1 Introduction

Computable General Equilibrium (CGE) models are widely used by different institutions (public administrations, academia, think tanks, and international organizations) to support policy evaluations and prospective analyses. They rely on a complex representation of the economic system, which allows for quantitatively determining through a numerical resolution the *ex-ante* effects resulting from an exogenous shock (e.g., a technical shock) or the implementation of a given policy (e.g., a carbon tax). The first empirically estimated macro-econometric model was constructed for the Dutch economy by Tinbergen in 1936 (Dhaene and Barten, 1989) and opened a field of research in applied macroeconomics. A CGE model that combines dynamic effects with a multi-sectoral representation of the economy was first proposed by Johansen (1960) following the strand of Input-Output analysis on inter-branch relations developed by Leontief. Their application has been revived by the climate change threat and the need to evaluate the economic impacts of sustainable long-term decarbonization strategies (Böhringer and Löschel, 2006).

However, CGE models have often been criticized because their results are highly sensitive to the value of exogenous parameters whose estimation is uncertain. In the energy transition scenarios analysis, their results are highly contingent on the substitutability of energy with other inputs (Németh et al., 2011). Due to the limited data availability, modelers frequently use either macroeconomic estimation of elasticity of substitution or econometric estimations on micro-data for specific sectors. Either way, it induces a bias because these estimations are inconsistent with the set of data employed for the CGE model's calibration or because they are based on a different functional form than the model's equation.

Jacoby et al. (2006) demonstrate this impact in their MIT EPPA model[1]: setting different values of the elasticity of substitution between energy and non-energy commodities would dramatically change the costs of a mitigation policy case, the Kyoto protocol. The conclusions were similar regarding the rebound effect: the value of the elasticity directly impacts its magnitude (Jaccard and Bataille, 2000). The values of the elasticities of substitution in the production function play a central role in the dynamic of CGE models, especially regarding price-based instruments such as implementing carbon or energy taxes. Okagawa and Ban (2008), for instance, found that conventional parameter distribution could overestimate the carbon price required for a given targeted level of emissions reduction by 44%. Landa Rivera et al. (2016) using a CGE analysis and simulating an

[1]The Emissions Prediction and Policy Analysis (EPPA) model is a recursive-dynamic multi-regional general equilibrium model of the world economy that is part of the MIT Integrated Global Systems Model (IGSM) simulating the social systems.

energy transition scenario with a carbon tax policy in Mexico, show that the change of the elasticity between capital and energy (from 1.5 to 0) leads to a 20% difference in GHG emissions reduction by 2050[2].

Modeling communities have attempted to tackle this issue using econometric estimation of these parameters. Albeit, due to the limited data availability, empirical estimations of the parameters of the production function at a sectoral level are rather limited[3]. Another point of debate remains in the choice of the production function specification to conduct the econometric estimations. Relying on a CES has the advantage of being consistent with the macroeconomic theory but imposes important constraints on the possibility of substitutions between inputs. The Translog specification[4] popularity in the 1980s comes from its higher flexibility. However, it relies on an approximation of the production function by a second-order Taylor-expansion, and the well-behaved properties of the production function prove difficult to impose (Diewert and Wales, 1987; Ryan and Wales, 2000). Despite continuous work to provide selection criteria on the form to adopt, there is still no consensus in the research community on which specification of the production function to favor. The same is true regarding the nested-CES structure that accurately fits data.

In this study, we perform empirical estimations of elasticities of substitution for a KLEM[5] production function using Seemingly Unrelated Model (SUR) estimation procedures. More specifically, we use the VOE-CD specification as the standard case and test two alternative nested structures. The originality of this approach is twofold. First, we rely on an original and consistent panel dataset from the *WIOD 2016 Release* and from which all the variables (prices and quantities) used in the estimation are derived. Secondly, we introduce a new production function specification, which has not yet been tested in an empirical analysis.

