Multipliers and supermultipliers in a multisectoral framework: macroeconomic tools after all?

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Abstract

The paper aims to show that the common mistake of considering Keynesian income and/or employment multipliers as sector-specific, in a multisectoral framework, is taking its way back to economics. The phenomenon coincides with the recent debate about the macroeconomic Sraffian supermultiplier and generates some new miscommunication by using similar terminology to sectoral multipliers. A revival of the tradition of the multiplier as a matrix can be encountered in Mariolis (2018) despite the lack of references about the macroeconomic Sraffian supermultiplier. On the other hand, Dejuán (2014) presents the macroeconomic Sraffian supermultiplier as a set of vertically hyper-integrated sectors without making a truly macroeconomic connection between sectoral multipliers and that ones for the economy as a whole. In order to throw some light on this issue, the present paper emphasizes the differences between the traditional input-output multipliers and its Keynesian counterparts, showing also the required adaptations for supermultiplier representations. Keynesian multipliers and Sraffian supermultipliers emerge as typical macroeconomic concepts, in spite of the fact that the knowledge on production structure and consumption (and investment) patterns is mandatory to capture them in a multisectoral framework. Some estimates for multipliers and supermultipliers are also presented using data from the World Input-Output Database (WIOD).

1 Introduction

In Mariolis (2018) we can find a recent revival of the idea that a multiplier can be best represented as a matrix. It has been a long way since Goodwin's attempt to relax the Keynesian system's 'cruder aggregative aspects without too hopelessly complicating matters' (Goodwin, 1949, p. 537). Mariolis' inspiration, however, clearly comes more from a Classical-Sraffian standpoint, via Kurz (1985), than from the first attempts of Goodwin (1949) or Chipman (1950).

From a Sraffian macro-oriented perspective, though, another research strand has been paving its way towards the sedimentation of the idea of a Sraffian supermultiplier (Serrano, 1995; Freitas and Serrano, 2015). These two branches of research are not put together in Mariolis (2018), but a multisectoral representation of output and income Sraffian supermultipliers has appeared in Dejuán (2014) as a set of vertically hyper-integrated sectors (Pasinetti, 1988). Building on macroeconomic Sraffian supermultiplier references, Portella-Carbó (2016) presents input-output matrices for a multiregional gross output supermultiplier, and from there, for a multiregional employment supermultiplier. A truly combined multisectoral-macroeconomic Sraffian supermultiplier has not been presented yet.

Following Metcalfe and Steedman (1981), Mariolis (2008) talks about a 'super-multiplier' matrix or vector. In its turn, the reference in Metcalfe and Steedman (1981) is probably due to Hicks (1950), since it was made in the context of an investment growing endogenously. In Mariolis and Soklis (2018) and in Ntemiroglou (2016) we can find the terms 'Sraffian multiplier' or, sometimes, 'static Sraffian multiplier', denoting matrices linking an autonomous demand vector to final demand (net output). Empirical results in both papers for net output, import and employment multipliers show tables containing something very close to the traditional input-output total multipliers in the first two cases, and misconceived estimates in the employment case. They are neither *super*, in the sense of an endogenous investment, nor proper *multipliers*, in the original spirit of Kahn (1931). They are recognizably making use of vertically integrated coefficients (Pasinetti, 1973), not even of (a broad interpretation of) vertically hyper-integrated ones (Pasinetti, 1988) in order to endogenize investment, as in the supermultipliers suggested by Dejuán (2014).

This is not the first time in economics, however, that similar concepts are used meaning entirely different things. Even in this specific field and context. As in this Sraffian supermultipliers case, Keynesian multipliers have a long history of miscommunication. After setting the basic model in Section 2, a reassessment of Keynesian multipliers is presented in Section 3. Then, in Section 4 we will turn our focus to the Sraffian supermultipliers, specifying them in a multisectoral context. Section 5 discusses some related literature in the language of the multipliers and supermultipliers previously introduced, Section 6 presents some estimates for both multipliers and supermultipliers using data from the World Input-Output Database (WIOD), and Section 7 concludes the paper.

2 The basic model

As in a typical Leontief quantity side of an input-output model, let us take $\mathbf{x}$ as the gross output vector, $\mathbf{Z}$ as the intermediate inputs matrix, $\mathbf{f}$ as the final demand vector and $\mathbf{i}$ as a unit (sum) vector. Then, in common notation, as in Miller and Blair (2009, p. 12), the total output can be written as the sum of intermediate inputs and final demand:

$$\mathbf{x} = \mathbf{Zi} + \mathbf{f} \tag{1}$$

Representing the technical coefficients matrix as

$$\mathbf{A} = \mathbf{Z}\widehat{\mathbf{x}}^{-1} \tag{2}$$

equation (1) can be rewritten as

$$\mathbf{x} = \mathbf{Ax} + \mathbf{f} \tag{3}$$

provided that $\mathbf{Ax} = \mathbf{Zi}$. Then, the Leontief inverse, $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$, can be found:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{Lf} \tag{4}$$

Let us also define a vector of direct labour coefficients, $\mathbf{l}$, in a way that the total volume of employment can be written as

$$\mathbf{l}'(\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{l}'\mathbf{x} \tag{5}$$

and a value added vector as the difference, *per* output unit, between gross output and costs with intermediate inputs:

$$\mathbf{v} = \widehat{\mathbf{x}}^{-1}[\mathbf{x} - (\mathbf{i}'\mathbf{A}\widehat{\mathbf{x}})'] \tag{6}$$

