Resource allocation model to estimate indirect economic damage of Tokyo Epicentral Earthquake.

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Abbreviation

GDE	Gross Domestic Expenditure
IIOA	International Input−Output Association
TEE	Tokyo Epicentral Earthquake
TM	Tokyo Metropolitan

Abstract

This research deals with supply constraints of intermediate goods such as raw materials and components due to Tokyo epicentral earthquakes. While Shimoda and Fujikawa (2022) estimated indirect earthquake damage as a decrease in production when a bottleneck occurs in the input of intermediate goods, this research analyzes the case where supply constraints are resolved by adjusting somehow the final demand side.

Mathematical programming can be applied to this kind of problem. The demand menu that an economy can choose form is called a demandable region in mathematical planning. the occurrence of a supply constraint means a shrink of the demandable region. After the shrink of the demandable region, which part of the new demandable region should be selected depends on the policy priority, or the objective function of policy authority.

Incidentally, in the conventional input−output model, a final demand is the exogenous variable and production volume to supply the final demand is the endogenous variable, while in the resource allocation model, the production volume is the exogenous variable, and the optimal final demand is the endogenous variable depending on the objective function. In the model of Shimoda and Fujikawa (2022), the indirect damage of an earthquake is estimated as changes of the production volume, while in the model of this research, the indirect damage of the earthquake is estimated as the change of the objective function.

In this research, the author considers “maximization of GDE” and “minimization of the change in final demand” as examples of the objective functions in mathematical programming. As a matter of course, it was confirmed that the optimum solution differs depending on the objective function in this research. More concretely, the damage is limited inside of Tokyo in the resource allocation to maximize GDE, while the damage extends outside of Tokyo in the resource allocation to minimize the change in final demands. What kind of resource allocation should be chosen depends on the object setting of policy authorities? This idea has rarely been introduced as a method for estimating indirect damage caused by earthquakes, and the author thinks it makes sense to introduce this at IIOA.

1. Introduction

Earthquakes directly cause human and property damage, and these direct damages lead to indirect economic damage such as supply constraints. However, since earthquakes occur in a specific area, the immediate issue is how to share the immediate economic damage among domestic regions. In this research, the author estimates what kind of resource allocation should be made when there are supply constraints in various industries and a certain policy decision was made.

I would like to introduce Nagaoka (1976) as a study dealing with a supply constraint. Nagaoka (1976) was written in an era when Japanese economy faced supply constraints due to the oil crisis in 1973. Most of the economic forecasting models at that time were of demand−driven type, but Nagaoka(1976) was aware of the problem that this type of models are not necessarily effective. Although the model of Nagaoka(1976) is based on equilibrium production model of input−output analysis, the causality direction is the opposite of the conventional usual input−output analysis

In a conventional model, the final demand is used as the exogenous variable and the production volume to satisfy the demand is calculated as the endogenous variable. On the other hand, the major feature of the Nagaoka(1976) is that domestic production (with supply constraints) is the exogenous variable and final demand is the endogenous variable as a solution of optimization problem in mathematical planning.

In this paper borrows to estimate the indirect damage of the Tokyo epicentral earthquake by solving the optimization problem under the supply constraint. The structure of this research is as follows. Section 2 introduces the analysis methods in this paper by illustrating the differences of the model in this research and the conventional input−output model. Section 2 also explains the assumed policy decisions. Section 3 discusses the simulation results, and finally, section 4 presents the conclusions.

2. Model

2−1 Illustration of the model

First, we explain the difference between the model in this paper and the traditional industrial linkage analysis model method.

(1) Setting of the exogenous variables

In the conventional input output analysis, the final demand f is the exogenous variable and the domestic production is the endogenous variable. The analyst assumes some change in the final demand f and calculate the domestic production x that satisfies this final demand using the Leontief inverse matrix.

On the other hand, in this paper, it is assumed that domestic production is restricted by an earthquake, and the final demand is calculated based on some criteria under this supply constraint as is shown in in Figure 1. In this paper, given the production constrained by the earthquake (exogenous variable), the final demand is calculated as the endogenous variable using the Leontief inverse matrix.

