Title: An alternative for tracing the path between supply and use tables in current and constant prices

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Abstract:

When measured in current prices across time, supply and use Tables (SUTs) reflect changes in the cost structure of an economy. If technological changes are the focus of an inquiry, SUTs should be in constant prices. This is the case of most applications of SUT time series. In such cases, researchers apply commodity-specific deflators to SUTs. From an economics perspective, deflators are undoubtedly cell-specific since exchanges of a commodity occur in different markets and institutional contexts. RAS can be used to calculate such cell-specific deflators. But deflating SUTs via RAS also can taint the reliability of some known information. In this investigation, we revisit Path-RAS and apply it to price deflation. It enables cell-specific deflators while lowering information requirements. Additional information about specific industries, products or aggregated published figures can be included if available and non-conflicting. We provide an empirical application based on ten European Union countries to explore the accuracy of the estimations obtained considering different information scenarios.

Key words: price deflation; limited information; input-output.

JEL codes: C89, D57, E31

1. Introduction

1.1. Supply and use tables: the accounting framework

Supply and use tables (SUT) play a central role in the United Nations' (2018, 1968) system national accounts. They provide the basis for the construction of input-output (IO) tables. As a framework, they ensure (i) systematic bookkeeping of aggregated and disaggregated macroeconomic data, (ii) consistency between the production of goods and services, as well as income accounts, and (iii) coherent gross domestic product (GDP) figures, not only within a nation from both production and expenditure perspectives but also across nations.

Since they report linkages from commodities to industries and vice versa, SUTs enable links between other economic datasets that report information either by commodity or industry. In this vein, SUTs can be used to relate national accounts to jobs and occupations, land use, energy consumption, pollution, waste generation, water usage, among a wider range of possibilities**.** They also can be used as foundational information in the construction of a social accounting matrices (SAMs). As SAMs, SUTs are frequently linked to various dimensions of social life (Round, 2003) as well as flows of funds (Tsujimura & Mizoshita, 2003).

Table 1. Supply and Use table with disaggregated domestic and imported flows[1].

	Products	Industries	Final demand	Sum
Products		$\mathbf{U}^d$	$\mathbf{F}^d$	$\mathbf{q}$
Products		$\mathbf{U}^m$	$\mathbf{F}^m$	$\mathbf{m}$
Value added		$\mathbf{W}$		$\mathbf{w}$
Industries	$\mathbf{V}$			$\mathbf{g}$
Imports	$\mathbf{M}$			$\mathbf{o}$
Sum	$\mathbf{q}' + \mathbf{m}'$	$\mathbf{g}'$	$\mathbf{f}$	

Source: Own elaboration.

A full SUT framework consists of a supply table in basic prices combined with a transformation matrix to purchaser's prices. A SUT also includes a use table measured in both basic and purchaser's prices. Gross value added (GVA) is measured in basic prices

[1] Matrices are denoted in upper-case bold font; vectors in lower-case bold font; and scalars are denoted in italic font. Vectors are columns by definition. Superscript $'$ indicates transposition. A bar above the variable, $\bar{\mathbf{x}}$, denotes constant prices. A circumflex, $\hat{\mathbf{x}}$, indicates that the vector has been transformed into a square diagonal matrix, i.e., one with elements on the main diagonal and zeros elsewhere. A summation vector of ones is denoted by $\mathbf{i}$.

to complete the accounting framework. For our purposes, the SUT framework only considers the supply and use tables in basic prices. For k products and l industries a SUT can be summarised as in table 1.

Matrices $\mathbf{U}^d = \{\mathbf{u}_{\mathbf{ij}}^d\}$ and $\mathbf{U}_{\mathbf{ij}}^m = \{\mathbf{u}_{\mathbf{ij}}^m\}$ represent intermediate consumptions of products $(\mathrm{i} = 1, \cdots, \mathrm{k})$ by industries $(\mathrm{j} = 1, \cdots, \mathrm{l})$ domestically produced (d) and imported (m) respectively. Matrices $\mathbf{F}^d = \{\mathbf{f}_{\mathbf{ij}}^d\}$ and $\mathbf{F}^m = \{\mathbf{f}_{\mathbf{ij}}^m\}$ represent domestically produced and imported commodity shipments to final demand. They have dimensions $(\mathrm{k} \times \varphi)$ were φ represents the number of final demand components. Matrix $\mathbf{W} = \{\mathbf{w}_{ij}\}$ stands for value added and has dimensions $(\mathrm{l} \times \rho)$ where ρ stands for the number value-added components. Matrix $\mathbf{V} = \{\mathbf{v}_{\mathbf{ij}}\}$ represents the supply that each industry provides for each commodity and has dimensions $(\mathrm{l} \times \mathrm{k})$. Matrix $\mathbf{M} = \{\mathbf{m}_{ij}\}$ stands for the commodity flows imported from different origins and has dimensions $(\mathrm{k} \times \mathrm{o})$ where o denotes the number of import origins.

In addition, vectors $\mathbf{q}$ and $\mathbf{m}$ denote total supply and total use (domestic and imported) by commodity. Vector $\mathbf{g}$ represents gross output by industry. Vectors $\mathbf{f}$ and $\mathbf{w}$ stand for the sum of each component of final demand and value-added matrices. Finally, $\mathbf{o}$ contains total imports by origin.

The following basic accounting equalities must hold for the SUT to be balanced:

$$\begin{gathered} \mathbf{q} = \mathbf{U}^d\mathbf{i} + \mathbf{F}^d\mathbf{i} = \mathbf{V}'\mathbf{i} = \mathbf{q} \\ \mathbf{m} = \mathbf{U}^m\mathbf{i} + \mathbf{F}^m = \mathbf{M}'\mathbf{i} = \mathbf{m} \\ \mathbf{g} = \mathbf{U}^{d'}\mathbf{i} + \mathbf{U}^{m'}\mathbf{i} + \mathbf{W}'\mathbf{i} = \mathbf{V}\mathbf{i} = \mathbf{g} \end{gathered} \tag{1}$$

Total commodity supply, both domestically produced and imported, must equal total use of each commodity. Total inputs by industry must equal total outputs by industry.

Table 2. Industry and commodity technology matrices

Industry technology			Commodity technology	
	$\mathbf{A}^d$			$\mathbf{B}^d$
	$\mathbf{A}^m$			$\mathbf{B}^m$
$\boldsymbol{C}$			$\mathbf{D}$	

Source: Own elaboration.

To facilitate the reader and following section 2 developments, we explicitly specify the industry and commodity structures for both purchases and sales in a SUT and relate them to balancing equations (1). Four matrices can be defined. On the one hand, using industry technology, we consider matrices $\mathbf{A}^d = \{\mathbf{a}_{\mathbf{ij}}^d = \mathbf{u}_{\mathbf{ij}}^d / \mathbf{g}_{\mathbf{j}}\}$, $\mathbf{A}^m = \{\mathbf{a}_{\mathbf{ij}}^m = \mathbf{u}_{\mathbf{ij}}^m / \mathbf{g}_{\mathbf{j}}\}$ and $\mathbf{C} = \{\mathbf{v}_{\mathbf{ij}} / \mathbf{g}_{\mathbf{j}}\}$. On the other hand, regarding commodity technology, we can define matrices $\mathbf{B}^d = \{\mathbf{u}_{\mathbf{ij}}^d / \mathbf{q}_{\mathbf{i}}\}$, $\mathbf{B}^m = \{\mathbf{u}_{\mathbf{ij}}^m / \mathbf{m}_{\mathbf{i}}\}$ and $\mathbf{D} = \{\mathbf{d}_{\mathbf{ij}} = \mathbf{v}_{\mathbf{ij}} / \mathbf{q}_{\mathbf{i}}\}$. Table 2 illustrates the relationship between these matrices and the SUT accounting framework. Substituting $\mathbf{A}^d$, $\mathbf{A}^m$, $\mathbf{C}$, $\mathbf{B}^d$, $\mathbf{B}^m$ and $\mathbf{D}$ in (1) we get:

$$\begin{gathered} \mathbf{q} = \mathbf{A}^d\mathbf{g} + \mathbf{F}^d\mathbf{i} = \mathbf{C}'\mathbf{g} = \mathbf{q} \\ \mathbf{m} = \mathbf{A}^m\mathbf{g} + \mathbf{F}^m\mathbf{i} = \mathbf{M}'\mathbf{i} = \mathbf{m} \\ \mathbf{g} = \mathbf{B}^{d\prime}\mathbf{q} + \mathbf{B}^{m\prime}\mathbf{m} + \mathbf{W}'\mathbf{i} = \mathbf{D}\mathbf{q} = \mathbf{g} \end{gathered} \tag{2}$$

As in (1), total commodity supply, both domestically produced and imported, must equal total use by commodity. Total inputs by industry must equal total industry outputs. In this way, SUTs remain balanced.