The remainder of the paper is organized as follows. We first introduce the VOE-CD specification of the production function and derive the estimated equations. We then describe the dataset construction in a third section and the econometric strategy we apply in Section 4. Section 5 presents our estimation results, discussing which nesting structure fits the dataset best. Section 6 concludes and discusses policy implications.

[2]This difference can be interpreted as the contribution of energy efficiency measures to the total variation of GHG emissions.

[3]The first econometric estimation of these parameters from input-output data was done by Burniaux et al. (1992) for the CGE model GREEN, using OECD data on a sample of 12 countries and seven industries.

[4]The translog function is based on a second-order linear approximation of production function and is characterized by input symmetry and Hicks neutrality.

[5]This acronym stands for the inputs considered separately into the production function where the Value-Added is decomposed between Capital (K) and Labor (L) and the intermediate consumption between Energy (E) Materials (M).

2 The model specification

A production function describes a process of transforming a certain quantity of inputs into a quantity of output. In CGE models, the Cobb-Douglas, the CES, and the Translog functions are the primary functional forms used. The modeling of the producer's behavior generally relies on three main assumptions:

- The firm produces only one output
- The production function is homogeneous of degree one, meaning that the returns to scale are constant
- The substitutability between production inputs is limited

The CES production function introduced by Solow (1956) and formalized by Arrow et al. (1961) has become widely used in the CGE modeling community. It has the advantage of allowing for representing a continuum of substitution possibilities between the inputs, from the Leontief production function where the Elasticity of Substitution (ES) is 0 (strict complementarity) to the linear production function where the ES is infinite (perfect substitution). The Cobb-Douglas function (unitary ES) is also a particular case of the CES function. However, the CES function limits the possibilities of substitution. As its name says, it imposes a constant ES along the isoquant. As shown by Uzawa (1962) and McFadden (1963), it constrains the elasticity to be equal across every pair of inputs, which may prove very limiting in the case of more than two inputs. To circumvent these limits, Sato (1967) proposed a nested form of the function. For instance, in a case with three inputs (X_1, X_2, X_3), a system of a nested CES function can be written:

$$Y = \left(\alpha X_1^{\frac{\eta_{X_1,Z}-1}{\eta_{X_1,Z}}} + (1-\alpha) Z^{\frac{\eta_{X_1,Z}-1}{\eta_{X_1,Z}}} \right)^{\frac{\eta_{X_1,Z}}{\eta_{X_1,Z}-1}} \tag{1}$$

$$Z = \left(\beta X_2^{\frac{\eta_{X_2,X_3}-1}{\eta_{X_2,X_3}}} + (1-\beta) X_3^{\frac{\eta_{X_2,X_3}-1}{\eta_{X_2,X_3}}} \right)^{\frac{\eta_{X_2,X_3}}{\eta_{X_2,X_3}-1}} \tag{2}$$

Equation (1) states that Input X_1 is substitutable to the composite input Z in the production of output Y with an ES of $\eta_{X_1,Z}$ whereas equation (2) states that X_1 and X_2 are two substitutable inputs in the production of the composite input Z.

Although this approach has been widely used in the literature, it is subject to criticism. As argued by van der Werf (2008), there is no theoretical reason to favor a nested structure over another. The choice of the nested structure is therefore left to the modelers' discretion. In one of the earliest works on this literature, Prywes (1986) on US manufacturing industries assumed a three-level-CES production function with a [[[KE]L]M][6] nested structure without providing theoretical nor empirical justifications.

Several studies attempt to provide approaches to determine the correct nested structure. In a study on German manufacturing sectors, Kemfert (1998) proposed a data-driven approach to discriminate between different nested structures. The strategy she uses is to estimate the different combinations of nested structures and select the model with the highest R^2 statistics. However, this criterion appears to be statistically inadequate to compare non-linear models since it assumes that the underlying model being fit is linear (Spiess and Neumeyer, 2010; Lagomarsino, 2020). Despite becoming popular in the CGE literature, it has also been questioned by some authors of this field who argue that in the case of an indirect method based on conditional factor demand is not recommended because the final comparison is made between models based on different dependent and explanatory variables (Baccianti, 2013; Dissou et al., 2015).

