Relative Price Changes and the Structural Decomposition Analysis of the Brazilian Economy from 2000 to 2015

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ABSTRACT
The paper aims to evaluate the effect of relative prices changes in structural decomposition analysis in the context of economies such as the Brazilian one in which these changes can be significant. Using an Input-Output updating methodology, we created a series of I-O matrices for the Brazilian economy from 2000 to 2015. The sources of information used in the updating process were the structural characteristics of official 2010 I-O matrix and the partial information for the totals available from the annual supply and use tables at current and last year prices. As a result, we obtained a series of I-O matrices valued at current and last year prices, which allowed us to obtain price and volume indices for each cell of these matrices. The latter indices were used to obtain a series of constant prices I-O matrices. However, the volume figures obtained in this way (i.e., by the use of chained indices) are characterized by the well-known problem of non-additivity. In order to overcome the latter problem in structural decomposition analysis we used the method adopted by Hillinger, Reich, Balk and Diewert to address the non-additivity problem. In our specific application of the method, we deflated the whole series of estimated matrices at current and last year prices by the price index of total gross-output. Then, we isolated the contribution of relative price changes and obtained a more accurate assessment of the real contributions of the factors involved in the gross-output structural decomposition exercise. Comparing the latter contributions to the ones obtained without the isolation of relative price changes, we were able to evaluate the effect of relative price changes. Our investigation revealed that relative price changes have indeed a relatively significant effect on the results of the structural decomposition exercise for the period consider.

Key-words: Relative prices. Structural decomposition analysis. Brazilian Economy.

1. Introduction

Multisectoral long-term analysis may involve not only the prices changes over time, but also the changes at the relative prices of each sector in relation to total prices. This generates the problem of non-additivity in the process of deflating input-output tables in a large time series. This effect must be stronger in a country where there is a historical of higher inflation and exchanges rates changes. Although many studies analyzes the effect of the relative prices in the context of developed countries, which are less susceptible to these variations, this may not be the case of developing countries, such as Brazil.

In this context, the paper aims to evaluate the effect of relative prices changes in structural decomposition analysis in the context of economies such as the Brazilian one in which these changes can be significant. Using an Input-Output updating methodology applied by Passoni and Freitas (2018a; 2018b), a series of I-O tables valued at constant prices for the Brazilian economy from 2000 to 2015, it is possible to develop an input-output model that includes the effect of relative prices in the decomposition. Using the series of I-O tables valued at current and last-year prices, it is possible to obtain price and volume indices for each cell of these matrices. The latter indices were used to obtain a series of constant prices I-O matrices. However, the volume changes obtained in this way (i.e., using chained indices) are characterized by the well-known problem of non-additivity. To overcome the latter problem in structural decomposition analysis we used the method adopted by Hillinger, Reich, Balk and Diewert to address the non-additivity problem.

This paper, besides this introduction, has five sections. The first one presents the problem of additivity in deflated IO tables. The next section presents the methodology used in our work, such as the database, the Input-Output model with the effect of relative prices, and the structural decomposition proposed. Section Four presents the results, and in the following one, we present our final considerations.

2. Deflacting input-output tables: the problem of additivity

Additivity, in the national accounts context, means the property of deflation and aggregation operations being interchangeable. In other words, to first deflate and then aggregate the values should arrive at the same result as do the same operations in the reverse order (Balk and Reich, 2008). Traditionally, to calculate and disclose the national accounts, it used to be used a direct Laspeyres volume index, which uses a fixed price vector of a determined base-year, to arrive at the volume increase of a certain sub-aggregate (Balk and Reich, 2008). As this procedure maintains the prices constant, index additivity was straightforward, and although it would be lost whenever an update in the relative price vector was made, this happened only after 5 or 10-year time span (Diewert, 1995).

During the 90's, though, with the rapid technological change and, with it, the rapid price vector changes, the countries started to favor the chaining method, in which, each year, a new vector price is adopted (Hillinger, 2002). This method consists of a multiplication of the quantities of a period by a quantity index, using current price vector as weights (Hillinger, 1999). This methodological modification led to the emergence of an academic discussion about the role of price-vector and additivity in national accounts (Hillinger, 2002). With the chaining method and the ever-changing of price vectors, the additivity propriety holds for the immediate-previous year, but it is lost in the analysis of chained indices for different periods. To put it differently, if we consider a time series of volume and prices indexes, it became necessary to choose between using the same indexes for the aggregate and sub-aggregates, which naturally distort the results, and using separate indexes, which renders a more reliable series, but without the additivity propriety. (Hillinger, 2002).

Different authors have taken different views about this problem. Eheman, Katz, and Moulton (2002), for instance, argue that additivity is not a problem at all for a national account system, and it is not even desirable, as it misguides crossover economic analysis by using outdated relative price vectors. In other words, non-additivity would be a (low) cost to be paid for a more reliable accounting system (Balk and Reich, 2008). Other authors tried to establish procedures to distribute in a reasonable way the additivity discrepancy, defined as the difference between the sum of the sub-aggregate indexes and the aggregate index. Casler (2006), for example, propose that the additivity discrepancy, in his words the interaction term, be distributed proportionally between each source of change (price and quantity). His suggestion is to use as weights the growth rate of prices and quantity, respectively. Accounting for the price change, therefore, starts to have here a role in the determination of the series of national accounts.

An alternative approach has been put forward in the last two decades by authors such as Hilinger, Reich, Balk and Diewert, who have been suggesting methodological solutions that combine deflation of values for longer periods with the additivity property. This alternative consists of, initially, deflating all the price vectors by the most aggregate deflator, which, in the national accounts context, can be the total gross-output. This is equivalent to render the series constant in relation to the general price movements. It is as if all the prices of the series were prices of a single chosen year, in a way to eliminate all the inflationary effect. After that, there remains the price variation of one sector in relation to the whole economy, which is called relative price, or real price (Balk and Reich, 2008). Taking into account this last real price effect will render volume indexes with additivity property holding for multiple-year periods.

Additivity and the importance of considering the relative price's influence are not a question limited to the index number theory. They have also spread to the study of value added, for example, as shown in Diewert (1995) and Reich (2010). This happens first because value added is a concept that applies naturally both to sectors and the whole economy, and the total value added of the economy should be equal to the sum of sectoral values added. More importantly, though, is the fact that, as it is calculated by the difference between gross output and the intermediate consumption, value added have a more significant additivity and relative prices problem.

