The limiting distributions of (country) responsibilities*

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Abstract

This paper simultaneously examines the circular flows of incomes and emissions for an open economy to reveal the conceptual correspondences between the widely used macroeconomic aggregates and input-ouput (IO) responsibility measures. This approach clearly demonstrates the interlinkages, uses and limitations of the considered IO responsibility indicators. As a by-product of this analysis, a *disposable income-based accounting* as a new responsibility allocation scheme is introduced.

Next, it is shown that there are two alternative and entirely legitimate ways of computing both consumption- and income-based emissions, which are not discussed and used in the literature. Here two types of allocation matrices play critical role, which we refer to as the *"factor to final users allocation matrices"* and *"factor to primary suppliers allocation matrices"*. The properties of these matrices are explored.

Finally, we study the issue of *infinite reallocation* of responsibilities between final (output) users and primary (input) suppliers as, among other reasons, these agents are the ultimate beneficiaries of the whole production process. It is shown analytically that the process of such repeated redistribution of responsibilities is finite. In particular, the *limiting or "stationary" distribution* of a country's final users' [resp. primary suppliers'] responsibility is found to be proportional to global emissions, with the proportion being equal to its gross national expenditure (GNE) [resp. gross national income (GNI)] share in the world GNE/GNI. If the residence of primary suppliers is not distinguished in a global multi-regional IO table, the limiting responsibility of a country's primary suppliers is given in terms of gross domestic product (GDP) rather than GNI. The implications of the discovered limiting responsibilities are discussed in the context of international cooperation on climate and in relation to the equitable burden-sharing frameworks used by climate scientists.

Keywords: circular flows of incomes/payments and of emissions, factor to final users allocation matrices, factor to primary suppliers allocation matrices, limiting distribution of responsibilities

JEL Classification Codes: C67, D57, F64, Q56, E01

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1 Introduction

There is a large and ever growing input-output (IO) literature on how countries need to be made responsible for the damage caused and/or the benefits gained from their production activities worldwide and the resulting emissions (for an overview, see e.g. Lenzen and Murray, 2010; Tukker et al., 2020).Depending on who is ultimately held responsible, there exist different approaches to a proper allocation of environmental burden. Often the literature distinguishes between such economic agents as producers of goods and services, final users (also referred to as final consumers or final demanders) of goods and services, and suppliers of primary inputs (such as workers and capital owners). A *full responsibility* approach calls for only one of the three types of agents to take on the entire environmental burden. In contrast, a *shared responsibility* framework recognizes that ultimately all agents are responsible for emissions generation and thus takes a position that, at least, two types of agents have to share the burden of production consequences.[1]

This paper takes a (somewhat) different approach to understanding and extending the IO responsibility measures. First, we look for the conceptual correspondences between a few well-understood and widely used aggregate economic performance measures and the most salient IO responsibility indicators. Such simultaneous examination of the circular flows of incomes and emissions for an open economy is shown to be insightful in that it clearly demonstrates the uses and limitations of each of the considered responsibility accounting mechanisms. As a by-product of this initial part of the analysis, a *disposable income-based accounting* as a new responsibility allocation scheme is introduced.

Second, we show that there exist two entirely legitimate ways of computing both the consumption- and income-based emissions, although the literature, to our knowledge, discusses and applies only one of such alternative expressions in each case. Here two types of allocation matrices play critical role, which we refer to as "*factor to final users allocation matrices*" and "*factor to primary suppliers allocation matrices*". The properties of these matrices are explored in some detail.

Finally, we study the issue of *infinite redistribution* of responsibilities between final (output) users and primary (input) suppliers on the ground that namely these agents are the ultimate beneficiaries of the whole production process in terms of consumption and earned income. They also directly and/or indirectly manage and operationalize production units and determine future production expansion strategies and plans. We show an-

[1] Selected references of this literature includes Proops et al. (1993); Munksgaard and Pedersen (2001); UNFCCC (1997); Ahmad and Wyckoff (2003); Ferng (2003); Gallego and Lenzen (2005); Rodrigues et al. (2006); Peters (2008); Peters and Hertwich (2008); Hertwich and Peters (2009); Davis et al. (2011); Peters et al. (2012); Marques et al. (2012); Wiebe and Yamano (2016); Steininger et al. (2016); Temursho and Miller (2020); Yamano and Guilhoto (2020).

alytically that the process of repeated reallocation of responsibilities between final users and primary suppliers is finite. In particular, we find that the *limiting or "stationary" distribution* of a country's final users responsibility is given by its gross national expenditure (GNE) share in the world GNE, and that of primary suppliers' responsibility by its gross national income (GNI) share in the world GNI. If one does not distinguish the countries of origin (i.e. residence) of primary suppliers in the value added quadrant of a global multi-regional IO table, the latter limiting distribution is given by the proportion of the country gross domestic product (GDP) in world GDP. The implications of the discovered limiting distributions of environmental responsibilities are discussed in the context of international cooperation on climate and in relation to the equitable burden-sharing frameworks widely used by climate scientists.

For a better readability of the followup material, in this introductory section we define most of the variables used throughout this paper and, if deem necessary, explain their meaning right away to avoid further definitional discussions.[2] Without loss of generality, it is assumed that within the considered global multi-region IO framework there are N countries, each with n industries.

- $\boldsymbol{\imath}$ summation vector of ones of appropriate length, $\boldsymbol{\imath} = [1, 1, \dots, 1]'$
- $\mathbf{I}$ identity matrix of appropriate dimension
- $\mathbf{x}^r$ $n \times 1$ vector of gross output of country r, with its typical element x_i^r indicating total output (in value terms) produced by industry i in country r
- $\mathbf{x}$ $Nn \times 1$ vector of gross outputs, i.e. $\mathbf{x} = \left[(\mathbf{x}^1)', (\mathbf{x}^2)', \dots, (\mathbf{x}^N)'\right]'$
- $\mathbf{Z}^{rs}$ $n \times n$ interindustry transactions matrix, wherein z_{ij}^{rs} refers to the value of intermediate deliveries from industry i in country r to industry j in country s
- $\mathbf{Z}$ $Nn \times Nn$ matrix of intermediate deliveries, with $\mathbf{Z}^{rs}$ being its typical rs-th block
- $\mathbf{y}^{rs}$ $n \times 1$ final demand vector, with y_i^{rs} giving the deliveries from industry i in country r to final users in country s. Final demand includes final consumption expenditures by households and non-profit organisations serving households, private investments, final consumption expenditure by government, gross fixed capital formation, and changes in inventories and valuables.
- $\mathbf{y}^r$ $Nn \times 1$ vector of final demands of country r for final products produced in and delivered from all countries, i.e. $\mathbf{y}^r = \left[(\mathbf{y}^{1r})', (\mathbf{y}^{2r})', \dots, (\mathbf{y}^{Nr})'\right]'$
- $\mathbf{Y}$ $Nn \times n$ matrix of final demands of all countries, i.e. $\mathbf{Y} = \left[\mathbf{y}^1 \;\; \mathbf{y}^2 \;\; \cdots \;\; \mathbf{y}^N\right]$
- $\mathbf{v}^{rs}$ $n \times 1$ vector of gross value added (at producers' prices), with v_j^{rs} indicating the total value added generated by industry j in country s that is earned by primary inputs'

[2] We use the well-established mathematical notation in the input-output literature. Matrices are given in bold, capitals; vectors in bold, lower cases; and scalars in italicized, lower case letters. Vectors are columns by definition, row vectors are obtained by transposition, indicated by a prime. $\hat{\mathbf{x}}$ denotes a diagonal matrix with the entries of $\mathbf{x}$ on its main diagonal and zeros elsewhere.

owners resident in country r. Value added includes returns to labor (compensation of employees), capital (gross operating surplus, including mixed income), and government (taxes on production and imports less subsidies).

- $\mathbf{v}^r$ $Nn \times 1$ vector of total value added earned worldwide by providers of primary inputs that are residents of country r, i.e. $\mathbf{v}^r = \left[(\mathbf{v}^{r1})', (\mathbf{v}^{r2})', \ldots, (\mathbf{v}^{rN})'\right]'$
- $\mathbf{V}$ $n \times Nn$ matrix of (global) value added, i.e. $\mathbf{V}' = \begin{bmatrix}\mathbf{v}^1 & \mathbf{v}^2 & \cdots & \mathbf{v}^N\end{bmatrix}$
- $\mathbf{A}$ $Nn \times Nn$ direct input coefficients matrix, $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$. Its element $a_{ij}^{rs} = z_{ij}^{rs}/x_j^s$ shows the intermediate deliveries (inputs) from industry i in country r per unit of output of the receiving industry j in country s.
- $\mathbf{L}$ $Nn \times Nn$ Leontief inverse or total input requirements matrix, $\mathbf{L} \equiv (\mathbf{I} - \mathbf{A})^{-1}$. The typical entry of its rs-th block $\mathbf{L}^{rs}$, indicated by l_{ij}^{rs}, shows how much output from industry i in country r is directly and indirectly required per unit of final output produced by industry j in country s. Alternatively, l_{ij}^{rs} indicates the total output of industry i in country r that is embodied in one unit of final output of industry j in country s.
- $\mathbf{B}$ $Nn \times Nn$ direct output coefficients matrix, $\mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{Z}$. Its typical entry $b_{ij}^{rs} = z_{ij}^{rs}/x_i^r$ shows per-output intermediate sales of industry i in country r (flowing) to industry j in country s
- $\mathbf{G}$ $Nn \times Nn$ Ghosh inverse, $\mathbf{G} \equiv (\mathbf{I} - \mathbf{B})^{-1}$. Its typical entry g_{ij}^{rs} gives the total input of industry j in country s that is enabled – or supported directly and indirectly – by a unitary primary inputs supplied by all factor providers to industry i in country r
- $\mathbf{e}^r$ $n \times 1$ vector of (production-based) emissions (expressed in e.g. kilotons) that are generated by industries in country r
- $\mathbf{e}$ $Nn \times 1$ vector of industry emissions worldwide, i.e. $\mathbf{e} = \left[(\mathbf{e}^1)', (\mathbf{e}^2)', \ldots, (\mathbf{e}^N)'\right]'$
- $\mathbf{e}_c^r$ $n \times 1$ vector of direct emission coefficients of industries in country r, $e_{ci}^r = e_i^r/x_i^r$ or $\mathbf{e}_c^r = (\hat{\mathbf{x}}^r)^{-1}\mathbf{e}^r$. These are also referred to as emission intensities.[3]
- $\mathbf{e}_c$ $Nn \times 1$ vector of industry (production-based) emission intensities of all countries, i.e. $\mathbf{e}_c = \left[(\mathbf{e}_c^1)', (\mathbf{e}_c^2)', \ldots, (\mathbf{e}_c^N)'\right]'$
- $\mathbf{c}$ $Nn \times 1$ vector of emissions embodied in final demand, also referred to as consumption-based accounting or upstream emission responsibilities
- $\mathbf{n}$ $Nn \times 1$ vector of emissions enabled by the sale of primary inputs, also referred to as income-based accounting or downstream emission responsibilities.

