The Effects of Technology and Division of Labor on Value Added Rates —An Analysis Based on Input-Output Model

(preliminary draft)

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Abstract: This paper discusses how division of labor and technology progress affects value added rates. First, an input-output model for analyzing value added rates is built, and used to investigate the relations between division of labor and value added rates theoretically. Then by using world input-output tables in current and previous year prices, the effects of international division and specialization are analyzed empirically. The main results are as follows. First, division of labor without technology progress and efficiency improvement will certainly cause decreased in value added rates. However, division of labor accompanying technology advance and efficiency increases has different effects: for the industry where division of labor originates, its value added rate does not necessarily decrease, and may increase in some occasions; for other industries, their value added rates will increase. The empirical results show that from 1996 to 2007, international divisions and specializations with efficiency improvements lead to increases in some countries' manufacture value added rates, such as US Japan, India and European Union. But for China, the manufacture value added rate actually decrease under international specialization. The main factor causing the decreases of value added rates in US and Japan is price changes, and the main factors for the decreases of value added rates in EU and Canada are non price factors except for international division of labor and price changes.

Key words: value added rate; division of labor; technology; input-output technique

1.Introduction

The international intermediate trades grow fast recently, along with the production globalization, international specialization and fragmentation. It leads to the big differences between the measurements of gross trade and the actual income or value added of a country. Then, the concept of value added embodied in trade and its measurement has been investigated extensively, and there are a good number of literatures in related issues. (For example, Koopman, et al, 2010, 2014; Johnson & Noguera, 2012; Yang et al, 2015; and Los et al, 2016). In computing the value added embodied in trade, the value added rates of a country's industries are parameters with great importance. Currently, in global economy, the value added rates in each country has decreased distinctly. Particularly, the value added rates of China are lower than other big economies, such as US and Japan, and moreover, decrease more hardly than other countries (Xia and Zhang, 2015; Yu and Chang, 2015.) So it is necessary to examine what factors affect the change of value added rates, and how to explain the trends of decreasing of value added rates in most countries.

There are some researches on this kind of issues. Xia and Zhang (2015) analyze the implications and the comparative static features of value added rates based on input-output model, and suggest that the promotion of technology and efficiency will cause the increases in value added rates, and the factors of distribution will affect value added rates as well. Yu and Chang (2015) investigate how the division of labor influence value added rates, and they conclude that division of labor will definitely make value added rates decrease. Additionally, some works explain the reasons of the decreasing of value added rate, from alternative angles. For example, Shen (2009) argues that the reasons of the low overall value added rate in China are industrial structure and industrial value added rates. Wang and Wang (2012) point out that industrial structure and international specialization result in the low value added rates of manufacture in China. Zhang (2013) analyzes the micro data at firm level, and considers exports as the main factor that restrains value added rates of Chinese firms. And so on and so forth. This paper constructs an input-output framework for analyzing value added rates theoretically. First, we focus on how division of labor and technology progress affect value added rates, and discusses whether division of labor will definitely cause decreases in value added rates or not theoretically. And then, we do some empirical analysis based on world input-output tables, to investigate the effects of international specialization and technology progress on value added rates of all countries.

2.An input-output framework for analyzing value added rates

In input-output model, the value added rate of an industry is defined as the ratio of its value added to its total output, representing the proportion of its own income gained in its production in the total contribution of the industry. Meanwhile, value added rate is also the ratio of the primary input of the industry to its total input, and it is similar to technology coefficients, related with technology.

In order to investigate the relations division of labor and technology and value added rates, we construct an input-output price model for analyzing value added rates, from the physical input-output table with two products.

Table 1 The physical input-output table with two products

		1	2	Final demand	Total output
intermediate inputs	1	q_{11}	q_{12}		q_1
	2	q_{21}	q_{22}		q_2
Value added(primary inputs)		V_1	V_2		

q_{ij} denotes the amount of product i used in the production process of product j, in physical unit. V_j is the value added or primary input in the production of j, and q_j represents the total output of j in physical unit. The value added rate of product j is defined as the ratio of the value added (primary input) of j to its total output in monetary unit. Therefore, it requires the mechanism of price formation. The divisions of labor and technology progress affect prices through influencing the production cost, and then affect value added rates. In the following, we introduce the mechanism of price formation, based on input-output price model, as

$$\begin{aligned} p_1 a_{11} + p_2 a_{21} + a_{v_1} &= p_1 \\ p_1 a_{12} + p_2 a_{22} + a_{v_2} &= p_2 \end{aligned} \quad (1)$$

where a_{ij} is the input coefficient in physical unit, indicating the amount of product i required in producing one unit of product j; a_{vj} denotes the primary input coefficient of product j, indicating the amount of primary input (in monetary unit) required in producing one unit of product j (in physical unit). We have $a_{v_j} = V_j / q_j$. Solving equation (1), we find:

$$p = A_v (I - A)^{-1}$$

where $p = (p_1 \quad p_2)$, $A_v = (a_{v1} \quad a_{v2})$. Further, the value added rate of product j is $v_j = \frac{V_j}{p_j q_j} = \frac{a_{vj}}{p_j}$. Define the denotation $\frac{(\ )}{(\ )}$ as the operation of correspondingly division of the entries of two vectors. Then, we have

$$v = (v_1 \quad v_2) = \frac{A_v}{p} = \frac{A_v}{A_v (I - A)^{-1}} \tag{2}$$

3. The theoretical analysis of the effects of division of labor on value added rates

In this section, we begin with the input-output model for value added rates noted above, to analyze the effect of division of labor. First the case without progress in technology and efficiency, and then the case with technology progress and efficiency promotion.

3.1 The effects of division of labor without technology progress on value added rates

Suppose the production of product 1 requires product 1, product 2 and product 3. However, product 3 is just one step of product 1's production process. Firstly, product 3 is made by using product 1 and 2, in the production process of product 1, and then used in producing 1 in the next step of the process. It requires a_{31} units of product 3 to manufacture one unit of product 1, while to produce a_{31} units of product 3 requires intermediate input a_{11}^3 and a_{21}^3. We have $a_{v_1} = V_1 / q_1$, meaning the primary input needed in producing one unit of product 1, except for the primary input needed by product 3; $a_{v_{31}} = V_3 / q_1$, representing the primary input in producing a_{31} units of product 3, for the requirements product 1's production. The case is shown in table 2 as follows.

Table 2 The physical input-output table without division of labor

		1	2	Final demand	Total output
intermediate input	1	$a_{11} + a_{11}^3$	a_{12}		q_1
	2	$a_{21} + a_{21}^3$	a_{22}		q_2
Value added(primary input)		$a_{v1} + a_{v31}$	a_{v2}		

Actually, in table 2, product 1 represents the kind of products that use product 3, while product 2 represents the other kind of products that need not use product 3. To be simplified, we assume that product 3 do not use product 3 in its production. Then the price model without division of labor is

$$\begin{aligned} &p_1 a_{11} + p_2 a_{21} + p_1 a_{11}^3 + p_2 a_{21}^3 + a_{v_1} + a_{v_{31}} = p_1 \\ &p_1 a_{12} + p_2 a_{22} + a_{v2} = p_2 \end{aligned} \tag{3}$$

We find

$$\begin{pmatrix} p_1 & p_2 \end{pmatrix} = \begin{pmatrix} a_{v_1} + a_{v_3} & a_{v_2} \end{pmatrix} (I - A)^{-1}$$

where $A = \begin{pmatrix} a_{11} + a_{11}^3 & a_{12} \\ a_{21} + a_{21}^3 & a_{22} \end{pmatrix}$. Then the vector of value added rates is

$$\begin{pmatrix} v_1 & v_2 \end{pmatrix} = \frac{\begin{pmatrix} a_{v_1} + a_{v_{31}} & a_{v_2} \end{pmatrix}}{\begin{pmatrix} a_{v_1} + a_{v_{31}} & a_{v2} \end{pmatrix} (I - A)^{-1}} \tag{4}$$

Let $\bar{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, then

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} a_{11}^3 & 0 \\ a_{21}^3 & 0 \end{pmatrix} = \bar{A} + a_{31} \begin{pmatrix} a_{\cdot 3} & 0 \end{pmatrix}$$

$$(I - A)^{-1} = \left(I - \bar{A} - a_{31} \cdot \begin{pmatrix} a_{13} & 0 \\ a_{23} & 0 \end{pmatrix} \right)^{-1}$$

And we have

$$= \left(I - \bar{B} a_{31} \begin{pmatrix} a_{13} & 0 \\ a_{23} & 0 \end{pmatrix} \right)^{-1} \bar{B}$$

If division of labor for specialization appears and the production of product 3 is separated from the production process of product 1, the input-output relations will be the one shown in table 3 as follows.

Table 3 the physical input-output table with division of labor

		1	2	3	final demand	Total output
intermediate input	1	a_{11}	a_{12}	a_{13}		
	2	a_{21}	a_{22}	a_{23}		
	3	a_{31}	0	0		
value added (primary input)v		a_{v1}	a_{v2}	a_{v3}/a_{31}		