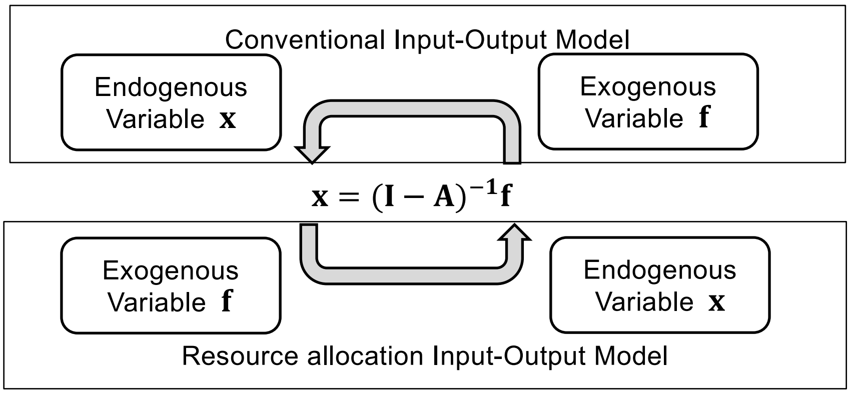

Figure 1 Differences between the conventional input−output analysis model and the model in this research

Source: Created by the author

(2) Setting of objective function

If the earthquake causes production constraints, the government has several policy options to take. For example, “maximization of GDE after the earthquake”, “minimization of changes in GDE before and after the earthquake”, or “maximization of the utility of households”.

(3) Method to estimate the damage caused by the earthquake

Let $\mathbf{x}$ be the production volume before the earthquake and $\mathbf{x}'$ be the production volume after the earthquake. As described above, by solving the mathematical programing model, the optimal final demand $\mathbf{f}'$ can be obtained based on the objective function. Since the distribution of the final demand to each industry is determined as the optimum solution regarding the objective function, this model can be called the “resource allocation model”.

In this model, the indirect economic damage caused by the earthquake is calculated as the difference between the value of the current objective function and the value of the objective function after the supply constraint is given.

2–2 Maximization of final demands

Human and physical damage after the earthquake will reduce production in the affected area. In this section, we will maximize GDE by changing the final demand of each sector including the areas that are not affected the disaster under the decreased production due to the earthquake. Let us explain this with two sector input output table.

The optimization problem can be written as follows.

$$Max.GDE = f_1 + f_2 \tag{1}$$

$$st.\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq [\mathbf{I} - \mathbf{A}]^{-1}\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{2}$$

We rewrite the Leontief inverse matrix as follow to illustrate.

$$[\mathbf{I} - \mathbf{A}]^{-1} = \begin{bmatrix} 1 - a_{11} & -a_{12} \\ -a_{21} & 1 - a_{22} \end{bmatrix}^{-1} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{3}$$

Then the optimization problem can be rewrite as follows.

$$Max. GDE = f_1 + f_2 \tag{4}$$

$$st. \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{5}$$

Furthermore, equation (5) can be rewritten as equation (6) and equation (7).

$$x_1 \geq b_{11} f_1 + b_{12} f_2, \text{ or } \ f_2 \leq -\frac{b_{11}}{b_{12}} f_1 + \frac{x_1}{b_{12}} \tag{6}$$

$$x_2 \geq b_{21} f_1 + b_{22} f_2, \text{ or } \ f_2 \leq -\frac{b_{21}}{b_{22}} f_1 + \frac{x_2}{b_{22}} \tag{7}$$

Figure 2 shows this mathematical optimization model.