It should be noted that although throughout the paper we focus on responsibilities in terms of emissions, the current analysis is valid for any other factor that can reasonably

[3] For simplicity and convenience, following Miller and Blair (2022), throughout this paper *per unit output* vectors are denoted by a subscript c that stands for "coefficients". For example, for a vector variable denoted by $\mathbf{t}$ (whatever its meaning), the adopted convention implies $\mathbf{t}_c = \hat{\mathbf{x}}^{-1}\mathbf{t}$.

be linked to industry-level outputs such as job creation, water use, biodiversity loss, etc. In such settings the meaning of the vectors $\mathbf{e}^r$, $\mathbf{e}_c^r$, $\mathbf{e}$, $\mathbf{e}_c$ and of the expressions involving these vectors should be reinterpreted accordingly.

2 Conceptual correspondences between macroeconomic aggregates and IO responsibility measures

In this section, we first detail the derivations of and relationships between production-, consumption- and income-based emissions. Then we explore the conceptual correspondences of these environmental responsibility indicators with the widely used macroeconomic aggregates. As a result of such equivalence examination of the measures of economic activities and their environmental consequences, a *disposable income-based accounting* as a new responsibility allocation principle is introduced.

2.1 Production-, consumption- and income-based allocation principles

Our starting point for computing (country-level) environmental responsibility within a global multi-region input-output (GMRIO) framework is a GMRIO table shown in Table 1.

Table 1: Global MRIO table

	Intermediate use				Final use				Total output
	1	2	$\cdots$	N	1	2	$\cdots$	N	
Output flows from:									
country 1	$\mathbf{Z}^{11}$	$\mathbf{Z}^{12}$	$\cdots$	$\mathbf{Z}^{1N}$	$\mathbf{y}^{11}$	$\mathbf{y}^{12}$	$\cdots$	$\mathbf{y}^{1N}$	$\mathbf{x}^1$
country 2	$\mathbf{Z}^{21}$	$\mathbf{Z}^{22}$	$\cdots$	$\mathbf{Z}^{2N}$	$\mathbf{y}^{21}$	$\mathbf{y}^{22}$	$\cdots$	$\mathbf{y}^{2N}$	$\mathbf{x}^2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
country N	$\mathbf{Z}^{N1}$	$\mathbf{Z}^{N2}$	$\cdots$	$\mathbf{Z}^{NN}$	$\mathbf{y}^{N1}$	$\mathbf{y}^{N2}$	$\cdots$	$\mathbf{y}^{NN}$	$\mathbf{x}^N$
Primary inputs from:									
country 1	$(\mathbf{v}^{11})'$	$(\mathbf{v}^{12})'$	$\cdots$	$(\mathbf{v}^{1N})'$					GNI_1
country 2	$(\mathbf{v}^{21})'$	$(\mathbf{v}^{22})'$	$\cdots$	$(\mathbf{v}^{2N})'$					GNI_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$					$\vdots$
country N	$(\mathbf{v}^{N1})'$	$(\mathbf{v}^{N2})'$	$\cdots$	$(\mathbf{v}^{NN})'$					GNI_N
Total inputs	$(\mathbf{x}^1)'$	$(\mathbf{x}^2)'$	$\cdots$	$(\mathbf{x}^N)'$	GNE_1	GNE_2	$\cdots$	GNE_N	

Note: All notations are explained in the introductory section.

Note that the country of origin and destination indices are also provided for the value added quadrant $\mathbf{V}$, colored in pink in Table 1. Given the definition of $\mathbf{v}^{rs}$, the columns of $\mathbf{V}$ refer to industries of income-generating countries and its rows refer to countries of residence of providers of primary inputs. Thus, $\mathbf{v}^{\bullet r} \equiv \mathbf{v}^{1r} + \mathbf{v}^{2r} + \cdots + \mathbf{v}^{Nr}$ gives the vector of *gross domestic product* (GDP) *by industry* in country r, which overall aggregates to the country GDP, i.e. $GDP_r = \mathbf{\imath}'\mathbf{v}^{\bullet r}$.[4] On the other hand, *gross national income* (GNI) of country r is obtained from the corresponding row total of $\mathbf{V}$, i.e. $GNI_r = (\mathbf{v}^r)'\mathbf{\imath} = (\mathbf{v}^{r\bullet})'\mathbf{\imath}$.[5] Although such detailed value-added data is not yet available due to the difficulty of tracing factor ownership and the corresponding inter-country income movements at industry level, for theoretical consistency we use the GMRIO framework presented in Table 1. In the final section of this paper, we will briefly discuss the implications for our analysis if instead of the latter framework one uses GMRIO table at basic prices.

Table 1 makes explicit the relationships between three macroeconomic aggregates of GDP, GNI and *gross national expenditure* (GNE). The latter indicator is defined as all home entities' total expenditures on final goods and services, irrespective of their production (or better, sale) origin. In terms of our notation, GNE of country r is equal to $GNE_r = \mathbf{\imath}'\mathbf{y}^r = \mathbf{\imath}'\mathbf{y}^{\bullet r}$. Given the GMRIO balancing identities of industry-level total inputs and total outputs, for each country r the following relationships between the three macroeconomic aggregates (or accounting identities) can be easily shown to hold true:

$$GDP_r = GNE_r + X_r^{G\&S} - M_r^{G\&S}, \tag{1a}$$

$$GNI_r = GDP_r + X_r^{FS} - M_r^{FS}, \tag{1b}$$

where $X_r^{G\&S}$ and $M_r^{G\&S}$ refer, respectively, to country r's total exports and total imports of both final and intermediate goods and services; X_r^{FS} indicates income receipts of residents of country r for their factor (labor, capital and land) services exported to the rest of the world; and M_r^{FS} refers to income payments made by the entities of country r for factor services imported from elsewhere. The net exports of products is also called *trade balance* (TB), i.e. $TB_r = X_r^{G\&S} - M_r^{G\&S} = \mathbf{\imath}'(\mathbf{Z}^{r\bullet} - \mathbf{Z}^{\bullet r})\mathbf{\imath} + \mathbf{\imath}'(\mathbf{y}^{r\bullet} - \mathbf{y}^{\bullet r})$. In the System of National Accounts 2008 (United Nations, 2009, p. 35) and the Balance of Payments and International Investment Position Manual (IMF, 2009, p. 184), net trade in factor services is referred to as *net primary income* (NPI), which is the balance of primary income account. In their earlier versions it was also known as *net factor income* (NFI). Since the usage of NFI is widespread in the (academic) literature, we will be using both terms interchangeably. Hence, one might also write $NPI_r = NFI_r = X_r^{FS} - M_r^{FS} = \mathbf{\imath}'(\mathbf{v}^{r\bullet} - \mathbf{v}^{\bullet r})$.

Now we turn to the derivation of IO-based responsibility measures. Satellite accounts, such as pollutant emissions, are (mostly) available at the level of industries. Hence,

[4] A bullet point (•) indicates summation over the respective index. For example, $\mathbf{Z}^{\bullet r} \equiv \sum_k \mathbf{Z}^{kr}$ gives the $n \times n$ matrix of total intermediate deliveries from both domestic and foreign industries to industries in country r.

[5] An aggregate country-level GDP-GNI matrix (outside a GMRIO table framework) was recently used in Bohn et al. (2021) to quantify "income exports" from the perspective of "trade in income" as opposed to the by-now more traditional "value-added exports" in the "trade in value added" literature.

to derive upstream emission responsibility, industry-level *total outputs embodied in the use of final outputs* are first – often implicitly – estimated. For this purpose, the demand-driven IO model (Leontief, 1936) in a global setting is employed, according to which production in country r that is embodied in all final uses of country s is obtained as

$$\mathbf{x}^{rs} = \sum_k \mathbf{L}^{rk}\mathbf{y}^{ks} = \begin{bmatrix}\mathbf{L}^{r1} & \mathbf{L}^{r2} & \cdots & \mathbf{L}^{rN}\end{bmatrix}\mathbf{y}^{s}. \tag{2}$$

The latter, when summed across all final demand destinations, results in total outputs of the output-supplying country r, i.e. $\mathbf{x}^r = \sum_s \mathbf{x}^{rs}$. Next, pre-multiplication of (2) by the emission intensities of country r gives total emissions generated in country r that are embodied in all final uses of country s:[6]

$$e^{rs} = (\mathbf{e}_c^r)'\mathbf{x}^{rs}. \tag{3}$$

The information derived in (3) is now sufficient to estimate emission responsibilities of each country according to the allocation principles of *production-based accounting* (PBA) and *consumption-based accounting* (CBA). As visualized in Figure 1, the PBA (minus direct emissions by households[7]) for country r is equal to all the emissions generated in country r irrespective of their final demand destination, i.e. $PBA_r = \sum_k e^{rk} = \sum_k (\mathbf{e}_c^r)'\mathbf{x}^{rk} = (\mathbf{e}_c^r)'\mathbf{x}^r = (\mathbf{e}^r)'\mathbf{1}$. On the other hand, the CBA (minus direct emissions by households) amounts to the sum of all emissions generated globally that are embodied in the final uses of the residents of country r. Thus, the CBA for country r is obtained as:

$$CBA_r = \sum_k e^{kr} = PBA_r - \sum_{k\neq r} e^{rk} + \sum_{k\neq r} e^{kr}. \tag{4}$$

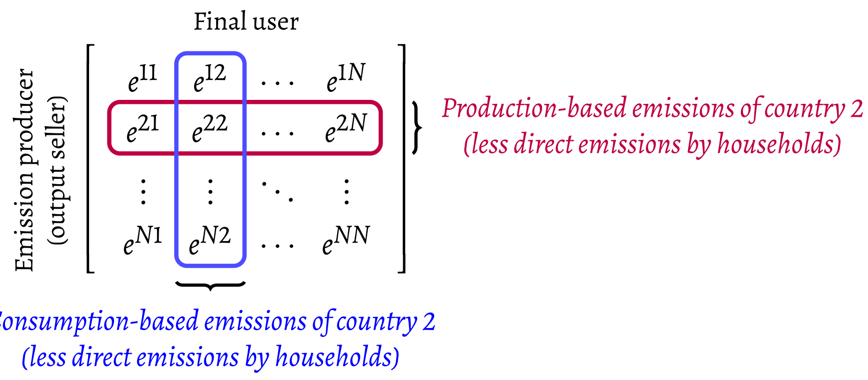
That is, for any country, consumption-based emissions equal its production-based emissions minus exports of domestic emissions plus imports of foreign emissions. Hence, not surprisingly, akin to the relationships of aggregate economic measures in (1a) and (1b), also for emissions accounting the fundamental supply-use accounting identity holds, i.e. *Production + Imports = Consumption + Exports*.