Reich (2010) investigates the consequences of the additivity problem to the calculation of value added, focusing in the Danish national accounts between 1970 and 2005. He arrives at an inconsistency of 22% in the industry's gross output value, and 16% for the intermediary consumption. However, as expected, the inconsistency in the computation of value added amounts to 32%, indicating that accounting for relative prices is a rather important factor in determining the real added value growth in the Danish economy.

Concerning decomposition methods, another field in which additivity and relative prices problem have been explored, Reich (2008) evaluates the importance of real prices in the decomposition of the Dutch input-output tables for 25 industrial sectors, from 1990 to 2000. His broad results are that real prices indeed play a meaningful role, in which contribution to growth often counterbalances the contribution of volume changes, acting as a sort of corrector of simple value growth.

Recently, Dietzenbacher and Temurshoev (2012) investigated the importance of relative prices for demand-shift impact on the economy within an input-output framework. For this, they also used Danish input-output tables between 2000 and 2007 to try to determine if using current or constant prices change the IO results significantly. The conclusion is that using current or constant IO tables change very little the economic impact forecast, pointing out that discussion of deflation methods is rather irrelevant in the context of input-output analysis.

When it comes to using deflators to arrive at the contribution to growth of different sources, the approach of computing the real prices leads not only to additive contributions but also to a much more reliable volume indexes. This is so because avoiding to deal with relative prices pollutes in a way the volume contribution. Not doing so is as if Casler's interaction term referred above was randomly and senselessly attributed to volume contribution.

In this context, the present article will address the issue of the importance of relative prices in the decomposition of gross output growth, by using a Structural Decomposition Analysis (SDA) of the Brazilian economy in an input-output framework in which, relative prices do play a meaningful role, and accounting for this effect render different and more reliable decomposed volume contributions to growth. We chose to analyze the Brazilian economy between the years of 2000 and 2015, as it was subject to volatile relative price movements in this period. In fact, the inflation process in the Brazilian economy was accompanied by significant relative prices changes. As minimum wage raised, labor-intensive sectors had sharper price increases, and imports of manufactured goods increased counterbalancing the first effect.

3. Methodology

3.1 Database

We utilize an annual series of I-O Matrices at current prices constructed by Passoni and Freitas (2018a) for the period 2000-2015. The construction of this series was based on IO updating technics and used structural information from the 2010 official I-O Matrix combined with partial information obtained from the Supply and Use tables of the Brazilian SNA.

Since our goal is to asses the importance of relative prices for SDA in the Brazilian case, it was necessary to build also, for each year, I-O tables valued at constant relative prices, as proposed in Casler (2006), Dietzenbacher and Temurshoev (2012), Hillinger (1999, 2002) and Reich (2008). First, the same updating methodology used to construct the current prices IO matrices series was utilized to obtain IO tables at previous year prices. With a series of IOTs at current and prevoius year prices it was possible to calculate cell-specific price indices. Those indices were then used to obtain the IO tables at current prices, resulting in a series IO tables at constant 2010 prices. However, the latter series does not have the additive property. To overcome this problem, the cell-specific price indices were divided by total gross output price index, which allowed the construction of cell-specific relative prices indices. These indices were then used to contruct a series of IOT at constant 2010 prices with the additive property.[1] This step allows us to decompose the change in total real gross output in terms of volume and relative prices effects while preserving the additivity property.[2.]

3.2 The IO Model and relative prices

We start from the basic expression on an I-O model:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A_n})^{-1}\mathbf{f_n} \qquad (1)$$

where $\mathbf{x}$ is the vector of sectoral gross output, $\mathbf{A_n}$ $(n \times n)$ is the matrix of domestic technical coefficients, $(\mathbf{I} - \mathbf{A_n})^{-1}$ $(n \times n)$ is the Inverse Leontief Matrix, $\mathbf{f_n}$ is the domestic final demand $(n \times 1)$, and *n* is the number of sectors in the economy.

The industry by industry IO coefficient matrix, $\mathbf{A_n}$, and the vector of final demand by industry $\mathbf{f_n}$ are obtained from the use table of national products at basic prices, where we have information on intermediate demand of commodities by industry and final demand of commodities. Since the Brazilian IO database is compiled according to the industry technology, we use a market-share matrix to transform demand of commodities into demand for industry output. The latter matrix can be defined as follows:

$$\mathbf{D} = \mathbf{V}\hat{\mathbf{q}} \qquad (2)$$

where $\mathbf{V}$ is the transposed production matrix from the Supply table, $\mathbf{q}$ is the vector of gross output by commodity $(k \times 1$, with k commodities) $(k \times 1)$ and $\mathbf{D}$ is the resulting market-share matrix in which each element in each column indicates the share of an industry in total commodity output $(n \times k)$.

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[1] Passoni and Freitas (2018b) paper provides a detailed account of the deflation methodology used in the construction of these tables. An early version of this methodology applied to period between 2000 and 2009 can be found in Neves (2013).

[2] Besides volume and relative price effects, Reich (2008) also talks about a component correcting for general inflation. However, as it is not an objective of this paper to analyze the inflation effect on the gross output decomposition, we will only consider the *relative price effect.*

Let $\mathbf{U_n}$ be the flow matrix of intermediate demand for national products at basic prices. Thus, we can obtain the commodity by industry matrix of national technical coefficients $\mathbf{B_n}$ as follows:

$$\mathbf{B_n} = \mathbf{U_n}\hat{\mathbf{x}}^{-1} \qquad (3).$$

Next, by premultiplying $\mathbf{B_n}$ by $\mathbf{D}$ we obtain $\mathbf{A_n}$.

$$\mathbf{A_n} = \mathbf{D}.\mathbf{B_n} \qquad (4)$$

Similarly, we obtain the vector of final demand by industry by premultiplying the vector of final demand for commodities ($\mathbf{d_{F_n}}$) by $\mathbf{D}$ such as:

$$\mathbf{f_n} = \mathbf{D}.\left(\mathbf{d_{F_n}}\right) \qquad (3).$$

Using equations (4) and (3), we may present the gross output as:

$$\mathbf{x} = \mathbf{D}.\mathbf{B_n}\mathbf{x} + \mathbf{D}.\mathbf{d_{F_n}} \qquad (4).$$

Now, when it comes to the changes in volume and prices, we have to rewrite the vector of sectoral gross output as:

$$\mathbf{x} = \widehat{\mathbf{p}}_{\mathbf{x}}\mathbf{x}^{\mathbf{v}} \qquad (5)$$

which is the basis of every deflation procedure (Reich, 2008), with $\mathbf{p_x}$ and $\mathbf{x^v}$ representing, respectively, the relative price and volume vectors related to sectoral gross output. As discussed in section 3.1 above, each element of the relative price vector is given by the ratio $p_{x\,j}/p$, where $p_{x\,j}$ is the industry *j* gross output price index and p the price index of total gross output.