1.2. Why should supply and use tables be measured in constant prices?

According to de Boer and Rodrigues (2020), interest in price deflation can be traced back as far as the 18th century. Dutot (1738) was a pioneer in calculating indexes, when he did so for several commodities by accounting for price variations as far back as 1515. Since then, a vast literature on indexes has emerged that suggests economist consider both price and quantity changes (Balk, 2008).

IO models are no exception. According to Leontief (1951), they were initially conceived from both a physical and a monetary perspective. In fact, the first precedent of a IO price model (Leontief, 1937) appeared soon after the first IO table was published (Leontief, 1936). The extant literature identifies three main reasons why IO frameworks should be measured also in constant prices. They are discussed in following paragraphs. Please note that the motivations listed are neither exhaustive nor mutually exclusive.

1.2.1. Measuring impacts and structural change

There has been a bit of a debate about the suitability and utility of using IO frameworks in constant prices for measuring structural change. Arto et al. (2015) suggest that some trade analysis related with global value chains can yield misleading results if performed in constant prices. Dietzenbacher and Termursho (2012) test the extent to which impact analyses differ in current versus constant prices. Using Danish IO tables from years 2001 to 2007, at a somewhat aggregate level they find that deflated and non-deflated IO tables yield quite similar results. They do point, however, that (i) sector-level differences are notable and (ii) their test used potentially peculiar data and, thus, should be taken with some caution. Moreover, other work on other countries (Tandon & Ahmed, 2016) shows substantial differences in impacts over time by industry .

Within IO economics, structural decomposition analysis (SDA) is arguably one of the most widely used family of techniques for measuring structural change and its drivers. See Rose & Casler (1996) for a historical overview and Oosterhaven (2021) for more recent comments and insights. Nowadays, most empirical SDA analyses use deflated IO data (Savona & Ciarli, 2019). Structural change consist of the relocation of economic activity across sectors (Herrendorf, Rogerson, & Valentinyi, 2014). If we consider current prices only, relative price changes could relocate value while the distribution of the volume of output follows a different path. Several studies show that differences can be quite substantial if researchers use either current or constant prices. For example, Kander (2005) and Henriques & Kander (2010), both of which evaluate the global transition towards the service economy, using both possibilities as many others have since then. Fix (2019) suggests that, to some extent, the spread of Baumol's (1967) disease within an economy is an illusion once real output changes are considered.

Data on prices and volumes are increasingly published at higher levels of disaggregation in developed countries. But data availability for developing nations and regions remains a prime constraint for the compilation of official IO statistics in constant prices (Tandon & Ahmed, 2016).

1.2.2. Linking physical and monetary IO tables

Since exchanges in an economy involve both a physical and a monetary dimension, two parallel IO models can be derived based on physical input-output (PIOT) and a monetary input-output (MIOT) tables (Miller & Blair, 2009, pp. 41–53). Economic-environmental analysis can be traced further back in IO literature (Leontief, 1970). Pioneering work on PIOT models include Isard, Chougill, Kissin, Seyfarth and Tatlock (1972) and Szyrmer and Ulanowicz (1987). However, it was not until the decade of 1990 when PIOT models started to be compiled in more regular basis (Giljum & Hubacek, 2004). Nowadays much of the information needed for PIOT construction is systematically collected with that for economic national accounts (United Nations, 2003).

Hubacek and Giljum (2003) and Giljum, Hubacek, & Sun (2004) initiated a debate on whether PIOT or MIOT should be used. They argued that PIOTs yield different and more accurate results since they better capture the physical reality of economic exchanges. In a reply, Suh (2004) argued that PIOT results might only tell us that some products are less expensive and others more costly per unit of physical measurement. He also pointed that PIOTs suffer from operational issues including statistical bias that attaches to sectoral aggregation as well as problems that arise from sectoral inconsistencies across tables over time. Hoen (2002) makes a similar point in defence of MIOT models in constant prices instead of PIOT. For all industries producing physical commodities, a bridge between the two models should exist and should not be hard to calculate since value equals mass multiplied by price per unit mass (Hoekstra & Van Den Bergh, 2006).

Establishing the equivalence between physical and monetary flows requires prices, either actual or estimated (Többen, 2017). Despite this straightforward relationship, Weisz and Duchin (2006) suggest that PIOT and MIOT tables cannot be translated from one to another using a single price for all deliveries of an industry or commodity. Cell-specific deflators are needed. Dietzenbacher (2005), however, notes that this is not the only issue one faces when linking PIOT and MIOT models. Appropriate waste treatment has also found to be fundamental when one links the two model types.

1.2.3. Institutional requirements

SUTs compiled in both current and constant prices ensure that both volume and price information contained in an SNA are coherent and consistent. The calculation of price and volume changes for the transactions of commodities is ideally supported through the use of SUT frameworks (Mahajan et al., 2018). In addition to statistical criteria, some policy decisions necessarily require perspectives in constant prices too. For example, the Stability and Growth Pact (SGP) suggests using volume growth rates, which require national accounts in constant prices (Eurostat, 2008). In any case, it should be clear by now that transparent and systematic approaches are needed during policy, and this means one should place figures in constant prices.

1.3. Alternatives for SUT deflation

1.3.1. Double deflation

Initially conceived to estimate real GDP (United Nations, 1973), the double-deflation method (DD) is still highly recommended for obtaining IO data in constant prices (Li & Kuroko, 2016). DD is based on the idea that it is difficult, if not impossible, to obtain price indices for the different GVA components (Ahmad, 1999). Some economic measures like gross output and imports often are published officially in both volume and monetary terms. Given the SUT framework in table 1, let $\boldsymbol{\pi}^{\mathbf{q}}$, $\boldsymbol{\pi}^{\mathbf{m}}$ and $\boldsymbol{\pi}^{\mathbf{g}}$ be the deflators associated with vectors $\mathbf{q}$, $\mathbf{m}$ and $\mathbf{g}$, respectively. Elements of vectors $\boldsymbol{\pi}^{\mathbf{q}}$ and $\boldsymbol{\pi}^{\mathbf{m}}$ are defined as $1/\mathrm{p_i}$ where $\mathrm{p_i}$ denotes the ratio of current (domestic and imports) price and the base year price for commodity i. Analogously, each element of $\boldsymbol{\pi}^{\mathbf{g}}$ is defined as $1/\mathrm{p_j}$ where $\mathrm{p_j}$ denotes the ratio of current price and the base year price for industry j.

Table 3. Supply and Use table in constant prices using double deflation.

