Investigating double counting terms in the value-added decomposition of gross exports

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1. Introduction

To better understand the fragmentation of production and trade in the context of global value chains (Gereffi and Fernandez-Stark, 2016), a series of papers propose a decomposition of gross exports in an inter-country input-output framework in order to identify the value-added contribution of all countries involved in the production process (Daudin et al., 2011; Johnson and Noguera, 2012; Koopman et al., 2014; Foster-McGregor and Stehrer, 2013; Los et al., 2016; Miroudot and Ye, 2017; Borin and Mancini, 2017; Johnson, 2018). One motivation for developing value-added measures of trade is to remove the 'double counting' in gross exports. In the input-output framework, the concept of 'double counting' comes from the measurement of intermediate inputs. Output is equal to (domestic) value-added plus intermediate inputs. But intermediate inputs are also produced with (domestic or foreign) value-added and other intermediate inputs. Double counting can be regarded as a subset of intermediate inputs in output decomposition.

Since gross exports correspond to the share of output sold to foreign consumers, there is also a 'double counting' involved. This double counting in intermediate inputs can be removed by looking at net trade (Trefler and Zhu, 2010) or by working with measures of value-added trade derived from final demand (Johnson and Noguera, 2012). But when authors start to introduce double counting terms in the decomposition of gross exports, things become more complicated since intermediate inputs are both part of exported goods and foreign inputs used in their production. Moreover, the concept of 'foreign value-added' in trade, which is the variable of interest to understand global production, leads to further questions on what is double counted. When looking at exports of all countries in the world, any foreign value-added is by definition double counted, since it is 'domestic value-added' in other countries. What authors try to define as double counting is therefore no longer the subset of intermediate inputs double counted in output but some share of value-added that would be counted several times from the point of view of the exporting economy, including in the foreign-value added term (something sometimes referred to as 'pure double counting').

There is no consensus yet on the definition of double counting terms in gross exports decompositions. Some authors, such as Koopman et al. (2014), Nagengast and Stehrer (2016) and Borin and Mancini (2017) propose to base the definition on the number of international border crossings. Also, Los and Timmer (2018) point out that the double counted domestic value-added is the sum of the bilateral domestic value-added across all partners minus the unilateral one (i.e. with partner world). Alternatively, Miroudot and Ye (2017) rely on a supply-side input-output model. In their framework, double counting terms can be measured by the second and later input rounds in the generation of value-added in exported goods using the Ghosh decomposition.

In this paper, we investigate more closely the concept of 'double counting' in the decomposition of gross exports. First, we show that while domestic value-added can be indeed 'double counted' in the domestic content of exports, the concept of foreign double counting is more complicated and does not always imply value-added counted twice from the point of view of the exporting economy. We review the existing literature and introduce a new decomposition framework (consistent with Los et al., 2016) to show that there are several possible answers to the definition of double counting in gross exports. Using numerical examples and calculations with the World Input-Output Database (WIOD), we suggest that these decompositions lead to a different economic interpretation and can answer different types of questions in relation to global production.

Section 2 discusses the concept of double counting in gross exports decompositions and how it was dealt with in previous papers. Section 3 introduces a new input-output framework that allows us to provide an alternative definition for double counting terms (consistent with Los et al., 2016). Section 4 develops numerical examples to illustrate how this decomposition compares to others in the literature and what we can learn through the double counting terms. Section 5 concludes.

2. Defining double counting terms in the decomposition of gross exports

In the framework developed by Koopman, Wang and Wei (2014), KWW hereafter, double counting is defined as the value-added that crosses international borders more than once. Therefore, all the foreign value-added is already double counted. It makes sense since the authors are interested in removing double counting from aggregate world trade statistics. In this case, any foreign value-added in exports is by definition domestic value-added in the exports of another country and double counted. In order to decompose gross exports of a specific country and to introduce a foreign value-added (FVA) term, the authors then refer to a 'pure' double counting, which is the difference between gross exports and the sum of domestic value-added (DVA) and FVA. This 'pure double counting' is then split between a domestic and foreign component so that at the end gross trade is decomposed into four terms: DVA, FVA, pure domestic double counting (DDC) and pure foreign double counting (FDC).[1] Defined as a residual, this pure double counting can be calculated but there is no clear interpretation of what it exactly measures. And since there is no underlying definition, one can also question why specific terms in the decomposition are interpreted as 'pure double counting'. We refer to the KWW approach for double counting as the 'first approach'.

Pointing out the issue with KWW, Borin and Mancini (2017) propose a different definition for the double counting. From the point of view of a specific exporting economy, double counting corresponds to the value-added that has crossed the country's border more than once. It is a better starting point but the issue with a definition of double counting based on the number of border crossings is that the input-output framework cannot tell us how many times value-added has crossed borders. The input-output matrix identifies international and domestic transactions but there are

[1] KWW have a total of 9 terms by further splitting DVA and FVA.

many paths through which value-added can reach final consumers and these paths are not known. They are summarized in a single input-output matrix that has collapsed the different production stages (Los and Timmer, 2018).

The definition that Borin and Mancini (2017) propose for double counting in the sense of value-added coming twice to the same economy is conceptually sound. But its implementation in the input-output framework is problematic. As we will formally show in the next Section, value-added ratios multiplied by the Leontief inverse can be used to measure value-added when it enters a specific country "for the first time" but before entering a specific country, this value-added has already crossed all possible borders according to the input-output table. Therefore, there is no clarity in terms of how many times borders are crossed. Moreover, the concept of 'border' is not the same when dealing with global exports (exports to the world) and bilateral exports. This further complicates the reference to border crossings in the definition of double counting.