As we aim to capture the influence of relative prices in the IO model components, we rewrite all variables disaggregating relative price and volume terms. The elements of the $\mathbf{U_n}$ matrix becomes:

$$u_{n_{ij}} = \frac{P_{U_{n_{ij}}}}{p} \times u_{n_{ij}}^{v} \qquad (6)$$

Where $P_{U_{n_{ij}}}$ is the relative price of commodity i used as an input by industry *j,* and $u_{n_{ij}}^{v}$ is the volume measure of commodity *i* used as an input by industry *j*.

Using (5) and (6) in **Error! Reference source not found.**, the elements of $\mathbf{B_n}$ are given by:

$$b_{n_{ij}} = \frac{\frac{P_{U_{n_{ij}}}}{p}}{\frac{p_{x\,j}}{p}} \times \frac{u_{n_{ij}}^{v}}{x_j^v} = \frac{P_{U_{n_{ij}}}}{p_{x\,j}} \times \frac{u_{n_{ij}}^{v}}{x_j^v} \qquad (7).$$

Let us define the elements of matrices $\mathbf{P_{B_n}}$ and $\mathbf{B_n^v}$ as follows:

$$P_{B_{n_{ij}}} = \frac{P_{U_{n_{ij}}}}{p_{x\,j}} \qquad (8)$$

$$b_{n_{ij}}^{v} = \frac{U_{n_{ij}}^{v}}{x_j^v} \qquad (9).$$

Thus, using the symbol $\otimes$ to denote the Hadamard product, the $\mathbf{B_n}$ matrix can be expressed in the following way:

$$\mathbf{B_n} = \mathbf{P_{B_n}} \otimes \mathbf{B_n^v}$$

where, $\mathbf{P_{B_n}}$ is the matrix of relative price indices and $\mathbf{B_n^v}$ is the matrix of national technical coefficients measured in volume terms, both related to the commodity by industry national technical coefficient matrix.

As to the final demand, let us define the relative price of an element of the vector of final demand of commodities and the corresponding we can express the final demand of commodities as follows:

$$p_{d_{F_{nj}}} = \frac{p_{d_{nj}}}{p}$$

Next, we can decompose the vector final demand in its relative price and volume components, obtaining the expression below:

$$\mathbf{d_{F_n}} = \hat{\mathbf{p}}_{\mathbf{d_{F_n}}} \mathbf{d_{F_n}}^{\mathbf{v}} \qquad (13)$$

Finally, for the market-share matrix, the approach was somewhat different. First, we calculated a *constant prices* (or volume) market-share matrix $\mathbf{D^v}$:

$$\mathbf{D^v} = \mathbf{V^v}\widehat{\mathbf{q^v}} \qquad (14).$$

where $\mathbf{V^v}$ is the transposed production matrix at constant prices and $\mathbf{q^v}$ is the constant price vector of gross output by commodity.

Then, we obtained the relative prices matrix related to $\mathbf{D}$ ($\mathbf{P_D}$) implicitly, by dividing each element of the current price market-share matrix by the corresponding element of the constant price matrix $\mathbf{D^v}$. That is, using the symbol $\oslash$ for the Hadamard division operator we have

$$\mathbf{P_D} = \mathbf{D} \oslash \mathbf{D^v} \qquad .$$

Therefore, we can express the current prices market-share matrix as follows:

$$\mathbf{D} = \mathbf{P_D} \otimes \mathbf{D^v} \qquad (15).$$

Now we can go back to equation (6) and use the previous results to obtain a version of it in which each variable involved is decomposed in its relative price and volume components:

$$\hat{\mathbf{p}}_\mathbf{x}\mathbf{x^v} = (\mathbf{P_D} \otimes \mathbf{D^v})(\mathbf{P_{B_n}} \otimes \mathbf{B_n^v})(\hat{\mathbf{p}}_\mathbf{x}\mathbf{x^v}) + (\mathbf{P_D} \otimes \mathbf{D^v}).\left(\hat{\mathbf{p}}_{\mathbf{d_{F_n}}}\mathbf{d_{F_n}}^{\mathbf{v}}\right) \qquad (16)$$

Solving the last equation for the vector of gross output in volume terms, we obtain the equivalent to the equation (1) above:

$$\mathbf{x^v} = \left[\left[\mathbf{I} - \left((\mathbf{P_D} \otimes \mathbf{D^v}).(\mathbf{P_{B_n}} \otimes \mathbf{B_n^v})\right)\right]\hat{\mathbf{p}}_\mathbf{x}\right]^{-1}\left((\mathbf{P_D} \otimes \mathbf{D^v}).\left(\hat{\mathbf{p}}_{\mathbf{d_{F_n}}}\mathbf{d_{F_n}}^{\mathbf{v}}\right)\right) \qquad (107)$$

This equation is interesting because it allows us to identify the volume contribution to changes on gross output (the *volume effect*), leaving aside the relative price contribution. The Leontief matrix becomes $\boldsymbol{Z}_{p_x} = \left[\left[\mathbf{I} - \left((\mathbf{P_D} \otimes \mathbf{D^v}).(\mathbf{P_{B_n}} \otimes \mathbf{B_n^v})\right)\right]\hat{\mathbf{p}}_\mathbf{x}\right]^{-1}$**,** and takes explicit account of the influence of relative prices.

3.3 Structural decomposition

The structural decomposition analysis (SDA) approach provides us with a technique that disaggregates the change in a variable into its various components quantitative contributions - disaggregating an identity into several components (Miller & Blair, 2009). Any economic variable can be decomposed into its elements, enabling a better understanding of the changes that occurred between two periods and allowing us to assess the importance and strength of each element.