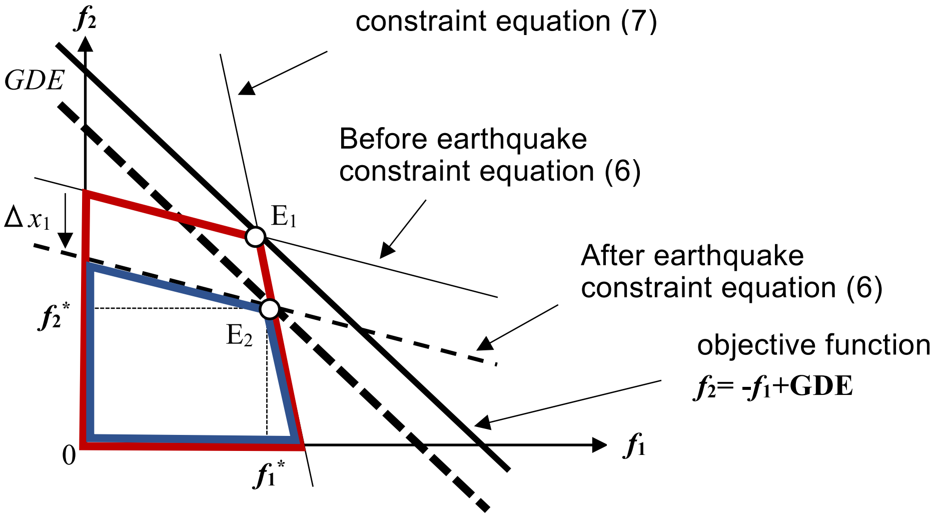

Figure 2 Illustration of GDE maximization

Source: drawn by the author

Point E_1 is the intersection of the constraint (6) and the constraint (7) and this is the point of maximizing GDE before the earthquake. When an earthquake occurs, the production volume of the first sector decreases, if the input coefficient does not change. Then, the constraint (6) shifts downward. On the other hand, the constraint (7) is invariant. The new equilibrium point is E_2 which is the intersection of the shitted new constraint (6) and the constraint (7).

The demandable region before the earthquake is expressed by the red square and that of after the earthquake is expressed by the blue square. The maximized GDE decreases accordingly.

2–3 Minimization of the total change in final demands

Human and physical damage after the earthquake will reduce production in the affected area. In this section, we minimize the changes in the final demands for each sector before and after the earthquake. Under the decreased domestic production after the disaster, we calculate the final demands for each sector including not affected area that minimizes the change in the final demand for each industry compared to before the earthquake.

The mathematical optimization model is expressed as follows.

$$Min. r^2 = (f_1 - f_{1o})^2 + (f_2 - f_{2o})^2 \tag{9}$$

$$st. \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq [\mathbf{I} - \mathbf{A}]^{-1} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{10}$$

Figure 3 is the illustration of this model..

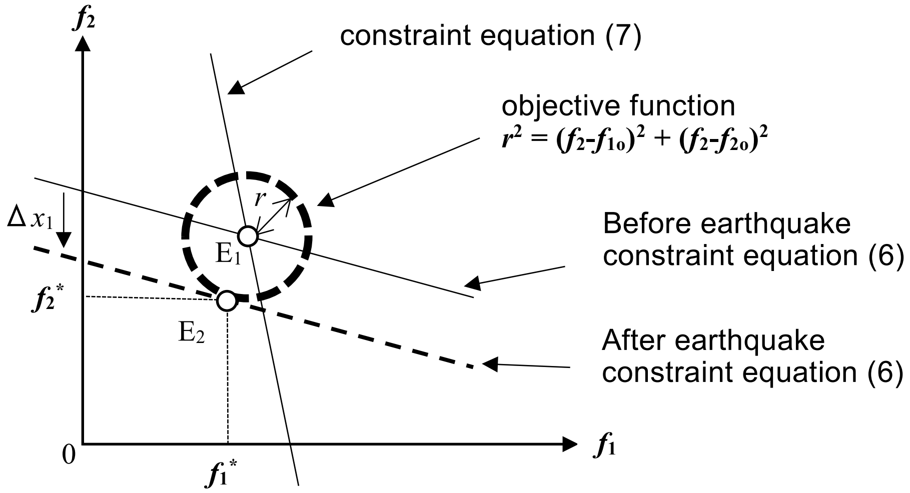

Figure 3 Illustration of minimize the total changes in the final demands

Source: drawn by the author

Point E_1 is the intersection of the constraint (6) and the constraint (7) and this is assumed to be the initial optimal point. When an earthquake occurs, the production volume of the first sector decreases, if the input coefficient does not change. Then, the constraint (6) shifts downward. On the other hand, the constraint (7) is invariant. The new equilibrium point is E_2 which is contact point between the constraint equation (6) and the circle centered on E_1 since the objective function is the radius of the circle centered on E_1.