Obviously, $a_{13} \cdot a_{31} = a_{11}^3$, $a_{23} \cdot a_{31} = a_{21}^3$.So the price model now is

$$\begin{aligned} p_1 a_{11} + p_2 a_{21} + p_3 a_{31} + a_{v1} &= p_1 \\ p_1 a_{12} + p_2 a_{22} + a_{v2} &= p_2 \\ p_1 a_{13} + p_2 a_{23} + a_{v3} / a_{31} &= p_3 \end{aligned} \tag{5}$$

and

$$\begin{pmatrix} p_1 & p_2 & p_3 \end{pmatrix} = \begin{pmatrix} a_{v1} & a_{v2} & a_{v3} / a_{31} \end{pmatrix} (I - A^{'})^{-1}$$

where, $A^{'} = \begin{pmatrix} \bar{A} & a_{\cdot 3} \\ a_{3\cdot} & 0 \end{pmatrix} = \begin{pmatrix} \bar{A} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & a_{\cdot 3} \\ a_{3\cdot} & 0 \end{pmatrix}$

then

$$\left(I-A'\right)^{-1}=\left(I-\begin{pmatrix}\bar{A} & a_{.3}\\ a_{3.} & 0\end{pmatrix}\right)^{-1}$$

$$\begin{pmatrix}I-\bar{A} & -a_{.3}\\ -a_{3.} & I\end{pmatrix}\begin{pmatrix}B_{11}' & B_{12}'\\ B_{21}' & B_{22}'\end{pmatrix}=\begin{pmatrix}I & 0\\ 0 & I\end{pmatrix}$$

$$\left(I-\bar{A}\right)B_{11}'-a_{.3}B_{21}'=I$$

$$\left(I-\bar{A}\right)B_{12}'-a_{.3}B_{22}'=0$$

$$-a_{3.}B_{11}'+B_{21}'=0$$

$$-a_{3.}B_{12}'+B_{22}'=I$$

To solve it, we find
$$B_{11}'=\left(I-\bar{A}-a_{.3}a_{3.}\right)^{-1}$$
$$B_{21}'=a_{3.}\left(I-\bar{A}-a_{.3}a_{3.}\right)^{-1}$$

$$B_{11}'=\left(I-\bar{B}a_{.3}a_{3.}\right)^{-1}\bar{B}$$
$$B_{21}'=a_{3.}\left(I-\bar{B}a_{.3}a_{3.}\right)^{-1}\bar{B}$$

Because $a_{.3}a_{3.}=\begin{pmatrix}a_{13}\\ a_{23}\end{pmatrix}\cdot\begin{pmatrix}a_{31} & 0\end{pmatrix}=\begin{pmatrix}a_{13}a_{31} & 0\\ a_{23}a_{31} & 0\end{pmatrix}=a_{31}\begin{pmatrix}a_{13} & 0\\ a_{23} & 0\end{pmatrix}$, $\left(I-A\right)^{-1}=B_{11}'$

The vector of prices of product 1 and 2 before division of labor is known as

$$\begin{pmatrix}p_1 & p_2\end{pmatrix}=\begin{pmatrix}a_{v1}+a_{v3} & a_{v2}\end{pmatrix}\left(I-\bar{B}a_{31}\begin{pmatrix}a_{13} & 0\\ a_{23} & 0\end{pmatrix}\right)^{-1}\bar{B}$$

while after division of labor, it will be

$$\begin{pmatrix}\begin{pmatrix}a_{v1} & a_{v2}\end{pmatrix} & a_{v3}/a_{31}\end{pmatrix}\begin{pmatrix}B_{11}'\\ B_{21}'\end{pmatrix}=\begin{pmatrix}\begin{pmatrix}a_{v1} & a_{v2}\end{pmatrix} & a_{v3}/a_{31}\end{pmatrix}\begin{pmatrix}\left(I-\bar{B}a_{.3}a_{3.}\right)^{-1}\bar{B}\\ a_{3.}\left(I-\bar{B}a_{.3}a_{3.}\right)^{-1}\bar{B}\end{pmatrix}$$

$$=\begin{pmatrix}a_{v1} & a_{v2}\end{pmatrix}\left(I-\bar{B}a_{.3}a_{3.}\right)^{-1}\bar{B}+(a_{v3}/a_{31})\begin{pmatrix}a_{31} & 0\end{pmatrix}\left(I-\bar{B}a_{.3}a_{3.}\right)^{-1}\bar{B}$$
$$=\begin{pmatrix}a_{v1}+a_{v3} & a_{v2}\end{pmatrix}\left(I-\bar{B}a_{.3}a_{3.}\right)^{-1}\bar{B}$$
$$=\begin{pmatrix}a_{v1}+a_{v3} & a_{v2}\end{pmatrix}\left(I-A\right)^{-1}$$

Namely, there is no change in the prices of product 1 and 2, with the division of labor, when there is no technology progress.

With division of labor, the vector of value added rates of product 1 and 2 will be

$$\begin{pmatrix} v_1^{'} & v_2^{'} \end{pmatrix} = \frac{\begin{pmatrix} a_{v1} & a_{v2} \end{pmatrix}}{\begin{pmatrix} a_{v1}+a_{v3} & a_{v2} \end{pmatrix}\left(I-A\right)^{-1}} \tag{6}$$

Clearly, the value added rate of product 1 decreases, while that of product 2 has no change. That is, the division of labor without technology progress and efficiency promotion will definitely causes the decreases in industrial value added rates.