	Products	Industries	Final demand	Sum
Products		$\bar{\mathbf{U}}^{d} = \hat{\boldsymbol{\pi}}^{\mathbf{q}}\mathbf{U}^{d}$	$\bar{\mathbf{F}}^{d} = \hat{\boldsymbol{\pi}}^{\mathbf{q}}\mathbf{F}^{d}$	$\bar{\mathbf{q}}$
Products		$\bar{\mathbf{U}}^{m} = \hat{\boldsymbol{\pi}}^{\mathbf{m}}\mathbf{U}^{m}$	$\bar{\mathbf{F}}^{\mathbf{m}} = \hat{\boldsymbol{\pi}}^{\mathbf{m}}\mathbf{F}^{m}$	$\bar{\mathbf{m}}$
Value added		$\bar{\mathbf{W}} = \mathbf{W}\hat{\boldsymbol{\pi}}^{\mathbf{w}}$		$\mathbf{w}$
Industries	$\bar{\mathbf{V}} = \hat{\boldsymbol{\pi}}^{\mathbf{g}}\mathbf{V}\hat{\boldsymbol{\pi}}^{\mathbf{q}}$			$\bar{\mathbf{g}} = \hat{\boldsymbol{\pi}}^{\mathbf{g}}\mathbf{g}$
Imports	$\bar{\mathbf{M}} = \mathbf{M}\hat{\boldsymbol{\pi}}^{\mathbf{m}}$			$\bar{\mathbf{o}}$
Sum	$\bar{\mathbf{q}}' + \bar{\mathbf{m}}'$	$\bar{\mathbf{g}}' = \mathbf{g}\hat{\boldsymbol{\pi}}^{\mathbf{g}}$	$\bar{\mathbf{f}}$	

Source: Own elaboration.

If coherent deflators can be derived from price information, matrices $\bar{\mathbf{V}}$ and $\bar{\mathbf{M}}$ remain balanced. Through balancing equations in (1), total GVA in constant prices by industry can be derived as a residual:

$$\mathbf{W}'\mathbf{i} = \bar{\mathbf{g}} - \bar{\mathbf{U}}^{d'}\mathbf{i} - \bar{\mathbf{U}}^{m'}\mathbf{i} \tag{3}$$

To obtain a full GVA matrix $\bar{\mathbf{W}}$, a vector of implicit GVA deflators $\boldsymbol{\pi}^{\mathbf{w}}$ can be calculated. Each element of $\boldsymbol{\pi}^{\mathbf{w}}$ is the difference between an industry's deflated gross output and its deflated intermediated consumption divided by that industry´s GVA in current prices:

$$\boldsymbol{\pi}^{\mathbf{w}} = \left(\bar{\mathbf{g}} - \bar{\mathbf{u}}_{\bullet\mathbf{j}}^{d} - \bar{\mathbf{u}}_{\bullet\mathbf{j}}^{m}\right)\left(\widehat{\mathbf{W}'\mathbf{\imath}}\right)^{-1} \tag{4}$$

DD is theoretically sound and preferred to single-deflation methods since it yields a balanced SUT and thus balanced real GDP figures from both income and spending perspectives (Oulton, Rincon-Aznar, Samek, & Srinivasan, 2018). But it does have some pitfalls. First, DD implicitly assumes that an industry or commodity category is linked to a single commodity that corresponds to the price index applied. The reality, however, is that any given element of a SUT presents a composite commodity that embodies a commodity mix unique to that specific industry and/or commodity transaction that is represented by the element. Thus, DD necessarily induces some degree of aggregation bias (Dietzenbacher & Hoen, 1999). DD also assumes that all exchanges of an industry or commodity have the same price dynamics; this neglects the fact that different market and institutional contexts undoubtedly affect the price changes of the composite commodity differently (Folloni & Miglierina, 1994). GVA estimates are particularly sensitive to the manner in which they are deflated, due to measurement errors inherent in $\bar{\mathbf{q}}$, $\bar{\mathbf{g}}$ and $\bar{\mathbf{m}}$ (Wolff, 1994). Moreover, from (3) and (4) one can readily see that DD can induce a sign flip in an industry if its deflated intermediate consumption exceeds its deflated gross output. In such cases as this, *ad hoc* adjustments are required.

1.3.2. Biproportional techniques

Dietzenbacher and Hoen (1998) suggest that deflating via RAS ultimately applies cell-specific deflators. They focus on the intermediate transaction matrix given a set of industry output, final demand and import vectors in constant prices. Despite their solid theoretical foundation and their promising empirical results, the scarcity of price indices has tended to prevent the use of RAS-based deflation approaches. The remaining of this subsection explores subsequent RAS extensions and applications to SUT framework.

Within IO analysis, basic RAS is a popular technique, if not the most popular, for matrix updating and balancing (Lahr & de Mesnard, 2004). Extensions to it include considerations for negative values (Günlük-Şenesen & Bates, 1988; Junius & Oosterhaven, 2003), for multiple subset and block-wise constraints (Gilchrist & St. Louis, 1999; Valderas Jaramillo & Rueda-Cantuche, 2021) and for data with different reliabilities (Dalgaard & Gysting, 2004; Lahr, 2001; Lenzen, Gallego, & Wood, 2009).

Note that several other approaches exist that can deal with problems to which RAS is typically applied (Jackson & Murray, 2004). Some have been used to balance SUTs (Nicolardi, 2013; Rampa, 2008). Temursho, Webb and Yamano (2011), however, suggest that GRAS (Günlük-Şenesen & Bates, 1988; Junius & Oosterhaven, 2003) presents the best balance between accuracy and speed/simplicity. Thus, in what remains of this paper, we consider GRAS to be the *status quo* of the state of the art in matrix balancing.[2]

To implement GRAS,[3] we split a benchmark matrix $\mathbf{Z}_{(0)}$ into two matrices $\mathbf{Z}_{(0)}^{+}$ and $\mathbf{Z}_{(0)}^{-}$. On the one hand, matrix $\mathbf{Z}_{(0)}^{+}$ contains only the positive elements of $\mathbf{Z}_{(0)}$. On the other, matrix $\mathbf{Z}_{(0)}^{-}$ contains the absolute values of all negative elements. Therefore, we have $\mathbf{Z}_{(0)} = \mathbf{Z}_{(0)}^{+} - \mathbf{Z}_{(0)}^{-}$. Vectors $\boldsymbol{\mu}$ and $\boldsymbol{\upsilon}$ are the row and column sum targets for the balanced matrix $\mathbf{Z}^{*}$. GRAS derivation arrives at a second-order equation. For rows and columns with positive and negative elements, the definition of coefficients $\mathrm{r_i}$ and $\mathrm{s_j}$ considers the positive root of the second-order equations. For the cases where no positive or negative elements are in a row or column, scalars are defined as in standard RAS (Temursho, Miller, & Bouwmeester, 2013). The algorithm runs iteratively until the conditions:

$$\begin{aligned} &|[\hat{\mathbf{r}}\mathbf{Z}^{+}\hat{\mathbf{s}} - (\hat{\mathbf{r}})^{-1}\mathbf{Z}^{-}(\hat{\mathbf{s}})^{-1}]\mathbf{i} - \boldsymbol{\mu}| < \varepsilon \\ &|[\hat{\mathbf{r}}\mathbf{Z}^{+}\hat{\mathbf{s}} - (\hat{\mathbf{r}})^{-1}\mathbf{Z}^{-}(\hat{\mathbf{s}})^{-1}]'\mathbf{i} - \boldsymbol{\upsilon}| < \varepsilon \end{aligned} \tag{5}$$

are fulfilled for a sufficiently small value of ε—a pre-determined level of tolerated error.

While originally developed for balancing and updating symmetric IO tables, GRAS and other biproportional techniques can also be applied to SUTs (Serpell, 2018). Pioneer works in this regard was presented by Timmer (2005) in the context of the EUKLEMS project. The biggest obstacle for GRAS implementation for SUT updating is that data on commodity vectors, $\mathbf{q}$ and $\mathbf{m}$**,** are rarely available.

