1, R_2 and R_3 and denote them as r_1, r_2 and r_3 respectively. After comparing these three values, we range three methods and select the most effective one for the projection of the matrix A_g to the $g+t$ by assuming that the lowest value of r_i corresponds to the most effective method *i*. If the values r_i and r_j occur to be equal we assume that the methods *i* and *j* have the same effectiveness.

5. Empirical results

Based on the obtained data on the base years, we made the following computations. For each country which has more than one benchmark SUT in the observed interval, we make separate projections of benchmark quadrants I, II, and united (I & II) of Use tables both in basic and purchaser's prices and quadrant I of Supply table from each benchmark year to the next benchmark year. For example, in the case of Germany the benchmark years are the following: 2000, 2005 and 2011. So, there are two pairs of neighboring benchmark years 2000-> 2005 and 2005-> 2011. Hence 14 forecasts were built (7 for each pair, that is, three for each pair for Use table in basic prices, three for each pair for Use table in purchaser's prices, and one for Supply table).

Finally, the described forecasts were constructed for 52 base year pairs. As a result of each of the forecasts we obtain a table of the following type (example for the Netherlands' data for the combined quadrants I and II of Use table in purchaser's prices for the projections 2000-> 2005), see table 2.

Table 2. The value of metrics for different methods of Use table projection in purchaser's prices (combined quadrants I and II) for the Netherlands for 2005 from the benchmark year 2000

Methods	Metrics												
	MAPE	R	SWAD	R	WAPE	R	PsiStat	R	RSQ	R	N0	R_all	CmR
GRAS	38,347	1	0,049	1	13,698	1	0,136	1	0,9944	1	52	10	1
Kuroda1	38,999	3	0,059	3	14,110	3	0,140	3	0,9931	3	52	0	3
INSD	38,613	2	0,050	2	13,871	2	0,138	2	0,9943	2	52	5	2

The rows of the table correspond to prediction methods, and the columns show the values of metrics on the obtained forecasts and corresponding rank (R) of methods. In addition to the metrics described above, the table also entered the value of the discrepancy ($Discr$), which is the maximum of absolute value of the difference between the sum of the row or column of the constructed table and the analogous sum in the table from WIOD, as well as $N0$ - the number of non-zero elements in the table from WIOD that correspond to zero values in the constructed table. The value in the column R_all is calculated as the sum of five summands, one for each of the first five metrics, equal to 2, 1 or 0, depending on whether the corresponding method was the first, second or third in the rank relative to the metric in question. In other words, the value R_all equals the difference between the maximum possible sum of ranks in the first five metrics (i.e. 15) and the observed sum of ranks in these metrics. The column CmR contains the cumulative rank on the basis of R_all, which we take as the main criterion in comparing the methods.

Tables 3 and 4 contain the numbers of forecasts in which each of the methods was the first, second and third in the cumulative rank. These tables are presented separately according to forecasts in the basic prices and in the purchaser's prices (for Use tables), as well as separately for the first quadrant, the second quadrant and for the combined matrix of the first and second quadrants of Use table. Note that, due to technical reasons, in 5 cases the forecasts for Supply tables were not calculated, so that the number of cases for Supply tables is 47 (see Table 3).

Table 3. Number of forecasts for cumulative ranks for tables in basic prices

	Use, quadrant I			Use, quadrant II			Use, quadrants I and II			Supply		
Cumulative rank	GRAS	Kuroda 1	INSD	GRAS	Kuroda 1	INSD	GRAS	Kuroda1	INSD	GRAS	Kuroda1	INSD
1	51	0	2	50	0	3	50	0	3	46	0	1
2	1	4	46	1	3	47	1	3	47	1	10	37
3	0	48	4	1	49	2	1	49	2	0	37	9

Table 4. The number of predictions for cumulative ranks for Use tables in purchaser's prices

	Use, quadrant I			Use, quadrant II			Use, quadrants I and II		
Cumulative Rank	GRAS	Kuroda1	INSD	GRAS	Kuroda1	INSD	GRAS	Kuroda1	INSD
1	50	0	2	48	3	2	50	0	3
2	1	3	48	1	11	40	2	5	44
3	1	49	2	3	38	10	0	47	5

Some tables cannot be exactly balanced by the RAS or GRAS method (for example, in the case of sparse matrices, see [Miller, Blair, 2009, 7.4.9]). Moreover, some tables cannot be balanced accurately by any sign-preserved method because, for example, new SNA categories are taken into account in the forecast year so that a nonzero row or column in Supply or Use table for this year corresponds to the zero one in the same table for the benchmark year. Another possible obstruction to accurate balancing is a change in the sign in some category of final demand. Therefore, the construction of an approximate table in which relatively small discrepancies are observed could also be of interest.