In this paper, we will focus our analysis on the change of the gross output ($\mathbf{x}$) in the Brazilian economy between 2000 and 2015. With it, we propose a two-level decomposition. The first one deals with the ecomposition of the change in gross output in terms of its relative price ($\mathbf{p_x}$) and volume ($\mathbf{x^v}$) components according to equation (7) above. The decomposition follows Dietzenbacher & Los (1998) and Miller & Blair (2009), using the average of the two extreme decomposition situations. Denote 0 and 1 as superscripts for the initial and final period, respectively. Hence, the change in gross output can be expressed as follows:

$$\mathbf{\Delta x} = \hat{\mathbf{p}}_{\mathbf{x}}{}^{\mathbf{1}}\mathbf{x}^{\mathbf{v1}} - \hat{\mathbf{p}}_{\mathbf{x}}{}^{\mathbf{0}}\mathbf{x}^{\mathbf{v0}} \tag{11}$$

$$\underbrace{\mathbf{\Delta x}}_{\substack{\textit{total gross output}\\ \textit{change}}} = \underbrace{\frac{1}{2}\left(\hat{\mathbf{p}}_{\mathbf{x}}{}^{\mathbf{1}} + \hat{\mathbf{p}}_{\mathbf{x}}{}^{\mathbf{0}}\right)\mathbf{\Delta x}^{\mathbf{v}}}_{\textit{volume change contribution}} + \underbrace{\frac{1}{2}\Delta\hat{\mathbf{p}}_{\mathbf{x}}\left(\mathbf{x}^{\mathbf{v1}} + \mathbf{x}^{\mathbf{v0}}\right)}_{\substack{\textit{relative prices}\\ \textit{changes contribution}}} \tag{19}$$

On the other hand, the second-level decomposition analyzes changes in gross output in volume ($\mathbf{x}^{\mathbf{v}}$) in terms of its component contributions given by equation (17). In this decomposition, we investigate the quantitative contribution of changes in the volume variables involved (i.e., $\mathbf{D}^{\mathbf{v}}$, $\mathbf{B}^{\mathbf{v}}_{\mathbf{n}}$ and $\mathbf{d}_{\mathbf{F}_{\mathbf{n}}}{}^{\mathbf{v}}$) approximately isolated from the prices variables contributions (i.e., $\mathbf{p}_{\mathbf{x}}$, $\mathbf{P}_{\mathbf{D}}$, $\mathbf{P}_{\mathbf{B}_{\mathbf{n}}}$ and $\mathbf{p}_{\mathbf{d}_{\mathbf{F}_{\mathbf{n}}}}$).

To this decomposition, the same principle of equation (19) is applied in order to find $\mathbf{\Delta x}^{\mathbf{v}}$ and its *volume and price effects contributions*:

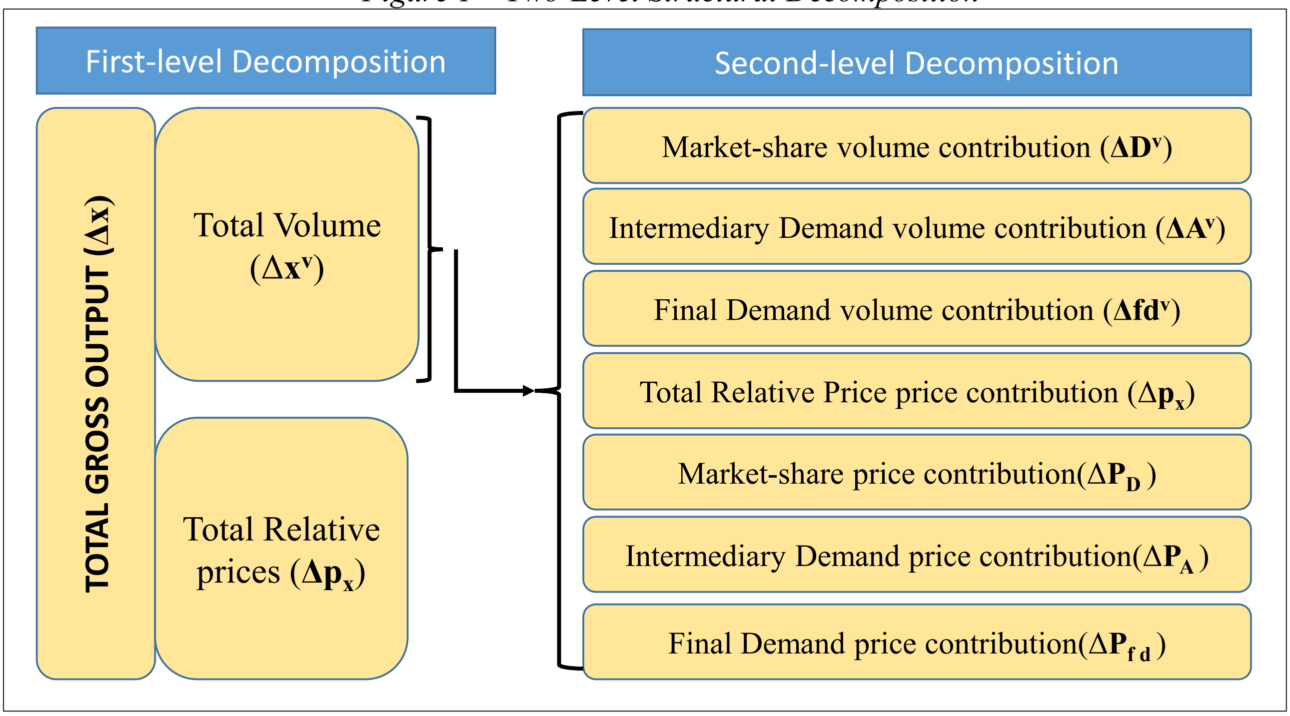
$$\mathbf{\Delta}(\hat{\mathbf{p}}_{\mathbf{x}}\mathbf{x}^{\mathbf{v}}) - \mathbf{\Delta}\left[\left(\mathbf{D}.\left(\mathbf{P}_{\mathbf{B}} \otimes \mathbf{B}^{\mathbf{v}} - \mathbf{P}_{\mathbf{B}_{\mathbf{m}}} \otimes \mathbf{B}^{\mathbf{v}}_{\mathbf{m}}\right)\right)\hat{\mathbf{p}}_{\mathbf{x}}\mathbf{x}^{\mathbf{v}}\right] = \mathbf{\Delta}\left[\mathbf{D}.\left(\hat{\mathbf{p}}_{\mathbf{d}_{\mathbf{F}}}\mathbf{d}_{\mathbf{F}}{}^{\mathbf{v}} - \hat{\mathbf{p}}_{\mathbf{d}_{\mathbf{F}_{\mathbf{m}}}}\mathbf{d}_{\mathbf{F}_{\mathbf{m}}}{}^{\mathbf{v}}\right)\right] \tag{12}$$