Zha and Zhou (2014) insert a Translog specification into the two-level CES production function to select the most appropriate nested structure. Similarly, Lagomarsino (2020) proposed in a meta-analysis on the nested-CES production function to proceed through the use of a Translog specification of each nested structure. A Wald test on the separability and homogeneity assumption for each Translog specification informs if the nested model is rejected or not statistically.

In a recent study on CGE models in China, Feng and Zhang (2018) surveyed the nesting structure of their production function specification and found that the [[KL]E] form has been mostly preferred in 75% of the cases, the [[KE]L] nest being chosen three times and [[EL]K] none. However, the choice is rarely motivated.

Some authors argued against taking the value-added variable as a composite variable from K and L in the upper level combined with E (referred to as a [[KL]E], whereas others claimed to adopt a [[KE]L] structure. It may reflect two visions of the functioning of the economy. The first one favors the income approach by combining Capital & Labor to form an added-value input in the production process. The second one puts emphasis on the physical relation between Capital (equipment) & Energy in the production process [7].

[6]For this case and the following of this paper, the brackets represent the organization of the nest. In this case, K and E are combined to produce KE, KE is combined with L to produce KEL, and KEL is combined with M to produce the output Y.

[7]At the microeconomic level, it is generally considered a Leontief production function between these two production

To overcome this limit, we take advantage of the VOE-CD specification of the production function (Reynès, 2019). It is a flexible form of the Cobb-Douglas production function, which provides a generalization of the CES functional form to the case where the Elasticity of Substitution (ES) between each pair of inputs is not equal. In this sense, it exhibits properties that are well-suited to the case of the multi-factors CES production function without assuming a specific nesting structure.

2.1 The VOE Cobb-Douglas function

Considering a general production function where output Y is produced from a combination of input X_i such as:

$$Y = Y(X_1, X_2, \ldots, X_i, \ldots, X_n) \tag{3}$$

and for which the standard assumptions apply: the production function is a continuous, twice differentiable function that is homogeneous of degree one; the output is increasing in inputs ($Y'(X_i) = \frac{\partial Y}{\partial X_i} > 0$) and strictly concave ($Y''(X_i) = \frac{\partial^2 Y}{\partial {X_i}^2} < 0$)

Using the Euler theorem, Reynès (2019) shows that equation (3) can be written in growth rate[8] (or similarly in logarithm first difference).

$$\dot{Y} = \sum_i \varphi_i \, \dot{X}_i \leftrightarrow d\ln Y = \sum_i \varphi_i \, d\ln X_i \tag{4}$$

where φ_i is the output elasticity, which measures a relative change in output induced by a relative change in input i. It is defined according to the following equation:

$$\varphi_i = \left[\sum_j \frac{Y'(X_j)\; X_j}{Y'(X_i)\; X_i} \right]^{-1} \tag{5}$$

The definition of the ES proposed by Hicks (1932) and Robinson (1933) measures the change in the ratio between two factors of production (i and j) due to a change in their relative marginal productivity. Formally this yields:

$$-\eta_{ij} = \frac{d\ln(X_i/X_j)}{d\ln(Y'(X_i)/Y'(X_j))} \leftrightarrow \dot{X}_i - \dot{X}_j = -\eta_{ij}\;(\dot{Y}'(X_i) - \dot{Y}'(X_j)) \tag{6}$$

factors.

[8]The first and second partial derivatives of the function Y with respect to X_i are respectively $Y'(X_i) = \partial Y/\partial X$ and $Y''(X_i) = \partial^2 Y/\partial X_i^2$. Variables in growth rate are referred to as $\dot{X} = dX/X = d(\ln X)/dX$. All parameters written in Greek letters are positive.