In Miroudot and Ye (2017), this issue is avoided by relying on the supply-side input-output model to define double counting. The Ghosh insight already refers to different rounds in the process of value generation. There is, embedded in the model, the concept of an initial round and value-added measured in all later rounds is by definition double counted. This provides a theoretically founded measure and definition of double counting which is straightforward when it comes to its implementation in the context of an inter-country input-output table (to derive a foreign double counting). Since the supply-side input-output model and its underlying assumptions are not always well accepted, Miroudot and Ye (2018) show that the same decomposition of gross exports can be achieved through an "hypothetical extraction" method (as in Los et al., 2016) or by relying on the Leontief model. The Ghosh insight remains however a more intuitive way of introducing the concept of double counting.

Something common to Borin and Mancini (2017) and Miroudot and Ye (2017) is a definition of double counting that assumes that there is a first country where value-added is generated (and exported) and that any time this value-added is measured somewhere else in the exports of another country, it has to be regarded as part of the double counting terms. We refer to this approach as the 'second approach'. It is explicit in Miroudot and Ye (2017) but maybe less clear in the context of Borin and Mancini (2017) since they refer to value-added crossing twice the border of the *same* country. But we will show in Section 4 that the decompositions by Miroudot and Ye (2017) and Borin and Mancini (2017) provide the same results. We can also call this second approach the 'source-based approach', referring to the work of Nagengast and Stehrer (2016).[2]

Lastly, the paper by Los et al. (2016) is the only one that does not introduce double counting terms. It also has no explicit formula (or hypothetical extraction) for the foreign value-added. Nevertheless, the methodology it applies to derive the domestic value-added in gross exports can also be used to estimate a foreign value-added.[3] The difference between the sum of DVA and FVA in such framework also creates a residual that can be interpreted as a double counting. Even more interesting is the fact that this double counting is different from the one calculated by KWW and by Borin and Mancini (2017) or Miroudot and Ye (2017). We believe that this residual corresponds to the value-added coming actually twice to the exporting economy (domestic or foreign), thus providing a third type of double counting. In the next Section, we develop a new framework to calculate a domestic double counting term and foreign double counting term with this third approach based on an hypothetical extraction method.

3. A new framework to decompose gross exports with double counting defined as value-added coming twice to the exporting economy

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[2] Nagengast and Stehrer (2016) define the source-based approach as the one taking the perspective of the country in which value added originates, as opposed to a sink-based approach where the perspective is from the country which ultimately absorbs the value added in its final demand. This distinction works well in the context of value-added trade balances (the topic of the paper by Nagengast and Stehrer) but is more difficult to implement with gross exports decompositions where the country of final absorption is not always well known. The KWW approach comes close to a sink-based approach but does not fully work this way, as also highlighted by Nagengast and Stehrer (2016).

[3] We thank Bart Los for sharing with us insights on how it can be done, something that the authors of the paper had developed but was not included in the published version.

We start with the standard Leontief (1936) input-output framework extended to G countries and N sectors in an inter-country input-output (ICIO) table, as it is usually done in the trade in value-added literature. The basic input-output relationship states that all gross output must be used either as an intermediate good or as a final good:

$$\mathbf{X} = \mathbf{AX} + \mathbf{Y} \tag{1}$$

where, $\mathbf{X}$ is the $NG \times 1$ gross output vector, $\mathbf{Y}$ is the $NG \times 1$ final demand vector, and $\mathbf{A}$ is the $NG \times NG$ I-O coefficients matrix.

As previously emphasized, gross exports is a subset of gross output. Focusing on exports of a given country *i,* we can split the output vector into an exports vector $\mathbf{E}$ that has the length G times N with the exports for all industries in country i corresponding to elements $\mathbf{e_i}$ and zeros elsewhere: $\mathbf{E=[0,\ldots,\ e_i,\ldots,0]}$) and remaining term $\mathbf{H}$ $\mathbf{(X=E+H}$).

Then, the following accounting equations can be obtained: $\mathbf{E} = \mathbf{A^I(E+H)+Y^I}$ and $\mathbf{H} = \mathbf{A^*(E+H)+Y^*}$, where $\mathbf{A^I}$ is the given export measurement matrix including the IO coefficients for the use of intermediate inputs from one country into another country and $\mathbf{A^*}$ is the corresponding extraction matrix, so that we have $\mathbf{A} = \mathbf{A^I} + \mathbf{A^*}$. $\mathbf{Y^I}$ is the foreign final demand for the given exports and Y^D is the extraction final demand matrix, so that $\mathbf{Y} = \mathbf{Y^I} + \mathbf{Y^*}$.

Here, to better understand the structure of the matrix and its extraction, we give a simple example to show how to split the original $\mathbf{A}$ matrix for different exports. In the three country case (country *i*, *j* and *k*), the intermediate inputs coefficients matrix can be given by

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_{ii} & \mathbf{A}_{ij} & \mathbf{A}_{ik} \\ \mathbf{A}_{ji} & \mathbf{A}_{jj} & \mathbf{A}_{jk} \\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{A}_{kk} \end{pmatrix}$$