If $\mathbf{c}^r$ denotes the $Nn \times 1$ vector of worldwide industry-specific CBA emissions of coun-

[6]Alternatively, the $Nn \times N$ matrix $\mathbf{E} \equiv \hat{\mathbf{e}}_c\mathbf{L}\mathbf{Y} = \hat{\mathbf{e}}_c\mathbf{X}$ compactly gives the detailed *industry-level* worldwide emissions embodied in the final uses of each of N countries, where $\mathbf{x}^{rs}$ is the typical rs-th country-pair vector (block matrix) of $\mathbf{X}$.

[7]In what follows, whenever we refer to a country responsibility accounting, an explicit expression for the *direct emissions by households* – residential emissions from e.g. burning natural gas and emissions from private road transport – is suppressed for simplicity, which, however, in practice have to be added to the corresponding industrial emissions. For an individual country, households' direct emissions are exactly equal under all responsibility allocation frameworks discussed in this paper. Hence, their suppression does not invalidate the relationships between the considered responsibility accounting principles.

Figure 1: Country-level (upstream) embodied emissions



Note: Country r's emissions embodied in all final uses (demands) of country s, e^{rs}, are defined by equations (2) and (3).

try r, then in a compact matrix form these embodied emissions are derived from

$$(\mathbf{c}^r)' = \mathbf{e}_c'\mathbf{L}\hat{\mathbf{y}}^r. \tag{5}$$

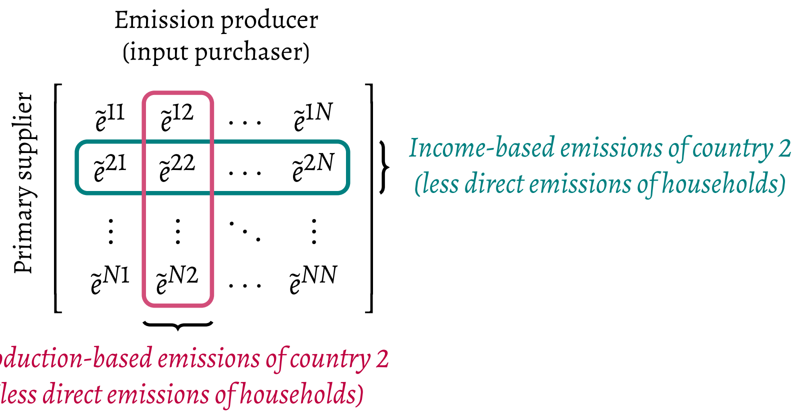
Let us now consider a supply-side perspective of the emission responsibility. According to the *income-based accounting* (IBA) principle, a country is held responsible for all emissions generated worldwide, whose generation is enabled by the supply of primary inputs from the residents of the country in question. These so-called *income-based emissions* are thus also referred to as "downstream enabled emissions". Here the point of departure is the supply-driven IO model (Ghosh, 1958) that is used in its *ex post* descriptive interpretation. To estimate such enabled emissions, one first needs to quantify (the value of) *total inputs enabled by the supply of primary inputs*. From total input perspective, industry production of country s that is supported directly and/or indirectly (or, equivalently, enabled) by total primary supply[8] from country r is calculated as follows:

$$(\tilde{\mathbf{x}}^{rs})' = \sum_k (\mathbf{v}^{rk})'\mathbf{G}^{ks} = (\mathbf{v}^r)' \begin{bmatrix} \mathbf{G}^{1s} \\ \mathbf{G}^{2s} \\ \vdots \\ \mathbf{G}^{Ns} \end{bmatrix}. \tag{6}$$

The latter, if summed across the (residence) origins of all primary supply, gives the total inputs (equivalently, outputs) of the input-purchasing country s, i.e. $\mathbf{x}^s = \sum_r \tilde{\mathbf{x}}^{rs}$. Note that although total industry outputs are equal to total inputs at the system-wide (i.e. global) level, production in country r embodied in the final uses of country s generally

[8] For simplicity, we use the concept of "primary supply" referring to the supply of primary inputs. This is akin to the notion of "final use" that refers to the use of (or demand for) final outputs.

Figure 2: Country-level (downstream) enabled emissions



Note: Country s's emissions enabled by all primary supplies from country r, $\tilde{e}^{rs}$, are defined by equations (6) and (7). "Primary supplier" refers to the owner and provider of factor services.

differs from the input-side production in country r that is enabled by all primary supplies from country s, i.e. $\mathbf{x}^{rs} \neq \tilde{\mathbf{x}}^{sr}$.

Akin to the derivations of (3), now post-multiplying (6) by the direct emission coefficients of country s gives the total emissions generated in country s that are enabled by all primary supplies from country r:[9]

$$\tilde{e}^{rs} = (\tilde{\mathbf{x}}^{rs})'\mathbf{e}_c^s. \tag{7}$$

As illustrated in Figure 2, the IBA (less direct emissions by households) for country r is equal to all the emissions generated worldwide that are enabled by primary inputs, owned and supplied directly and/or indirectly by the residents of country r. Given that $\sum_k \tilde{e}^{kr} = \sum_k (\tilde{\mathbf{x}}^{kr})'\mathbf{e}_c^r = (\mathbf{x}^r)'\mathbf{e}_c^r = \mathbf{1}'\mathbf{e}^r = PBA_r$, we thus have:

$$IBA_r = \sum_k \tilde{e}^{rk} = PBA_r - \sum_{k \neq r} \tilde{e}^{kr} + \sum_{k \neq r} \tilde{e}^{rk}. \tag{8}$$

Therefore, similar to (4), income-based emissions equal production-based emissions minus exports of domestic emissions plus imports of foreign emissions, with the difference that now emissions refer to downstream enabled emissions and those in (4) defined upstream embodied emissions.[10]

[9] Alternatively, the $N \times Nn$ matrix $\tilde{\mathbf{E}} \equiv \mathbf{VG}\hat{\mathbf{e}}_c = \tilde{\mathbf{X}}\hat{\mathbf{e}}_c$ compactly gives the detailed *industry-level* worldwide emissions that are enabled by all primary supplies of each of N countries, where $(\tilde{\mathbf{x}}^{rs})'$ is the rs-th country-pair row vector of $\tilde{\mathbf{X}}$.

[10] To correctly identify exports and imports of emissions, recall that while (country-)producers are defined by the first superscript of the embodied emissions e^{rs}, it is the second superscript in the enabled emissions $\tilde{e}^{rs}$ that identifies the location of production. Such notation choice is intentional and aimed at being consistent with the overall structure of the GMRIO table (see Table 1, Figure 1 and Figure 2).

If $\mathbf{n}^r$ denotes the $Nn \times 1$ vector of worldwide industry-specific IBA emissions of country r, then in compact matrix form these income-based emissions are given by:

$$\mathbf{n}^r = \hat{\mathbf{v}}^r \mathbf{G} \mathbf{e}_c. \tag{9}$$

Finally, it needs to be highlighted that the existing empirical studies on income-based emissions (effectively) use the value-added matrix of the form:

$$\mathbf{V}_{GDP} = \begin{bmatrix} (\tilde{\mathbf{v}}^{\bullet 1})' & \mathbf{0}' & \cdots & \mathbf{0}' \\ \mathbf{0}' & (\tilde{\mathbf{v}}^{\bullet 2})' & \cdots & \mathbf{0}' \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}' & \mathbf{0}' & \cdots & (\tilde{\mathbf{v}}^{\bullet N})' \end{bmatrix}, \tag{10}$$

whose rows (resp. industry columns) sum up to the corresponding country (resp. industry) GDP, excluding net taxes on final products (hence, the superscript "GDP"). This is of practical necessity since inter-country industry data $\mathbf{v}^{rs}$ for $r \neq s$ are still lacking, as mentioned earlier. As such the income-based emissions $\tilde{e}^{rs}$ with (10), i.e. when $\tilde{e}^{rs} = (\tilde{\mathbf{v}}^{\bullet r})' \mathbf{G}^{rs} \mathbf{e}_c^s$ as follows from (6) and (7), are emissions generated in country s that are enabled by primary inputs supplied by all resident and non-resident factor owners to country r's production. Thus, the income-based emissions obtained with the global value added matrix defined as in (10) are *emissions enabled by gross domestic income, GDI* (noting that here GDI ignores net taxes on final products) and not GNI as would have been the case with the full GDP-GNI data in $\mathbf{V}$ of the GMRIO table (Table 1). If the residency principle is chosen as a primary anchor of responsibility allocation, then the latter approach is to be preferred since then a country is made responsible for the emissions enabled by all incomes earned solely by its residents from their participation in production activities worldwide.