Defining the value added vector in that way guarantees that $\mathbf{v}'(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{i}'$, so, that aggregate income and aggregate final demand equals each other:

$$\mathbf{v}'\mathbf{x} = \mathbf{v}'(\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{i}'\mathbf{f} \tag{7}$$

3 Keynesian multipliers

Since the first page of the seminal paper written by Kahn (1931), the case for considering both direct and indirect employment required by an increased autonomous expenditure was made clear. The employment so obtained was called 'primary employment', as opposed to the 'secondary employment' — the employment required for the boosted production of consumption-goods[1], also considering direct and indirect repercussions. It is the ratio of total employment (created by both autonomous and induced expenditure) to primary employment that should be called employment multiplier in Keynesian macroeconomics. If a consumption vector, $\mathbf{f_c}$, is disentangled from final demand vector, the Keynesian employment multiplier can be represented as:

$$\varphi_l = \frac{\mathbf{l}'(\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}}{\mathbf{l}'(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{f} - \mathbf{f_c})} \tag{8}$$

In the input-output analysis, however, a vector of the so called employment multipliers can be obtained simply by $\mathbf{l}'(\mathbf{I} - \mathbf{A})^{-1}$. It should be noted that what this vector can deliver is the connection between units of final demand *per* economic activity and required employment in the economy as a whole, not something close to the ratio of total employment to primary employment. Induced consumption is not even mentioned in this context.

A second approach to obtaining the Keynesian employment multiplier could consist in dividing each component of the consumption vector by the total employment,

$$\mathbf{c} = \frac{\mathbf{f_c}}{\mathbf{l}'\mathbf{x}} \tag{9}$$

in a way that the system could be rewritten as[2]:

$$\mathbf{Ax} + (\mathbf{f} - \mathbf{f_c}) + \mathbf{f_c} = \mathbf{x} \tag{10}$$

$$\mathbf{Ax} + (\mathbf{f} - \mathbf{f_c}) + \mathbf{cl}'\mathbf{x} = \mathbf{x} \tag{11}$$

$$\mathbf{x} = [\mathbf{I} - (\mathbf{A} + \mathbf{cl}')]^{-1}(\mathbf{f} - \mathbf{f_c}) \tag{12}$$

and then the Keynesian employment multiplier as:

$$\varphi_l = \frac{\mathbf{l}'[\mathbf{I} - (\mathbf{A} + \mathbf{cl}')]^{-1}(\mathbf{f} - \mathbf{f_c})}{\mathbf{l}'(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{f} - \mathbf{f_c})} \tag{13}$$

[1]Boosted by the 'increased expenditure of wages and profits that is associated with the primary employment' (Kahn, 1931, p. 173).

[2]A similar procedure can be found in Trigg and Lee (2005) for employment and ten Raa (2005, p. 28) for income.

Now the expression $\mathbf{l}'[\mathbf{I}-(\mathbf{A}+\mathbf{cl}')]^{-1}$, in the numerator, stands for input-output total employment multipliers, considering direct, indirect and induced effects, as opposed to the direct and indirect effects captured by the simple employment multiplier in the denominator. An element-wise division between these two multipliers brings us back to the Keynesian multiplier, meaning that no matter which economic activity is chosen for the autonomous demand impulse, the multiplier effect is the same.

$$\mathbf{l}'[\mathbf{I}-(\mathbf{A}+\mathbf{cl}')]^{-1}[\widehat{\mathbf{l}'(\mathbf{I}-\mathbf{A})^{-1}}]^{-1} = \begin{bmatrix} \varphi_l & \varphi_l & \cdots & \varphi_l \end{bmatrix} \tag{14}$$

Summing up, provided that consumption basket proportions are preserved, the Keynesian employment multiplier can be restated as a macroeconomic concept[3].

Similar results can be shown for income multipliers. From the same notion of a ratio of total income to primary income, we get:

$$\varphi_v = \frac{\mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{f}}{\mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}(\mathbf{f}-\mathbf{f_c})} \tag{15}$$

By taking the proportions of income spent on consumption in each economic activity, *i.e.*, by computing disaggregated marginal (and average) propensities to consume, the same system written for employment can be expressed for income:

$$\mathbf{a} = \frac{\mathbf{f_c}}{\mathbf{v}'\mathbf{x}} \tag{16}$$

$$\mathbf{x} = [\mathbf{I}-(\mathbf{A}+\mathbf{av}')]^{-1}(\mathbf{f}-\mathbf{f_c}) \tag{17}$$

and so for the Keynesian income multiplier:

$$\varphi_v = \frac{\mathbf{v}'[\mathbf{I}-(\mathbf{A}+\mathbf{av}')]^{-1}(\mathbf{f}-\mathbf{f_c})}{\mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}(\mathbf{f}-\mathbf{f_c})} \tag{18}$$

[3]Interestingly enough, these results concerning division between total income multipliers and simple income multipliers first appear in economics as an empirical curiosity (Moore and Petersen, 1955; Hirsch, 1959; Sandoval, 1967). From there, Sandoval (1967), Bradley and Gander (1969) and ten Raa and Chakraborty (1983), for instance, have started to seek for an explanation through different mathematical approaches. Notwithstanding, none of these studies mention the Keynesian multiplier, including in this group the last edition of Miller and Blair (2009), which also acknowledges the fact that the ratio of total to simple 'income multipliers can be shown to be a constant across all sectors' (Miller and Blair, 2009, p. 254) — in that case, provided that the 'parallel between this measure [the type II] and the type I effect [...] is the same as that between the total and simple household income multipliers' (Miller and Blair, 2009, p. 253). On the other hand, the link with the Keynesian multiplier is explictly recognised in Miyazawa (1968, 1976) and, more recently, Trigg and Lee (2005) and ten Raa (2005).