In a number of cases (listed in [Piontkovsky, Sokolov, Starchikova, 2015]), the matrix of the forecast of the quadrant I of Use table did not turn out to be accurately balanced, i.e. the value of $Discr$ (the maximum absolute deviation of a column or row total from the exogenously given value) is at least 10 (i.e., at least \$10 million). Due to the presence of negative elements in the second quadrant of Use table and because the matrix of the second quadrant is usually sparse, such large values of $Discr$ occur more often in balancing separately the second quadrant and simultaneously the first and second quadrants, such cases are more common. In general, the comparative effectiveness of the methods in Tables 3 and 4 does not change if we restrict the statistical base to only sufficiently balanced tables (with $Discr < 10$).

So, we assume that the forecast tables are balanced if the value of $Inac$ is at most 10. Depending on the chosen version of the table (Supply table, or quadrants I, II, or combined I & II of Use table) and whether they are calculated in basic prices or in purchaser's prices, we get lists of balanced forecasts for different countries and years indicated in Table 5. In the cases listed in this table, the forecasts constructed by each of the three methods are balanced.

It makes sense to consider the results of calculations separately for balanced matrices. They are given in Tables 6 and 7, where the values of the cumulative rating CmR are given. The results of calculations for individual metrics can be found in [Piontkovsky, Sokolov, Starchikova, 2015].

Tables 6 and 7 below show that, according to the cumulative rank (i.e., by the criterion of minimizing the sum of ranks), the GRAS method is the most effective when projecting the first quadrant and the combined I and II quadrants.

For each of the seven versions of forecast, the problem of choosing the most efficient method from our data can be considered as a multicriteria optimization problem with five criteria. The criteria are the ranks of the considered methods that were calculated for each of the five quasi-metrics. Then the cumulative rank is an aggregated criterion. Analysis of detailed calculation results shows that in the projection of Supply tables as well as in the projection of the table of intermediate consumption (Use table, quadrant I) and the combined table of intermediate demand and final consumption (Use table, united quadrants I and II), both at base prices and in the purchaser's prices, the GRAS method surpasses the others for each of the five separate criteria. So, in these cases it is Pareto optimal. (For Use tables, this can be seen from the tables given in pp 4.5 and 4.6 of [Piontkovsky, Sokolov, Starchikova, 2015]).

Table 5. List of balanced forecasts by countries (marked with a plus sign)

Country	Years	Use, basic prices			Use, purchaser's prices			Supply
		I q.	II q.	I and II q.	I q.	II q.	I and II q.	I q.
Australia	2003 -> 2004	+	+	+	+	+	+	+
Bulgaria	1995 -> 2000	+	+	+	+	+	+	+
Brazil	1995 -> 1996	+	+	+	+	+	+	
Brazil	1996 -> 2000	+		+	+		+	
Brazil	2000 -> 2005		+			+		
Bulgaria	1995 -> 2000							+
Great Britain	2004 -> 2008	+			+			
Hungary	2005 -> 2010	+	+	+	+			
Germany	2000 -> 2005	+			+			
Germany	2005 -> 2011	+		+	+		+	
Greece	2005 -> 2011	+			+			
India	2003 -> 2008	+						
Indonesia	1995 -> 2000							+
Ireland	1995 -> 2000	+	+	+	+	+	+	
Italy	1995 -> 2000	+			+			
Italy	2000 -> 2005	+		+				
South Korea	1995 -> 2000							+
South Korea	2005 -> 2008	+	+	+	+	+	+	+
Luxembourg	2000 -> 2005	+			+			
Luxembourg	2005 -> 2010	+		+	+		+	
Portugal	2001 -> 2006	+	+	+	+	+	+	
Slovenia	2000 -> 2008	+			+			
Japan	2000 -> 2005	+	+	+	+	+	+	
Japan	2002 -> 2007	+			+			
Japan	2005 -> 2011	+		+	+		+	

Table 6. The quantities of forecasts for cumulative ranks for balanced tables in basic prices