To identify gross exports for country *i*, exports flows from *i* to other countries should be identified in $\mathbf{A}$ matrix, so for the gross measurement, $\mathbf{A}^{\mathbf{I}}=\begin{pmatrix}\mathbf{0} & \mathbf{A}_{ij} & \mathbf{A}_{ik}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{pmatrix}$ and $\mathbf{A}^{*}=\begin{pmatrix}\mathbf{A}_{ii} & \mathbf{0} & \mathbf{0}\\ \mathbf{A}_{ji} & \mathbf{A}_{jj} & \mathbf{A}_{jk}\\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{A}_{kk}\end{pmatrix}$. If we measure the bilateral exports between country i and j, $\mathbf{A}^{\mathbf{I}}=\begin{pmatrix}\mathbf{0} & \mathbf{A}_{ij} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{pmatrix}$ and $\mathbf{A}^{*}=\begin{pmatrix}\mathbf{A}_{ii} & \mathbf{0} & \mathbf{A}_{ik}\\ \mathbf{A}_{ji} & \mathbf{A}_{jj} & \mathbf{A}_{jk}\\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{A}_{kk}\end{pmatrix}$. Especially, if we measure the global exports, the corresponding matrixes should be $\mathbf{A}^{\mathrm{I}}=\begin{pmatrix}\mathbf{0} & \mathbf{A}_{ij} & \mathbf{A}_{ik}\\ \mathbf{A}_{ji} & \mathbf{0} & \mathbf{A}_{jk}\\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{0}\end{pmatrix}$ and $\mathbf{A}^{*}=\begin{pmatrix}\mathbf{A}_{ii} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}_{jj} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{kk}\end{pmatrix}$.

After re-arrangement, the accounting relationship between exports vector and the final demand in destination countries in the ICIO model can be expressed as:

$$\mathbf{E}=\tilde{\mathbf{A}}\mathbf{E}+\tilde{\mathbf{Y}} \tag{2}$$

with $\tilde{\mathbf{Y}}=\mathbf{Y}^{\mathbf{I}}+\tilde{\mathbf{A}}\mathbf{Y}^{*}$ and $\tilde{\mathbf{A}}=\mathbf{A}^{\mathbf{I}}(\mathbf{I}-\mathbf{A}^{*})^{-1}$.

Each element of the $\tilde{\mathbf{A}}$ matrix describes how domestic intermediate goods are sent abroad (or transported domestically) to produce one unit of given exports product in foreign countries (or in the domestic economy). For example, the element $\tilde{\mathbf{A}}_{ji}$ ($N\times N$ matrix) means that in order to produce one unit of exports in country i, country j needs to produce $\tilde{\mathbf{A}}_{ji}$ units of intermediate inputs that are then embodied in domestic sales in country j. $\tilde{\mathbf{A}}_{ji}\mathbf{e}_i$ ($N\times 1$ vector) means that country j needs to produce $\tilde{\mathbf{A}}_{ji}\mathbf{e}_i$ intermediate inputs for given exports measurement $\mathbf{e}_i$ ($N\times 1$ vector) in country i, so we can call $\tilde{\mathbf{A}}$ as the 'direct exports requirements matrix'. Re-arranging equation (2) above, we obtain $\mathbf{E} = \tilde{\mathbf{B}}\tilde{\mathbf{Y}}$, and $\tilde{\mathbf{B}} = (\mathbf{I} - \tilde{\mathbf{A}})^{-1}$, similar to $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$ in the IO model. We can define matrix $\tilde{\mathbf{B}}$ as the 'total exports requirements matrix'.

For $\mathbf{e}_i$ ($N \times 1$ vector), the exports in country *i*, all the intermediate inputs needed are $\sum_j^G \tilde{\mathbf{A}}_{ji}\mathbf{e}_i$. We can thus calculate the value-added in exports in country *i* as $\mathbf{VaE}(i)^{\mathrm{T}} = \mathbf{E}_i - \sum_j^G \tilde{\mathbf{A}}_{ji}\mathbf{E}_i$ ($\mathbf{VaE}(i)$ is $1 \times N$ vector). This value-added does not only include country *i*'s value-added (domestic value-added) but also other countries' value-added (foreign value-added). We can then express the value-added multiplier coefficients in domestic sales in the form of a $1 \times NG$ vector $\tilde{\mathbf{V}}$, defined as:

$$\tilde{\mathbf{V}} = \mathbf{u}(\mathbf{I} - \tilde{\mathbf{A}}) = \mathbf{u}(\mathbf{I} - \mathbf{A})(\mathbf{I} - \mathbf{A}^*)^{-1} = \mathbf{V}(\mathbf{I} - \mathbf{A}^*)^{-1} \tag{3}$$

where $\mathbf{V}$ is a $1 \times NG$, direct value-added coefficients vector. Each element of $\mathbf{V}_i$ ($1 \times N$ vector) gives the share of direct domestic value-added in total output. It is equal to one minus the intermediate input share from all countries (including domestically produced intermediates): $\mathbf{V}_i = \mathbf{u}[\mathbf{I} - \sum_j^G \mathbf{A}_{ji}]$, where $\mathbf{u}$ is a $1 \times N$ unit vector. If we use the notation $\mathbf{B}^* = (\mathbf{I} - \mathbf{A}^*)^{-1}$, we obtain the expression for value-added coefficients in exports for country *i*: $\tilde{\mathbf{V}}_i = \mathbf{V}_i\mathbf{B}^*_{ii} + \sum_{j \neq i}^G \mathbf{V}_j\mathbf{B}^*_{ji}$. They can be divided into two parts: the value-added from country *i* (domestic value-added) $\mathbf{V}_i\mathbf{B}^*_{ii}$ and the value-added from other countries (foreign value-added) $\sum_{j \neq i}^G \mathbf{V}_j\mathbf{B}^*_{ji}$ [4].

[4] The expression for the domestic and foreign value-added measurement is consistent with Los, Timmer and de Vries (LTV, 2016). It's noteworthy that if we measure the global exports in this framework, the concepts of 'domestic' and 'foreign' wouldn't exist.

Coming back to Borin and Mancini (2017), the above value-added ratios times corresponding extraction matrix Leontief inverse elements ($\mathbf{V}_i\mathbf{B}_{ii}^*$ and $\sum_{j\neq i}^{G}\mathbf{V}_j\mathbf{B}_{ji}^*$) formulate explicitly value-added when it enters a specific country border and is embodied in exports for the "first time". But as previously emphasised, before entering a specific country, this value-added has already crossed all possible borders according to the input-output table.