Again, an element-wise division would result in the same multiplier for each economic activity,

$$\mathbf{v}'[\mathbf{I}-(\mathbf{A}+\mathbf{a}\mathbf{v}')]^{-1}\left[\widehat{\mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}}\right]^{-1} = \begin{bmatrix} \varphi_v & \varphi_v & \cdots & \varphi_v \end{bmatrix} \tag{19}$$

and again the Keynesian multiplier can be reasserted as a 'true macroeconomic concept': 'Household consumption reinforces production effects irrespective of the source of the latter. The Keynesian multiplier acts indiscriminately' (ten Raa, 2005, p. 30). In this case, given equation (6) definition, the Keynesian income multiplier can be found simply by:

$$\mathbf{v}'[\mathbf{I}-(\mathbf{A}+\mathbf{a}\mathbf{v}')]^{-1} = \begin{bmatrix} \varphi_v & \varphi_v & \cdots & \varphi_v \end{bmatrix} \tag{20}$$

But the same definition allows a different derivation for the multiplier, closer to the macroeconomic ones. Provided that $\mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{f} = \mathbf{i}'\mathbf{f}$, we can write:

$$\mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{f} = \mathbf{i}'(\mathbf{f}-\mathbf{f_c}) + \mathbf{i}'\mathbf{f_c} \tag{21}$$

and, given the propensities to consume in (16),

$$\mathbf{v}'\mathbf{L}\mathbf{f} = \mathbf{i}'(\mathbf{f}-\mathbf{f_c}) + \mathbf{i}'\mathbf{a}\mathbf{v}'\mathbf{L}\mathbf{f} \tag{22}$$

in a way that the Keynesian income multiplier can be found between square brackets in the following equation, with aggregate income at the left-hand side and the sum of autonomous expenditures at the right-hand side:

$$\mathbf{v}'\mathbf{L}\mathbf{f} = \left[\frac{1}{1-\mathbf{i}'\mathbf{a}}\right]\mathbf{i}'(\mathbf{f}-\mathbf{f_c}) \tag{23}$$

Then, the Keynesian income multiplier, given the definition in (6), can be exhibited, as in Miyazawa (1968, p. 42), either by the reciprocal of the aggregate marginal propensity to save, as in macroeconomic textbooks[4], or by writing the full expression including the production structure of the economy subsumed in the Leontief inverse:

$$\varphi_v = \frac{1}{1-\mathbf{v}'\mathbf{L}\mathbf{a}} = \frac{1}{1-\mathbf{i}'\mathbf{a}} \tag{24}$$

Nonetheless, it is not the case for the Keynesian employment multiplier, which necessarily carries the labour requirements, through the vector of direct labour coefficients, as well as the production structure in the Leontief inverse:

$$\varphi_l = \frac{1}{1-\mathbf{l}'\mathbf{L}\mathbf{c}} \tag{25}$$

[4]It becomes crystal clear that the consumption is supposed to be completely induced, then, the Keynesian multipliers so obtained should be understood as ceilings in an explanation that assumes that consumption is the sole induced component of final demand.

4 Sraffian supermultipliers

Investigations in areas such as cyclical fluctuations and economic growth could be benefited from another multiplier definition. And there is nothing wrong about that since ‘it is not so important which multiplier is used as that it be matched with the appropriate multiplicand’ (Samuelson, 1942, p. 586)[5]. In this spirit, considerations about an induced investment could lead us to the concept of supermultiplier, as in Hicks (1950) or Serrano (1995).

More precisely, it would be appropriate to introduce a marginal propensity to invest, as in Samuelson (1942, p. 577) or, specifically for the Sraffian supermultiplier, in Freitas and Serrano (2015, p. 261)[6]. Just like in the case of marginal propensities to consume, we can take the proportions of income spent on investment in each economic activity as[7]

$$\mathbf{b} = \frac{\mathbf{f_i}}{\mathbf{v'Lf}} \tag{26}$$

where $\mathbf{f_i}$ is an investment vector. Then, a new equation similar to (22) can be written as:

$$\mathbf{v'Lf} = \mathbf{i'}(\mathbf{f} - \mathbf{f_c} - \mathbf{f_i}) + \mathbf{i'av'Lf} + \mathbf{i'bv'Lf} \tag{27}$$

in a way that the Sraffian income supermultiplier can be expressed between the square brackets:

$$\mathbf{v'Lf} = \left[\frac{1}{1 - \mathbf{i'}(\mathbf{a} + \mathbf{b})}\right] \mathbf{i'}(\mathbf{f} - \mathbf{f_c} - \mathbf{f_i}) \tag{28}$$

The analogous ratios of total income to primary income are still valid once we remember that the investment would pertain now to the secondary income:

$$\psi_v = \frac{1}{1 - \mathbf{v'L}(\mathbf{a} + \mathbf{b})} = \frac{\mathbf{v'}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}}{\mathbf{v'}(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{f} - \mathbf{f_c} - \mathbf{f_i})} \tag{29}$$

[5]‘It would be extremely unfortunate if the multiplicity of multipliers were to be regarded as a defect of the analysis, when in fact it is rather a tribute to the flexibility of the concept’ (Samuelson, 1942, p. 586).