After some manipulations of the above equation, we arrive at the fundamental equation for the structural decomposition analysis of the change in $\mathbf{x}^{\mathbf{v}}$:

$$\mathbf{\Delta x}^{\mathbf{v}} = \mathbf{\delta Z}_{\mathbf{px}}{}^{-\mathbf{1}}\left[\underbrace{\left(\mathbf{\Delta A}^{\mathbf{v}}_{\mathbf{n}} + \mathbf{\Delta f}_{\mathbf{nd}}{}^{\mathbf{v}} + \mathbf{\Delta D}^{\mathbf{v}}\right)}_{\textit{volume contribution } (v)} + \underbrace{\left(\mathbf{\Delta}\hat{\mathbf{p}}_{\mathbf{x}} + \mathbf{\Delta P}_{\mathbf{B}_{\mathbf{n}}} + \mathbf{\Delta}\hat{\mathbf{p}}_{\mathbf{f}_{\mathbf{nd}}} + \mathbf{\Delta P}_{\mathbf{D}}\right)}_{\textit{price contribution } (\rho)} + \mathbf{\Delta}\boldsymbol{s}\right] \tag{13}$$

$\mathbf{\delta}_{\mathbf{Z}_{\mathbf{px}}}{}^{-\mathbf{1}}$ is a Leontief matrix weight and $\mathbf{\Delta}\boldsymbol{s}$ is the contribution of inventories (from final demand) to volume variation, and it is included in the decomposition just to maintain the validity of the subjacent identity. From now on, national final demand, excluded inventories, is denoted as $\mathbf{f}_{\mathbf{nd}}$. The equations definitions are presented at Appendix, and the following figure helps to understand de two-level decomposition proposed in this paper.

Figure 1 – Two-Level Structural Decomposition



Source: Author's elaboration.

4. Results

The results of the first-level decomposition from equation 20 above are reported in the Table 1 below, where they are compiled as contribution to gross output growth of each sector of the economy. Contribution to growth is an interesting form of reporting results. First, because it is a synthetic indicator that combines the influence of sectoral dynamism, as captured by the rate of growth of sectoral gross output, with the weight of each sector in the production structure of an economy, as captured by the gross output share of the sector. Secondly, because it allows us to aggregate among sectors, which is crucial both for disclosure of information and data analysis.

Table 1: First-Level Decomposition: contribution to Brazilian gross output growth between 2000-2015. (in p.p.)

Sectors	Volume (A)	Relative Prices (B)	Total (A+B)=C
Agriculture, fishing and related	3.02	-0.52	2.51
Industral Commodities	4.18	1.42	5.59
Processed Agricultural Commodities	0.32	-0.22	0.10
Traditional Industry	1.62	1.41	3.02
Innovative industry	2.34	-0.54	1.80
Public utility	1.74	-0.18	1.56
Construction	2.49	-0.36	2.14
Trade, accommodation and food	5.93	3.73	9.66
Transport, storage and communication	5.40	-1.70	3.70
Financial intermediation, insurance and real estate services	8.55	-4.39	4.16
Community, social and personal services	10.41	1.17	11.58
Total	46.00	-0.18	45.82

Source: Authors' elaboration based on Passoni & Freitas (2018a; 2018b) and IBGE (2015; 2016).

Throughout this section, the volume contribution in A column will be compared to total contribution in column C, as this allows us to isolate the quantitative contribution of volume changes of sectoral gross output, without the influence of relative prices changes. Regarding the latter, it is worth noticing that it only makes sense to interpret it, as a causal factor, for the whole of the economy, as its factor-wise effects are meaningless from the economic point of view.

Let us first note that, for the economy as whole, the relative price effect is not important. In fact, the accumulated rate of growth of total gross output in the period (45,82%) can be explained almost completely by the contribution of its volume component (46 percentage points, hereafter denoted pp), while the relative price component has a minor negative contribution of - 0,18 pp. The whole exercise of volume/relative price decomposition here proposed would seem not to be justified. However, as it can be seen from table 1, it turns out that this is only due to a balancing out of important sector-wise price effects.

Turning to the analysis of specific sectors, the "processed agricultural commodities" is the one in which the relative price effect seems to have the most important impact. It has a total contribution to growth of 0,10 pp, but excluding the first-level price effect, this number goes to 0,32 pp. Thus, in this case, the volume contribution amounts to 314% of the total contribution, and the latter, therefore, underestimates the true contribution of the sector's gross output growth. In contrast, the "Traditional Industry" has the strongest opposite effect, with first-level volume contribution representing only 53% of total contribution of the sector. This time, the relative price effect makes the total contribution an overestimated indicator of the growth process.

Taking all the four industrial sectors, the contribution to total gross output growth of 10,51 pp decreases to 8,46 pp when the relative price effect are put away, an almost 20% fall. In the Brazilian case, in which there is a vast literature discussing if the country has passed through a deindustrialization process and the importance of industry for economic growth, spotting this overestimation of role of industry due to relative prices may represent an important contribution to the discussion.

In a broad view of the economy, the first-level relative price effect seems to play an important role in all sectors. In every case, the total contribution underestimates or overestimates the volume contribution by more than 10%, and in 7 sectors, total contribution underestimates a more precise volume contribution to growth. This points out to the fact that, for the Brazilian economy, relative prices are indeed an important factor affecting the sectoral growth performance, and we should recognize its influence if we want a more accurate decomposition analysis.

Passing to the analysis of the second-level decomposition, as established in equation 21 above, the results are reported in Table 2 below. Here, the scenario is somewhat different. Recall, looking at the equations 10, 13 and 16 in the previous section, that the prices contribution to the volume variation is understood as the influence of the cell-specific price deflator divided by the sectoral price deflator. Intuitively, the change in the relative price between the different products demanded by each factor of demand can contribute for a lower or higher volume variation, and this is the effect that we aim to capture in the second-level decomposition. Accounting for it allows us to provide a even more precise assessment of the volume contribution to growth.

Table 2: Second-Level decomposition to gross output growth. (in p.p.)

Sectors	Volume (D)			
	Volume Contribution (D)	Prices contribution (E)	Inventories (F)	Total (D+E+F=A)
Agriculture, fishing and related	2.98	0.10	-0.05	3.02
Industrial Commodities	4.89	-0.21	-0.50	4.18
Processed Agricultural Commodities	0.16	0.24	-0.08	0.32
Traditional Industry	1.89	0.07	-0.34	1.62
Innovative industry	1.93	0.60	-0.20	2.34
Public utility	1.83	-0.06	-0.04	1.74
Construction	2.47	0.03	-0.01	2.49
Trade, accommodation and food	5.93	0.08	-0.08	5.93
Transport, storage and communication	4.97	0.52	-0.09	5.40
Financial intermediation, insurance and real estate services	8.51	0.09	-0.05	8.55
Community, social and personal services	10.41	0.08	-0.08	10.41
Total	45.97	1.55	-1.52	46.00

Source: Authors' elaboration based on Passoni & Freitas (2018a; 2018b) and IBGE (2015; 2016).