To overcome this issue, a first alternative appeared when Beutel´s (2002, pp. 114–118) algorithm was adapted by Eurostat for their SUT framework (SUT-Euro). SUT-Euro endogenously generates industry output and commodity supply/use vectors considering GVA variations over time. However, SUT-Euro has some limitations that hinder its application to SUT deflation. First, SUT-Euro can only be applied to square matrices. This is a notable drawback since SUTs are often arranged in rectangular formats to show secondary production in greater detail (Temursho, 2021). Second, SUT-Euro makes decisive use of GVA data requiring at least $\mathbf{W}'\mathbf{i}$ to be known. While this is reasonable in

[2] For the sake of simplicity, we do not consider subsequent GRAS extensions (Lenzen et al., 2009; Lenzen, Moran, Geschke, & Kanemoto, 2014; among others).
[3] We standardize notation across all sections of this paper. As a result, we do not follow notation in preceding GRAS literature.

terms of current price updating, it unfortunately also is deflated GVA data that are least likely to be available.

SUT-RAS proposed by Temursho and Timmer (2011) solves some SUT-Euro limitations. It uses SUT balancing equations (1) to endogenously derive targets for $\mathbf{q}$ and $\mathbf{m}$ given industry output target and GVA by industry data. Further, it can be applied to rectangular matrices and manages negative values just like GRAS. SUT-RAS also can apparently adopt additional information as long as constraints do not conflict (Valderas Jaramillo, Rueda Cantuche, Olmedo, & Beutel, 2019). In fact, SUT-RAS and GRAS are apparently equivalent as long as $\mathbf{q}$ and $\mathbf{m}$ are exogenously set (Temursho, 2021). As in the case of SUT-Euro, SUT-RAS data requirements are reasonable when updating via current prices. But data on industry output (either in current or constant prices) are often not available for many countries and most regions. Like SUT-Euro, SUT-RAS also requires a deflated GVA vector when SUT deflation is the goal.

1.3.3. The aim of this paper: an alternative for tracing the path between SUTs in current and constant prices.

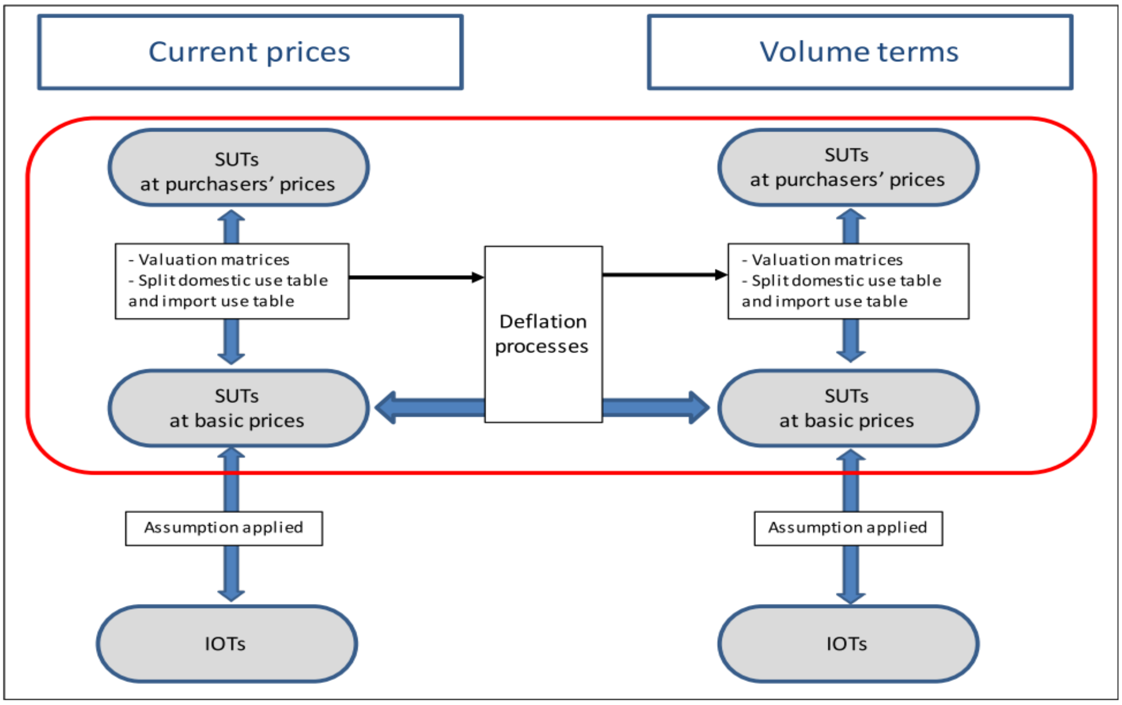
The "H-Approach" is recommended for SUTs compilation both in current and constant prices (Mahajan et al., 2018). The workflow of this approach is depicted in figure 1. This approach to compilation can be done either sequentially or simultaneously. Sequentially, the SUTs are balanced in current prices and then price deflated. For simplicity, this is the approach used in the present paper. This paper centres on the middle part of the H-Approach scheme, where SUTs in basic prices are converted from current to constant prices (in volume terms).

Our aim in this paper is to point out an alternative way for tracing the path between the two legs of the H depicted in figure 1. According to the improvement opportunities spotted in literature, via this methodological proposal, we seek to:

a) Obtain cell-specific deflators, as opposed to the one-price-fits-all market approach implicit in double deflation.
b) Reduce information requirements to enable SUT deflation where data are scarce and, yet permit the application of additional information if available and nonconflicting.
c) Transparently handle possible incoherencies that can arise during the deflation process in order to avoid *ad hoc* solutions.

To such ends, we revisit Path-RAS (Pereira López, Carrascal Incera & Fernández Fernández, 2013) as an alternative to SUT-Euro.

Figure 1. Schematic overview of the H-Approach for SUT compilation.



Source: Mahajan et al. (2018, p. 30).

In essence, we propose to derive targets endogenously for **g**, **o**, **q**, **m** and **f**, given a starting point with limited information. We achieve this iteratively using commodity and industry structures. While, like SUT-Euro and SUT-RAS, this approach requires GVA data, Path-RAS can be suitably transformed for SUT deflation. In this vein, we introduce major modifications in the original Path-RAS algorithm. This is the prime contribution of this paper. As such, we detail it in section 2. Still, it essentially remains a modification of Path-RAS (Pereira López, Carrascal Incera & Fernández Fernández, 2013; Pereira López & Rueda Cantuche, 2013).

2. Our methodological proposal: the modified Path-RAS

2.1. Minimum information requirements

Our method operates equivalently to GRAS. A benchmark matrix $\mathbf{Z}_{(0)}$ is modified to obtain a new matrix $\mathbf{Z}^*$ using targets up-to-date margins $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$. Different from GRAS, row and column targets are endogenously calculated during each iteration. They are however, some a minimum set of information is needed to start the balancing process:

- $\mathbf{q}^{*}_{\bullet\bullet}$ = Overall supply and use by products.
- $\mathbf{g}^{*}_{\bullet\bullet}$ = Overall input and output by industries.
- $\mathbf{f}^{*}_{\bullet\bullet}$ = Overall final demand.
- $\mathbf{m}^{*}_{\bullet\bullet}$ = Overall imports.
- $\mathbf{w}^{*}_{\bullet\bullet} = \mathbf{f}^{*}_{\bullet\bullet} - \mathbf{m}^{*}_{\bullet\bullet}$ = Overall gross value added.

2.2. Workflow

2.2.1. Step 0: defining a starting point

Let iterations $n = 0, 1, \cdots, N$ be indicated by subscript (n) associated with matrices and vectors. Superscripts A, C, B, D in vectors refer to the structures or paths followed by the algorithm to estimate new target vectors after every iteration[4].