[6]We shall not discuss the adjustment of the degree of capacity utilization towards a fully adjusted position, so there is no need to mention the changes of the marginal propensity to invest, as detailed in Freitas and Serrano (2015).

[7]As referred in footnote 4, supermultipliers obtained assuming completely induced investment (besides consumption) should be also understood as ceilings, in the face of the more realistic case of partially autonomous consumption and investment.

Doing the same for employment, the Sraffian employment supermultiplier can be shown to be:

$$\psi_l = \frac{1}{1 - \mathbf{l}'\mathbf{L}(\mathbf{c}+\mathbf{d})} = \frac{\mathbf{l}'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{f}}{\mathbf{l}'(\mathbf{I}-\mathbf{A})^{-1}(\mathbf{f}-\mathbf{f_c}-\mathbf{f_i})} \tag{30}$$

where $\mathbf{d}$ stands for the employment counterpart of $\mathbf{b}$:

$$\mathbf{d} = \frac{\mathbf{f_i}}{\mathbf{l}'\mathbf{L}\mathbf{f}} \tag{31}$$

But it is also true that these supermultipliers can be delivered by:

$$\psi_v = \frac{\mathbf{v}'\{\mathbf{I} - [\mathbf{A} + (\mathbf{a}+\mathbf{b})\mathbf{v}']\}^{-1}(\mathbf{f}-\mathbf{f_c}-\mathbf{f_i})}{\mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}(\mathbf{f}-\mathbf{f_c}-\mathbf{f_i})} \tag{32}$$

$$\psi_l = \frac{\mathbf{l}'\{\mathbf{I} - [\mathbf{A} + (\mathbf{c}+\mathbf{d})\mathbf{l}']\}^{-1}(\mathbf{f}-\mathbf{f_c}-\mathbf{f_i})}{\mathbf{l}'(\mathbf{I}-\mathbf{A})^{-1}(\mathbf{f}-\mathbf{f_c}-\mathbf{f_i})} \tag{33}$$

and then, as before, element-wise divisions would result in:

$$\mathbf{v}'\{\mathbf{I} - [\mathbf{A} + (\mathbf{a}+\mathbf{b})\mathbf{v}']\}^{-1}[\widehat{\mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}}]^{-1} = \begin{bmatrix} \psi_v & \psi_v & \cdots & \psi_v \end{bmatrix} \tag{34}$$

$$\mathbf{l}'\{\mathbf{I} - [\mathbf{A} + (\mathbf{c}+\mathbf{d})\mathbf{l}']\}^{-1}[\widehat{\mathbf{l}'(\mathbf{I}-\mathbf{A})^{-1}}]^{-1} = \begin{bmatrix} \psi_l & \psi_l & \cdots & \psi_l \end{bmatrix} \tag{35}$$

5 A brief discussion of some related literature

According to Kurz (1985, p. 130), its own analysis for employment effects could be ‘carried out starting from the premise of a given value of investment in terms of labour embodied, or what Kahn (1931) in his original formulation of the multiplier called “primary employment”’. As previously discussed, Kahn (1931) has approached the employment multiplier considering direct and indirect repercussions, and so did Keynes (1929, 1933, 1936) by using (and sometimes misusing) the primary / secondary split in his way towards the income multiplier.

Kurz (1985, p. 126) presents a matrix multiplier, $\mathbf{M}$, linking autonomous demand (investment, in that case) to final demand. Then, four special cases for this matrix were exhibited and the connection with employment was made by vertically integrated labour coefficients, in fact, the input-output employment multipliers, $\mathbf{l}'(\mathbf{I}-\mathbf{A})^{-1}$. Since the matrix $\mathbf{M}$ is not unique, it can be defined in terms of our previous analysis for the Keynesian multiplier as

$$\mathbf{M_v} = \mathbf{av}'[\mathbf{I} - (\mathbf{A} + \mathbf{av}')]^{-1} + \mathbf{I} \tag{36}$$

or

$$\mathbf{M_l} = \mathbf{cl}'[\mathbf{I} - (\mathbf{A} + \mathbf{cl}')]^{-1} + \mathbf{I} \tag{37}$$

in a way that the following relationship can be written:

$$\mathbf{f} = \mathbf{M_v}(\mathbf{f} - \mathbf{f_c}) = \mathbf{M_l}(\mathbf{f} - \mathbf{f_c}) \tag{38}$$

and, then, the ratio between total employment and primary employment, as in (8):

$$\varphi_l = \frac{\mathbf{l}'(\mathbf{I} - \mathbf{A})^{-1}\mathbf{M_l}(\mathbf{f} - \mathbf{f_c})}{\mathbf{l}'(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{f} - \mathbf{f_c})} \tag{39}$$

The secondary employment could be written as $\mathbf{l}'\mathbf{L}(\mathbf{M_l} - \mathbf{I})(\mathbf{f} - \mathbf{f_c})$ and we are fully aware that a given value for the primary employment 'is compatible with a range of different physical compositions of investment [autonomous] demand which are generally associated with different levels of total employment' (Kurz, 1985, p. 130), but we are not endorsing the view that, in general, the measure of beneficial repercussions is not unequivocal[8].