As discussed previously, Inventories are not considered in this study. Therefore, for the assessment of the importance of this second-level price effect, we will compare the D column to the sum of columns D and E, leaving aside the inventories (column F) contribution. Observe also, that the "Total" column connects Table 2 to Table 1.

However, before going on, it is worthwhile noticing that the aggregate effect is somewhat stronger than the first-level effect, 1.55 pp of contribution of the former compared to a - 0.18 pp contribution of the latter. Nevertheless, it remains minimum when compared to total volume contribution of 45.82 pp. In addition to that, the sector-wise effects are much smaller than the first-level one, and much more limited.

Following table 2 we can verify that there are two sectors in which the relative price effect is the main influence. First, the "Processed Agricultural Commodities" has a 0,16 pp volume contribution, compared to a total contribution of 0,40 pp. Thus, the latter contribution overestimates the volume contribution in almost 60%. Secondly, in the "Innovative industry" there is also an important overestimation of the volume contribution, the volume contribution being equal to 76,18% of the total one.

Apart from these two examples, though, the broad result is of a factor limited in importance, as in 8 of the 11 sectors the volume contribution represents a less than 5% difference from the total result. The scenario changes if we consider the input-output SDA for specific demand factors. In this case, the relative price adjustment is more relevant.

To see that, we concentrate now in the decomposition given by equation 21, in which the volume and price contributions of the second-level consider three sources of demand: Market-share, Final Demand and Intermediate Demand. In relation to their interpretation, Market-share factor is understood as impact that changes in the proportions of products in each sector have in the total production. Final and Intermediate Demand contribution to growth have rather straightforward interpretations. The results, as contribution to growth, are summarized in Table 3.

Table 3: Second-level decomposition for demand factos. (in p.p.)

Sectors	Intermediate			Final			Market Share		
	Volume Contribution	Prices contribution	Subtotal	Volume Contribution	Prices contribution	Subtotal	Volume Contribution	Prices contribution	Subtotal
Agriculture, fishing and related	0.03	-0.48	-0.45	2.96	0.11	3.07	-0.01	-0.07	-0.07
Industrial Commodities	-1.52	1.47	-0.05	6.21	-0.29	5.91	0.20	0.01	0.22
Processed Agricultural Commodities	-0.53	0.23	-0.30	0.64	-0.09	0.55	0.05	-0.12	-0.07
Traditional Industry	-1.40	0.67	-0.73	3.43	1.00	4.43	-0.14	-0.22	-0.35
Innovative industry	-1.07	0.67	-0.40	3.39	-0.78	2.61	-0.39	0.16	-0.23
Public utility	0.25	-0.12	0.13	1.48	-0.12	1.37	0.09	-0.01	0.08
Construction	-0.09	0.17	0.07	2.63	-0.54	2.09	-0.06	0.04	-0.02
Trade, accommodation and food	-0.59	2.03	1.44	6.53	1.59	8.12	-0.01	0.19	0.18
Transport, storage and communication	-0.20	-0.34	-0.55	5.34	-0.98	4.37	-0.17	0.13	-0.04
Financial intermediation, insurance and real estate services	-0.27	-1.12	-1.39	8.74	-3.22	5.52	0.04	0.01	0.05
Community, social and personal services	-0.61	-0.07	-0.68	10.83	1.47	12.30	0.19	-0.18	0.01
Total	-6.01	3.11	-2.90	52.19	-1.84	50.35	-0.21	-0.04	-0.25

Source: Authors' elaboration based on Passoni & Freitas (2018a; 2018b) and IBGE (2015; 2016).

Decomposed in this way, second-level relative prices effect has a much more widespread relevance, which becomes clear only now because it is balanced out between the different demand factors. For Intermediate Demand factor, the consideration of second-level price effect alters de contribution of all sectors by more than 10% difference. For Final Demand contribution, the same is true for 8 sectors and, in the case do Market-share contribution, this happens to 10 sectors. In addition, the scale of the differences are much more pronounced. Intermediate demand for "Industrial Commodities" production, for instance, has a contribution to growth of only -0,05 pp, but when second-level relative price effect is put away, this number grows to -1,52 pp, almost a thirty-fold increase.

Another meaningful characteristic is the changes that occur in the signal of the volume effect when we consider de price change effect. This fact may lead to qualitative misinterpretations of the role of individual sectors to the overall growth process, of a positive contribution of volume to growth turning negative because of influence of relative prices, and vice-versa. It turns out that this happens 5 times throughout the demand-factor decomposition, which means that in 15% of the cases, the volume contribution of first-level decomposition shows the opposite effect that it actually has, that is growth instead of decrease, and vice-versa.

Accounting for price effects seems to be relevant also to understand the importance of different sectors to the growth process, as it influences sectors heterogeneously. For instance, the total contribution of "Industrial Commodities" is the 3rd more important, but when first-level price effect is put aside, it becomes only the 5th more important. The same occurs to "Traditional

Industry", where it goes from 6th position to 10th. Now, the "Public Utility" case is an interesting one when it comes down to the importance of a full relative price contribution analysis. If we stopped the analysis in the first-level stage, it would show a rather irrelevant sector, only the 10th, becoming an important 3rd place sector in growth contribution. When we account for the second-level price effect, though, the previous increase vanishes and the sector goes back to the 10th position.

5. Concluding remarks

This paper assessed the importance of relative prices in the context of IO Structural Decomposition Analysis. Although there are some evidence in the literature pointing to the irrelevance of the question, this does not necessarily hold for economies with strong price movements such as the Brazilian.

For measuring this influence, we suggested a two-level decomposition for considering away relative price contribution: the first one applied to sectoral gross output, while the second one considered the influence of relative prices within the volume contribution to growth. In a second moment, this second influence was broken down in three IO demand components: Market-share, Intermediary Demand and Final Demand. It was found that, even though the relative price effect has a small aggregate impact, the first-level relative price decomposition has meaningful effects in every sector of the economy. Concerning the second-level decomposition, on the other hand, sector-wise effects found were much smaller and more limited. However, decomposing it further into the IO demand components brought different results, with second-level relative price effects having widespread and heterogeneous relevance among the sectors and demand components.

In general, it was found that, in an IO framework, not considering relative prices in growth analysis may lead to important misunderstanding of growth process. Even though the paper focused in the Brazilian economy, the result found may apply for all economies in which occur important relative price fluctuation.