The process initiates with the obtention of output by industry $\mathbf{g}^{(0)}_{(1)}$, final demand components $\mathbf{f}^{(0)}_{(1)}$ and total imports by origin $\mathbf{m}^{(0)}_{(1)}$ estimates that will be considered as the starting point. To do so, we define a diagonal matrix $\widehat{\boldsymbol{\pi}}^{(0)}_{(1)}$ with dimensions $(\mathrm{l} + \varphi + \mathrm{o}) \times (\mathrm{l} + \varphi + \mathrm{o})$. This matrix contains deflators defined as ratios between the given pieces of information in constant prices and their counterparts in matrix $\mathbf{Z}_{(0)}$. As an example, for the minimum information scenario[5]:

$$\boldsymbol{\pi}^{(0)}_{(1)} = \begin{bmatrix} \mathbf{g}^{*}_{\bullet\bullet} \\ \mathbf{f}^{*}_{\bullet\bullet} \\ \mathbf{m}^{*}_{\bullet\bullet} \end{bmatrix} \operatorname{diag} \begin{bmatrix} \sum \mathbf{g}_{(0)} \\ \sum \mathbf{f}_{(0)} \\ \sum \mathbf{o}_{(0)} \end{bmatrix}^{-1} \tag{6}$$

Deflators can be specific for some industries, products, final demand components, import origins or calculated according to aggregated information. The starting point of the balancing process is defined as:

$$\begin{bmatrix} \bar{\mathbf{g}}^{(0)}_{(1)} \\ \bar{\mathbf{f}}^{(0)}_{(1)} \\ \bar{\mathbf{o}}^{(0)}_{(1)} \end{bmatrix} = \widehat{\boldsymbol{\pi}}^{(0)}_{(1)} \begin{bmatrix} \mathbf{g}_{(0)} \\ \mathbf{f}_{(0)} \\ \mathbf{o}_{(0)} \end{bmatrix} \tag{7}$$

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[4] Superscript 0 indicates the starting point.

[5] We use *diag* as equivalent to our prior use of a circumflex. We apply it to composite vectors for notational clarity.

2.2.2. Step 1. Path AC

2.2.2.1. Industry balancing

The first step balances industries using the starting-point targets in equation (7). For the supply matrix, row targets $\boldsymbol{\mu}_{(1)}^{(0)}$ are:

$$\boldsymbol{\mu}_{(1)}^{(0)} = \begin{bmatrix} \bar{\mathbf{g}}_{(1)}^{(0)} \\ \bar{\mathbf{o}}_{(1)}^{(0)} \end{bmatrix} \tag{8}$$

For the use matrix, column targets $\boldsymbol{\upsilon}_{(1)}^{(0)}$ are:

$$\boldsymbol{\upsilon}_{(1)}^{(0)} = \begin{bmatrix} \bar{\mathbf{g}}_{(1)}^{(0)} \\ \bar{\mathbf{f}}_{(1)}^{(0)} \end{bmatrix} \tag{9}$$

By industry balancing, we mean that matrices $\mathbf{V}$ and $\mathbf{M}$ are row-scaled. Conversely, matrices $\mathbf{U}^{d}$, $\mathbf{U}^{m}$ and $\mathbf{W}$ are column-scaled. Formally:

$$\begin{gathered} \begin{bmatrix} \mathbf{V}_{(1)} \\ \mathbf{M}_{(1)} \end{bmatrix} = \hat{\mathbf{r}} \begin{bmatrix} \mathbf{V}_{(0)}^{+} \\ \mathbf{M}_{(0)}^{+} \end{bmatrix} + (\hat{\mathbf{r}})^{-1} \begin{bmatrix} \mathbf{V}_{(0)}^{-} \\ \mathbf{M}_{(0)}^{-} \end{bmatrix} \\ \begin{bmatrix} \mathbf{U}_{(1)}^{d} \\ \mathbf{U}_{(1)}^{m} \\ \mathbf{W}_{(1)} \end{bmatrix} = \begin{bmatrix} [\mathbf{U}_{(0)}^{d}]^{+} \\ [\mathbf{U}_{(0)}^{m}]^{+} \\ \mathbf{W}_{(0)}^{+} \end{bmatrix} \hat{\mathbf{s}} + \begin{bmatrix} [\mathbf{U}_{(0)}^{d}]^{-} \\ [\mathbf{U}_{(0)}^{m}]^{-} \\ \mathbf{W}_{(0)}^{-} \end{bmatrix} (\hat{\mathbf{s}})^{-1} \end{gathered} \tag{10}$$

Coefficients vectors $\mathbf{r}$ and $\mathbf{s}$ are calculated using the GRAS algorithm.

2.2.2.2.Commodity and value-added component targets

To conclude step 1, the algorithm endogenously calculates the targets $\mathbf{q}$, $\mathbf{m}$ and $\mathbf{w}$. Substituting $\bar{\mathbf{g}}_{(1)}^{(0)}$ in (2) we get:

$$\begin{gathered} \mathbf{q}_{(2)}^{(A)} = \mathbf{A}_{(1)}^{d} \bar{\mathbf{g}}_{(1)}^{(0)} + \mathbf{F}_{(1)}^{d}\mathbf{i} = \mathbf{U}_{(1)}^{d}\mathbf{i} + \mathbf{F}_{(1)}^{d}\mathbf{i} \\ \mathbf{m}_{(2)}^{(A)} = \mathbf{A}_{(1)}^{m} \bar{\mathbf{g}}_{(1)}^{(0)} + \mathbf{F}_{(1)}^{m}\mathbf{i} = \mathbf{U}_{(1)}^{m}\mathbf{i} + \mathbf{F}_{(1)}^{m}\mathbf{i} \\ \mathbf{q}_{(2)}^{(C)} = \mathbf{C}_{(1)}' \bar{\mathbf{g}}_{(1)}^{(0)} = \mathbf{V}_{(1)}'\mathbf{i} \\ \mathbf{m}_{(2)}^{(C)} = \mathbf{M}_{(1)}'\mathbf{i} \end{gathered} \tag{11}$$

We also obtain estimates for total components of value added $\mathbf{w}_{(2)}^{(A)}$:

$$\mathbf{w}_{(2)}^{(A)} = \mathbf{W}_{(1)}\mathbf{i} \tag{12}$$

Since $\bar{\mathbf{g}}_{(1)}^{(0)}$ has been substituted in different structural equations, it is most likely that $\mathbf{q}_{(2)}^{(A)} \neq \mathbf{q}_{(2)}^{(C)}$ and $\mathbf{m}_{(2)}^{(A)} \neq \mathbf{m}_{(2)}^{(C)}$. Target vectors for $\mathbf{q}$ and $\mathbf{m}$ are derived as convex combination between vectors obtained through paths A and C. Formally:

$$\begin{gathered}\mathbf{q}_{(2)}^{(AC)} = \alpha\mathbf{q}_{(2)}^{(A)} + (1-\alpha)\mathbf{q}_{(2)}^{(C)} \\ \mathbf{m}_{(2)}^{(AC)} = \alpha\mathbf{m}_{(2)}^{(A)} + (1-\alpha)\mathbf{m}_{(2)}^{(C)} \\ \text{with} \quad 0 \leq \alpha \leq 1\end{gathered} \tag{13}$$

Values for α can be interpretated as the degree of reliability assigned to the information contained in the $\mathbf{A}^{d}$, $\mathbf{A}^{m}$ and $\mathbf{C}$ matrices.

Vectors $\mathbf{q}_{(2)}^{(AC)}$ and $\mathbf{m}_{(2)}^{(AC)}$ might need to be corrected to ensure balances defined in equation (1). If additional information is available, this correction can require the introduction of a subset of constraints to be applied to the target vectors during the next iteration. Following the example given in step 0, new deflators can be calculated:

$$\begin{gathered}\boldsymbol{\pi}_{(2\boldsymbol{\upsilon})}^{(AC)} = \frac{\mathbf{q}_{\bullet\bullet}^{*} + \mathbf{m}_{\bullet\bullet}^{*}}{\sum\mathbf{q}_{(2)}^{(AC)} + \sum\mathbf{m}_{(2)}^{(AC)}} \\ \boldsymbol{\pi}_{(2\boldsymbol{\mu})}^{(AC)} = \begin{bmatrix}\mathbf{q}_{\bullet\bullet}^{*} \\ \mathbf{m}_{\bullet\bullet}^{*} \\ \mathbf{w}_{\bullet\bullet}^{*}\end{bmatrix} \operatorname{diag}\begin{bmatrix}\sum\mathbf{q}_{(2)}^{(AC)} \\ \sum\mathbf{m}_{(2)}^{(AC)} \\ \sum\mathbf{w}_{(2)}^{(A)}\end{bmatrix}^{-1}\end{gathered} \tag{14}$$