In the Keynesian employment multiplier presented here, it would still be true that:

$$\mathbf{l}'(\mathbf{I} - \mathbf{A})^{-1}\mathbf{M_l}[\widehat{\mathbf{l}'(\mathbf{I} - \mathbf{A})^{-1}}]^{-1} = \begin{bmatrix} \varphi_l & \varphi_l & \cdots & \varphi_l \end{bmatrix} \tag{40}$$

which does not imply that we can compute an employment multiplier without any knowledge of the production structure and of the labour direct requirements, as showed in equation (25). We couldn't agree more with Kurz (1985, p. 135) 'that a larger volume of investment could be associated with smaller levels of total income and employment'. After all, 'a different bill of goods [...] will require a different combination of industrial outputs and result in different employment figures for various industries' (Leontief, 1944, p. 303), but it does not changes the fact that the ratios of total to simple multipliers / supermultipliers are the same. If the consumption basket proportions are preserved, Keynesian employment and income multipliers are still insensitive to autonomous expenditure proportions, despite the fact that different levels of total employment and income would be found from different autonomous expenditure proportions.

[8]Neither for the ratio of secondary to primary employment, as in Kurz (1985, p. 130), nor for the multiplier relating total to primary employment.

As in Mariolis (2018, p. 477) special case with an unique consumption pattern and an aggregate savings ratio, the dominant eigenvalues of $\mathbf{M_l}$ and $\mathbf{M_v}$ are given by the respective Keynesian multipliers, φ_l and φ_v. But there is more: the right eigenvectors corresponding to the dominant eigenvalues have the same proportions of the consumption baskets, $\mathbf{c}$ and $\mathbf{a}$. Then, the respective eigensystems can be written as:

$$\mathbf{M_l c} = \varphi_l \mathbf{c} \tag{41}$$

$$\mathbf{M_v a} = \varphi_v \mathbf{a} \tag{42}$$

meaning that induction effects can be captured by a single multiplier, so, there is no need for a matrix multiplier if the consumption basket proportions are the same. In other words, the Keynesian multiplier is a scalar.

Considering these eigensystems and given the proportions of the right eigenvectors, the consumption vector can be expressed either from primary employment or from primary income totals, regardless of whether operations are performed by multiplier matrices or scalars:

$$\mathbf{f_c} = \mathbf{M_l c}[\mathbf{l}'(\mathbf{I}-\mathbf{A})^{-1}(\mathbf{f}-\mathbf{f_c})] = \varphi_l \mathbf{c}[\mathbf{l}'(\mathbf{I}-\mathbf{A})^{-1}(\mathbf{f}-\mathbf{f_c})] \tag{43}$$

$$\mathbf{f_c} = \mathbf{M_v a}[\mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}(\mathbf{f}-\mathbf{f_c})] = \varphi_v \mathbf{a}[\mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}(\mathbf{f}-\mathbf{f_c})] \tag{44}$$

The terms between square brackets represent, respectively for equations (43) and (44), the aggregates for primary employment and primary income. These aggregates (scalars) multiplied by their respective consumption basket proportions, $\mathbf{c}$ or $\mathbf{a}$, result in a 'first round' consumption vector. The induction process behind the multipliers towards the actual consumption vector can be represented by a matrix or by a scalar — precisely by the dominant eigenvalues of those matrices, the Keynesian multipliers.

In addition to that, column sums of the income multiplier matrix, $\mathbf{M_v}$, result in a vector of Keynesian income multipliers, and every single column of $(\mathbf{M_v} - \mathbf{I})$ equals the dominant eigenvalue of $\mathbf{M_v}$ times the vector of the propensities to consume, *i.e.*, equals $\varphi_v \mathbf{a}$, as could be deduced from the definition in (6) resulting in equation (20). But as soon as a Miyazawa (1968, 1976) system with more than one income group and more than one consumption basket is introduced, those nice properties related to the dominant eigenvalues and their associated eigenvectors are gone.

From the 'static matrix multiplier' found in Mariolis (2018, p. 476), we can go back to Mariolis and Soklis (2018) and Ntemiroglou (2016), where the terms Sraffian multiplier and static Sraffian multiplier can be encountered.

Definitions for the output multiplier and for the import multiplier, intendend to link the autonomous demand for each commodity to the money value of net output (final demand) and of imports, can be found in Ntemiroglou (2016, p. 6) and in Mariolis and Soklis (2018, p. 121). Then, empirical results are presented not considering the primary / secondary split for the targeted variables, but showing commodity-specific multipliers *per* unit of autonomous demand, which resemble traditional input-output total mulipliers (considering direct, indirect and induced repercussions), notwithstanding the fact that rectangular multi-product Source and Use Tables are utilised for estimates.