Using the profit maximization behavior from the producer, we can derive the demand function by minimizing the production cost (7).

$$C = \sum_{i} P_i^X \, X_i \tag{7}$$

From the first-order conditions, the ratio between the marginal productivities of two inputs equals the ratio between prices ($Y'(X_i)/Y'(X_j) = P_i^X/P_j^X$). Combining the first-order conditions with equation (5), the OE of input i corresponds to the cost share of input i:

$$\varphi_i = \frac{P_i^X \, X_i}{\sum_j P_j^X \, X_j} \tag{8}$$

Finally, combining the first-order conditions, the definition of the ES (6) and the production function (4) gives the demand function for each factor as a positive function of the output and a negative function of the relative prices between inputs:

$$\dot{X}_i = \dot{Y} - \sum_{j=1} \eta_{i,j} \varphi_j (\dot{P}_i^X - \dot{P}_j^X) \tag{9}$$

3 Data

Our econometric estimation is based on panel data.It allows for considering a more apparent distinction between input substitution and technological change than time-series (Baccianti, 2013).

The ES estimation requires prices and quantities for all the economic variables used in the economic regression. For the construction of the final database, we use the following data sources:

- WIOD Socio-Economic Account (WIOD SEA)
- WIOD National Supply-Use Tables (NIOT)
- WIOD World Input-Output Tables (WIOT)

These data sets belong to the World Input-Output Database Project (WIOD) (Timmer et al., 2015), a consistent regional input-output dataset with a detailed sectoral granularity of the world economy. In its latest version (2016 Release), the dataset covers the period from 2000 to 2014 and distinguishes 42 countries (plus the rest of the world) and 56 sectors (see Table 1 in Appendix A). Since the WIOD tables are both provided in current prices (CP) and previous year prices (PYP), we can distinguish, for each variable, value (in current price) and volume (nominal price) using the chained-price method[9]. The examples of other panel data sources employed in the literature include Eurostat's National Accounts and COMEXT (Németh et al., 2011), the IEA Energy Balances and the OECD International Sectoral Database (Saito, 2004; van der Werf, 2008) as well as the OECD International Trade by Commodities Statistics and the OECD Input-Output Database (Sato, 2014).

From the WIOT dataset, we extract for each sector their aggregate intermediate consumption of energy goods[10] and non-energy goods.

The price growth rate of the input X used in the sector i is computed according to the following equation:

$$\dot{P}_{i,t}^{X} = \frac{X_{i,t}^{CP}}{X_{i,t}^{PYP}} - 1 = \frac{X_{i,t}\ P_{i,t}^{X}}{X_{i,t}\ P_{i,t-1}^{X}} - 1 = \frac{P_{i,t}^{X}}{P_{i,t-1}^{X}} - 1 \tag{10}$$

Furthermore, taking a unitary value for the price at the base year (2000 = 1) allows us to calculate the price index :

[9] In previous studies on the estimation of the ES of a KLEM production function, authors used the 2013 release of WIOD, which do not provide previous year prices national accounts. They, therefore, adopted an alternative source of data to construct the price series (see Baccianti (2013); Koesler and Schymura (2015); Antoszewski (2019)).

[10] The intermediate energy consumption aggregates the *Manufacture of coke and refined petroleum products* (C19) and *Electricity, gas, steam and air conditioning supply* (D35).

$$P^X_{i,t} = \prod_t (1 + \dot{P}^X_{i,t})\ P^X_{i,0} \tag{11}$$

Finally, using it as a deflator on the input series expressed in current price allows for expressing these series in real terms:

$$X_{i,t} = \frac{X^{CP}_{i,t}}{P^X_{i,t}} \tag{12}$$

The capital and labor price and volume series are constructed from the WIOD SEA database following series: Total hours worked by employees (in millions) (H_EMPE), compensation of employees[11] ($COMP$), capital compensation (CAP) and nominal capital stock (K^{VAL}). By default, the series are expressed in nominal value and in national currencies[12].