	Use, quadrant I			Use, quadrant II			Use, quadrants I and II			Supply		
Cumulative Rank	GRAS	Kuroda1	INSD	GRAS	Kuroda1	INSD	GRAS	Kuroda1	INSD	GRAS	Kuroda1	INSD
1	18	0	3	5	2	2	11	0	2	6	0	0
2	3	1	17	3	4	3	2	3	10	0	2	4
3	0	20	1	1	3	4	0	10	1	0	4	2

Table 7. The quantities of forecasts for cumulative ranks for balanced Use tables in consumer's prices

	Use, quadrant I			Use, quadrant II			Use, quadrants I and II		
Cumulative Rank	GRAS	Kuroda1	INSD	GRAS	Kuroda1	INSD	GRAS	Kuroda1	INSD
1	17	0	2	2	4	2	9	0	2
2	2	2	15	5	1	3	2	2	8
3	0	17	2	1	3	3	0	9	1

6. Concluding remarks

The results of our computations show that the GRAS method is the most effective of the three projection methods considered. The advantage of this method is total in the projection of the first quadrant of Supply table (production matrix) and Use table (intermediate use matrix), on which the method is reduced to the RAS method, and also in the projection of the united first and second quadrants of Use table with known totals by column and totals by row. This advantage is observed with the projection tables in both purchasers' prices and basic prices. Nevertheless, with the projection of the final demand table (Use table, quadrant II), all our three methods give comparable results: although the GRAS method is still slightly more effective on average, there is no reason for unambiguous recommendation.

Note that our results for Spain and the Netherlands are similar to the results mentioned in the introduction from [Temurshoev, Webb, Yamano, 2011] obtained on the basis of more detailed national SUTs, see, for example, Table 3. However, our study shows that the cases of relative effectiveness of quadratic methods discovered there are quite rare in comparison with GRAS and can be considered as a statistical fluctuation.

The confirmation of the effectiveness of proportional methods in the projection of input-output tables serves as an argument for using exactly these methods when Use tables for Russia in the 2000s were constructed.

For the projection of the intermediate table (quadrant I) as well as for entire Use table both in basic prices and in purchasers' prices, the INSD method was took the second rank in comparative efficiency. Perhaps the reason for the relatively high comparative effectiveness of this method is purely mathematical. As noted in [Huang, Kobayashi, Tanji, 2008], the objective function (5) of the INSD method is a second-order Taylor approximation of the objective function (3) of the GRAS method. Therefore, it is natural that the matrices constructed by these methods are close to each other (especially in the case of projections to small time intervals of 2-5 years, when the changes in the coefficients are small, that is, their ratios are close to 1).

The comparatively low efficiency of Kuroda's method was a surprise for us. Probably this means that the previous relatively high efficiency of this method on Use tables is based on random fluctuations. Since the results of our calculations for Spain and the Netherlands are close to the results from [Temurshoev, Webb, Yamano, 2011], it is unlikely that the reason of this low efficiency is any particular feature of the aggregation of national tables in WIOD tables. However we see that this method is relatively effective in measuring the results by the MAPE metric. This allows us to recommend the forecasts based on it in problems where only relative and not absolute values of the changes in separate indicators are important.

Another area of possible application of the quadratic projection methods Kuroda 1 and INSD is found. We have observed that in some cases the table cannot be balanced by GRAS method due to a significant change in the locations of zero elements between the benchmark table and the forecasted one (especially in the case of sparse matrices). In our computational experiments, in the most cases the Supply tables were balanced by the quadratic methods Kuroda 1 and INSD but not by GRAS. In the intermediate use table where the RAS method is the most effective, this occurred in 31 cases out of 52 for calculations in basic prices and in 33 cases out of 52 in purchasers' prices (see Section 4). At the same time, in 25 cases of these 31 (respectively, in 27 out of 33), i.e. in 80% of cases, the Kuroda 1 and INSD methods gave satisfactory results. Thus, in the case when the RAS method does not provide a balanced matrix, one could recommend the quadratic methods of projection of SUTs, that is, INSD and the Kuroda method.

Our comparative research methodology could be improved to obtain more accurate results. One possible reason for the inaccuracy in our calculations is a noticeable number of unbalanced tables. More delicate work with data should exclude most of such cases and clarify the results of computations. The exclusion of import (known from SNA) from the table of final uses can slightly change the metric values on the forecasts for the second quadrant. Another reason for the possible inaccuracy (although not the main one) is the dependence of the result of a quadratic method application on the chosen mathematical algorithm of conditional quadratic optimization. In particular, we have found that the results differ somewhat depending on whether we use standard tools of such optimization implemented in the MATLAB system or we apply the barrier method implemented in the GUROBI library which is more efficient on this task.