The employment multipliers case is more delicate. Inspired by Kahn (1931) and making use of matrix multipliers as proposed by Kurz (1985), Ntemiroglou (2016, p. 4) and Mariolis and Soklis (2018, p. 119) describe aggregate primary and secondary employment effects. After that, the suggested empirical framework considers n vertically integrated sectors in order to evaluate employment multipliers. And the primary employment effects from the increase of one unit of the autonomous demand for each commodity seem uncontroversial, corresponding to the input-output simple employment multipliers, $\mathbf{l}'(\mathbf{I}-\mathbf{A})^{-1}$.

But the equations in Mariolis and Soklis (2018, p. 123–125) and in Ntemiroglou (2016, p. 9) for total and secondary employment imply that the entire income generated[9] is supposed to be spent on only one consumption good, the same one of the initial autonomous demand shock. This step towards n employment multipliers was not taken by Kurz (1985) and suggests that if we are assuming an autonomous demand impulse for (*e.g.*) agricultural products, the average consumption basket for the economy as a whole must be composed only by agricultural products. It is this misconceived assumption that guarantees different multipliers for each economic activity, as compared with the case of fixed consumption basket proportions resulting in a macroeconomic Keynesian employment multiplier.

We could go back further to Mariolis (2008), where was defined a 'matrix of "super-multipliers" linking exports to gross output', as well as a 'vector of super-multipliers linking exports to total employment' (Mariolis, 2008, p. 659–660). However, apart from the terminology issues involved, there is not much to add to the present discussion. To be sure, there is no relationship between the super-multipliers so defined and the macro-oriented Sraffian supermultipliers presented in Section 4. Still, matrices like $\mathbf{M_v}$ and $\mathbf{M_l}$ for

[9]Regardless of whether the induction process comes from the income or from the employment. The description of an income being spent was adopted here because it seems to be more appealing.

the Sraffian income and employment supermultipliers can be defined in an analogous way:

$$\mathbf{S_v} = (\mathbf{a}+\mathbf{b})\mathbf{v}'\{\mathbf{I}-[\mathbf{A}+(\mathbf{a}+\mathbf{b})\mathbf{v}']\}^{-1}+\mathbf{I} \tag{45}$$

$$\mathbf{S_l} = (\mathbf{c}+\mathbf{d})\mathbf{l}'\{\mathbf{I}-[\mathbf{A}+(\mathbf{c}+\mathbf{d})\mathbf{l}']\}^{-1}+\mathbf{I} \tag{46}$$

and the respective Sraffian supermultipliers, ψ_v and ψ_l, can also be found from the dominant eigenvalues[10] of matrices $\mathbf{S_v}$ and $\mathbf{S_l}$.

Having in mind the macroeconomic concept, Dejuán (2014, p. 4) presents the Sraffian supermultiplier 'as a variant of Pasinetti's vertically hyper-integrated sectors'. Adopting a broad interpretation of the process of vertical hyper-integration (Pasinetti, 1988) and assuming that, relative to the process of vertical integration, the 'essential difference' relies on 'including in each hyper-subsystem *all* gross investments' (Pasinetti, 1988, p. 127)[11], the supermultipliers proposed by Dejuán (2014, p. 10) would be close to a set of input-output multipliers augmented by an induced investment, besides the endogenized consumption as in standard closed models:

$$\mathbf{v}'\{\mathbf{I}-[\mathbf{A}+(\mathbf{a}+\mathbf{b})\mathbf{v}']\}^{-1} = \mathbf{v}'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{S_v} \tag{47}$$

$$\mathbf{l}'\{\mathbf{I}-[\mathbf{A}+(\mathbf{c}+\mathbf{d})\mathbf{l}']\}^{-1} = \mathbf{l}'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{S_l} \tag{48}$$

If instead of pre-multiplying the inverse matrices above by income and employment coefficient row-vectors we do the same operations using diagonalised vectors $\hat{\mathbf{v}}$ and $\hat{\mathbf{l}}$, some version of the supermultiplier matrices suggested by Portella-Carbó (2016) would make its appearance[12]. As shown at the end of Section 4, truly macroeconomic Sraffian supermultipliers can be obtained from equations (34) and (35).

6 Some estimates

Using data from the World Input-Output Database (WIOD), both income and employment[13] multipliers and supermultipliers have been computed for

[10] The associated right eigenvectors now presenting the same proportions of the sum of consumption and investment coefficients, respectively $\mathbf{a}+\mathbf{b}$ and $\mathbf{c}+\mathbf{d}$.

[11] Acknowledging that the 'key concept behind hyper-integration is the induced character of (fixed and circulating) investment expenditures' (Garbellini and Wirkierman, 2014, p. 163).

[12] Actually, Dejuán (2014) has not presented a vector for employment supermultipliers and Portella-Carbó (2016) has not presented an income supermultiplier matrix.

[13] In fact, the chosen variable in the Socio Economic Accounts was the number of persons engaged in economic activities, not the number of employees.

2014. From the original list of 43 countries, the results are shown for 32 countries after aggregating economic activities from 56 to 47. The aggregation procedure was done in order to solve compatibility issues, since some countries do not report positive values for all the 56 economic activities. Notwithstanding, it was still not possible to compute multipliers and supermultipliers for 11 countries in that new 47 activities level of aggregation.