2.2 Aggregate economic measures vis-à-vis responsibility indicators

A nation's income flow is measured by such aggregate economic indicators as GNE, GDP, GNI and *gross national disposable income* (GNDI). The relations between the first three of these measures were already explained in the previous section, see (1a) and (1b). However, economists ofter prefer GNDI as a measure of national income because "it most closely corresponds to the resources available to the nation's households, and national welfare depends most closely on this accounting measure" (Feenstra and Taylor, 2014, p. 578). GNDI equals GNI plus *net unilateral transfers* (NUT), which are non-market international transfers of goods, services and income. These "unrequited transfers" or "gifts" include such items of current transfers as income remittances made/received by resident households to/from non-resident households, current international cooperation between the governments of different countries or between governments and international organizations (e.g. official development aid, food aid, emergency aid, volunteer medical services), and pension payments to citizens living abroad. Nowadays NUT are also referred to as *net secondary income* (NSI) or net current transfers, which is the balance of secondary income account (see United Nations, 2009, p. 35; IMF, 2009, pp.207, 223). In what follows, we will use NUT and NSI interchangeably.

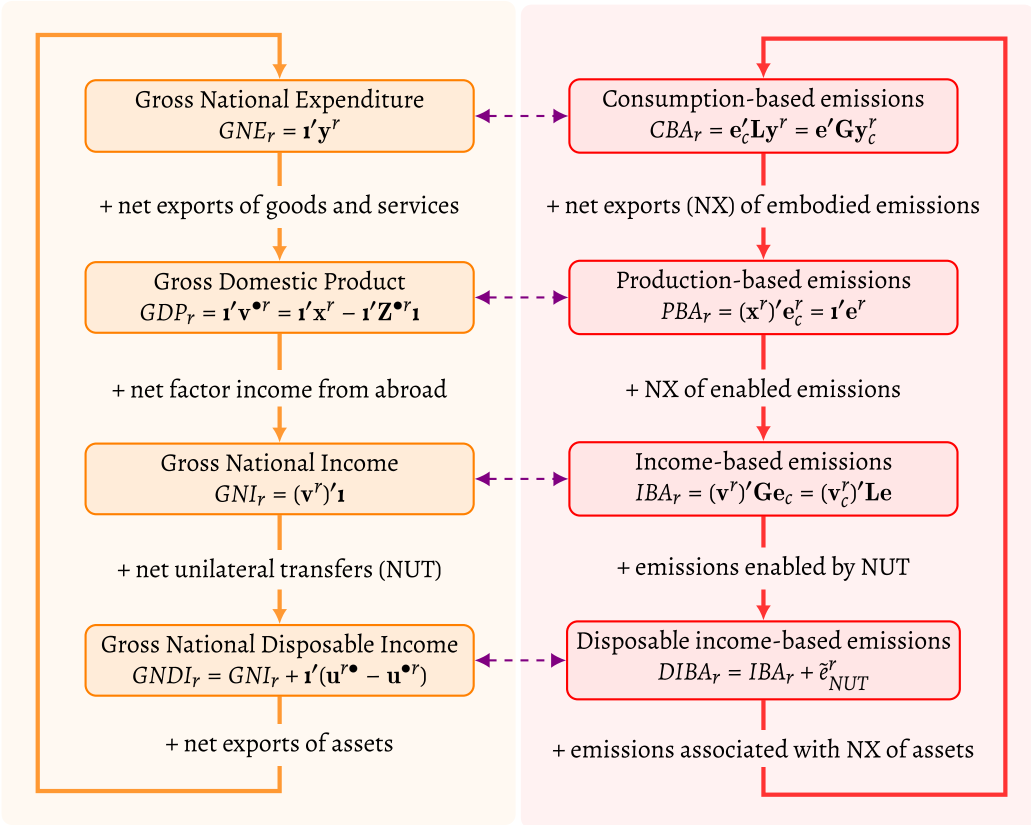
The links between the above-mentioned four economic measures are visualized on the left-hand side of Figure 3, which shows the *circular flow of payments or incomes* for a country when moving from one indicator to another.[11] In addition, in Figure 3 we spell out the conceptual equivalence of these indicators to the corresponding environmental responsibility measures. Thus, the *circular flow of emissions* (or of emission responsibilities) are presented on the right-hand side of the figure. For the sake of completeness, the mathematical expressions, using our IO notation, of both types of indicators are also presented in this summary flow-chart of the economic and environmental responsibilities.

To start with, Figure 3 shows that GNE is conceptually equivalent to the CBA measure as both focus on the inquired outcomes of national consumption of final products. That is, GNE shows the aggregate monetary value of all – both domestically and foreign produced – final goods and services consumed by final users of a country, while total emissions generated worldwide to satisfy exactly these final outputs are given by the corresponding CBA indicator. Hence, whereas GNE of country r is simply the "arithmetic sum" of the elements in $\mathbf{y}^r$, the CBA for r is instead equal to the "weighted sum" of the same final demand elements, with weights indicating the corresponding global emission multipliers, $\mathbf{L}'\mathbf{e}_c$. Evidently, these "weights" are the required conversion factors that translate the value of final uses to the corresponding physical amounts of emissions.

In addition, not surprisingly, the circular flows of the aggregate economic and responsibility measures are conceptually closely related. For example, as follows from (4), a country's CBA emissions plus exports of its domestically produced emissions embodied in foreign final demands minus imports of emissions generated elsewhere that are embodied in the country's final uses gives its PBA emissions. Thus, adding net exports of embodied emissions to the CBA for country r gives its PBA emissions. This is exactly similar to the definition of GDP in that adding net exports of products to GNE results in GDP of the country concerned. These latter derivations indicate to the conceptual link between the GDP and PBA indicators. Indeed, as these measures refer, respectively, to the production-side of economic activity and its environmental consequences based on

[11]This part of the figure is adopted from Feenstra and Taylor (2014, p. 571), which further elaborates on the relevant connections to the items in a country's balance of payments account.

Figure 3: Conceptual correspondences between aggregate economic indicators and input-output responsibility measures



Note: To account for full emission responsibility, direct emissions by households should be added to the CBA, PBA, IBA and DIBA expressions, which are suppressed for space consideration. The two-sided arrows (colored in violet) stand for "conceptual correspondence".

the residence principle in the System of National Accounts, the two (i.e. GDP and PBA) are conceptually equivalent.[12]

Moving further down along the income and (environmental) responsibility circular flows in Figure 3, one identifies a conceptual correspondence between the GNI and IBA measures. If looked from the perspective of primary suppliers, the rs-th element of intercountry enabled emissions in (7) and in Figure 2, i.e. $\tilde{e}^{rs} = \sum_k (\mathbf{v}^{rk})'\mathbf{G}^{ks}\mathbf{e}_c^s$, can be alternatively interpreted as *emissions enabled by the direct and indirect exports of factor services from country r to country s* for $r \neq s$. Take e.g. $(\mathbf{v}^{rr})'\mathbf{G}^{rs}\mathbf{e}_c^s$ as one component of $\tilde{e}^{rs}$ with $r \neq s$: these emissions are fully indirect as primary inputs of residents of country r are used in the production process in their home country (i.e. there is no direct exports of factor ser-

[12] For example, in United Nations et al. (2009) it is stated that "GDP is also equal to the sum of primary incomes distributed by *resident producer units*" (p. 34, italics added). In the same vein, PBA emissions of a country include world-wide emissions that are embodied in final products consumed by its *resident final users*.

vices in $\mathbf{v}^{rr}$). However, through the global production (namely, input demand) chains – fully captured by the corresponding submatrix of the Ghosh inverse, $\mathbf{G}^{rs}$ – the domestic primary supplies enable production and thus generate emissions in another country s. Therefore, $(\mathbf{v}^{rr})'\mathbf{G}^{rs}\mathbf{e}_c^s$ shows emissions in country s enabled by the indirect exports of factor services from r to s.

Now, as follows from (8) and Figure 2, adding the emissions enabled by (direct and indirect) net exports of factor services to the PBA emissions results in the IBA-based emissions. This is akin to the definition of GNI, which is obtained by adding to GDP the country's net factor income received from abroad, i.e. income receipts from factor (labor, capital and land) services exported abroad minus income payments to foreign entities for importing their factor services. Once again, indeed GNI and IBA are conceptually equivalent as they deal, respectively, with the economic and environmental consequences of national income formation that accounts for all international *market* transactions in goods, services, and factor services.[13]

Although GNDI is often considered to be a preferred measure of national income as it "better measures how well off citizens are" (Stiglitz et al., 2008, p. 95),[14] so far, to our knowledge, there does not exist a conceptually equivalent environmental responsibility measure in the literature. Such *disposable income-based accounting* (DIBA) of emissions can be introduced as an extension of the IBA principle, which additionally accounts for the emissions enabled by NUT between countries. That is, the IBA accounting has to be modified in such a way that a country is additionally made responsible for the global emissions enabled by the part of primary income generated elsewhere that is ultimately received as unilateral transfers by the country in question minus global emissions enabled by domestic income that is unilaterally sent abroad as current transfers.[15]

[13]Mathematically, whereas GNI of country r equals the "arithmetic sum" of the elements in $\mathbf{v}^r$, the IBA for r is instead identical to the "weighted sum" of the same primary supply entries, with weights indicating the corresponding global emission multipliers, $\mathbf{Ge}_c$. These "weights" play the role of conversion factors that translate the value of primary supplies to the corresponding physical amounts of emissions.

[14]The World Bank classifies countries into four income groups based on GNI per capita, using so-called *Atlas* conversion factors (instead of simple exchange rates) to reduce the impact of exchange rate fluctuations. In this regard, however, Capelli and Vaggi (2016) argue that "[i]f the aim is to assess the standard of living of a population and its ability to consume and to save, then GNDI per capita should be adopted. If, on the other hand, the thresholds are meant to capture the 'strengths and weaknesses' of a country in the world economy, then GDP per capita would be a better tool" (p. 234).