Finally, targets for iteration $n = 2$ are derived. For the supply table, column targets $\boldsymbol{\upsilon}_{(2)}^{(AC)}$:

$$\boldsymbol{\upsilon}_{(2)}^{(AC)} = \left[\bar{\mathbf{q}}_{(2)}^{(AC)} + \bar{\mathbf{m}}_{(2)}^{(AC)}\right] = \hat{\boldsymbol{\pi}}_{(2\boldsymbol{\upsilon})}^{(AC)}\left[\mathbf{q}_{(2)}^{(AC)} + \mathbf{m}_{(2)}^{(AC)}\right] \tag{15}$$

For the use table, row targets $\boldsymbol{\mu}_{(2)}^{(AC)}$:

$$\boldsymbol{\mu}_{(2)}^{(AC)} = \begin{bmatrix}\bar{\mathbf{q}}_{(2)}^{(AC)} \\ \bar{\mathbf{m}}_{(2)}^{(AC)} \\ \bar{\mathbf{w}}_{(2)}^{(A)}\end{bmatrix} = \hat{\boldsymbol{\pi}}_{(2\boldsymbol{\mu})}^{(AC)}\begin{bmatrix}\mathbf{q}_{(2)}^{(AC)} \\ \mathbf{m}_{(2)}^{(AC)} \\ \mathbf{w}_{(2)}^{(A)}\end{bmatrix} \tag{16}$$

These vectors combine information contained in the $\mathbf{A}^d$, $\mathbf{A}^m$ and $\mathbf{C}$ matrices. They also ensure balance in the SUT framework. In addition, subset constraints can be included.

2.2.3. Step 2. Path BD

2.2.3.1. Commodity balancing

By commodity balancing we mean that matrices $\mathbf{V}$ and $\mathbf{M}$ are column-scaled. Conversely, matrices $\mathbf{U}^d$, $\mathbf{U}^m$ and $\mathbf{W}$ are row-scaled. Formally:

$$\begin{bmatrix}\mathbf{V}_{(2)}\\ \mathbf{M}_{(2)}\end{bmatrix} = \begin{bmatrix}\mathbf{V}^{+}_{(1)}\\ \mathbf{M}^{+}_{(1)}\end{bmatrix}\hat{\mathbf{s}} + \begin{bmatrix}\mathbf{V}^{-}_{(1)}\\ \mathbf{M}^{-}_{(1)}\end{bmatrix}(\hat{\mathbf{s}})^{-1}$$
$$\begin{bmatrix}\mathbf{U}^{d}_{(2)}\\ \mathbf{U}^{m}_{(2)}\\ \mathbf{W}_{(2)}\end{bmatrix} = \hat{\mathbf{r}}\begin{bmatrix}[\mathbf{U}^{d}_{(1)}]^{+}\\ [\mathbf{U}^{m}_{(1)}]^{+}\\ \mathbf{W}^{+}_{(1)}\end{bmatrix} + (\hat{\mathbf{r}})^{-1}\begin{bmatrix}[\mathbf{U}^{d}_{(1)}]^{-}\\ [\mathbf{U}^{m}_{(1)}]^{-}\\ \mathbf{W}^{-}_{(1)}\end{bmatrix} \quad (17)$$

Coefficients vectors $\mathbf{r}$ and $\mathbf{s}$ are calculated using the GRAS algorithm.

2.2.3.2. Industry, final demand component, and imports by origin targets

To conclude step 2, targets for $\mathbf{g}$, $\mathbf{f}$ and $\mathbf{o}$ are calculated endogenously. Substituting $\overline{\mathbf{q}}^{(AC)}_{(2)}$ and $\overline{\mathbf{m}}^{(AC)}_{(2)}$ in (2) we get:

$$\mathbf{g}^{(B)}_{(3)} = \mathbf{B}^{d\,\prime}_{(2)}\overline{\mathbf{q}}^{(AC)}_{(2)} + \mathbf{B}^{m\,\prime}_{(2)}\overline{\mathbf{m}}^{(AC)}_{(2)} + \mathbf{W}'_{(2)}\mathbf{i}$$
$$\mathbf{g}^{(D)}_{(3)} = \mathbf{D}_{(2)}\overline{\mathbf{q}}^{(AC)}_{(2)} \quad (18)$$

We also obtain estimates for total final demand components and total imports by origin:

$$\mathbf{f}^{(B)}_{(3)} = \mathbf{F}^{d\prime}{}_{(2)}\mathbf{i} + \mathbf{F}^{m\prime}{}_{(2)}\mathbf{i}$$
$$\mathbf{o}^{(D)}_{(3)} = \mathbf{M}_{(2)}\mathbf{i} \quad (19)$$

Since $\overline{\mathbf{q}}^{(AC)}_{(2)}$ and $\overline{\mathbf{m}}^{(AC)}_{(2)}$ have been substituted in different structural equations, it is most likely that $\mathbf{g}^{(B)}_{(3)} \neq \mathbf{g}^{(D)}_{(3)}$. Target vector for $\mathbf{g}$ is derived as convex combination between vectors obtained through paths B and D. Formally:

$$\mathbf{g}^{(BD)}_{(3)} = \beta\mathbf{g}^{(B)}_{(3)} + (1-\beta)\mathbf{g}^{(D)}_{(3)}$$
$$\text{with} \quad 0 \leq \beta \leq 1 \quad (20)$$

where β can be interpretated the degree of reliability assigned to the information contained to the information in the $\mathbf{B}^d$, $\mathbf{B}^m$ and $\mathbf{D}$ matrices.

Vectors $\mathbf{g}_{(3)}^{(BD)}$ might need to be rectified to ensure balances defined in equation (1). If additional information is available, this rectification can be used to introduce subset constraints for next iteration. Following the example given in step 0, new deflators can be calculated:

$$\begin{gathered}\boldsymbol{\pi}_{(3\boldsymbol{\mu})}^{(BD)} = \begin{bmatrix}\mathbf{g}_{\bullet\bullet}^{*} \\ \mathbf{m}_{\bullet\bullet}^{*}\end{bmatrix} \operatorname{diag}\begin{bmatrix}\sum\mathbf{g}_{(3)}^{(BD)} \\ \sum\mathbf{o}_{(3)}^{(D)}\end{bmatrix}^{-1} \\ \boldsymbol{\pi}_{(3\boldsymbol{\upsilon})}^{(BD)} = \begin{bmatrix}\mathbf{g}_{\bullet\bullet}^{*} \\ \mathbf{f}_{\bullet\bullet}^{*}\end{bmatrix} \operatorname{diag}\begin{bmatrix}\sum\mathbf{g}_{(3)}^{(BD)} \\ \sum\mathbf{f}_{(3)}^{(B)}\end{bmatrix}^{-1}\end{gathered} \tag{21}$$

Subsequently, targets for iteration $n = 3$ are derived. For the supply table, we define row targets $\boldsymbol{\mu}_{(3)}^{(BD)}$ as:

$$\boldsymbol{\mu}_{(3)}^{(BD)} = \begin{bmatrix}\bar{\mathbf{g}}_{(3)}^{(BD)} \\ \bar{\mathbf{o}}_{(3)}^{(D)}\end{bmatrix} = \boldsymbol{\pi}_{(3\boldsymbol{\mu})}^{(BD)} \begin{bmatrix}\mathbf{g}_{(3)}^{(BD)} \\ \mathbf{o}_{(3)}^{(D)}\end{bmatrix} \tag{22}$$

For the use table, column targets $\mathbf{v}_{(3)}^{(BD)}$ are defined as:

$$\mathbf{v}_{(3)}^{(BD)} = \begin{bmatrix}\bar{\mathbf{g}}_{(3)}^{(BD)} \\ \bar{\mathbf{f}}_{(3)}^{(B)}\end{bmatrix} = \boldsymbol{\pi}_{(3\boldsymbol{\upsilon})}^{(BD)} \begin{bmatrix}\mathbf{g}_{(3)}^{(BD)} \\ \mathbf{f}_{(3)}^{(B)}\end{bmatrix} \tag{23}$$

These vectors combine information contained in the $\mathbf{B}^d$, $\mathbf{B}^m$ and $\mathbf{D}$ matrices. They also ensure balance in the SUT framework. In addition, subset constraints can be included.