To measure double counting as value-added coming twice to the exporting economy (the initial ambition in Borin and Mancini), we can use both the Leontief insight and the Ghosh insight.
In the Leontief insight, the total value-added coefficient (**VB**) matrix, or the total value-added multiplier as named in the input-output literature:

$$\mathbf{VB} = \tilde{\mathbf{V}}\tilde{\mathbf{B}} = \mathbf{VB}^*\tilde{\mathbf{B}} = \mathbf{VB}^*(\mathbf{I} + \mathbf{A}^{\mathbf{I}}\mathbf{B}) \tag{4}$$

The detailed proof of equation (4) is provided in the Appendix I. This equation explains the value-added distribution in our new framework: we already have the value-added measurement coefficient $\mathbf{VB}^*$, and then the residual term $\mathbf{VB}^*(\mathbf{A}^{\mathbf{I}}\mathbf{B})$. The implication of the residual term is straightforward: because $\mathbf{A}^{\mathbf{I}}$ is the extracted elements matrix of corresponding exports, which means ICIO coefficients for the use of intermediate inputs from one country into another country. Meanwhile, it can be used to introduce the concept of 'country borders' for the measurement of exports in the ICIO: the borders between the exporting country and other countries (while for bilateral exports, it means the border between two given countries). The coefficient $\mathbf{A}^{\mathbf{I}}\mathbf{B}$ points at flows crossing the same exporting economy twice. Therefore, the coefficient $\mathbf{VB}^*(\mathbf{A}^{\mathbf{I}}\mathbf{B})$ can be understood as value-added that has crossed the given country's border more than once, which is already accounted in the $\mathbf{VB}^*$ expression.

The same expression can be derived using the Ghosh insight. In the supply-side IO model, output coefficients are defined as $l_{ij} = x_{ij} / x_i$. An output coefficient gives the percentage of output of industry *i* that is sold to industry *j*. The accounting equation can be rewritten as:

$$\mathbf{X}^{\mathbf{T}} = \mathbf{VA}^{\mathbf{T}} + \mathbf{X}^{\mathbf{T}}\mathbf{L} = \mathbf{VA} \times \mathbf{G} \tag{5}$$

where $\mathbf{G} = (\mathbf{I} - \mathbf{L})^{-1}$ is the Ghosh inverse; meanwhile, in $\mathbf{G} = \hat{\mathbf{X}}^{-1}\mathbf{B}\hat{\mathbf{X}}$, $\hat{\mathbf{X}}$ is a $NG \times NG$ diagonal matrix with output on the diagonal.

Transposing the model to the 'export ICIO table' we have described above, exports can be written as $\mathbf{E}^{\mathbf{T}} = \mathbf{VaE}^{\mathbf{T}} + \mathbf{E}^{\mathbf{T}}\tilde{\mathbf{L}} = \mathbf{VaE}^{\mathbf{T}} \times \tilde{\mathbf{G}}$. Here $\tilde{\mathbf{G}} = \hat{\mathbf{E}}^{-1}\tilde{\mathbf{B}}\hat{\mathbf{E}}$, $\tilde{\mathbf{L}} = \hat{\mathbf{E}}^{-1}\tilde{\mathbf{A}}\hat{\mathbf{E}}$.

To illustrate the relationship between exports measurement and value-added, we can refer to the Taylor expansion.

$$\mathbf{E}^{\mathbf{T}} = \mathbf{VaE}^{\mathbf{T}}(\mathbf{I} + \tilde{\mathbf{L}} + \tilde{\mathbf{L}}^2 + \tilde{\mathbf{L}}^3 + \mathbf{L}) = \mathbf{VB}^*(\mathbf{I} + \mathbf{A}^{\mathbf{I}}\mathbf{B})\hat{\mathbf{E}} \tag{6}$$

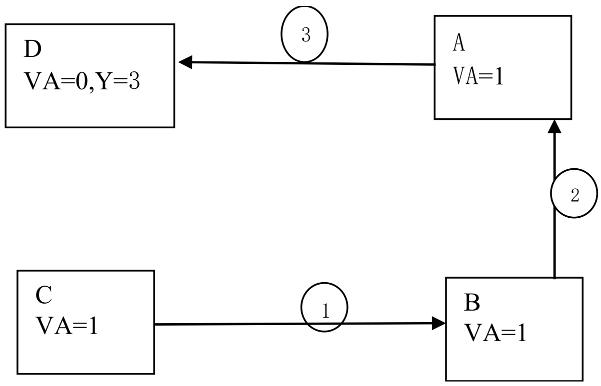
As before, we use the traditional concepts of input-output analysis linking output and value-added, transposed to the relationship between gross exports and value-added. The export value $\mathbf{E}^{\mathbf{T}}$ can be decomposed into different rounds where value is added. In particular, we can distinguish three value-added inputs: an initial input $\mathbf{VaE}^{\mathbf{T}}$, and indirect inputs in subsequent rounds amounting to $\mathbf{VaE}^{\mathbf{T}}(\tilde{\mathbf{L}} + \tilde{\mathbf{L}}^2 + \tilde{\mathbf{L}}^3 + \cdots)$.

The proof of equation (6) and other details of derivation are in the Appendix II. The above equation shows the consistency with the result obtained with the Leontief insight. It should be noted that the initial round already provides the domestic and foreign value-added in exports. Also, the Ghosh insight offers an alternative interpretation for the 'residual' or why we have further value-added in the measurement and why we can reasonably call it 'double counting'. Since the initial rounds have already exhausted the domestic and foreign value-added in the measurement of exports, what we measure as domestic value-added and foreign value-added in the later rounds of equation (6) -when continuing the Taylor expansion- is something that was already measured in the initial round.