3. Simulation

3–1 Data and simulation scenario

The simulation is implemented under the assumption that the production in Tokyo Metropolitan (TM) decreases due to the TEE. We assume that the decrease in production was 10% for the headquarters in Tokyo, and that the decreasing rates in production in other sectors are equivalent to the decrease in production in Kobe City due to the Great Hanshin Awaji Earthquake in 1995 based on Shimoda and Fujikawa (2022). This is shown in Table 1 below.

Table 1 Assumed production decrease in TM caused by TEE (Billion yen)

	Sectors	Pre TEE production in TM	Assumed damage rate in TM by TEE	Assumed damage in TM by TEE
1	Agriculture, forestry and fisheries	101.7	-4.5%	-4.6
2	Mining	9.5	-5.2%	-0.5
3	Food and beverage	1,161.9	-16.3%	-189.2
4	Textile and garment	82.9	-5.0%	-4.1
5	Pulp, paper and wood products	285.1	-1.7%	-4.9
6	Chemical products	511.2	-20.7%	-106.1
7	Petroleum and coal products	29.7	-38.2%	-11.3
8	Plastic and rubber products	200.5	-21.4%	-43.0
9	Ceramics and stone products	163.1	0.0%	0.0
10	Iron and Steel	166.0	-8.0%	-13.2
11	Non-ferrous metals	74.6	-8.0%	-5.9
12	Metal products	231.3	-7.2%	-16.5
13	Machinery for general use	268.5	-6.0%	-16.1
14	Production machinery	358.7	-6.0%	-21.5
15	Business machinery	543.5	-6.0%	-32.6
16	Electronic components	306.8	-5.4%	-16.4
17	Electric machinery	671.2	-5.4%	-35.9
18	Information and communication equip.	564.3	-5.4%	-30.2
19	Transport machinery	1,939.1	-8.7%	-169.6
20	Other manufacturing products	1,298.6	-21.4%	-278.2
21	Construction	8,346.0	0.0%	0.0
22	Electricity, gas and heat supply	1,393.2	-15.5%	-216.1
23	Water services	652.8	-15.5%	-101.3
24	Waste disposal	459.2	-15.5%	-71.2
25	Commerce	24,142.9	-10.3%	-2,480.8
26	Finance and insurance	11,151.6	-7.9%	-877.8
27	Real estate	14,788.4	-5.1%	-753.5
28	Transportation and postal services	7,550.3	-10.8%	-815.3
29	Telecommunications	22,538.6	-10.8%	-2,433.9
30	Public affairs	6,669.9	-0.1%	-5.0
31	Education and research	7,041.6	-3.3%	-230.5
32	Medical and welfare services	7,279.5	-3.3%	-238.3
33	Membership organization	589.6	-3.3%	-19.3
34	Business services	25,682.7	-3.3%	-840.8
35	Personal services	10,032.8	-3.3%	-328.5
36	Office supplies	289.8	0.0%	0.0
37	Not else classified	750.4	0.0%	0.0
38	Headquarters	30,707.5	-10.0%	-3,070.7
	Total	189,035.1	-7.1%	-13,483.2

Source: Shimoda and Fujikawa (2022)

The input−output table used in this research is the “2015 Tokyo Metropolitan Input−Output Table”. This table has 38 sectors, of which 37 sectors are normal industries and the headquarters is listed separately as a sector. The format of this table is an inter−regional table composed of two areas, inside Tokyo and outside Tokyo.