[15]Since unilateral transfers take not only the form of money (income), but also goods and services (such as food aid), estimating their environmental impacts is not so straightforward. For the latter case, assuming transfers take the form of final products, their associated emissions could be estimated using the CBA approach. However, in the presence of unilateral transfers in both products and income, it would be inconsistent to mix both the CBA and IBA approaches as one method is based on the demand- and the other on the supply-side perspectives of responsibilities. Therefore, we suggest first to estimate value added generated in the production of goods and services ultimately sent as transfers abroad, and then use these estimates jointly with the remaining monetary transfers to calculate the environmental outcome of all unilateral transfers. In this way, similar to IBA, the DIBA emissions remain an income-based approach, as are

Denote u^{rs} as the total monetary value of unilateral transfers received by country r from country s when $r \neq s$. By definition, $u^{rr} = 0$ for all r. The first step in derivation of DIBA emissions is allocation of u^{rs} over industries of the primary income-generating countries s, resulting in the $n \times 1$ vector $\mathbf{u}^{rs}$ with $\mathbf{1}'\mathbf{u}^{rs} = u^{rs}$. The presence of industry basis as the income-generating origin of secondary income ensures that the emission intensities, both direct and indirect along the corresponding global production chains, of activities as the source of transfers are appropriately accounted for.[16] One possibility could be to use the structure of the corresponding international transactions in factor services (or primary incomes), i.e.

$$\mathbf{u}^{rs} = \frac{u^{rs}}{\mathbf{1}'\mathbf{v}^{rs}}\mathbf{v}^{rs}$$

as long as $\mathbf{1}'\mathbf{v}^{rs} \neq 0$. If, however, $\mathbf{1}'\mathbf{v}^{rs} = 0$ but $u^{rs} > 0$, the above approach would not work, and thus $\mathbf{u}^{rs}$ has to be estimated in some other way.[17] But this approach might not be satisfactory since it has been found that empirically "the NSI flows are much more stable [over time] than the NPI flows" (Capelli and Vaggi, 2016, p. 231). Obviously, there are other (more practical) possibilities, depending on the availability of data. In any case, the estimation of $\mathbf{u}^{rs}$, or for that matter of $\mathbf{v}^{rs}$, falls beyond the scope of the current paper.

Now using the downstream responsibility IO framework, *total net emissions enabled by net unilateral transfers* received by country r can be quantified as follows:

$$\tilde{e}^r_{NUT} = \underbrace{\sum_s \sum_{k\neq r} (\mathbf{u}^{rk})'\mathbf{G}^{ks}\mathbf{e}^s_c}_{\substack{\text{Global emissions enabled by all} \\ \text{unilateral transfers recieved by } r}} - \underbrace{\sum_s \sum_{k\neq r} (\mathbf{u}^{kr})'\mathbf{G}^{rs}\mathbf{e}^s_c}_{\substack{\text{Global emissions enabled by all} \\ \text{unilateral transfers sent abroad by } r}}, \quad (11)$$

which in matrix form can be compactly written as

$$\tilde{e}^r_{NUT} = (\mathbf{u}^r)'\mathbf{G}\mathbf{e}_c - (\mathbf{u}^{\bullet r})' \begin{bmatrix} \mathbf{G}^{r1} & \mathbf{G}^{r2} & \cdots & \mathbf{G}^{rN} \end{bmatrix} \mathbf{e}_c.$$

Thus, the DIBA emissions for country r can be obtained by adding (11) to the country's

both their economic counterparts of GNI and GNDI.

[16]Think of two persons that are residents of the same country A and make transfers of the same amount, say, 100 euros each, to their families living in the same city of country B. Though the monetary value of their transfers is the same, the environmental consequences of these transfers depend on the production activities, where the corresponding primary incomes have been generated. For example, the associated emissions would be entirely different if one of the transfer senders is working for a fuel extraction company and the other is teaching in a primary school due to the differences in the production structures and thus enabled emissions of these two activities.

[17]For example, in the given formula instead of $\mathbf{v}^{rs}$ use $\tilde{\mathbf{x}}^{rs}$ from (6).

income-based emissions, i.e.

$$DIBA_r = IBA_r + \tilde{e}^r_{NUT}. \tag{12}$$

Notice that, as for any proper allocation scheme, the global DIBA emissions sum up to the world emissions as net emissions enabled by NUT nullify at the global level, i.e. it follows from (11) that $\sum_r \tilde{e}^r_{NUT} = 0$.

Compared to IBA, it is expected that countries largely dependent on workers' remittances from abroad would be assigned higher DIBA responsibility, while those that are major sources of such remittances would be made responsible for lower share of global emissions. Whether this approach is fair or not, is a matter of discussion. Though it is clear that GNDI better reflects national welfare than GNE, GDP and GNI, the question arises of whether it is indeed reasonable to "punish" with higher environmental burden poor countries that receive high NSI (as proportion of their GDP or GNI) from abroad? On the one hand, such reallocation of environmental burden might seem rather unacceptable, say on the moral grounds, as namely large number of citizens of those countries due to the lack of domestic job opportunities are forced to work elsewhere. On the other hand, this mechanism might be another instrument towards serious consideration of the issues of good governance and more equitable (re)distribution of income within the countries concerned (see e.g. Sumner, 2012a,b).

Lastly, the economic circular flow in Figure 3 is "closed" when moving from GNDI back to GNE, which happens when additionally international transactions in *assets* of the country in question are accounted for. Here the notion of assets include real assets (e.g. land and structures), financial assets (e.g. bonds, loans and equity), nonfinancial and nonproduced assets (e.g. patents, copyright, trademarks and franchises) and capital transfers (e.g. debt forgiveness and investment grants). Akin to the earlier discussed economic relationships, adding all emissions associated with the net exports of assets of a country to its DIBA emissions should result in its CBA emissions, thus closing the overall circular flow of emissions. In practice, however, data availability and quality issues on international transactions in assets is much more severe. Hence, instead of direct estimation of the assets-related emissions, these could be better derived as a "residual", i.e.

$$\tilde{e}^r_{NXA} = CBA_r - DIBA_r, \tag{13}$$

where $\tilde{e}^r_{NXA}$ denotes total net emissions associated with or supported by the net exports of assets of country r.

3 Alternative expressions for the CBA and IBA responsibilities

In Figure 3 two alternative expressions for both the CBA and IBA accounting approaches are presented. The CBA emissions are traditionally quantified using equation (5), which is derived from its underlying Leontief demand-driven IO model. However, the presented alternative Ghosh-based CBA emission formulation, to our best knowledge, is not discussed/used in the literature. This might have to do with the fact that the alternative CBA formulation is based on the supply-driven IO model, whose feasibility has been an issue of debate for a long time now (see e.g. Oosterhaven, 1988, 1989, 2012; Gruver, 1989; Rose and Allison, 1989; Dietzenbacher, 1997; De Mesnard, 2009; Guerra and Sancho, 2011; Aroche and Marquez, 2021). In addition, the supply-side IO framework might have not been considered relevant in the discussions and computations of CBA emissions as the latter focus on the demand (consumption) side of an economy.

However, a closer look at the Ghosh-based expressions indicates to their equally important relevance for the purposes of a general responsibility allocation. Let us go back to (2) that shows country r's industry outputs embodied in final uses of country s. In its expanded version, the typical element $\left(\mathbf{L}\hat{\mathbf{y}}^r\right)_{ij}^{kt} = l_{ij}^{kt} y_j^{tr}$ gives *the (value of) total output of industry i in country k that is required directly and indirectly to meet the demand of residents of country r for final deliveries from industry j in country t*. This latter $Nn \times Nn$ matrix can be alternatively rewritten as:

$$\mathbf{L}\hat{\mathbf{y}}^r = \hat{\mathbf{x}}\hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{y}}^r = \hat{\mathbf{x}}\underbrace{\hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}}}_{\mathbf{G}}\underbrace{\hat{\mathbf{x}}^{-1}\hat{\mathbf{y}}^r}_{\hat{\mathbf{y}}_c^r} = \hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{y}}_c^r, \tag{14}$$

where $\mathbf{y}_c^r$ is the vector of *final output coefficients* of country r, with its typical element $y_{ci}^{sr} = y_i^{sr}/x_i^s$ indicating the output share of industry i in country s that is consumed by final users in country r.[18] In deriving (14) we have used the following well-known relationship between the Leontief and Ghosh inverses (Miller and Blair, 2022) :

$$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = \left(\mathbf{I} - \mathbf{Z}\hat{\mathbf{x}}^{-1}\right)^{-1} = \left[\hat{\mathbf{x}}\left(\mathbf{I} - \hat{\mathbf{x}}^{-1}\mathbf{Z}\right)\hat{\mathbf{x}}^{-1}\right]^{-1} = \hat{\mathbf{x}}(\mathbf{I} - \mathbf{B})^{-1}\hat{\mathbf{x}}^{-1} = \hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{x}}^{-1}. \tag{15}$$

Thus, (14) shows that to find outputs embodied in final uses one can also *allocate* the observed industry total outputs to final uses in country r using the matrix $\mathbf{G}\hat{\mathbf{y}}_c^r$. This turns out to be valid for any policy variable of interest that can be linked to industry outputs. For example, the $Nn \times Nn$ matrix of CBA responsibilities of country r, i.e. a detailed version

[18] Recall that subscript c implies that the corresponding variable is given in its "coefficients" form, indicating that the variable of interest is expressed per unit of industry output.

of (5), maybe written as

$$\hat{\mathbf{e}}_c\mathbf{L}\hat{\mathbf{y}}^r = \hat{\mathbf{e}}\hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{y}}^r = \hat{\mathbf{e}}\,\underbrace{\hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}}}_{\mathbf{G}}\,\underbrace{\hat{\mathbf{x}}^{-1}\hat{\mathbf{y}}^r}_{\hat{\mathbf{y}}_c^r} = \hat{\mathbf{e}}\mathbf{G}\hat{\mathbf{y}}_c^r, \tag{16}$$

whose overall (two-sided) sum gives exactly the second expression shown within the CBA box in Figure 3, i.e. $CBA_r = (\mathbf{c}^r)'\mathbf{1} = \mathbf{e}'\mathbf{G}\mathbf{y}_c^r$. Thus, as follows from (16), allocation or assignment of world-wide producers' (production-based) emissions $\mathbf{e}$ to final users in country r with the help of the allocation matrix $\mathbf{G}\hat{\mathbf{y}}_c^r$ results in r's CBA responsibility.