2.2.4. The iterative process

In each iteration, let $\mathbf{z}_{\mathbf{i}\bullet}$ and $\mathbf{z}_{\bullet\mathbf{j}}$ stand for the SUT row and column sum vectors. In addition, let vectors $\boldsymbol{\mu}_{(n)}$ and $\boldsymbol{\upsilon}_{(n)}$ be defined as:

$$\begin{gathered}\boldsymbol{\mu}_{(n)} = \begin{bmatrix}\boldsymbol{\mu}_{(n)}^{(AC)} \\ \boldsymbol{\mu}_{(n)}^{(BD)}\end{bmatrix} \\ \boldsymbol{\upsilon}_{(n)} = \begin{bmatrix}\boldsymbol{\upsilon}_{(n)}^{(AC)} \\ \boldsymbol{\upsilon}_{(n)}^{(BD)}\end{bmatrix}\end{gathered} \tag{24}$$

To achieve a unique solution, steps 1 and 2 are repeated iteratively until $n = N$ when:

$$\begin{aligned} \max\left|\boldsymbol{\mu}_{(N)} - \mathbf{z}_{\mathbf{i}\bullet}\right| < \varepsilon \\ \max\left|\boldsymbol{\upsilon}_{(N)} - \mathbf{z}_{\bullet\mathbf{j}}\right| < \varepsilon \end{aligned} \tag{25}$$

is fulfilled for a sufficiently small ε.

3. Empirical application

3.1. Methods and data

The main challenge faced when empirical testing deflation alternatives is the absence of "real" IO data measured in constant prices. In this paper, we circumvent this difficulty by using survey-based data in current prices. Our rationale for this choice follows. If a balancing method can accurately update a matrix $\mathbf{Z}_{(0)}$ to obtain matrix $\mathbf{Z}^*$ given a set of marginal totals, we can expect the result to be as accurate as if this same information is given in constant prices. In other words, we assume if a method is good for updating, it will be good for deflating. We understand this is a strong assumption. But then no true full set of IO data in current prices are ever available since a complete census of establishments is never cost-effective (Lahr, 1993), not even in the case of national statistical agencies.[6] In any case, we understand that that our findings are imperfect from this perspective, but it is the best that one can do, given the resources at hand. Thus, our findings in this section should be absorbed with appropriate caution.

Our dataset includes 2010 and 2015 SUTs for ten European Union (EU) countries.[7] Supply $\mathbf{V}$, and use $\mathbf{U}^d, \mathbf{U}^m$ matrices account for $\mathrm{k} = 65$ for both products and industries. Import matrices $\mathbf{M}$ has $\mathrm{o} = 2$ import origins: imports from EU member states and imports from non-member of the EU. Final demand matrices $\mathbf{F}^d, \mathbf{F}^m$ have $\varphi = 7$ components: (i) household consumption, (ii) collective consumption, (iii) government spending, (iv) gross fixed capital formation and (v) inventory variations, (vi) export to EU member states and (vii) exports to non-member of the EU. Matrices $\mathbf{W}$ have $\rho = 3$ different rows: (a)

[6] Governments interpolate information for firms that do not reply and also answers to some questions that establishments fail to supply. Published government data are far from perfect, despite our hopes and beliefs.
[7] All data used in this paper was retrieved from: https://ec.europa.eu/eurostat/web/esa-supply-use-input-tables/data/database

compensation of employees, (b) gross operating surplus and (c) other net taxes on production. To simplify how results are reported, let:

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}^d \\ \mathbf{U}^m \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} \mathbf{F}^d \\ \mathbf{F}^m \end{bmatrix} \tag{26}$$

SUTs are in basic prices and organised following the scheme depicted in table 1. We use years 2010 and 2015 since those are years in which EU member countries must report symmetric IO tables alongside SUTs (Eurostat, 2014). This opens the door to further research beyond SUTs such as, for example, impact analysis or multiplier analysis. Countries and their corresponding codes are listed in table 4.

Table 4. EU member country codes

Code	Country
AT	Austria
BE	Belgium
DK	Denmark
EL	Greece
ES	Spain
HR	Croatia
HU	Hungary
NL	Netherlands
PT	Portugal
RO	Romania

Source: own elaboration

To measure the accuracy of our estimates, we use the weighted average percentage error (WAPE) (Mínguez, Oosterhaven, & Escobedo-Cardeñoso, 2009). This measure is defined as follows. Let $\mathbf{X}^* = \{\mathbf{x}^*_{\mathbf{ij}}\}$ be a subset of target matrix $\mathbf{Z}^*$ (e.g., $\mathbf{X}^* = \mathbf{V}^*$). Let t stand for a specific deflation alternative (e.g., double deflation). WAPE is calculated as:

$$\omega^{(t)} = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{|x^*_{ij}|}{\sum_i \sum_j x^*_{ij}}\right)\frac{\left|x^{(t)}_{ij} - x^*_{ij}\right|}{|x^*_{ij}|} \times 100 \tag{27}$$

To facilitate comparisons between methodologies across countries, we consider the initial distance $\omega^{(0)}$ defined as the WAPE between $\mathbf{Z}_{(0)}$ and $\mathbf{Z}^*$ matrices. The accuracy gains of a deflation alternative t with respect to the initial distance is defined as:

$$\Delta^t = \frac{\omega^{(0)} - \omega^{(t)}}{\omega^{(0)}} \times 100 \qquad (28)$$

Hence, the closer we get to 100%, the better the result will be.

3.2.Deflation alternatives: two Path-RAS settings, double deflation and GRAS

We evaluate our methodological proposal for two different information settings. The first (Path-RAS-1) makes use of the minimum information requirements as stated in section 2.1. For the second setting (Path-RAS-2), we assume output by industry and total imports by origin to be fully known. In both cases we arbitrarily set $\alpha = \beta = 0.5$.

Table 5. Information used for each deflation alternative.

	$\mathbf{g}^*$	$\mathbf{o}^*$	$\mathbf{q}^*$	$\mathbf{m}^*$	$\mathbf{f}^*$	$\mathbf{w}^*$
Path-RAS-1	Sum	Sum	Sum	Sum	Sum	Sum
Path-RAS-2	Vector	Vector	Sum	Sum	Sum	Sum
DD	Vector	Vector	Vector	Vector	Sum	Sum
GRAS	Vector	Vector	Vector	Vector	Vector	Vector

Source: own elaboration.