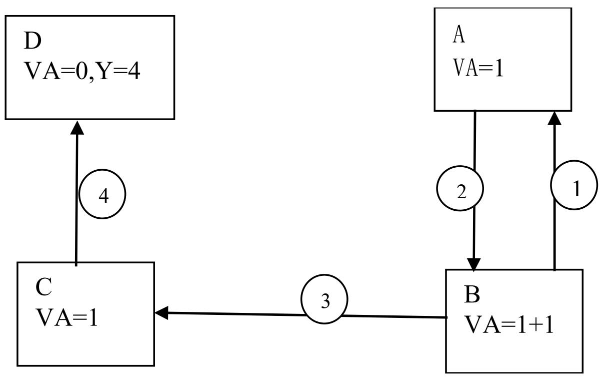
4. Numerical examples and empirical analysis

In this section, we first provide several simple numerical examples to compare the various frameworks we have reviewed and the three approaches we identified when it comes to the definition of double counting. We work with 3 examples of global value chains described as follows:

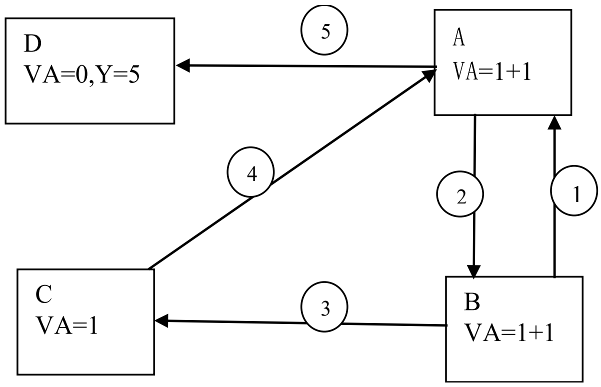
Case 1: country C exports 1 unit to country B, then B exports 2 units to country A (using as input the production of country C), then A exports 3 units to country D (using as input the production of country B) that are finally absorbed by D. The value chain can be represented as below:



Case 2: country B exports 1 unit to country A at the beginning, then A exports 2 units back to country B, then B re-exports 3 units to country C, then C exports 4 units to country D, finally absorbed by D.



Case 3: this case is similar to the previous one but with a simple modification. For the fourth step in the value chain, country C now exports 4 units back to country A again, then A exports 5 units to country D, finally absorbed by D.



The ICIO tables corresponding to these examples are provided in the Appendix III. Next, we show results for the decomposition of gross exports grouped into the three approaches previously identified. The first approach is the one found in KWW (close to a 'sink-based' approach). The second approach is the 'source-based' approach from Borin and Mancini (2017) and Miroudot and Ye (2017). Johnson (2018) has also equations that would fall under this category but his paper does not include a full decomposition of gross exports for G countries (but only for two countries). The third approach is the one presented in this paper and based on value-added crossing twice or more the same exporting country. It is also consistent with Los et al. (2016) or an extended version that would include a foreign value-added term.

Table 1: Decomposition of Case1

	The first approach				The second approach				The third approach			
Gross exports	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC
3	1	0	2	0	1	0	1	1	1	0	2	0
2	1	0	0	1	1	0	1	0	1	0	0	0
1	1	0	0	0	1	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Table 2: Decomposition of Case2

	The first approach				The second approach				The third approach			
Gross exports	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC
2	1	0.33	0	0.67	1	0.33	0.5	0.17	1	0.33	0.5	0.17
4	2	0.67	0	1.33	2	0.67	1	0.33	2	0.67	1	0.33
4	1	0	3	0	1	0	1.5	1.5	1	0	3	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3: Decomposition of Case3

	The first approach				The second approach				The third approach			
Gross exports	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC
7	2	0.8	3	1.2	2	0.8	1.5	2.7	2	0.8	3	1.2
4	2	0.8	0	1.2	2	0.8	0.57	0.63	2	0.8	0.86	0.34
4	1	0.3	0	2.7	1	0.3	1.5	1.2	1	0.3	2.08	0.62
0	0	0	0	0	0	0	0	0	0	0	0	0

The results illustrate the advantages and disadvantages of the various decompositions. First, it should be noted that all decompositions agree on the calculation of domestic value-added and its double counting term. The results are the same across the three approaches. However, the allocation of the rest of gross exports to foreign value-added and foreign double counting is very different from one approach to another.

As the first approach (KWW) is close to a 'sink-based approach', the measurement of FVA and FDC depends more on the country of absorption. When value added crosses more than one country and is still not finally absorbed, KWW counts this value-added as double counting. This is reflected in the gross exports decomposition of country B in all cases and of country C in case 3. Because the export flow is not absorbed by the direct export target country, the value of FVA is 0 in the KWW framework. As such, this approach leads to counter-intuitive results with high values in the foreign double counting terms.

The second approach (source-based) also leads to high values for FDC, but in this case the explanation is clearer and more logical. When value added has already crossed a border and is measured a second time in the exports of another country, this value added contributes to the foreign double counting even if it has never crossed the exporting economy in which this double counting is identified. This is illustrated with the decomposition of gross exports in country A in case 1, country C in case 2, or countries B and C in case 3. From these examples, we can see that the definition is not about value added crossing twice the same border but more about value added being measured twice in the value added generation.

The third approach (presented in this paper and consistent with Los et al., 2016) is the one that actually takes the perspective of the exporting economy for which gross exports are decomposed and where the double counting term is really about value added coming twice to this same economy. In case 1, country A's exports have 2 units of FVA with the third approach because inputs coming from C never crossed the border with A. it was not the case with the source-based approach (second approach) where VA was split between FVA and FDC.

Additional results using the WIOD database

Simple numerical examples are useful to understand differences across decompositions but one could argue that actual GVCs are more complex and that maybe these differences are exaggerated using the simple above examples. But this is not the case. In Table 4, we provide results according to the three approaches in the context of the full WIOD tables with 44 countries for the year 2014 (Timmer et al., 2015).