It is worth to highlight the following three features of the matrices $\mathbf{G}\hat{\mathbf{y}}_c^r$ for all countries r. First, they are indeed allocation matrices since their row totals equal unity, i.e.

$$\Big(\sum_r \mathbf{G}\hat{\mathbf{y}}_c^r\Big)\mathbf{1} = \mathbf{G}\Big(\sum_r \mathbf{y}_c^r\Big) = \mathbf{G}\mathbf{Y}_c\mathbf{1} = \mathbf{G}(\mathbf{I}-\mathbf{B})\mathbf{1} = \mathbf{1}, \tag{17}$$

where we have used the output coefficients-based accounting identity $\mathbf{B}\mathbf{1} + \mathbf{Y}_c\mathbf{1} = \mathbf{1}$, with $\mathbf{Y}_c \equiv \begin{bmatrix}\mathbf{y}_c^1 & \mathbf{y}_c^2 & \cdots & \mathbf{y}_c^N\end{bmatrix}$. The *unit additivity property* (17) ensures that the world figure for the factor of interest – e.g. world gross output $\mathbf{x}'\mathbf{1}$ in an output allocation scheme in (14), or the assignment of global CO_2 emissions $\mathbf{e}'\mathbf{1}$ to final users in (16) – is fully allocated without being under- or over-estimated under the new allocation principle.

Second, the allocation matrices $\mathbf{G}\hat{\mathbf{y}}_c^r$ for all r or their country-level (at the final use side) equivalent matrix $\mathbf{G}\mathbf{Y}_c$ in (17) distribute a worldwide factor of interest, detailed at country-industry basis, from producers as output sellers to final users taking into full account the complex (direct and indirect) inter- and intra-country/industry output sales linkages that represent *global output supply chains*. This explains the appearance of intermediate and final output coefficients matrices $\mathbf{B}$ and $\mathbf{Y}_c$ (or $\mathbf{y}_c^r$) as part of the allocation matrices, as e.g. $\mathbf{G}\mathbf{Y}_c = (\mathbf{I}+\mathbf{B}+\mathbf{B}^2+\cdots)\mathbf{Y}_c$. As such we will refer to $\mathbf{G}\hat{\mathbf{y}}_c^r$ for each country r and $\mathbf{G}\mathbf{Y}_c$ as *factor to final users allocation matrices*.

Third, the elements of the mentioned allocation matrices are not necessarily all non-negative because of a possible negative final demand due to large negative changes in inventories and valuables. Though "genuine" shares are often thought to be non-negative, within a framework of CBA responsibility assignment having *negative shares* is reasonable owing to the demand-driven nature of the underlying IO model. That is, as industry outputs and related variables are considered the outcomes of a country's final demand, then the country should be given less CBA responsibility due to its negative, environmentally favorable, final use element(s). This interpretation remains valid for other responsibility measures, such as e.g. jobs embodied in final demand.

Alternatively, we can look at the factor to final user allocation matrices from a supply-side IO model's perspective. Post-multiplication of the Ghosh inverse by the final output coefficients converts the country-industry total inputs (or production) enabled by unitary primary supply to final demands. Thus, the typical element of the allocation matrix $\mathbf{G}\hat{\mathbf{y}}_c^r$ in (14) and (16), i.e. $\left(\mathbf{G}\hat{\mathbf{y}}_c^r\right)_{ji}^{kt} = g_{ij}^{kt} y_{cj}^{tr}$, gives *the amount (in monetary terms) of country r's final demand for products of industry j in country t that is enabled by unitary primary supply provided to industry i in country k*. Hence, the r-th column of the $Nn \times N$ matrix $\mathbf{GY}_c$ in (17) gives country r's final uses supported directly and indirectly through the global supply chains by unitary primary inputs supplied to industries and countries shown along the rows of the allocation matrix $\mathbf{GY}_c$.

This latter perspective clarifies the economic interpretation of the unit additivity property (17). For the world as a whole, all four aggregate economic measures of GNE, GDP, GNI and GNDI must be identical (see Table 1), that is:[19]

$$\text{World GNE} = \text{World GDP} = \text{World GNI} = \text{World GNDI} = \mathbf{1}'\mathbf{V}\mathbf{1} = \mathbf{1}'\mathbf{Y}\mathbf{1}.$$

Hence, if we take a unitary primary income vector, i.e. take $\mathbf{v}' = \mathbf{1}'\mathbf{V} = (\boldsymbol{\varnothing}_i^r)'$ consisting of all zeros except for one non-zero entry equal unity for industry i in country r, then this 1 unit of primary income must appear as an overall sum of final demand values on the expenditure side of the world economy. Namely, the sum of the elements along the row corresponding to industry i in country r of the matrix $\hat{\boldsymbol{\varnothing}}_i^r \mathbf{GY}_c$ should equal one, which is indeed the case as $\hat{\boldsymbol{\varnothing}}_i^r \mathbf{GY}_c \mathbf{1} = \boldsymbol{\varnothing}_{r,i}$. That is, the original 1 unit of primary income should re-appear in an "enabled form" somewhere in the final uses block of the GMRIO setting.

With this background, we can now ask the following question: how the allocation of final uses of producers' emissions would look like if we assumed that the worldwide industry-level primary incomes are equal to these production-based emissions (say, assuming unitary prices of emissions), $\mathbf{v}' = \mathbf{1}'\mathbf{V} = \mathbf{e}'$? The answer is, of course, given by $\hat{\mathbf{e}}\mathbf{G}\hat{\mathbf{y}}_c^r$ for country r, i.e. the last expression in (16), or by $\mathbf{e}'\mathbf{GY}_c$ showing the total CBA emissions of each country. That is, by realizing that one could choose as an "exogenous variable" any factor of interest instead of primary income, an application of the Ghosh model – extended to final uses – results in a new allocation of the considered factor from final use (consumption) perspective.[20] Importantly, such choice of exogenous variable in this "counter-factual" case essentially boils down to assuming that the total industry-level emission responsibilities of producers as input purchasers are equal to such responsibilities of the same producers as output sellers. This should be valid as the industry-base

[19] At the world level, all the four aggregate economic measures are equal as so far there are no *interplanetary* transactions with the Earth's inhabitants in products, factor services, current transfers and assets. This, however, might change (hopefully) in the coming future!

[20] Note again that the unitary additivity property (17) ensures that global consumers' emissions are equal to the global producers' emissions.

identity of total inputs and total outputs imply the equivalence of total emission responsibilities of a producer as output seller and input purchaser. Thus, this approach also justifies referring to $\mathbf{G}\hat{\mathbf{y}}_c^r$ and $\mathbf{GY}_c$ as matrices of factor allocation to final users.

Next we turn to the derivation of the second expression for IBA emissions in Figure 3. In the literature the IBA-based emissions are obtained using equation (9), which is based on the supply-side IO model. However, no reference/use is made to/of their alternative Leontief-based expressions such as the one shown in Figure 3. In all likelihood, this has to do with the fact that the IBA emissions represent emissions enabled by the supply of primary inputs, whose value is an exogenous or driving element in the Ghosh IO model. Thus, one might again think that it is irrelevant to use a demand-driven IO model to calculate IBA that is a "supply-side concept".

Recall that (6) shows country s's industry-level total inputs (or input-side production amounts) enabled by primary supply from country r. In its expanded version, the typical element $(\hat{\mathbf{v}}^r\mathbf{G})_{ij}^{kt} = v_i^{rk} g_{ij}^{kt}$ gives *the (value of) total inputs of industry j in country t that is enabled directly and indirectly by factor services of the residents of country r supplied to industry i in country k*. Using (15), this latter $Nn \times Nn$ matrix can be alternatively rewritten as:

$$\hat{\mathbf{v}}^r\mathbf{G} = \hat{\mathbf{v}}^r\mathbf{G}\hat{\mathbf{x}}^{-1}\hat{\mathbf{x}} = \underbrace{\hat{\mathbf{v}}^r\hat{\mathbf{x}}^{-1}}_{\hat{\mathbf{v}}_c^r}\underbrace{\hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{x}}^{-1}}_{\mathbf{L}}\hat{\mathbf{x}} = \hat{\mathbf{v}}_c^r\mathbf{L}\hat{\mathbf{x}}, \tag{18}$$

where $\mathbf{v}_c^r$ is the vector of *primary input coefficients* of country r, with its typical element $v_{ci}^{rs} = v_i^{rs}/x_i^s$ indicating the value of primary inputs of country r supplied to industry i in country s per unit of its output.

Thus akin to the output allocation from producers to final users in (14), equation (18) shows that to calculate production enabled by primary supply one can alternatively *allocate* the observed industry total outputs to primary input suppliers from country r using the matrix $\hat{\mathbf{v}}_c^r\mathbf{L}$. Again this turns out to be valid for any policy variable that may be linked to industry-level production. For example, the $Nn \times Nn$ matrix of IBA responsibilities of country r, i.e. a disaggregated version of (9), is given by

$$\hat{\mathbf{v}}^r\mathbf{G}\hat{\mathbf{e}}_c = \hat{\mathbf{v}}^r\mathbf{G}\hat{\mathbf{x}}^{-1}\hat{\mathbf{e}} = \underbrace{\hat{\mathbf{v}}^r\hat{\mathbf{x}}^{-1}}_{\hat{\mathbf{v}}_c^r}\underbrace{\hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{x}}^{-1}}_{\mathbf{L}}\hat{\mathbf{e}} = \hat{\mathbf{v}}_c^r\mathbf{L}\hat{\mathbf{e}}, \tag{19}$$

whose overall sum gives exactly the second expression shown within the IBA box in Figure 3, i.e. $IBA_r = (\mathbf{n}^r)'\mathbf{\iota} = (\mathbf{v}_c^r)'\mathbf{Le}$. Thus, as follows from (19), by simply distributing the world producers' (production-based) emissions $\mathbf{e}$ to primary suppliers in country r with the help of the allocation matrix $\hat{\mathbf{v}}_c^r\mathbf{L}$ gives r's IBA responsibility.