Product 3's production is contained in the production process of product 1. Therefore, the value added rate of product 3 should be the ratio of the value added occurred for product 3 in the process of producing product 1 to the output of product 3 in monetary unit, which is

$$v_3 = \frac{a_{v3}}{p_1 a_{13} a_{31} + p_2 a_{23} a_{31} + a_{v3}}$$

After division of labor, the production of product 3 is separated from the production of product 1. Hence its value added rate will be

$$v_3^{'} = \frac{a_{v3}^{'}}{p_3^{'}} = \frac{a_{v3}/a_{31}}{p_1^{'} a_{13} + p_2^{'} a_{23} + a_{v3}/a_{31}} = \frac{a_{v3}}{p_1^{'} a_{13} a_{31} + p_2^{'} a_{23} a_{31} + a_{v3}} \tag{7}$$

Because the prices of product 1 and 2 have no change compared with that before division of labor, that is, $p_1^{'} = p_1, p_2^{'} = p_2$, we get $v_3 = v_3^{'}$, and the value added rate of product 3 has no change as well. If the value added rate of product 3 is pretty low, then if the production of the product is moved to another region, it will decrease the overall value added rate of that region. However, if there is no improvement in technology, the value added rate of product 1 itself will also decrease.

Here we can come to the first conclusion: without technology progress, a sector or a county/region cannot increase its value added rate by moving out the section with low value added rate in its production process.

However, generally , there will be technology progress and efficiency promotion along with the division of labor. How will the value added be? We discuss it in the following part.

3.2 The effect of division of labor with technology progress on the value added rates

Suppose other factors such as income distribution keep invariant, while the efficiency of the newly separated product 3 increases. It should be noted that we can also suppose that the efficiency of initial product (product 1) increases or both of the efficiencies of the two products increase. Because the methodology and procedure of the analysis are similarly, and the results are the same, we just omit these two cases.

In the following part, we discuss the effects of technology progress on value added rates from two types of efficiency improvement.

The first type of efficiency improvement: more outputs are obtained through the same amount of inputs, that is, the input coefficients of product 3 reduce in the same proportion, which is

$$\begin{pmatrix} a_{13}^{'} \\ a_{23}^{'} \end{pmatrix} = s \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix}, a_{v3}^{'} = s a_{v3} / a_{31}$$

where s is the parameter showing how the efficiency improves, and 0<s<1.

Table 4 The physical input-output table with division of labor with technology progress (1)

	1	2	3	Final demand	Total output

Intermediate input	1	a_{11}	a_{12}	sa_{13}		
	2	a_{21}	a_{22}	sa_{23}		
	3	a_{31}	0	0		
Primary input		a_{v1}	a_{v2}	sa_{v3}/a_{31}		

Then the price model is

$$\begin{aligned} & p_1 a_{11} + p_2 a_{21} + p_3 a_{31} + a_{v1} = p_1 \\ & p_1 a_{12} + p_2 a_{22} + a_{v2} = p_2 \\ & sp_1 a_{13} + sp_2 a_{23} + sa_{v3} / a_{31} = p_3 \end{aligned} \quad (8)$$

We find

$$\left(I - A^{'}\right)^{-1} = \left(I - \begin{pmatrix} \bar{A} & sa_{\cdot 3} \\ a_{3\cdot} & 0 \end{pmatrix}\right)^{-1}$$

$$B_{11}^{'} = \left(I - s\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1} \bar{B}$$

$$B_{21}^{'} = a_{3\cdot}\left(I - s\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1} \bar{B}$$

The vector of prices of product 1 and 2 is

$$\begin{aligned} \left(p_1^{'} \quad p_2^{'}\right) &= \left((a_{v1} \quad a_{v2}) \quad sa_{v3} / a_{31}\right) \begin{pmatrix} \left(I - s\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1} \bar{B} \\ a_{3\cdot}\left(I - s\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1} \bar{B} \end{pmatrix} \\ &= \left(a_{v1} + sa_{v3} \quad a_{v2}\right)\left(I - s\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1} \bar{B} \end{aligned}$$

The vector of value added rates of product 1 and 2 will be

$$\left(v_1^{'} \quad v_2^{'}\right) = \frac{\left(a_{v1} \quad a_{v2}\right)}{\left(a_{v1} + sa_{v3} \quad a_{v2}\right)\left(I - s\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1} \bar{B}} \quad (9)$$

Meanwhile, the vector of value added rates before division of labor is

$$\left(v_1 \quad v_2\right) = \frac{\left(a_{v1} + a_{v3} \quad a_{v2}\right)}{\left(a_{v1} + a_{v3} \quad a_{v2}\right)\left(I - \bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1} \bar{B}}$$