Table 4: Decomposition of gross exports, % (WIOD, 2014)

		The first approach				The second approach				The third approach			
	Gross exports (million USD)	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC
AUS	287161.82	85.83	0.14	10.08	3.95	85.83	0.14	10.47	3.56	85.83	0.14	14.01	0.02
AUT	210995.34	63.86	0.29	23.24	12.61	63.86	0.29	24.7	11.15	63.86	0.29	35.65	0.21
BEL	383013.8	53.96	0.39	30.81	14.84	53.96	0.39	32.71	12.94	53.96	0.39	45.21	0.44
BGR	31698.3	61.81	0.03	25.51	12.65	61.81	0.03	28.02	10.14	61.81	0.03	38.13	0.03
BRA	270262.89	87.16	0.06	9.69	3.09	87.16	0.06	9.69	3.09	87.16	0.06	12.77	0.01
CAN	563511.41	75.77	0.42	20.29	3.52	75.77	0.42	19.03	4.77	75.77	0.42	23.68	0.12
CHE	352569.59	74.48	0.2	19.96	5.37	74.48	0.2	18.29	7.03	74.48	0.2	25.23	0.09
CHN	2425464.4	83.15	0.94	12.69	3.22	83.15	0.94	11.68	4.23	83.15	0.94	15.69	0.23
CYP	9346.89	71.94	0.04	17.14	10.87	71.94	0.04	20.12	7.9	71.94	0.04	28	0.02
CZE	161569.69	54.02	0.33	30.34	15.31	54.02	0.33	30.73	14.92	54.02	0.33	45.36	0.29
DEU	1682252.9	71.85	1.39	19.22	7.53	71.85	1.39	18.77	7.98	71.85	1.39	26.12	0.63
DNK	170292.92	62.47	0.17	28.99	8.37	62.47	0.17	27.31	10.05	62.47	0.17	37.26	0.1
ESP	389005.3	68.87	0.26	23.02	7.84	68.87	0.26	22.56	8.3	68.87	0.26	30.71	0.16
EST	18266.2	56.55	0.09	30.77	12.59	56.55	0.09	28.83	14.53	56.55	0.09	43.28	0.08
FIN	100453.27	64.97	0.12	24.01	10.9	64.97	0.12	25.83	9.07	64.97	0.12	34.82	0.09
FRA	759654.36	72.28	0.46	19.96	7.3	72.28	0.46	19.44	7.82	72.28	0.46	27.06	0.2
GBR	751599.24	80.74	0.29	13.7	5.27	80.74	0.29	13.84	5.13	80.74	0.29	18.89	0.08
GRC	56260.59	69.58	0.04	22.61	7.77	69.58	0.04	23.19	7.19	69.58	0.04	30.35	0.02
HRV	23268.55	72.68	0.05	19.36	7.91	72.68	0.05	19.37	7.9	72.68	0.05	27.25	0.02
HUN	116445.03	48.13	0.16	35.84	15.87	48.13	0.16	35.46	16.25	48.13	0.16	51.51	0.2
IDN	210599.3	82.74	0.11	13.15	3.99	82.74	0.11	12.61	4.54	82.74	0.11	17.13	0.02
IND	369456.46	79.28	0.11	15.78	4.82	79.28	0.11	16.13	4.47	79.28	0.11	20.57	0.04

IRL	262751.15	50.65	0.13	39.39	9.83	50.65	0.13	41.7	7.53	50.65	0.13	49.12	0.1
ITA	588585.23	73.63	0.32	18.94	7.11	73.63	0.32	18.5	7.56	73.63	0.32	25.91	0.14
JPN	817514.18	76.41	0.32	17.19	6.09	76.41	0.32	17.89	5.38	76.41	0.32	23.15	0.12
KOR	697935.06	64.79	0.35	26.03	8.84	64.79	0.35	26.74	8.13	64.79	0.35	34.65	0.22
LTU	32722.53	64.29	0.05	24.9	10.76	64.29	0.05	27.42	8.24	64.29	0.05	35.61	0.05
LUX	118439.4	33.96	0.08	49.29	16.67	33.96	0.08	57.23	8.72	33.96	0.08	65.79	0.16
LVA	14718.87	68.98	0.1	21.87	9.04	68.98	0.1	20.78	10.14	68.98	0.1	30.87	0.05
MEX	368185.26	66.44	0.26	29.7	3.59	66.44	0.26	25.43	7.86	66.44	0.26	33.17	0.12
MLT	13420.37	34.51	0.03	51.53	13.93	34.51	0.03	44.67	20.79	34.51	0.03	65.39	0.07
NLD	575067.62	63.15	0.8	23.84	12.2	63.15	0.8	26.22	9.83	63.15	0.8	35.6	0.45
NOR	188130.65	82.96	0.25	10.88	5.91	82.96	0.25	12.16	4.64	82.96	0.25	16.75	0.04
POL	251641.57	69.04	0.27	20.82	9.87	69.04	0.27	21.52	9.18	69.04	0.27	30.56	0.13
PRT	76632.96	68.84	0.09	22.42	8.65	68.84	0.09	21.47	9.6	68.84	0.09	31.01	0.06
ROU	77647.74	73.31	0.07	18.17	8.46	73.31	0.07	18.35	8.28	73.31	0.07	26.59	0.03
RUS	493789.09	92.36	0.14	4.86	2.64	92.36	0.14	5.27	2.22	92.36	0.14	7.49	0.01
SVK	82119.46	51.86	0.2	33.75	14.18	51.86	0.2	30.87	17.06	51.86	0.2	47.72	0.22
SVN	30812.48	62.63	0.08	25.29	12	62.63	0.08	25.15	12.15	62.63	0.08	37.24	0.05
SWE	235353.74	71.2	0.28	19.81	8.71	71.2	0.28	20.75	7.77	71.2	0.28	28.38	0.14
TUR	249783.18	71.47	0.13	22.02	6.39	71.47	0.13	19.31	9.1	71.47	0.13	28.35	0.06
TWN	369923.22	58.17	0.4	28.08	13.35	58.17	0.4	29.87	11.56	58.17	0.4	41.15	0.29
USA	1927091.5	87.15	0.7	8.84	3.32	87.15	0.7	9.45	2.71	87.15	0.7	12.04	0.12
ROW	3833149.2	73.53	1.68	17.88	6.91	73.53	1.68	20.83	3.96	73.53	1.68	24.24	0.55