Figure 1: Income multipliers and supermultipliers

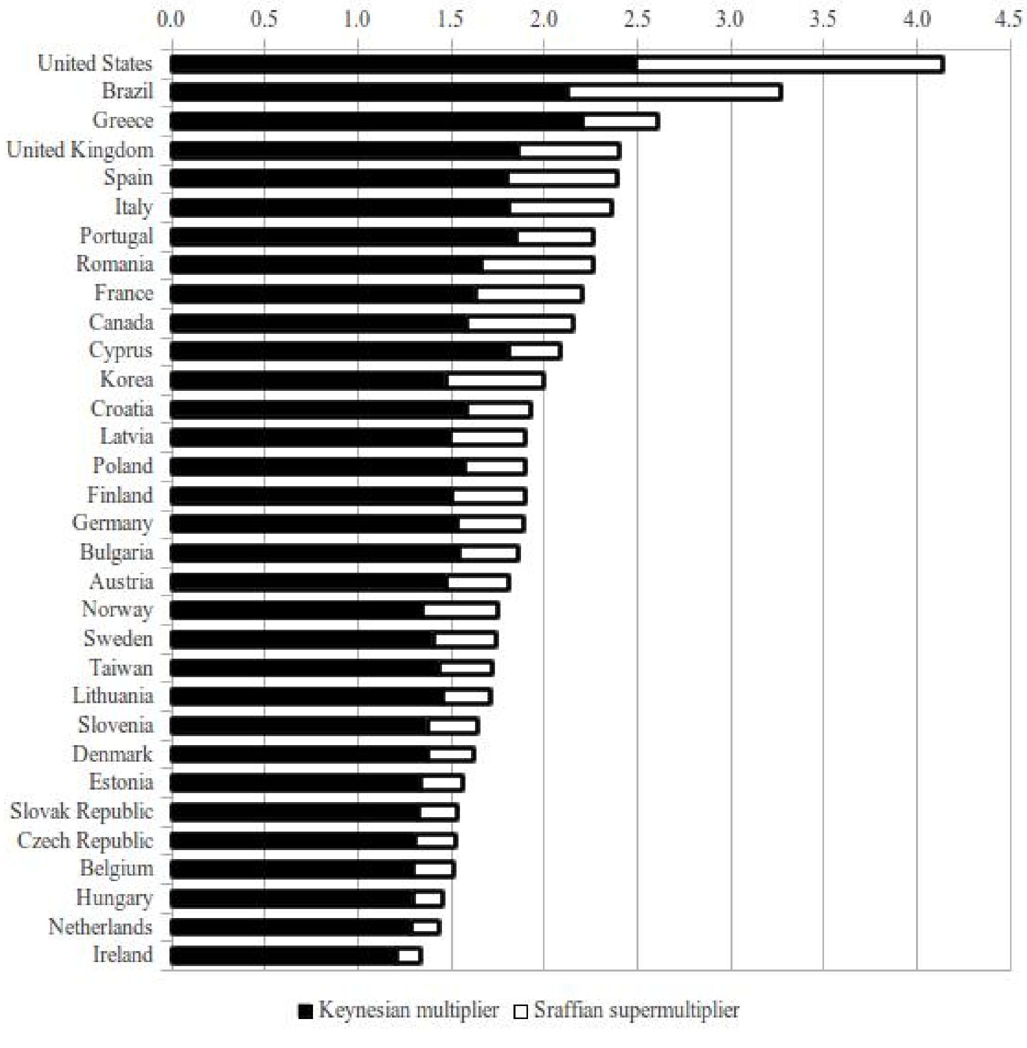
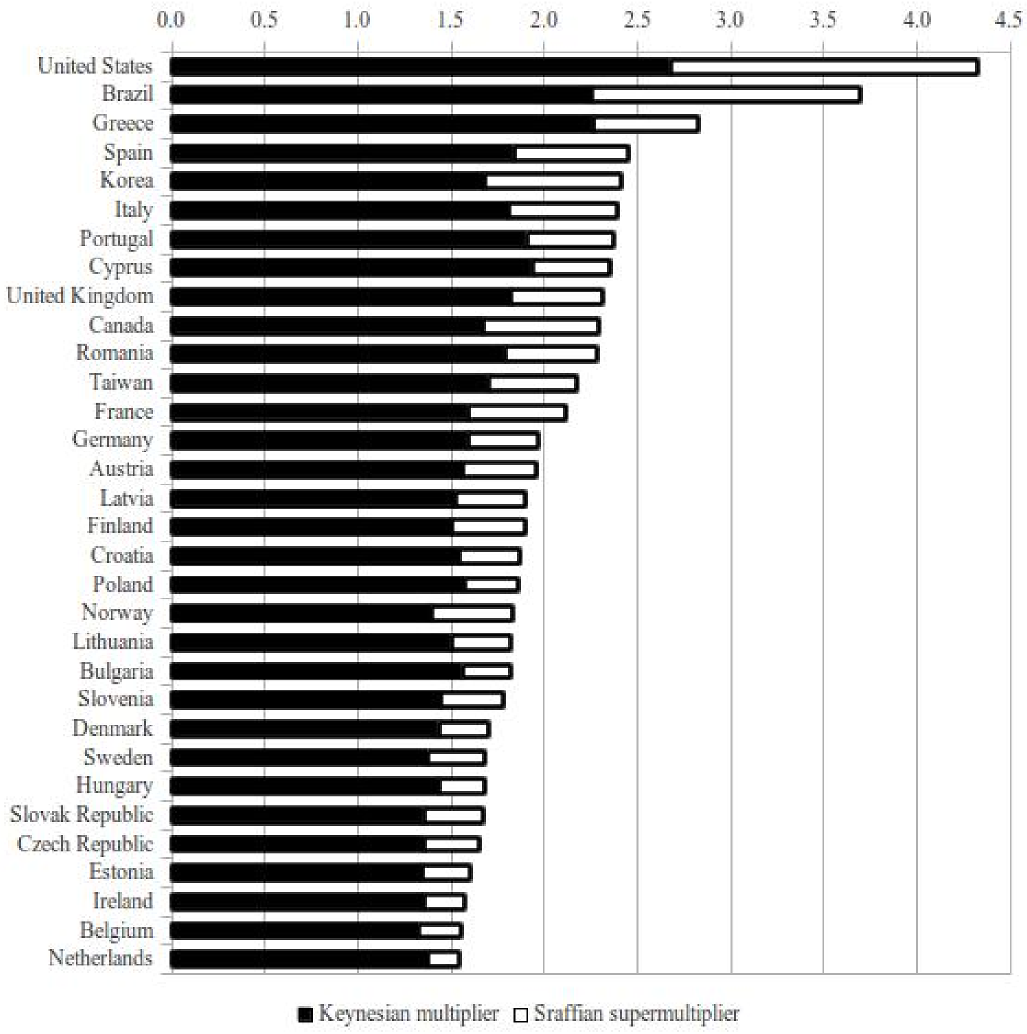