So, how are the value added rates affected by the efficiency improvement (represented by s)? What are the relations of the value added rates before and after the change? We analyze it in the following.

$$\left(v_1^{'} \quad v_2^{'}\right) = \frac{\left(a_{v1} \quad a_{v2}\right)}{\left(a_{v1} + sa_{v3} \quad a_{v2}\right)\left(I - s\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1} \bar{B}} = \frac{\left(a_{v1} \quad a_{v2}\right)}{\sum_{i=0}^{\infty} s^i \left(a_{v1} + sa_{v3} \quad a_{v2}\right)\left(\bar{B}a_{\cdot 3}a_{3\cdot}\right)^i \bar{B}}$$

Let

$$\left(\bar{B}a_{\cdot 3}a_{3\cdot}\right)^i \bar{B} = T^i = \begin{pmatrix} T_{11}^i & T_{12}^i \\ T_{21}^i & T_{22}^i \end{pmatrix}$$

And we have

$$\left(a_{v1}+sa_{v3} \quad a_{v2}\right)\left(\bar{B}a_{\cdot 3}a_{3\cdot}\right)^i \bar{B} = \left(a_{v1}+sa_{v3} \quad a_{v2}\right)\begin{pmatrix} T_{11}^i & T_{12}^i \\ T_{21}^i & T_{22}^i \end{pmatrix} = \begin{pmatrix} (a_{v1}+sa_{v3})T_{11}^i + a_{v2}T_{21}^i \\ (a_{v1}+sa_{v3})T_{12}^i + a_{v2}T_{22}^i \end{pmatrix}^T$$

$$\left(v_1^{'} \quad v_2^{'}\right) = \frac{\left(a_{v1} \quad a_{v2}\right)}{\sum_{i=0}^{\infty} s^i \left(\left(a_{v1}+sa_{v3}\right)T_{11}^i + a_{v2}T_{21}^i, \quad \left(a_{v1}+sa_{v3}\right)T_{12}^i + a_{v2}T_{22}^i\right)}$$

$$= \left(\frac{a_{v1}}{\sum_{i=0}^{\infty} s^i \left[\left(a_{v1}+sa_{v3}\right)T_{11}^i + a_{v2}T_{21}^i\right]} \quad \frac{a_{v2}}{\sum_{i=0}^{\infty} s^i \left[\left(a_{v1}+sa_{v3}\right)T_{12}^i + a_{v2}T_{22}^i\right]} \right)$$

then

$$\frac{dv_1^{'}}{ds} = -\frac{a_{v1}}{\left(\sum_{i=0}^{\infty} s^i \left[\left(a_{v1}+sa_{v3}\right)T_{11}^i + a_{v2}T_{21}^i\right]\right)^2}\left(a_{v3}T_{11}^0 + \sum_{i=1}^{\infty} is^{i-1}\left[\left(a_{v1}+sa_{v3}\right)T_{11}^i + a_{v2}T_{21}^i\right] + \sum_{i=1}^{\infty} s^i a_{v3}T_{11}^i\right) < 0$$

$$\frac{dv_2^{'}}{ds} = -\frac{a_{v2}}{\left(\sum_{i=0}^{\infty} s^i \left[\left(a_{v1}+sa_{v3}\right)T_{12}^i + a_{v2}T_{22}^i\right]\right)^2}\left(a_{v3}T_{12}^0 + \sum_{i=1}^{\infty} is^{i-1}\left[\left(a_{v1}+sa_{v3}\right)T_{12}^i + a_{v2}T_{22}^i\right] + \sum_{i=1}^{\infty} s^i a_{v3}T_{12}^i\right) < 0$$

Therefore, the value added rates of product 1 and 2 are all monotone decreasing function of s. Along with the decreasing in s (the efficiency increasing in production of 3), their value added rates are increasing. We also find

$$\lim_{s\to 0} \frac{\left(a_{v1} \quad a_{v2}\right)}{\sum_{i=0}^{\infty} s^i \left(a_{v1}+sa_{v3} \quad a_{v2}\right)\left(\bar{B}a_{\cdot 3}a_{3\cdot}\right)^i \bar{B}} = \frac{\left(a_{v1} \quad a_{v2}\right)}{\left(a_{v1} \quad a_{v2}\right)\bar{B}}$$

$$\lim_{s\to 1} \frac{\left(a_{v1} \quad a_{v2}\right)}{\sum_{i=0}^{\infty} s^i \left(a_{v1}+sa_{v3} \quad v_2\right)\left(\bar{B}a_{\cdot 3}a_{3\cdot}\right)^i \bar{B}} = \frac{\left(a_{v1} \quad a_{v2}\right)}{\left(a_{v1}+a_{v3} \quad a_{v2}\right)\left(I-\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1}\bar{B}}$$

$$\le \frac{\left(a_{v1}+a_{v3} \quad a_{v2}\right)}{\left(a_{v1}+a_{v3} \quad a_{v2}\right)\left(I-\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1}\bar{B}}$$