Table 4 confirms that there is a consensus for the calculation of DVA and that all the frameworks provide the same DDC, which is generally a small percentage of gross exports (most of the time below 1%). When it comes to FVA, we find important differences across the three approaches, as it was the case with the simple numerical examples. For example, KWW (first approach), or Borin and Mancini (2017) and Miroudot and Ye (2017) (second approach) have a foreign double counting equal to about 15% for the Czech Republic. This foreign double counting is only 0.29% with the third approach that we have proposed in this paper.

The third approach has results for the foreign double counting in line or symmetric with the domestic double counting. It confirms that it measures some foreign value-added coming twice to the exporting economy as part of some circular trade, the same way that the domestic double counting measures domestic value-added coming back embodied in imports of foreign inputs. At the country-level, this circular trade is rather rare. Table 4 also highlights that this value-added coming back to the same exporting economy is even smaller for the foreign value added.

The definition of double counting in the third approach seems closer to the initial objective of the literature, which is to disentangle what is domestic and foreign value added in the exports of a given country and to remove what is double counted, from the point of view of this exporter. As such, authors interested in removing double counting to have measures of domestic and foreign value-added consistent with the production of inputs in the different countries should rather follow the third approach. It is also the case when the foreign value-added is used to look at its content (e.g. C02 emissions).

But the source-based approach is also interesting as it identifies some component of FVA that has been part of more complex value chains than the direct import of foreign inputs. For example, the high foreign double counting in the exports of the Czech Republic highlights that a high share of the foreign content comes from vertical trade upstream in the value chain. We can even subtract FDC from the third approach from FDC in the second approach and obtain a measure of vertical trade in inputs embodied in the exports of the Czech Republic.

Concluding remarks

This paper has further investigated the concept of double counting in the decomposition of gross exports and found that differences in definitions and approaches to the measurement of double counting can explain why several decompositions are proposed in the literature with results that are the same for the domestic value added and domestic double counting but quite different when it comes to foreign value added and foreign double counting.

In addition, the paper has developed a new framework to measure the foreign double counting defined as value-added coming twice (or more) to the same exporting economy, which was the initial objective in several papers but not yet achieved. This approach is consistent with the 'hypothetical extraction' proposed by Los et al. (2016) but adds two new terms: domestic and foreign double counting, together with the FVA term that was missing in Los et al. (2016).

Although it is not shown in this paper, the framework can be extended to decompose bilateral gross exports. It should however be noted that there is a difference between double counting in bilateral exports and exports with world. As mentioned in the section 3 of this paper, the bilateral exports would be accounted by a different export measurement and extraction matrix, the implication of the 'border' in the bilateral exports has been transformed into the 'border' between two given countries, then the double counting measurement means the value-added crossed the two given countries 'border' more than once. It's also noteworthy that the decomposition of bilateral exports is NOT the mapping gross exports decomposition into the bilateral level. Still, if we consider the global exports, the concept of 'borders' would not exist, this decomposition just would be decomposed into the value-added term and intermediate input term, which's value-added term is equal to the sum of all countries' domestic value-added.

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Appendix I

Lemma A1：With respect to 'total exports measurement requirements matrix' $\tilde{\mathbf{B}}$, we have

$$\begin{aligned}\tilde{\mathbf{B}} &= (\mathbf{I}-\tilde{\mathbf{A}})^{-1} = [\mathbf{I}-\mathbf{A}^{\mathbf{I}}(\mathbf{I}-\mathbf{A}^{*})^{-1}]^{-1} = [(\mathbf{I}-\mathbf{A}^{*})(\mathbf{I}-\mathbf{A}^{*})^{-1} - \mathbf{A}^{\mathbf{I}}(\mathbf{I}-\mathbf{A}^{*})^{-1}]^{-1} \\ &= [(\mathbf{I}-\mathbf{A}^{*}-\mathbf{A}^{\mathbf{I}})(\mathbf{I}-\mathbf{A}^{*})^{-1}]^{-1} \\ &= (\mathbf{I}-\mathbf{A}^{*})\mathbf{B} = (\mathbf{I}-\mathbf{A}+\mathbf{A}^{\mathbf{I}})\mathbf{B} = \mathbf{I}+\mathbf{A}^{\mathbf{I}}\mathbf{B}\end{aligned}$$

Lemma A2：In the exports measurement accounting framework, we have

$$\mathbf{B}^{*}\tilde{\mathbf{B}} = \mathbf{B}$$

Here, $\mathbf{B}^{*} = (\mathbf{I}-\mathbf{A}^{*})^{-1}$, is the 'extraction matrix Leontief inverse'. B is the 'total requirements matrix' in the ICIO table which is $\mathbf{B} = (\mathbf{I}-\mathbf{A})^{-1}$.