Figure 1 presents income multipliers and supermultipliers in descending order of Sraffian supermultipliers. The dark portions of the bars stand for Keynesian multipliers, showing the traditional multiplier effects assuming completely induced consumption, and the white portions correspond to the difference from the assumption of completely induced investment, resulting in Sraffian supermultiplier total effects. Figure 2 does the same for employment multipliers and supermultipliers.

Figure 2: Employment multipliers and supermultipliers



The mean for Keynesian income multipliers is 1.58 and for Sraffian supermultipliers it is 1.99, above the medians of 1.50 and 1.89 respectively. These mean results are clearly very influenced by dicrepantly high figures for supermultipliers in the United States (4.14) and Brazil (3.27), and, to a lesser degree, in Greece (2.60). The top five income supermultipliers is completed with the United Kingdom (2.40) and Spain (2.39). A few inversions in the ranking would be noticed if the criterion of Keynesian multipliers was taken, since Greece (2.21) presents a higher multiplier than Brazil (2.12) or Portugal (1.85) presents a higher multiplier than Italy (1.81) and even than Spain (1.80). The United States (2.49) would still have the highest Keynesian multiplier[14], though. In the low end, with Sraffian supermultipliers below 1.5 and Keynesian multipliers below 1.3, we can find Ireland, the Netherlands and Hungary.

Employment multipliers and supermultipliers slightly differ from their income counterparts. Averages (1.64 and 2.11) are still above the medians (1.56 and 1.90) and the three highest supermultipliers were computed for the United States (4.33), Brazil (3.69) and Greece (2.82), now with Spain (2.45) and Korea (2.41) closing the top five. The Netherlands, Belgium and Ireland have presented the lowest employment supermultipliers, with Belgium, Estonia and the Slovak Republic showing the lowest Keynesian multipliers for the selected countries.

7 Conclusion

Old and new attempts of representing multipliers as matrices were addressed. There must be something very seductive in generalising scalar concepts by using a matrix, but there are some cases in which a matrix is just redundant, even though the recommended path to achieve this conclusion is by using matrix algebra. The Leontief inverse is the ultimate example that a matrix is needed in order to connect final demand to gross output. The opposite happens in the Keynesian multiplier and Sraffian supermultiplier cases: there is no need for a matrix.

In this paper, we have tried to show that Keynesian multipliers and Sraffian supermultipliers can be actually acknowledged as macroeconomic tools. This conclusion, however, is not intended to mean that induction effects are independent of the technical conditions of production, summarised by the Leontief inverse, or of income distribution and consumption patterns. In fact, assuming that we have an unique consumption proportions vector is quite different of the assumption that these proportions are invariant to income distribution. Changes in income distribution must alter an aggregate (by income group, not by economic activity) consumption basket.

In the supermultiplier case, as soon as the investment becomes induced,

[14]Since consumption and investment were assumed to be completely induced for all countries, the results for the United States should be considered distorted only if there is plausible evidence that the autonomous portions of consumption and investment, via institutional channels for finance, for instance, are above the average of the other countries.

we should also consider investment composition in estimating supermultipliers. In determining both income and employment Keynesian multipliers and Sraffian supermultipliers, the proportions of the autonomous expenditures vector are the ones that could be considered irrelevant, in spite of the fact that income and employment levels are not insensitive to these proportions.

The multiplier and supermultiplier concepts are not purely theoretical, though. Then, by using data from the World Input-Output Database (WIOD) and estimating the Keynesian income multiplier for a 32 countries sample, we were able to find results ranging from 1.2 (Ireland) to 2.5 (United States). For the same two countries, the Sraffian income supermultiplier ranges from 1.3 to 4.1. In the employment case, the Keynesian multiplier reaches the lowest value for Belgium (1.3) and has a maximum of 2.7 for the United States. The highest (4.3) Sraffian employment supermultiplier was also found for the United States and the lowest one (1.5) was computed for the Netherlands.

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