Using this Tokyo Table, we simulated the optimal resource allocation in the following three cases: (1) direct damage only to the headquarters, (2) direct damage to normal sectors other than the headquarters, and (3) direct damage to all sectors. And we considered the following two objective functions for the resource allocation estimation: (1)

maximizing GDE and (2) minimizing change in final demands.

3–2 Results of simulation

(1) Maximization of final demands

Table 2 below shows the calculation results when “maximization of final demands” is taken as one of the policy options. The result is that the final demand amount is calculated so as to maximize GDE under the production constraint after the TEE and the indirect economic damage is measured as the reduction in final demands total in Japan.

1) Decrease in GDE

The order of the degree of the final demand decrease is “all industries”, “headquarters only” and “other than headquarters” in descending order. Since the amount of production decrease estimated for damage is the largest in the “all industries”, this result is reasonable. However, “headquarters only”, which is the minimum damage assumption, had the second largest decrease in final demand after “all industries”. This suggests that the damage to the headquarters in TM may have a great impact comparing to other sectors. The rate of decrease in final demands for the “all industries” was calculated as −3.5%. This means that Japan has to be prepared 3.5% GDE loss as indirect economic damages of TEE.

2) Decrease in GDE by region

What is common to the three cases of “all industries”, “headquarters only” and “other than headquarters” is reduction in GDE in TM and increase in GDE in outside TM. This means that in the event of a disaster in Tokyo, the policy of maximizing GDE will be realized by a decrease in the final demand in TM and an increase in final demand in outside TM. In this assumption, the indirect economic damage is concentrated in TM.

3) Decrease in GDE by industrial sector

“Commerce” was the industrial sector with the largest decrease in final demand and the largest contribution rate in “all industries”. Damage to “Commercial” significantly large in “headquarters only”, while that in “other than headquarters” is relatively marginal. The “Service” is an industrial sector that occupies the largest share of final demand in “Tokyo”. The “Service” had the second largest damage in final demand after “Commerce” in “all industries”. Since the share of final demand in the “Service” is high, the level of decrease is also large, but the rate of decrease is no more than 12%. These two industries account for most of the indirect damage.

This simulation result can also be interpreted as follows. GDE loss of 3.5% In the “all industries” can be interpreted as the indirect economic damage in case there is no backup system for the headquarters function. On the other hand, GDE loss in “other than headquarters” is 1.7%, which can be interpreted as the indirect economic damage when the headquarters function has a complete backup system. In other words, if there is a backup system for the headquarters function, indirect damage of TEE may be reduced to about a half.

Table 2 Estimated result of maximization of final demands

		Pre TEE FD	FD changes by TEE (Billion yen)			Rate of change(%)			Contribution rate(%)		
			Only headquarters	All sectors except headquarters	All sectors including headquarters	Only headquarters	All sectors except headquarters	All sectors including headquarters	Only headquarters	All sectors except headquarters	All sectors including headquarters
Tokyo	Agriculture / mining	52	29	-2	9	56.4	-3.9	18.1	0.2	-0.0	0.0
	Manufacturing	4,770	-1,073	-3,483	-3,092	-22.5	-73.0	-64.8	-5.6	-33.3	-14.3
	Construction	7,730	44	-1,181	72	0.6	-15.3	0.9	0.2	-11.3	0.3
	Electricity, gas, etc.	850	476	-439	-51	56.0	-51.6	-6.0	2.5	-4.2	-0.2
	Commerce	15,659	-15,659	-2,258	-15,659	-100.0	-14.4	-100.0	-82.2	-21.6	-72.2
	Service	61,290	-7,022	-6,965	-7,383	-11.5	-11.4	-12.0	-36.9	-66.6	-34.0
	Headquarters	0	0	1,070	0	-	-	-	0.0	10.2	0.0
Outside of Tokyo	Agriculture / mining	3,610	396	65	225	11.0	1.8	6.2	2.1	0.6	1.0
	Manufacturing	134,025	600	1,005	891	0.4	0.8	0.7	3.1	9.6	4.1
	Construction	49,407	4	5	6	0.0	0.0	0.0	0.0	0.0	0.0
	Electricity, gas, etc.	8,911	360	203	333	4.0	2.3	3.7	1.9	1.9	1.5
	Commerce	47,281	620	357	479	1.3	0.8	1.0	3.3	3.4	2.2
	Service	283,901	422	529	724	0.1	0.2	0.3	2.2	5.1	3.3
	Headquarters	0	1,751	636	1,760	-	-	-	9.2	6.1	8.1
A. TM Total		90,351	-23,204	-13,257	-26,104	-25.7	-14.7	-28.9	-121.8	-126.8	-120.4
B. Outside of TM		527,135	4,155	2,800	4,418	0.8	0.5	0.8	21.8	26.8	20.4
C. Japan Total		617,486	-19,050	-10,457	-21,686	-3.1	-1.7	-3.5	-100.0	-100.0	-100.0