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APPENDIX A
Model decomposition

To this decomposition, we apply the difference between all the variables at the final and at initial point on (18) to find $\mathbf{\Delta x^v}$ and its *volume* and *price effect contribution*. So we have:

$$\Delta(\hat{\mathbf{p}}_{\mathbf{x}}\mathbf{x^v}) - \Delta\left[\left(\mathbf{D}.\left(\mathbf{P_B}\otimes\mathbf{B^v}-\mathbf{P_{B_m}}\otimes\mathbf{B^v_m}\right)\right)\hat{\mathbf{p}}_{\mathbf{x}}\mathbf{x^v}\right] = \Delta\left[\mathbf{D}.\left(\hat{\mathbf{p}}_{\mathbf{d_F}}\mathbf{d_F}^{\mathbf{v}}-\hat{\mathbf{p}}_{\mathbf{d_{F_m}}}\mathbf{d_{F_m}}^{\mathbf{v}}\right)\right]$$

The structural decomposition is found after doing all the methodological procedures, following Dietzenbacher & Los (1998) and Miller & Blair (2009), and can be seen at the following equation:

$$\mathbf{\Delta x^v} = \boldsymbol{\delta}_{Z_{px}}{}^{-1}\left[(\mathbf{\Delta A^v_n}+\mathbf{\Delta f_{nd}}^{\mathbf{v}}+\mathbf{\Delta D^v})+\left(\Delta\hat{\mathbf{p}}_{\mathbf{x}}+\mathbf{\Delta P_{A_n}}+\Delta\hat{\mathbf{p}}_{\mathbf{f_{nd}}}+\mathbf{\Delta P_D}\right)+\mathbf{\Delta s}\right]$$

where:

- **Leontief ponderation:**

$$\boldsymbol{\delta}_{Z_{px}} = \left\{\frac{1}{2}\left(\hat{\mathbf{p}}_{\mathbf{x}}{}^{\mathbf{1}}+\hat{\mathbf{p}}_{\mathbf{x}}{}^{\mathbf{0}}\right)-\frac{1}{2}\left[\left(\left(\mathbf{P^1_D}\otimes\mathbf{D^{v1}}\right).\left(\mathbf{P^1_{B_n}}\otimes\mathbf{B^{v1}_n}\right)\right)\hat{\mathbf{p}}^{\mathbf{1}}_{\mathbf{x}}+\left(\mathbf{P^0_D}\otimes\mathbf{D^{v0}}.\left(\mathbf{P^0_{B_n}}\otimes\mathbf{B^{v0}_n}\right)\right)\hat{\mathbf{p}}^{\mathbf{0}}_{\mathbf{x}}\right]\right\}$$

VOLUME EFFECT

- **National input coefficients**

  The changes on the matrix of national coefficients ($\mathbf{\Delta A^v_n}$) is expressed by:

$$\mathbf{\Delta A^v_n} = \frac{1}{2}\left\{\frac{1}{2}\left[\left(\left(\mathbf{P^1_D}\otimes\mathbf{D^{v1}}\right)+\left(\mathbf{P^0_D}\otimes\mathbf{D^{v0}}\right)\right)\left(\frac{\mathbf{1}}{\mathbf{2}}\left(\left(\mathbf{P^1_{B_n}}+\mathbf{P^0_{B_n}}\right)\otimes\mathbf{\Delta B^v_n}\right)\right)\right]\left(\hat{\mathbf{p}}^{\mathbf{1}}_{\mathbf{x}}\mathbf{x^{v^1}}+\hat{\mathbf{p}}^{\mathbf{0}}_{\mathbf{x}}\mathbf{x^{v^0}}\right)\right\}$$

  Notice that we denote the changes in $\mathbf{A^v_n}$ but as the transitional matrix are expressed in the dimension commodity-by-sector, it in fact represents the chance in $\mathbf{B^v_n}$.

- **Final demand**

  National final demand is the may be expressed by:

$$\mathbf{\Delta f_{nd}}^{\mathbf{v}} = \frac{1}{2}\left[\left(\left(\mathbf{P^1_D}\otimes\mathbf{D^{v1}}\right)+\left(\mathbf{P^0_D}\otimes\mathbf{D^{v0}}\right)\right)\left(\frac{\mathbf{1}}{\mathbf{2}}\left(\left(\hat{\mathbf{p}}^{\mathbf{1}}_{\mathbf{d_{f_{nd}}}}+\hat{\mathbf{p}}^{\mathbf{0}}_{\mathbf{d_{f_{nd}}}}\right)\mathbf{\Delta d_{f_{nd}}}^{\mathbf{v}}\right)\right)\right]$$

  Notice that we denote $\mathbf{f^v_{nd}}$ but as the transitional matrix are expressed in the dimension commodity-by-sector, it in fact represents the chance in $\mathbf{d^v_{f_{nd}}}$.

- **Market share matrix**

  As all the transitional matrix are expressed at commodity-by-sector dimension, the variation of the Market share matrix includes its variation sized by all the variables on the model (change on intermediate and final demand, excluded inventories). As this matrix does not have an important economic meaning, its change is not open by national and imported.

$$\mathbf{\Delta D^v} = \frac{1}{2}\left\{\frac{1}{2}\left[\left(\frac{1}{2}\left(\left(\mathbf{P^1_D}+\mathbf{P^0_D}\right)\otimes\mathbf{\Delta D^v}\right)\right)\left(\left(\mathbf{P^1_{B_n}}\otimes\mathbf{B^{v1}_n}\right)+\left(\mathbf{P^0_{B_n}}\otimes\mathbf{B^{v0}_n}\right)\right)\right]\left(\hat{\mathbf{p}}^{\mathbf{1}}_{\mathbf{x}}\mathbf{x^{v^1}}+\hat{\mathbf{p}}^{\mathbf{0}}_{\mathbf{x}}\mathbf{x^{v^0}}\right)\right\}$$
$$+\frac{1}{2}\left[\left(\frac{1}{2}\left(\left(\mathbf{P^1_D}+\mathbf{P^0_D}\right)\otimes\mathbf{\Delta D^v}\right)\right)\left(\left(\hat{\mathbf{p}}^{\mathbf{1}}_{\mathbf{d_{f_{nd}}}}\mathbf{d_{f_{nd}}}^{\mathbf{v1}}\right)+\left(\hat{\mathbf{p}}^{\mathbf{0}}_{\mathbf{d_{f_{nd}}}}\mathbf{d_{f_{nd}}}^{\mathbf{v0}}\right)\right)\right]$$

PRICE EFFECT (ρ)

- **Total prices**

  Represents the effect of total relative prices ($\hat{\mathbf{p}}_{\mathbf{x}}$) in volume contribution to volume gross output.