Similar to the factor to final user responsibility allocation matrices, the matrices $\hat{\mathbf{v}}_c^r\mathbf{L}$

for all r and $\mathbf{V}_c\mathbf{L}$, where $\mathbf{V}_c = \mathbf{V}\hat{\mathbf{x}}^{-1}$ or $\mathbf{V}_c' \equiv \begin{bmatrix}\mathbf{v}_c^1 & \mathbf{v}_c^2 & \cdots & \mathbf{v}_c^N\end{bmatrix}$, are also characterized by analogous three features. First, as required for any proper redistribution procedure, the latter allocation matrices satisfy the unit additivity property akin to (17), i.e.

$$\boldsymbol{\imath}'\Big(\sum_r \hat{\mathbf{v}}_c^r\mathbf{L}\Big) = \Big(\sum_r (\mathbf{v}_c^r)'\Big)\mathbf{L} = \boldsymbol{\imath}'\mathbf{V}_c\mathbf{L} = \boldsymbol{\imath}'(\mathbf{I}-\mathbf{A})\mathbf{L} = \boldsymbol{\imath}', \qquad (20)$$

where we have used the input coefficients-based accounting identity $\boldsymbol{\imath}'\mathbf{A} + \boldsymbol{\imath}'\mathbf{V}_c = \boldsymbol{\imath}'$.

Second, the allocation matrices $\hat{\mathbf{v}}_c^r\mathbf{L}$ for all r or their country-level (at the primary supply side) equivalent matrix $\mathbf{V}_c\mathbf{L}$ in (20) transform a global factor of interest, with country and industry disaggregation, from producers as input purchasers to primary suppliers taking full account of the complex (direct and indirect) inter- and intra-country/industry input purchase linkages that represent *global input demand chains*.[21] This explains why now the input coefficients matrices $\mathbf{A}$ and $\mathbf{V}_c$ (or $\mathbf{v}_c^r$), and not their output coefficients counterparts, constitute the latter allocation matrices, as in e.g. $\mathbf{V}_c\mathbf{L} = \mathbf{V}_c(\mathbf{I}+\mathbf{A}+\mathbf{A}^2+\cdots)$. As such we will refer to $\hat{\mathbf{v}}_c^r\mathbf{L}$ for all countries r and $\mathbf{V}_c\mathbf{L}$ as *factor to primary suppliers allocation matrices*.

And third, the elements of the factor to primary suppliers allocation matrices are not necessarily all non-negative due to a possibility of having negative primary income for certain industries with highly subsidized production. Responsibility assignment based on a possibility of having negative shares is again reasonable owing to the supply-driven nature of the underlying IO model. Since industry total inputs (outputs) and related satellite impacts are considered the outcome of a country's primary supply, then the country should be given less IBA responsibility due to its negative, but environmentally favorable, primary income element(s). This interpretation remains valid for other responsibility measures, such as e.g. the number of jobs enabled by primary supply.

Analogously, one can examine the factor to primary suppliers allocation matrices from its underlying demand-side IO model's perspective. Pre-multiplication of the Leontief inverse by the primary input coefficients converts the country-industry total outputs (production) embodied in unitary final demands to primary incomes. Thus, the typical element of the allocation matrix $\hat{\mathbf{v}}_c^r\mathbf{L}$ in (18) and (19), i.e. $(\hat{\mathbf{v}}_c^r\mathbf{L})_{ij}^{kt} = v_{ci}^{rk} l_{ij}^{kt}$, gives *country r's primary income earned from production of industry i in country k that is embodied in unitary final demand for products of industry j in country t*. Consequently, the r-th row of the $N \times Nn$ matrix $\mathbf{V}_c\mathbf{L}$ in (20) gives country r's primary incomes generated directly and indirectly through the global input demand chains by unitary final outputs demanded *from* industries and countries shown along the columns of the allocation matrix $\mathbf{V}_c\mathbf{L}$.[22]

[21] For the importance of distinguishing between the output supply chains vs. input demand chains, see Miller and Temurshoev (2017).

[22] This demand-side perspective gives an economic interpretation to the unit additivity property (20),

We can now raise the following "unconventional" question: how the allocation of primary supplies of producers' emissions would look like if we assumed that the industry-level global final demands are equal to the corresponding production-based emissions (again, say, assuming unitary prices of emissions), $\mathbf{y} = \mathbf{Y}\boldsymbol{\imath} = \mathbf{e}$? The answer is given by $\hat{\mathbf{v}}_c^r\mathbf{L}\hat{\mathbf{e}}$ for country r, i.e. the last expression in (19), or by $\mathbf{V}_c\mathbf{L}\mathbf{e}$ showing the total IBA emissions of each country. That is, by realizing that one may choose as an "exogenous variable" any factor of interest instead of final demand, an application of the demand-driven Leontief model – extended to primary incomes (or supplies) – results in a new allocation of the considered factor from primary supply (income) perspective. All in all, this later approach also validates our reference to $\hat{\mathbf{v}}_c^r\mathbf{L}$ and $\mathbf{V}_c\mathbf{L}$ as the matrices of factor allocation to primary suppliers.

As a final note it is important to highlight that *at the most detailed country-industry level, income earned by primary suppliers from country r due to the demand of final users in country s is exactly equal to the value of final use in country s enabled by primary supply from country r*, as follows from

$$\underbrace{\hat{\mathbf{v}}_c^r\mathbf{L}\hat{\mathbf{y}}^s}_{FU(s)-to-PI(r)} = \hat{\mathbf{v}}^r\hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}}\hat{\mathbf{x}}^{-1}\hat{\mathbf{y}}^s = \underbrace{\hat{\mathbf{v}}^r\mathbf{G}\hat{\mathbf{y}}_c^s}_{PI(r)-to-FU(s)}, \tag{21}$$

where $FU(s)$ and $PI(r)$ refer, respectively, to final use in country s and primary income of country r. The *FU(s)-to-PI(r)* and *PI(r)-to-FU(s)* matrices in (21) are basically the (global) multi-regional extensions of the "gross value added-final use (GVA-FU)" and (the transpose of the) "final use-gross value added (FU-GVA)" matrices introduced in Cai and Leung (2020). The latter study seems to be the first contribution that makes explicit the link between GVA and FU at the industry level.[23]

4 The limiting distributions of responsibilities

In the previous section we have derived and explained the importance of the factor to final users and factor to primary suppliers allocation matrices. These matrices provide ready tools for a different approach to responsibility determination as compared to the allocation mechanisms currently used in the literature. In particular, it may be argued that only final users and primary suppliers need to be considered as responsibility targets,

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similar to the related discussions earlier using the income- and expenditure-based identity for the world GNE/GDP/GNI/GNDI. Here one can also easily show that 1 monetary unit of final demand must ultimately show up as the sum of elements along the corresponding column of $\mathbf{V}_c\mathbf{L}$, while the related detailed income contributions are shown along the same column of matrices $\hat{\mathbf{v}}_c^r\mathbf{L}$ for each country r.

[23] The authors interpret the *ij*-th element of the GVA-FU matrix as "the amount of sector *i*'s GVA attributable to sector *j*'s final use" and the *ji*-th entry of the FU-GVA matrix as "the amount of sector *i*'s GVA embedded in sector *j*'s final use" (Cai and Leung, 2020, p. 432). In its multi-regional extension (21), additionally the countries (of residence) of final users and primary suppliers need to be distinguished.

as namely these agents are the ultimate beneficiaries of the whole production process in terms of their (final) consumption and earned income. Among other environmentally consequential activities, primary suppliers and final users also either directly and/or indirectly manage and operationalize production units, are actively involved in marketing and advertising that affect final consumption patterns, and determine future production expansion plans and strategies.

Let us consider the following problem of responsibility reallocation between final users and primary suppliers.

Responsibility reallocation between final users and primary suppliers

Starting point: The starting distribution is the $Nn \times 1$ vector of production-based emissions, $\mathbf{e}$.

Iterations: At each iteration step $k = 1, 2, \ldots$, calculate the $Nn \times 1$ vectors of emission responsibilities of final users $\mathbf{c}^r_{(k)}$ and of primary suppliers $\mathbf{n}^r_{(k)}$ from (i.e. resident in) country $r = 1, 2, \ldots, N$, using the following responsibility reallocation rules:

Reallocation 1: Setting $\mathbf{n}^r_{(0)} = \mathbf{e}$, iteratively calculate

$$\mathbf{c}^r_{(k)} = \hat{\mathbf{y}}^r_c \mathbf{G}' \mathbf{n}_{(k-1)}, \quad \mathbf{c}_{(k)} = \sum_r \mathbf{c}^r_{(k)} = \hat{\mathbf{y}}_c \mathbf{G}' \mathbf{n}_{(k-1)} \tag{22a}$$

$$\mathbf{n}^r_{(k)} = \hat{\mathbf{v}}^r_c \mathbf{L} \mathbf{c}_{(k)}, \quad \mathbf{n}_{(k)} = \sum_r \mathbf{n}^r_{(k)} = \hat{\mathbf{v}}_c \mathbf{L} \mathbf{c}_{(k)}, \tag{22b}$$

where $\mathbf{c}^r_{(1)} = \hat{\mathbf{y}}^r_c \mathbf{G}' \mathbf{e}$ is the CBA emissions for country r (see (5) and (16)).

Reallocation 2: Setting $\tilde{\mathbf{c}}^r_{(0)} = \mathbf{e}$, iteratively calculate

$$\tilde{\mathbf{n}}^r_{(k)} = \hat{\mathbf{v}}^r_c \mathbf{L} \tilde{\mathbf{c}}_{(k-1)}, \quad \tilde{\mathbf{n}}_{(k)} = \sum_r \tilde{\mathbf{n}}^r_{(k)} = \hat{\mathbf{v}}_c \mathbf{L} \tilde{\mathbf{c}}_{(k-1)} \tag{23a}$$

$$\tilde{\mathbf{c}}^r_{(k)} = \hat{\mathbf{y}}^r_c \mathbf{G}' \tilde{\mathbf{n}}_{(k)}, \quad \tilde{\mathbf{c}}_{(k)} = \sum_r \tilde{\mathbf{c}}^r_{(k)} = \hat{\mathbf{y}}_c \mathbf{G}' \tilde{\mathbf{n}}_{(k)}, \tag{23b}$$

where $\tilde{\mathbf{n}}^r_{(1)} = \hat{\mathbf{v}}^r_c \mathbf{L} \mathbf{e}$ is the IBA emissions for country r (see (9) and (19)).