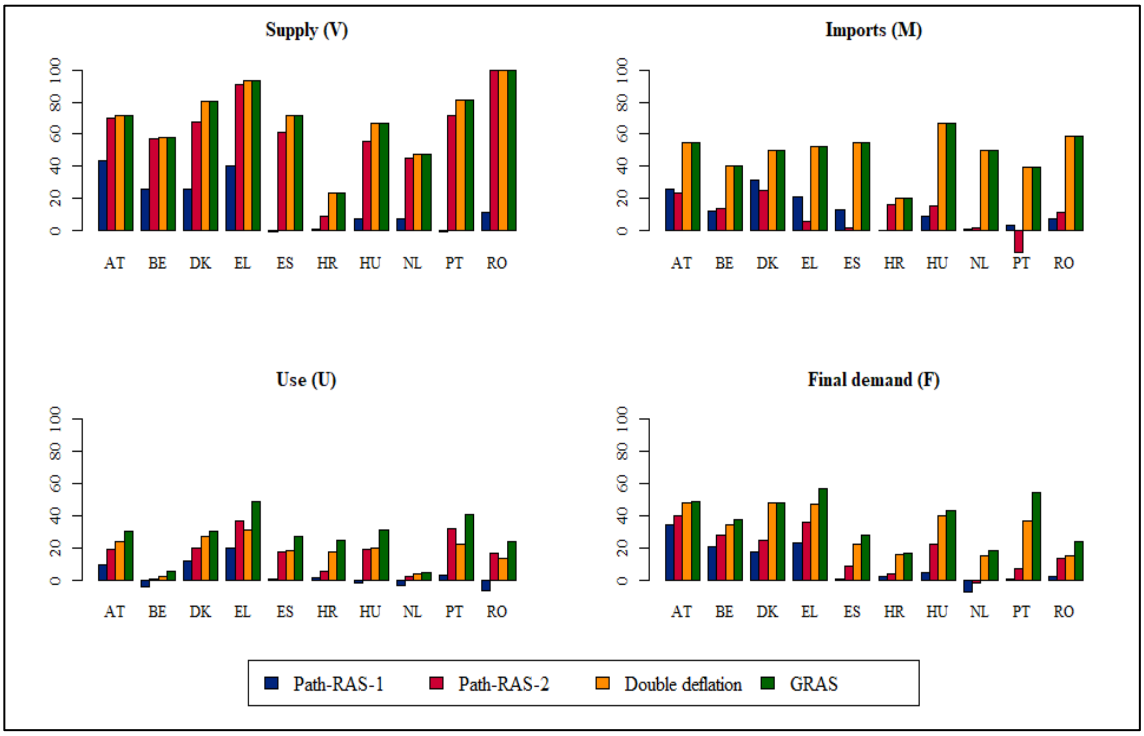
To contrast Path-RAS performance, we applied DD to our dataset following the developments of section 1.3.1. In addition, we approximately reproduce the “column-row-column” deflation practice reported by Eurostat (2008, pp. 247–250). To do so, we use GRAS. This situation (deflation using standard GRAS) is hardly reproducible from the user´s point of view, especially for countries/regions with less available data. Nevertheless, by using GRAS we simulate a reference point and observe how close we can get when making use of less information.

Table 5 summarises the information sets used by each deflation alternative. All target information is from 2015 SUTs. On the one hand, label “Sum” is used when only the overall sum of a vector is known. On the other hand, label “Vector” means that all elements of that vector are considered.

3.3. Empirical outcomes

Figures 2 and 3 illustrate the results. Two clarifying comments must be made before we start discussing the findings. First, double deflation yields by definition the same results as GRAS for the **M** and **V** matrices. This is because a set of consistent targets is imposed for all products, industries and for total imports by origin (see section 1.3.1). Second. in the case of Romania, the supply matrix **V** is a diagonal matrix with no secondary production. Therefore, all alternatives that consider exogenously given $\mathbf{g}^*$ targets (Path-Ras-2, DD and GRAS) precisely generate the 2015 matrix.

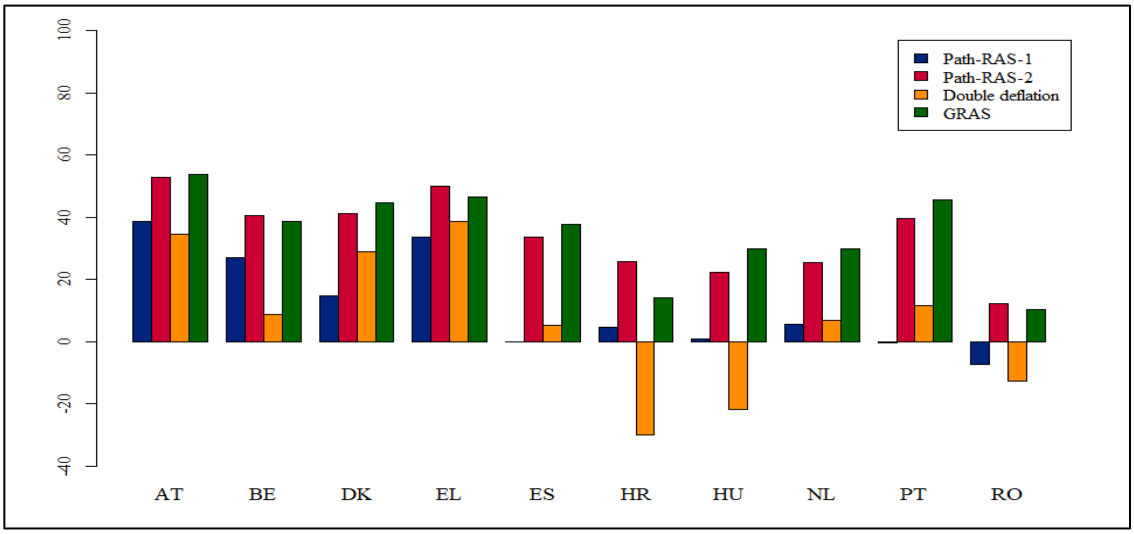
Figure 2. Accuracy gains for each deflation alternative. Matrices V, M, U and F.



Source: own elaboration

On the one hand, Path-RAS-1 yields, as expected, the poorest performance for matrices **V**, **M**, **U** and **F**. Using only the minimum information requirements, this alternative ensures that estimated tables respect the limited set of targets. But, in some cases, the overall error is greater after balancing than before. On the other hand, Path-RAS-2 generally performed as well as DD and GRAS. Thus, our results suggest, despite fewer information requirements, that Path-RAS is as accurate as alternatives that are more data-demanding. Path-RAS-2 seems to perform better for **V** and **U** matrices. This is in line with Dietzenbacher and Hoen's (1998) findings that intermediate transactions with cell-specific deflation would be substantially superior. Overall results are likely to improve as additional information constrains matrices **M** and **F**. Note that volume data on final demand and imports tend to be more widely available than are data for intermediate demand or intermediate inputs.

Figure 3. Accuracy gains for each deflation alternative. Value-added matrix (W).



Source: own elaboration

GVA estimates deserve a separate comment. For seven out of ten countries that we analysed, Path-RAS-1 outperformed DD in estimating GVA. Indeed, Path-RAS-2 yielded better results in all cases; accuracy gains are in line with those via GRAS. This result is highlighted since prior algorithms require GVA to be established exogenously. While the exogenous specification of GVA is reasonable when updating matrices in current prices, it is nigh unto impossible for matrix price deflation. Thus, our modification of Path-RAS appears to be a truly viable innovation in this regard.

4. Conclusions

In this paper, we develop an alternative way to estimate supply and use tables (SUT) in constant prices. Our proposal modifies the Path-RAS approach for SUT updating. It requires less information than do known predecessor approaches. Most importantly, we reduce needs for gross value added (GVA) prices. The approach yields cell-specific deflators, capturing IO price dynamics more realistically. Moreover, our approach does not introduce *ad hoc* adjustments, providing constraints are nonconflicting.

Even though Path-RAS demands comparatively modest amounts of data, it yields promising results. Admittedly, when available information is minimal, Path-RAS performs no better than its “competition.” But it appears to outshine double deflation when it comes to estimating GVA. Moreover, when industry output is constrained, Path-RAS performs as well as GRAS, which requires GVA to be exogenously defined. This suggests out modification to Path-RAS yields results that are sufficiently accurate despite using less information. Estimates for $\mathbf{V}$ and $\mathbf{U}$ appear to be relatively more accurate. This could be confirming the appropriateness of cell-specific deflators to measure intermediate transactions in constant prices.

The prime limitation of our work is the empirical test. Thus, our approach needs further empirical assessments. A broader coverage of countries could reduce biases associated with the use of peculiar data. We also hope to extend our analysis by deriving IO tables from the deflated SUTs. We, thus, should be able to analyse how different deflation methods relate to such standard work as, for example, impact analysis or multiplier analysis. A possible extension of the research presented here could be the inclusion of techniques that systematically handle conflicting information. Finally, we hope future research will identify optimal reliability values (α, β) associated with the industry and commodity technology matrices.

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