That is

$$\lim_{s\to 0} v_1^{'} = \frac{a_{v1}}{a_{v1}\bar{B}_{11} + a_{v2}\bar{B}_{21}}, \quad \lim_{s\to 0} v_2^{'} = \frac{a_{v2}}{a_{v1}\bar{B}_{12} + a_{v2}\bar{B}_{22}}$$

Because $\left(I-\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1}\bar{B} > \bar{B}$, $a_{v1}+a_{v3} > a_{v1}$, we can certainly get $\lim_{s\to 0} v_2^{'} > v_2$, and $\lim_{s\to 1} v_2^{'} = v_2$ .Therefore, it is always true that $v_2^{'} > v_2$.

For product 1, if $\lim_{s\to 0} v_1^{'} = \dfrac{a_{v1}}{a_{v1}\bar{B}_{11} + a_{v2}\bar{B}_{21}} > \dfrac{a_{v1} + a_{v3}}{(a_{v1} + a_{v3})M_{11} + a_{v2}M_{21}} = v_1$, according to mid - value theorem, there inevitable exists s^* that makes the value added rate after the change equal to that before the change. When $s<s^*, v_1^{'} > v_1$, and the value added rate of product 1 after the change is increased compared with that before the change. If $\lim_{s\to 0} v_1^{'} = \dfrac{a_{v1}}{a_{v1}\bar{B}_{11} + a_{v2}\bar{B}_{21}} < \dfrac{a_{v1} + a_{v3}}{(a_{v1} + a_{v3})M_{11} + a_{v2}M_{21}} = v_1$, we have $v_1^{'} < v_1$. Namely, for product 1 in which the division of labor occurs, if there is technology progress, it is not sure that how its value added rate change.

To be concluded, for product 1 and 2, if there is no efficiency improvement, division of labor will definitely causes the decreases in value added rates. However, with technology progress, division of labor may not lead to the decreases. On the contrary, the value added rates may increase, if the efficiency is sufficiently high.

For product 3, the value added rate after division of labor is

$$v_3^{'} = \frac{a_{v3}^{'}}{p_3^{'}} = \frac{sa_{v3} / a_{31}}{sp_1^{'}a_{13} + sp_2^{'}a_{23} + sa_{v3} / a_{31}} = \frac{a_{v3}}{p_1^{'}a_{13}a_{31} + p_2^{'}a_{23}a_{31} + a_{v3}} \tag{10}$$

For

$$\begin{aligned}\left(p_1^{'} \quad p_2^{'}\right) &= \left(a_{v1} + sa_{v3} \quad a_{v2}\right)\left(I - s\bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1}\bar{B} \\ &< \left(a_{v1} + a_{v3} \quad a_{v2}\right)\left(I - \bar{B}a_{\cdot 3}a_{3\cdot}\right)^{-1}\bar{B} = \left(p_1 \quad p_2\right)\end{aligned}$$

we have $v_3 < v_3^{'}$, and the value added rate of product 3 is higher than that before division of labor.

The second type of technology progress: some input is decreased for the same amount of output, shown by the decrease in the corresponding input coefficient.

Table 5 The physical input-output table with division of labor with technology progress (2)

		1	2	3	Final demand	Total output
Intermediate input	1	a_{11}	a_{12}	a_{13}		
	2	a_{21}	a_{22}	sa_{23}		
	3	a_{31}	0	0		
value added (primary input)		a_{v1}	a_{v2}	a_{v3}/a_{31}		

$$\begin{aligned} &p_1a_{11} + p_2a_{21} + p_3a_{31} + a_{v1} = p_1 \\ &p_1a_{12} + p_2a_{22} + a_{v2} = p_2 \\ &p_1a_{13} + sp_2a_{23} + a_{v3} / a_{31} = p_3 \end{aligned} \tag{11}$$

Here the prices of product 1 and 2 are

$$\begin{pmatrix} p_1^{'} & p_2^{'} \end{pmatrix} = \left(\begin{pmatrix} a_{v1} & a_{v2} \end{pmatrix} \quad a_{v3}/a_{31} \right) \begin{pmatrix} \left(I - \bar{B}a_{.3}^{'}a_{3.}\right)^{-1} \bar{B} \\ a_{3.}\left(I - \bar{B}a_{.3}^{'}a_{3.}\right)^{-1} \bar{B} \end{pmatrix}$$

$$= \begin{pmatrix} a_{v1} + a_{v3} & a_{v2} \end{pmatrix} \left(I - \bar{B}a_{.3}^{'}a_{3.}\right)^{-1} \bar{B}$$