Final allocations: If the sequence of responsibilities in (22a)-(22b) and (23a)-(23b) is finite, find these limiting or "stationary" distributions of responsibilities, i.e. for each country r determine

$$\mathbf{c}^r_\infty = \lim_{k\to\infty} \mathbf{c}^r_{(k)}, \quad \mathbf{n}^r_\infty = \lim_{k\to\infty} \mathbf{n}^r_{(k)}, \quad \tilde{\mathbf{n}}^r_\infty = \lim_{k\to\infty} \tilde{\mathbf{n}}^r_{(k)}, \quad \text{and} \quad \tilde{\mathbf{c}}^r_\infty = \lim_{k\to\infty} \tilde{\mathbf{c}}^r_{(k)}.$$

Hence, in the reallocation scheme 1, we first compute consumption-based emissions for all countries using the corresponding factor to final users allocation matrices $\hat{\mathbf{y}}_c^r\mathbf{G}'$, as follows from (22a). Then the sum of these detailed CBA emissions are reallocated back to primary suppliers using the corresponding factor for primary suppliers allocation matrices $\hat{\mathbf{v}}_c^r\mathbf{L}$, as indicated by (22b). However, the process of responsibility redistribution does not terminate here: in the second reallocation round, the sum of the latter detailed IBA emissions are again redistributed back to final users using the same allocation principle underlying (22a). In theory, such responsibility reallocation process between final users and primary suppliers is then repeated infinitely.

The reallocation scheme 2 in (23a)-(23b) differs from the reallocation rule 1 in (22a)-(22b) only with respect to its initial step (or initial distribution point). With (23a)-(23b), the first allocation computes the IBA emissions for each country and then redistributes their (detailed) global amounts back to final users, and then this redistribution process continues infinitely. In both cases, our main interest lies on whether the process of such infinitely repeated reallocation is finite and, if so, whether the nature of the ultimate distributions depend on the choice of the initial distribution point.

Before we delve into the technical details of the reallocation mechanics, it is interesting to observe that the first-step IBA emissions according to the reallocation rule 1, i.e. $\mathbf{n}_{(1)}^r = \hat{\mathbf{v}}_c^r\mathbf{L}\mathbf{c}_{(1)}$, is related to the so-called "value added-based responsibility" due to Piñero et al. (2019). In this latter paper, the CBA responsibility is used "with the single aim to then re-allocate it again to all entities participating in the supply chain according to their value-added shares" so that the approach "holds accountable all profiteers (value generators) along the entire supply chains" (p. XX). Aside from differing interpretations, $\mathbf{n}_{(1)}^r$ in our setting boils down exactly to the value-added responsibility when the value-added coefficients vector $\hat{\mathbf{v}}_c^r$ is defined as in Table 1, i.e. when the countries of residence of primary suppliers are explicitly distinguished within the primary income quadrant of the global MRIO table.

With consecutive substitutions of each-step responsibility expressions, the iterative relations in (22a)-(22b) can be more compactly written in the following alternative form:

$$\mathbf{c}_{(k)}^r = \hat{\mathbf{y}}_c^r\mathbf{G}' \left[\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{y}}_c\mathbf{G}'\right]^{k-1}\mathbf{e} \tag{24a}$$

$$\mathbf{n}_{(k)}^r = \hat{\mathbf{v}}_c^r\mathbf{L}\hat{\mathbf{y}}_c\mathbf{G}' \left[\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{y}}_c\mathbf{G}'\right]^{k-1}\mathbf{e}, \tag{24b}$$

Similarly, the iterative relations of responsibilities in (23a)-(23b) boil down to:

$$\tilde{\mathbf{n}}_{(k)}^r = \hat{\mathbf{v}}_c^r\mathbf{L} \left[\hat{\mathbf{y}}_c\mathbf{G}'\hat{\mathbf{v}}_c\mathbf{L}\right]^{k-1}\mathbf{e}, \tag{25a}$$

$$\tilde{\mathbf{c}}_{(k)}^r = \hat{\mathbf{y}}_c^r\mathbf{G}'\hat{\mathbf{v}}_c\mathbf{L} \left[\hat{\mathbf{y}}_c\mathbf{G}'\hat{\mathbf{v}}_c\mathbf{L}\right]^{k-1}\mathbf{e} \tag{25b}$$

It follows from (24a)-(25b) that the existence and nature of the searched limiting distributions depend on the convergence of powers of the matrices $\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{y}}_c\mathbf{G}'$ and $\hat{\mathbf{y}}_c\mathbf{G}'\hat{\mathbf{v}}_c\mathbf{L}$. The first matrix $\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{y}}_c\mathbf{G}'$, if read from the end to its beginning, indicates that a factor responsibility is first allocated to the world final users (captured by $\hat{\mathbf{y}}_c\mathbf{G}'$), which is then immediately reallocated to the world primary suppliers (using $\hat{\mathbf{v}}_c\mathbf{L}$). Thus, the power matrix $\left(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{y}}_c\mathbf{G}'\right)^{k-1}$ in (24a)-(24b) with k approaching infinity simply means that the process of global reallocation of responsibilities between final users and primary suppliers is implemented, in theory, indefinitely. In the same vein, the second power matrix $\left(\hat{\mathbf{y}}_c\mathbf{G}'\hat{\mathbf{v}}_c\mathbf{L}\right)^{k-1}$ with $k \to \infty$ used in (25a)-(25b) signifies a similar in(de)finite reallocation of responsibilities between the two agents globally, which however starts with attributing responsibilities first to primary suppliers.

`[Leave technical details for now, and summarize the main findings of this section.]`

If the global emissions figure is denoted by $e_w = \mathbf{1}'\mathbf{e}$, then the limiting distributions of responsibilities at the detailed country-industry levels are found to equal:

$$\mathbf{c}^r_\infty = \mathbf{y}^r\frac{e_w}{\mathbf{1}'\mathbf{v}} \quad \text{and} \quad \mathbf{n}^r_\infty = \mathbf{v}^r\frac{e_w}{\mathbf{1}'\mathbf{v}}, \tag{26a}$$

$$\tilde{\mathbf{c}}^r_\infty = \mathbf{y}^r\frac{e_w}{\mathbf{1}'\mathbf{y}} \quad \text{and} \quad \tilde{\mathbf{n}}^r_\infty = \mathbf{v}^r\frac{e_w}{\mathbf{1}'\mathbf{y}}. \tag{26b}$$

Since all the four aggregate economic indicators are equal at the world level, implying in particular $\mathbf{1}'\mathbf{v} = \mathbf{1}'\mathbf{y}$, then $\mathbf{c}^r_\infty = \tilde{\mathbf{c}}^r_\infty$ and $\mathbf{n}^r_\infty = \tilde{\mathbf{n}}^r_\infty$. Thus, the limiting distributions of responsibilities of final users and primary suppliers are independent from the initial allocation step that distinguishes the two reallocation schemes (22a)-(22b) and (23a)-(23b). Moreover, one can write the country-level limiting (distributions of) responsibilities simply in terms of global emissions and GNE or GNI as:

$$c^r_\infty = \frac{GNE_r}{GNE_w}e_w \quad \text{and} \quad n^r_\infty = \frac{GNI_r}{GNI_w}e_w. \tag{27}$$

If the primary income matrix $\mathbf{V}$ used in the repeated reallocation of responsibilities is constrained by the current data availability issues, i.e. when the value-added matrix $\mathbf{V}_{GDP}$ of the form (10) is used instead, then the country-level limiting responsibility of primary suppliers is given in terms of GDP rather than GNI:

$$n^r_\infty = \frac{GDP_r}{GDP_w}e_w. \tag{28}$$

`Things/issues to be added:`

- Provide the full proof and describe the conditions (e.g. in terms of countries' connectivity) for equations (26a)-(28);
- Discuss the limiting responsibilities for a hypothetical case with two (or more) groups of countries with no trade (direct and indirect) whatsoever between the groups;
- Detail the general conditions for the existence of $\lim_{k\to\infty}\left(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{y}}_c\mathbf{G}'\right)^k$ and $\lim_{k\to\infty}\left(\hat{\mathbf{y}}_c\mathbf{G}'\hat{\mathbf{v}}_c\mathbf{L}\right)^k$, and discuss the required semi-convergence in terms of the existence of negative elements in $\mathbf{y}_c$ and $\mathbf{v}_c$;
- $\lim_{k\to\infty}\left(\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{y}}_c\mathbf{G}'\right)^k$ is the projector onto the null space of $\mathbf{I}-\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{y}}_c\mathbf{G}'$ along the range (column space) of $\mathbf{I}-\hat{\mathbf{v}}_c\mathbf{L}\hat{\mathbf{y}}_c\mathbf{G}'$;
- Similarly, $\lim_{k\to\infty}\left(\hat{\mathbf{y}}_c\mathbf{G}'\hat{\mathbf{v}}_c\mathbf{L}\right)^k$ is the projector onto $N\left(\mathbf{I}-\hat{\mathbf{y}}_c\mathbf{G}'\hat{\mathbf{v}}_c\mathbf{L}\right)$ along $R\left(\mathbf{I}-\hat{\mathbf{y}}_c\mathbf{G}'\hat{\mathbf{v}}_c\mathbf{L}\right)$;
- Briefly discuss the implications of the use of global MRIO data in basic prices on the limiting responsibilities: then net taxes on final products are excluded from the limiting responsibility formulas;
- Discuss in some detail the implications of the discovered limiting distributions of responsibilities in the context of international cooperation on climate
- Advantages of the limiting responsibilities: readily available data, less uncertainties (more reliable data), robust to (dis)aggregation, hence no (dis)aggregation bias;
- Limiting responsibilities as composite ‘‘responsibility-capability" indicators, if seen from the perspective of equitable burden-sharing frameworks used by climate scientists (e.g. the dimensions of equity principles often include responsibility, capability, equality, and cost-effectiveness.)
- Explain (justify) why the discovered limiting responsibilities in relative terms are valid for *any* production ‘‘externality";
- Add a short concluding section.

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