$$\Delta\hat{\mathbf{p}}_{\mathbf{x}} = \frac{\mathbf{1}}{\mathbf{2}}\left[\left(\mathbf{P^1_D}\otimes\mathbf{D^{v1}}.\left(\mathbf{P^1_{B_n}}\otimes\mathbf{B^{v1}_n}\right)\right)\Delta\hat{\mathbf{p}}_{\mathbf{x}}\mathbf{x^{v0}}\right]+\frac{\mathbf{1}}{\mathbf{2}}\left[\left(\mathbf{P^0_D}\otimes\mathbf{D^{v0}}.\left(\mathbf{P^0_{B_n}}\otimes\mathbf{B^{v0}_n}\right)\right)\Delta\hat{\mathbf{p}}_{\mathbf{x}}\mathbf{x^{v1}}\right]$$
$$-\frac{\mathbf{1}}{\mathbf{2}}\Delta\hat{\mathbf{p}}_{\mathbf{x}}\left(\mathbf{x^{v1}}+\mathbf{x^{v0}}\right)$$

- **National input coefficients prices ($\mathbf{\Delta P_{A_n}}$)**

Notice that we denote $\mathbf{P}_{\mathbf{A}_\mathbf{n}}$ but as the transitional matrix are expressed in the dimension commodity-by-sector, it in fact represents the chance in $\mathbf{P}_{\mathbf{B}_\mathbf{n}}$.

$$\Delta\mathbf{P}_{A_n} = \frac{1}{2}\left\{\frac{1}{2}\left[\left(\left(\mathbf{P}_\mathbf{D}^1 \otimes \mathbf{D}^{v1}\right) + \left(\mathbf{P}_\mathbf{D}^0 \otimes \mathbf{D}^{v0}\right)\right)\left(\frac{1}{2}\left(\Delta\mathbf{P}_{\mathbf{B}_\mathbf{n}} \otimes \left(\mathbf{B}_\mathbf{n}^{v1} + \mathbf{B}_\mathbf{n}^{v0}\right)\right)\right)\right]\left(\hat{\mathbf{p}}_\mathbf{x}^1\mathbf{x}^{v^1} + \hat{\mathbf{p}}_\mathbf{x}^0\mathbf{x}^{v^0}\right)\right\}$$

- **Final demand prices**

$$\Delta\hat{\mathbf{p}}_{\mathbf{f}_{\mathbf{nd}}} = \left\{\frac{1}{2}\left[\left(\left(\mathbf{P}_\mathbf{D}^1 \otimes \mathbf{D}^{v1}\right) + \left(\mathbf{P}_\mathbf{D}^0 \otimes \mathbf{D}^{v0}\right)\right)\left(\frac{1}{2}\left(\Delta\hat{\mathbf{p}}_{\mathbf{d}_{\mathbf{f}_{\mathbf{nd}}}} \otimes \left({\mathbf{d}_{\mathbf{f}_{\mathbf{nd}}}}^{v1} + {\mathbf{d}_{\mathbf{f}_{\mathbf{nd}}}}^{v0}\right)\right)\right)\right]\right\}$$

- **Market share matrix prices**

$$\Delta\mathbf{P}_\mathbf{D} = \frac{1}{2}\left\{\frac{1}{2}\left[\left(\frac{1}{2}\left(\Delta\mathbf{P}_\mathbf{D} \otimes \left(\mathbf{D}^{v1} + \mathbf{D}^{v0}\right)\right)\right)\left(\left(\mathbf{P}_{\mathbf{B}_\mathbf{n}}^1 \otimes \mathbf{B}_\mathbf{n}^{v1}\right) + \left(\mathbf{P}_{\mathbf{B}_\mathbf{n}}^0 \otimes \mathbf{B}_\mathbf{n}^{v0}\right)\right)\right]\left(\hat{\mathbf{p}}_\mathbf{x}^1\mathbf{x}^{v^1} + \hat{\mathbf{p}}_\mathbf{x}^0\mathbf{x}^{v^0}\right)\right\}$$
$$+ \frac{1}{2}\left[\left(\frac{1}{2}\left(\Delta\mathbf{P}_\mathbf{D} \otimes \left(\mathbf{D}^{v1} + \mathbf{D}^{v0}\right)\right)\right)\left(\left(\hat{\mathbf{p}}_{\mathbf{d}_{\mathbf{f}_{\mathbf{nd}}}}^1 \otimes {\mathbf{d}_{\mathbf{F}_{\mathbf{nd}}}}^{v1}\right) + \left(\hat{\mathbf{p}}_{\mathbf{d}_{\mathbf{f}_{\mathbf{nd}}}}^0 \otimes {\mathbf{d}_{\mathbf{f}_{\mathbf{nd}}}}^{v0}\right)\right)\right]$$

***INVENTORIES* (Δs)**

To do an empirical adjustment on the decomposition, inventories are calculated separately. Here we have the volume and price effect, and the chance in the Market share matrix associated with this final demand component.

$$\Delta\mathbf{s} = \left\{\frac{1}{2}\left[\left(\left(\mathbf{P}_\mathbf{D}^1 \otimes \mathbf{D}^{v1}\right) + \left(\mathbf{P}_\mathbf{D}^0 \otimes \mathbf{D}^{v0}\right)\right)\left(\frac{1}{2}\left(\Delta\hat{\mathbf{p}}_{\mathbf{d}_\mathbf{s}} \otimes \left({\mathbf{d}_\mathbf{s}}^{v1} + {\mathbf{d}_s}^{v0}\right)\right)\right)\right]\right\}$$
$$+ \frac{1}{2}\left[\left(\frac{1}{2}\left(\left(\mathbf{P}_\mathbf{D}^1 + \mathbf{P}_\mathbf{D}^0\right) \otimes \Delta\mathbf{D}^v\right)\right)\left(\left(\hat{\mathbf{p}}_{\mathbf{d}_\mathbf{s}}^1 {\mathbf{d}_\mathbf{s}}^{v1}\right) + \left(\hat{\mathbf{p}}_{\mathbf{d}_\mathbf{s}}^0 {\mathbf{d}_\mathbf{s}}^{v0}\right)\right)\right]$$
$$+ \frac{1}{2}\left[\left(\frac{1}{2}\left(\Delta\mathbf{P}_\mathbf{D} \otimes \left(\mathbf{D}^{v1} + \mathbf{D}^{v0}\right)\right)\right)\left(\left(\hat{\mathbf{p}}_{\mathbf{d}_s}^1 \otimes {\mathbf{d}_{\mathbf{F}_\mathbf{n}}}^{v1}\right) + \left(\hat{\mathbf{p}}_{\mathbf{d}_\mathbf{s}}^0 \otimes {\mathbf{d}_\mathbf{s}}^{v0}\right)\right)\right]$$