And

$$\bar{B}a_{.3}^{'}a_{3.} = \begin{pmatrix} \bar{B}_{11} & \bar{B}_{12} \\ \bar{B}_{21} & \bar{B}_{22} \end{pmatrix} \begin{pmatrix} a_{13}a_{31} & 0 \\ sa_{23}a_{31} & 0 \end{pmatrix} = \begin{pmatrix} \bar{B}_{11}a_{13}a_{31} & 0 \\ \bar{B}_{21}a_{13}a_{31} & 0 \end{pmatrix} + s \begin{pmatrix} \bar{B}_{12}a_{23}a_{31} & 0 \\ \bar{B}_{22}a_{23}a_{31} & 0 \end{pmatrix} = D_1 + sD_2$$

Then the value added rates of product 1 and 2 after division of labor are

$$\begin{pmatrix} v_1^{'} & v_2^{'} \end{pmatrix} = \frac{\begin{pmatrix} a_{v1} & a_{v2} \end{pmatrix}}{\begin{pmatrix} a_{v1} + a_{v3} & v_2 \end{pmatrix} \left(I - \bar{B}a_{.3}^{'}a_{3.}\right)^{-1} \bar{B}} = \frac{\begin{pmatrix} a_{v1} & a_{v2} \end{pmatrix}}{\begin{pmatrix} a_{v1} + a_{v3} & v_2 \end{pmatrix} \left(I - D_1 - sD_2\right)^{-1} \bar{B}} \qquad (12)$$

$$= \frac{\begin{pmatrix} a_{v1} & a_{v2} \end{pmatrix}}{\begin{pmatrix} a_{v1} + a_{v3} & v_2 \end{pmatrix} \left(I - sM_1D_2\right)^{-1} M_1\bar{B}}$$

$$M_1 = (I - D_1)^{-1}$$

where

Meanwhile, the value added rates before division of labor are

$$\begin{pmatrix} v_1 & v_2 \end{pmatrix} = \frac{\begin{pmatrix} a_{v1} + a_{v3} & a_{v2} \end{pmatrix}}{\begin{pmatrix} a_{v1} + a_{v3} & a_{v2} \end{pmatrix} \left(I - \bar{B}a_{.3}a_{3.}\right)^{-1} \bar{B}} = \frac{\begin{pmatrix} a_{v1} & a_{v2} \end{pmatrix}}{\begin{pmatrix} a_{v1} + a_{v3} & v_2 \end{pmatrix} \left(I - D_1 - D_2\right)^{-1} \bar{B}}$$

$$= \frac{\begin{pmatrix} a_{v1} + a_{v3} & a_{v2} \end{pmatrix}}{\begin{pmatrix} a_{v1} + a_{v3} & a_{v2} \end{pmatrix} \left(I - M_1D_2\right)^{-1} M_1\bar{B}}$$

Obviously, the value added rates are monotone decreasing function of s.We find that

$$\lim_{s \to 1} \begin{pmatrix} v_1^{'} & v_2^{'} \end{pmatrix} = \frac{\begin{pmatrix} a_{v1} & a_{v2} \end{pmatrix}}{\begin{pmatrix} a_{v1} + a_{v3} & a_{v2} \end{pmatrix} \left(I - M_1D_2\right)^{-1} M_1\bar{B}}$$

When s is close to 1, the limitation of the value added rate of product 1 is smaller than that before the division of labor, and the limitation of the value added rate of product 2 equal that before division of labor.

$$\lim_{s \to 0} \begin{pmatrix} v_1^{'} & v_2^{'} \end{pmatrix} = \frac{\begin{pmatrix} a_{v1} & a_{v2} \end{pmatrix}}{\begin{pmatrix} a_{v1} + a_{v3} & a_{v2} \end{pmatrix} M_1\bar{B}}$$

When s is close to 0, the limitation of the value added rate of product 2 is bigger than that before the division of labor, and the limitation of the value added rate of product 1 may be smaller or bigger than that before the division of labor.

For product 1, if

$$\lim_{s \to 0} v_1^{'} > v_1$$

According to the mid-value theorem, there exists s^*, which makes the value added rate of product 1 bigger than that before division of labor, when $s<s^*$. If

$$\lim_{s\to 0} v_1^{'} < v_1$$

In this case the value added rate of product 1 is smaller than that before the division of labor. Therefore, in general, it is not certain that the value added rate of product 1 will decrease, and it may increase.

For product 3, we find

$$v_3^{'} = \frac{a_{v3}^{'}}{p_3^{'}} = \frac{a_{v3}/a_{31}}{p_1^{'}a_{13} + sp_2^{'}a_{23} + a_{v3}/a_{31}} = \frac{a_{v3}}{p_1^{'}a_{13}a_{31} + sp_2^{'}a_{23}a_{31} + a_{v3}} \quad (13)$$

For

$$\begin{pmatrix} p_1^{'} & p_2^{'} \end{pmatrix} = \begin{pmatrix} a_{v1} + a_{v3} & v_2 \end{pmatrix} \left(I - sM_1D_2\right)^{-1} M_1\bar{B}$$

$$< \begin{pmatrix} a_{v1} + a_{v3} & v_2 \end{pmatrix} \left(I - M_1D_2\right)^{-1} M_1\bar{B} = \begin{pmatrix} p_1 & p_2 \end{pmatrix}$$

Inevitably, $v_3 < v_3^{'}$. The value added rate of product 3 is bigger than that before the division of labor.