Dividing $COMP$ by H_EMPE gives the hourly wage W for each period, country, and sector. We then compute the labor economic volume variable L, as the total work expressed in hours multiplied by the hourly wage base year value $W_{i,0}$

$$L_{i,t} = W_{i,0}\ H\_EMPE_{i,t} \tag{13}$$

The price-variation of labor $\dot{P}^L_{i,t}$ is directly derived from the wage growth rate $(W_{i,t}/W_{i,t-1} - 1)$, from which we directly derive the labor price index.

$$P^L_{i,t} = \prod_{t=1} (1 + \dot{P}^L_{i,t}) \tag{14}$$

Regarding the distinction between quantities and prices for capital, we can not use the same approach for labor because there is no variable expressed in volume in the dataset that would allow for calculating a price deflator. To estimate the volume of capital stock, we use the standard approach of the Perpetual Inventory Method (PIM), which consists in deriving the capital stock from data on investment flows[13] The capital accumulation equation can be written in value or volume metrics:

$$K^{VAL}_{i,t} = K^{VAL}_{i,t-1}\ (1 - \delta_{i,t}) + P^I_{i,t}\ I_{i,t} \tag{15}$$

[11] The WIOD SEA database provides an alternative metric for labor compensation (LAB) that we did not consider because it includes self-employed workers.

[12] We convert the economic values in $ currency using the same exchange rates table used in WIOD to construct the international Supply-Use Tables.

[13] Some authors, such as Lee (2005) and (Soytas and Sari, 2007) use directly investment data as a proxy for capital stock. This approach underestimates the capital stock since it does not consider the lifespan of capital.

$$K_{i,t} = K_{i,t-1}\left(1 - \delta_{i,t}\right) + I_{i,t} \tag{16}$$

Inverting equation (15) and using the definition of the growth rate of capital, $\dot{K}_{i,t}^{VAL} = \frac{K_{i,t}^{VAL}}{K_{i,t-1}^{VAL}} - 1$, allows for deriving a relation for the depreciation rate:

$$\delta_{i,t} = \frac{P_{i,t}^{I} I_{i,t}}{K_{i,t-1}^{VAL}} - \dot{K}_{i,t}^{VAL} \tag{17}$$

This equation is used to derive the depreciation ratio from the WIOD database, which contains time series for the capital stock and investment in value. The depreciation rate is used to estimate the capital stock in volume thanks to equation (16). Finally, the nominal to real capital stock ratio provides a capital price index.

$$P_{i,t}^{K} = \frac{K_{i,t}^{VAL}}{K_{i,t}} \tag{18}$$

As in Antoszewski (2019), this price index will be used as a proxy for the cost of the capital input. This specification has the advantage of simplicity. Its main drawback is that it does not account for the opportunity cost related to investment. For several reasons, the capital cost specification remains controversial (Jorgenson and Griliches, 1967; Hall and Jorgenson, 1969; Hudson and Jorgenson, 1974; Levinsohn and Petrin, 2003; Collard-Wexler and Loecker, 2016) in the literature, among which the difficulties in distinguishing between physical and financial capital or between the user cost, opportunity cost, or desired rate of return. Addressing these issues goes largely beyond the scope of this paper. Hence we keep the impact of alternative specifications on the cost of capital for further research.

The final panel dataset gathers the following variables in volume (Y, K, L, E, M) and prices (pY, pK, pL, pE, pM). We also compute their respective growth rates ($\dot{Y}, \dot{K}, \dot{L}, \dot{E}, \dot{M}, \dot{pY}, \dot{pK}, \dot{pL}, \dot{pE}, \dot{pM}$) from which we perform the econometric estimations presented in the next section.