Note: The contribution rate is calculated so that the total is negative one hundred (−100).

Source: created by the author

(2) Minimization of the total change in final demands

Table 3 below shows the calculation results assuming "minimization of the total change in final demand". This assumption envisions a policy that minimizes the shocks between pre-disaster expenditure baskets and that in post-disaster. The optimal solution is obtained under the production constraint due to TEE as stated in the previous section.

Incidentally, the objective function in this calculation is the weighted total of squared differences between the final demands before and after the TEE, where the pre-disaster production share is used as a weight. The intention of using weighted total is to mitigate the influence of the size of the final demands between TM and outside TM.

1) Decrease in GDE

Not surprisingly, the impact on GDE in this case is greater than in the previous simulation, where the objective function is GDE maximization.

The order of the decrease in final demand change is also "all industries", "headquarters only" and "other than headquarters" in descending order as in the previous simulation. However, the difference between “headquarters only” and “all industries" is marginal. This suggests that the damage to the headquarters in TM has a great impact on the decrease in GDE nationwide. A same trend as the previous simulation is confirmed here as well.

2) Decrease in GDE by region

What is common to the three cases in the simulation is that the final demand amount not only in TM but also in outside TM decreases. This is very different from the results in the previous simulation of “maximization of GDE” where the indirect economic damage is limited in TM

Looking at the "all industries", the amount of decrease in final demand is about 47 trillion yen. This decrease is twice as larger as that in the previous simulation. On the other hand, focusing on the decrease in final demand in TM, the damage is 16 trillion yen in this simulation while 26 trillion yen in the previous simulation of maximizing GDE. In other words, objective function of “minimizing the total change in final demand” gives larger impacts to Japan but gives limited impacts on TM compared to the previous simulation.

3) Decrease in GDE by industrial sector

It can be seen that the "Manufacturing" and the "Service" have a great influence on the decrease in the final demand looking at the contribution rate in the case of "all industries". On the other hand, paying attention to "Comme", where indirect economic damage was remarkable in previous simulation, the impact on "Comme" is extremely limited. In addition, as mentioned earlier, the decrease in final demands has occurred in all industrial sectors in TM and outside TM. Looking at the contribution rate, it is characteristic that "Manufacturing" and "Service" contribute significantly to the decrease in the final demand.

In summary, according to this policy decision, the damage in TM has resulted in a decrease in the final demands both "inside" and "outside". In addition, the final demand outside TM decreases significantly compared to that in TM even though the initial damage is occurred in TM.