References

1. Baranov E. F., Kim I. A., Starytsina E. A. (2011) Metodologitcheskie voprosy rekonstruktsii sistemy tablits «zatraty–vypusk» Rossii za 2003 i posleduyshie gody v structure OKVJeD - OKPD [Constructing Retrospective Time Series of Russian Input-Output Accounts Based on the NACE/CPA Classifications] // Voprosy Statistiki. No. 12. P. 3-8. (In Russ.).
2. Baranov E. F., Kim I., Piontkovski D., Staritsyna E. A. (2014) Voprosy postroeniya tablits «zatraty–vypusk» Rossii v mezhdunarodnyh klassifilkatorah [Problems of Constructing Russian Input-Output Tables into the International Classifications] // HSE Economic Journal. Vol. 18, no. 1, pp. 7–42. (In Russ.)
3. Erumban, A. E., Gouma, R., Timmer, M., de Vries, G., de Vries, K. (2012) Sources for National Supply and Use Table Input files. World Input-Output Database (WIOD). April, 2012. http://www.wiod.org/publications/source_docs/SUT_Input_Sources.pdf
4. Eurostat Manual of Supply, Use and Input-Output Tables. Eurostat.– 2008.
5. Friedlander D. (1961) A technique for estimating a contingency table, given the marginal totals and some supplementary data. //Journal of the Royal Statistical Society, Series A (General). Vol. 124. P. 412-420.
6. Günlük-Şenesen G., Bates J. M. (1988) Some experiments with methods of adjusting unbalanced data matrices. // Journal of the Royal Statistical Society, Series A (Statistics in Society). Vol. 151. No. 3 P. 473–490
7. Huang W., Kobayashi S., Tanji H. (2008) Updating an input–output matrix with sign-preservation: some improved objective functions and their solutions.// Economic systems research. Vol. 20. No. 1. P. 111–123.
8. Kuroda, M. (1988) A Method of Estimation for the Updating Transaction Matrix in the Input-Output Relationships // Statistical Data Bank Systems. K. Uno and S. Shishido (eds.). Amsterdam: North Holland.
9. Kuznetsov, S,Yu., Piontkovski, D.I., Sokolov, D.D., Starchikova, O.S. (2016) Empiricheskoe sravnenie matematicheskikh metodov prognozirovamiya tablic "zatraty-vypusk" na osnove bazy dannykh WIOD. [An empirical comparison of the Mathematical Projection Methods of IO accounts on the database of the WIOD project.] HSE Economic Journal, 2016, vol. 20, no 4, pp. 711–730. (In Russ.)
10. Lenzen, M., Gallego, B., Wood, R. (2009) Matrix balancing under conflicting information.// Economic Systems Research. Vol. 21. No. 1. P.23–44.

11. Miller R. E., Blair P. D. (2009) Input-output analysis: foundations and extensions. Cambridge University Press.
12. Piontkovski D.I., Sokolov D.D., Starchikova O.S. (2015) Sravnenie matematicheskikh metodov prognozirovamiya tablic “zatraty-vypusk” na osnove bazy dannykh WIOD. [A Comparison of the Mathematical Projection Methods of IO accounts on the database of the WIOD project.] Working paper WP2/2015/07. Moscow: Higher School of Economics Publ. House, Series WP2 “Quantitative Analysis of Russian Economy”, 37 p. (in Russ.)
13. Stone R. A. (1961) Input-output accounts and national accounts //Organization for European Economic Cooperation, Paris.
14. Temurshoev U., Miller R. E., Bouwmeester M.C. (2013) A note on the GRAS method //Economic Systems Research. Vol. 25. No. 3. P. 361–367.
15. Temurshoev U., Timmer M. P. (2011) Joint estimation of supply and use tables //Papers in Regional Science. Vol. 90. No. 4. P. 863-882.
16. Temurshoev U., Webb C., Yamano N. (2011) Projection of supply and use tables: methods and their empirical assessment //Economic systems research. Vol. 23. No. 1. P. 91–123.
17. Wilcoxen P. J. (1989) Kuroda’s method for constructing consistent input-output data sets. Impact Research Centre, University of Melbourne.

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