Proof: Expanding the expression of $\mathbf{B}^{*}$ and $\tilde{\mathbf{B}}$, we obtain:

$$\begin{aligned}\mathbf{B}^{*}\tilde{\mathbf{B}} &= (\mathbf{I}-\mathbf{A}^{*})^{-1}(\mathbf{I}-\tilde{\mathbf{A}})^{-1} = [(\mathbf{I}-\tilde{\mathbf{A}})(\mathbf{I}-\mathbf{A}^{*})]^{-1} = \{[\mathbf{I}-\mathbf{A}^{\mathbf{I}}(\mathbf{I}-\mathbf{A}^{*})^{-1}](\mathbf{I}-\mathbf{A}^{*})\}^{-1} \\ &= \{[(\mathbf{I}-\mathbf{A}^{*})(\mathbf{I}-\mathbf{A}^{*})^{-1} - \mathbf{A}^{\mathbf{I}}(\mathbf{I}-\mathbf{A}^{*})^{-1}](\mathbf{I}-\mathbf{A}^{*})\}^{-1} \\ &= [(\mathbf{I}-\mathbf{A}^{*}-\mathbf{A}^{\mathbf{I}})(\mathbf{I}-\mathbf{A}^{*})^{-1}(\mathbf{I}-\mathbf{A}^{*})]^{-1} \\ &= (\mathbf{I}-\mathbf{A})^{-1} = \mathbf{B}\end{aligned}$$

Appendix II

Going deep to the country accounting level, the measurement become more complicated. Firstly, we can give the full decomposition for specific country i's exports measurement:

$$\mathbf{e}_i^{\mathbf{T}} = \mathbf{VaE}(i)^{\mathbf{T}} + \mathbf{VaE}(i)^{\mathbf{T}} \tilde{\mathbf{L}}_{ii} + \mathbf{VaE}(i)^{\mathbf{T}} [\tilde{\mathbf{L}}]^2_{ii} + \mathbf{VaE}(i)^{\mathbf{T}} [\tilde{\mathbf{L}}]^3_{ii} + \cdots \qquad \text{(A1)}$$

The above expression provides an explicit interpretation of the decomposition of exports measurement for the specific country i. The initial input $\mathbf{VaE}(i)^{\mathbf{T}}$, which is already explicated above, is correspondent to the original value-added term in the framework. The remain terms are the intermediate appearance of value-added labelled before, can be seen as the value-added double counting term.

In the first round input, the input term $\mathbf{VaE}(i)^{\mathbf{T}} \tilde{\mathbf{L}}_{ii}$, it means that the value-added term which is already accounted in the initial round propagates through the matrix $\tilde{\mathbf{L}}_{ii} = \hat{\mathbf{e}}_i^{-1} \tilde{\mathbf{A}}_{ii} \hat{\mathbf{e}}_i$, re-writing this term, we have country i's value-added input is equal to:

$$\mathbf{VaE}(i)^T \tilde{\mathbf{L}}_{ii} = \tilde{\mathbf{V}}_i \hat{\mathbf{e}}_i \cdot \hat{\mathbf{e}}_i^{-1} \overline{\mathbf{A}}_{ii} \hat{\mathbf{e}}_i = \tilde{\mathbf{V}}_i \overline{\mathbf{A}}_{ii} \hat{\mathbf{e}}_i \qquad \text{(A2)}$$

Having in mind that $\tilde{\mathbf{A}}_{ii} = \sum_k \mathbf{A}^{\mathbf{I}}_{ik} \mathbf{B}^*_{ki}$, this term clearly explain how the value-added in exports flow propagate through the borders via the borders identification matrix $\mathbf{A}^{\mathbf{I}}_{ik}$ then come back the export country i via extraction matrix Leontief inverse $\mathbf{B}^*_{ki}$, which means the value-added crossed all the possible borders except given country i's export borders.

In the other rounds, the additional value-added has a similar interpretation, if we sum up all the terms, the double counted value-added expression is:

$$\begin{aligned} &\mathbf{VaE}(i)^{\mathbf{T}} \tilde{\mathbf{L}}_{ii} + \mathbf{VaE}(i)^{\mathbf{T}} [\tilde{\mathbf{L}}]^2_{ii} + \mathbf{VaE}(i)^{\mathbf{T}} [\tilde{\mathbf{L}}]^3_{ii} + \cdots = \\ &\tilde{\mathbf{V}}_i (\tilde{\mathbf{A}}_{ii} + \sum_j^G \tilde{\mathbf{A}}_{ij} \tilde{\mathbf{A}}_{ji} + \sum_k^G \sum_j^G \tilde{\mathbf{A}}_{ij} \tilde{\mathbf{A}}_{jk} \tilde{\mathbf{A}}_{ki} + \cdots) \hat{\mathbf{e}}_i = \tilde{\mathbf{V}}_i (\tilde{\mathbf{B}}_{ii} - \mathbf{I}) \hat{\mathbf{e}}_i = \tilde{\mathbf{V}}_i \sum_k \mathbf{A}^{\mathbf{I}}_{ik} \mathbf{B}_{ki} \hat{\mathbf{e}}_i \end{aligned} \qquad \text{(A3)}$$

Merging equation (A1), (A3) and Lemma A1, we can get equation (6) in this paper.

Appendix III

Table A1: IO table for case1

	A	B	C	D	A	B	C	D
A	0	0	0	0	0	0	0	3
B	2	0	0	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0
VA	1	1	1	0				

Table A2: IO table for case2

	A	B	C	D	A	B	C	D
A	0	2	0	0	0	0	0	0
B	1	0	3	0	0	0	0	0
C	0	0	0	0	0	0	0	4
D	0	0	0	0	0	0	0	0
VA	1	2	1	0				

Table A3: IO table for case3

	A	B	C	D	A	B	C	D
A	0	2	0	0	0	0	0	5
B	1	0	3	0	0	0	0	0
C	4	0	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0
VA	2	2	1	0				