We discussed the effects of division of labor with technology progress on value added rates in the above. The results show that: for the sectors in which the division of labor initially occurs (like product 1 in our proof), their value added rates are not necessarily decreased, and division of labor with efficiency improvement may lead to the increases in their value added rates; the value added rates of other sectors will increase. Therefore, if a country/region move its sectors with low value added rates out to other country or region, with the efficiency improvement, the value added rates of other country/region may not decrease.

4. The empirical analysis: the effects of technology and international specialization on the value added rates of each country

When we analyze the effects of technology and international specialization on value added rates in real economy, we are faced with two kinds of problems. First is the influence of price level. We need to isolate the part affected by price change from the changes of value added rates. For this problem, we can deal with it by using the input-output tables in constant price. The second problem is that, in current input-output data, it is very hard for us to split division of labor and technology change completely. Therefore, what we analyze here is the aggregate effect of technology change and international specialization.

In the following, we use the world input-output tables in current prices and in previous year prices in WIOD database (Timmer et al, Dietzenbacher et al,) to analyze how the international specialization with technology change influence the value added rates of each country. Firstly, we deduce the price indices based on world input-output tables in current prices and in previous year prices, and then based on the price indices, we transform the WIOTs in 2007 in current prices into 2007 WIOTs in constant prices in 1995 . Then the price changes during 1996 to 2007 are eliminated. It should be noted that because it is hard to distinguish the division of labor and technology change, we have to compute the effect of division of labor with and without technology change simultaneously. So, accurately speaking, the effect of international specialization that we analyzed is the aggregated effects of division of labor and technology progress.

The deepening of international specialization is shown in the rising of international trade, and the intermediate inputs imported from others countries are increased. From 1996 to 2007, in almost all countries, the intermediate import coefficients in constant prices keep going up. For example, in aggregate level, the intermediate import coefficient in China increases 0.0797, while in US, the number is 0.0233, and in Japan, it is 0.0134, and for German, it is 0.0621. Therefore, we use the changes of intermediate input coefficients to simulate the effect of international specialization. The main results are shown in table 6.

Table 6 The factors affecting the value added rates in some countries and regions

	Total change	The effect of technology and division of labor	The effect of other non price factor	The effect of price
ASTRALIA	-0.0273	0.0138	-0.0073	-0.0337
CANADA	-0.0321	0.0075	-0.0289	-0.0107
CHINA	-0.0584	-0.0665	-0.0281	0.0363
GERMAN	-0.0472	0.0059	-0.0343	-0.0188
UK	-0.0020	0.0277	0.0085	-0.0382
INDIA	-0.0202	0.0073	-0.0197	-0.0078
JAPAN	-0.0535	0.0241	0.0127	-0.0903
KOREA	-0.0405	-0.0001	-0.0123	-0.0281
RUSSIA	-0.0338	0.0442	-0.0738	-0.0042
TAIWAN	-0.0832	-0.0832	-0.0596	0.0596
US	-0.0085	0.0317	0.0427	-0.0829
EU27	-0.0307	0.0040	-0.0181	-0.0166

From table 6, we can see that from 1996 to 2007, although the value added rates in main economies in the world are all decreased, the main factors that cause their decreases are different. The technology and international specialization have different effects on the countries or regions. For most developed countries, the technology progress and division of labor bring increases in their value added rates. Although the value added rate of manufacture in Japan decreases 0.0535, international specialization with technology progress makes it increase 0.0241, and the main factor that causes the decrease is price change from 1996 to 2007. The situation is similar for US. Technology and international specialization cause the value added rate of US increase 0.0317, while the price changes lead to decrease in it. For Korea, international specialization brings decrease in its value added rate, but in a small degree. For India, as a developing country, technology and international specialization bring a little bit increase in its value added rate.

The situation for China is totally different with US and Japan. It shows that international specialization and technology change lead to a big decrease in the value added rate in China, and it is the main factor for the value added rate decrease in China.

5. Conclusion

Does division of labor definitely cause the decreases in value added rates? This paper investigates this issue. First, we construct an input-output model for value added rate analysis, and based on this model, we analyze how division of labor affects value added rates theoretically. The main results are: first, division of labor without technology progress will definitely bring the decrease in value added rates; however, the division of labor with technology progress may cause the increase in value added rates. Then we empirically analyze the reasons of the decreases in many countries/regions' value added rates, by using WIOTs in current and constant prices.

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