Table 3 Estimated result of minimizing the total change in final demands

		Pre TEE FD	FD changes by TEE (Billion yen)			Rate of change(%)			Contribution rate(%)		
			Only headquarters	All sectors except headquarters	All sectors including headquarters	Only headquarters	All sectors except headquarters	All sectors including headquarters	Only headquarters	All sectors except headquarters	All sectors including headquarters
Tokyo	Agriculture / mining	52	-6	-13	-10	-10.6	-25.2	-18.4	-0.0	-0.1	-0.0
	Manufacturing	4,770	-604	-1,177	-801	-12.7	-24.7	-16.8	-1.3	-4.5	-1.7
	Construction	7,730	-518	-1,007	-949	-6.7	-13.0	-12.3	-1.1	-3.9	-2.0
	Electricity, gas, etc.	850	-111	-362	-296	-13.1	-42.6	-34.8	-0.2	-1.4	-0.6
	Commerce	15,659	-3,257	-2,019	-2,953	-20.8	-12.9	-18.9	-7.0	-7.8	-6.3
	Service	61,290	-8,286	-8,536	-11,118	-13.5	-13.9	-18.1	-17.9	-32.8	-23.7
	Headquarters	0	0	0	0	-	-	-	0.0	0.0	0.0
Outside of Tokyo	Agriculture / mining	3,610	-376	-135	-328	-10.4	-3.8	-9.1	-0.8	-0.5	-0.7
	Manufacturing	134,025	-11,063	-8,384	-11,061	-8.3	-6.3	-8.3	-23.8	-32.2	-23.6
	Construction	49,407	-1,921	-916	-1,860	-3.9	-1.9	-3.8	-4.1	-3.5	-4.0
	Electricity, gas, etc.	8,911	-894	-422	-906	-10.0	-4.7	-10.2	-1.9	-1.6	-1.9
	Commerce	47,281	-4,776	-519	-3,894	-10.1	-1.1	-8.2	-10.3	-2.0	-8.3
	Service	283,901	-14,575	-2,534	-12,638	-5.1	-0.9	-4.5	-31.4	-9.7	-27.0
	Headquarters	0	0	0	0	-	-	-	0.0	0.0	0.0
A. TM Total		90,351	-12,782	-13,113	-16,126	-14.1	-14.5	-17.8	-27.6	-50.4	-34.4
B. Outside of TM		527,135	-33,604	-12,912	-30,687	-6.4	-2.4	-5.8	-72.4	-49.6	-65.6
C. Japan Total		617,486	-46,386	-26,024	-46,813	-7.5	-4.2	-7.6	-100.0	-100.0	-100.0

Note: The contribution rate is calculated so that the total is negative one hundred (−100).

Source: created by the author

4. Concluding remarks

This paper focuses on resource allocation that adjusts demand as a method of dealing with supply constraints caused by an earthquake. The feature of this analysis method is that unlike the conventional input−output analysis, the constrained production is used as the exogenous variable and the final demand is calculated as the endogenous variable. By solving a mathematical optimization model, assuming a policy decision (objective function) to deal with supply constraints due to the earthquake, the final demand, which is an endogenous variable, is obtained. In other words, the distribution of final demand is determined as the optimal solution of the objective function under production constraints.

And, the amount of damage caused by TEE (indirect economic damage amount) is calculated by the difference between the final demand before and after the disaster. Although a direct estimated damage amount is set as a supply constraint, it is also a major feature that the indirect economic damage amount is calculated based on the policy decision to deal with it.

We used the following two objective functions (criteria for the policy decision); (1) Maximization of final demands and (2) Minimization of the total change in final demands. These two policy decisions lead to contrasting results on earthquake damage. In the case of “maximization of final demands”, the impact on final demands was almost limited to “in Tokyo”. On the other hand, in the case of “minimization of the total change in final demand”, the effects reach to outside Tokyo as well as inside Tokyo.

There are various possible policies for dealing with the earthquake and it is possible to clarify what kind of indirect economic damage will occur by assuming various policy decisions under supply constraints. The analytical method presented in this paper can be evaluated as a tool for evaluating various policy decisions.

Finally, let us talk about the remaining challenges. The first is that this research does not take changes in the input coefficient into account. We assumed the input coefficient is unchanged before and after the earthquake, but it would be natural to think that the input output structure would change after the earthquake. The second is the handling of imports. The final demand in this research is assumed to be domestic demand only and does not consider imports. We think that a more realistic simulation will be implemented by setting the final demand in consideration of imports.

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