4 Econometric strategy

Our empirical analysis considers a four inputs production function, often known as KLEM: Capital (K), Labor (L), Energy (E), and non-energy intermediate inputs (M). The function parameters to estimate are determined for each sector s specified in the WIOD database.

In the panel, we distinguish 13 periods t and 44 countries or regions r.

4.1 Literature review

The literature has proposed three approaches to estimating a nested CES production function: The direct approach based on its non-linear estimation, the indirect approach based on a cost minimization program, and the approximation based on its Kmenta's linearization.

The direct approach consists in using non-linear least squares estimation based on *ad-hoc* non-linear optimization algorithms[14]. However, their use is intricate because of the need to find a proper starting value to achieve a numerical convergence[15]. Since the CES production function is not-linear in its parameters, it implies that their values cannot be directly estimated with a standard OLS estimator.

The indirect approach has been often used to estimate nested CES production function (Prywes, 1986; Okagawa and Ban, 2008; Antoszewski, 2019). It relies on the assumption of the maximizing behavior of the supply-side (either through a cost-minimization or a profit-maximization problem) and therefore involves collecting data on prices, besides quantities.

An alternative approach to non-linear estimation is the one proposed by Kmenta (1967). The outcome is a restricted form of the general Translog function. It uses a linear approximation of the CES function to estimate its parameters. This approximation is a linear Taylor series expansion when the ES is around 1. This method has been criticized by Thursby and Lovell (1978), arguing that the Kmenta's approximation only converges to the underlying CES function in the region of convergence determined by the true parameters of the CES function. For these reasons, the linearization method proposed by Kmenta was rarely chosen [16].

The specification we test is derived from the demand function determined by the VOE-CD

[14] A version of this algorithm developed by Henningsen and Henningsen (2012) has been made available for empirical applications in the package *micEconCES*.

[15] According to Henningsen and Henningsen (2012), results obtained through this method should be taken with caution since they were not able to replicate the original findings from the (Kemfert, 1998) article adopting this approach.

[16] Koesler and Schymura (2015) compare the estimates obtained from the Kmenta's approximation with a non-linear estimation and conclude that the former performs less well in terms of statistics fit.

production function as stated in 9.

Since the economic framework assumes a constant return to scale, and to avoid endogeneity in the estimation, we take as explained variable the difference between the growth rates of input j and output Y. We also consider time and country fixed effects. We regress our model on a sub-panel independently defined for each sector.

Regarding our strategy, we want to take into account the advantage of the general form of the VOE-CD to perform a regression on the system of equations that defines the production process. Since the economic framework assumes a constant return to scale, and to avoid endogeneity in the estimation, we take as explained variable the difference between the growth rates of input j and output Y. We also consider time and country fixed effects. We perform regressions of our model independently for each sector.

4.2 Estimation approaches

We adopt a Seemingly Unrelated Regression (SUR) approach originally developed by Zellner (1962) and extended to panel data analysis by Avery (1977) and Baltagi (1980). It allows for accounting for potential correlations between the errors from equations of the system. Moreover, in order to take into account the assumption of symmetry of the ES between inputs ($\eta_{ij} = \eta_{ji}$), we have to impose cross-constraints restriction of the system of equations (19). Having derived the inputs demand (9) for a system of four inputs, the estimated system is:

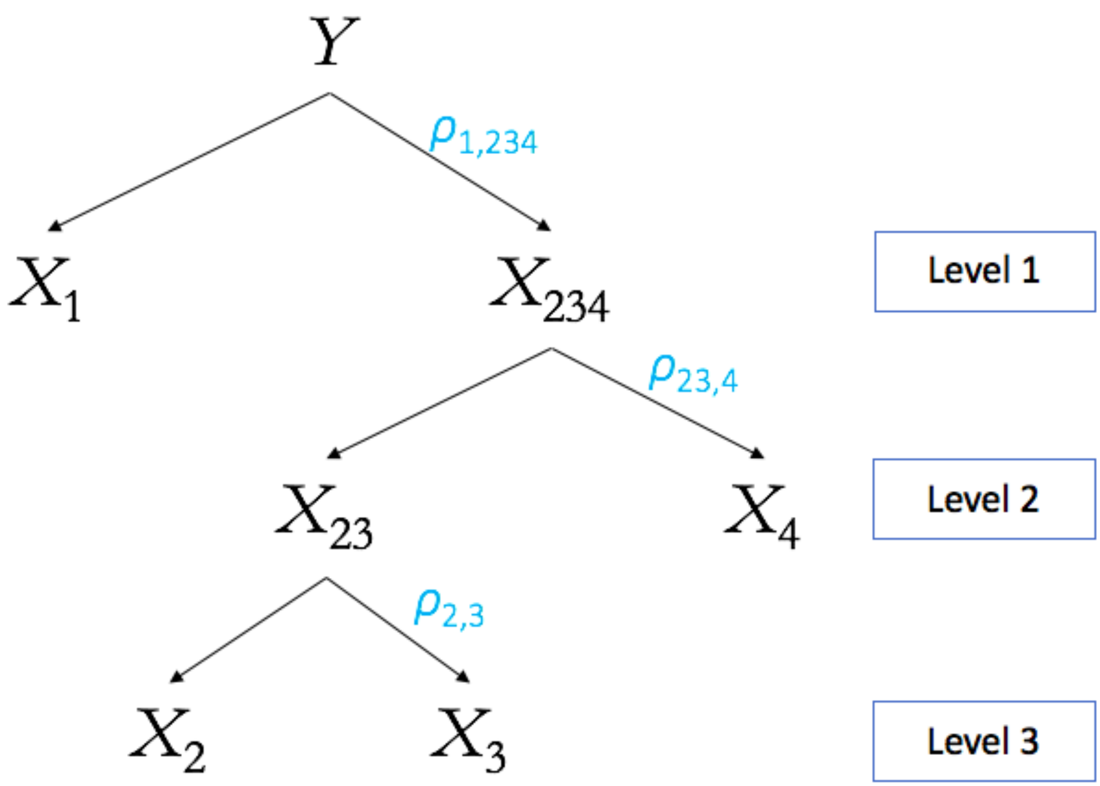
$$\begin{cases} \dot{K}_{r,t} - \dot{Y}_{r,t} = \alpha^K + \eta^{K,L}\ \varphi^L_{r,t-1}\ (\dot{P}^K_{r,t} - \dot{P}^L_{r,t}) + \eta^{K,E}\ \varphi^E_{r,t-1}\ (\dot{P}^K_{r,t} - \dot{P}^E_{r,t}) + \\ \qquad\qquad \eta^{K,M}\ \varphi^M_{r,t-1}\ (\dot{P}^K_{r,t} - \dot{P}^M_{r,t}) + \mu^K_t + \mu^K_r + \epsilon^K_{r,t} \\ \dot{L}_{r,t} - \dot{Y}_{r,t} = \alpha^L + \eta^{L,K}\ \varphi^K_{r,t-1}\ (\dot{P}^L_{r,t} - \dot{P}^K_{r,t}) + \eta^{L,E}\ \varphi^E_{r,t-1}\ (\dot{P}^L_{r,t} - \dot{P}^E_{r,t}) + \\ \qquad\qquad \eta^{L,M}\ \varphi^M_{r,t-1}\ (\dot{P}^L_{r,t} - \dot{P}^M_{r,t}) + \mu^L_t + \mu^L_r + \epsilon^L_{r,t} \\ \dot{E}_{r,t} - \dot{Y}_{r,t} = \alpha^E + \eta^{E,K}\ \varphi^K_{r,t-1}\ (\dot{P}^E_{r,t} - \dot{P}^K_{r,t}) + \eta^{E,L}\ \varphi^L_{r,t-1}\ (\dot{P}^E_{r,t} - \dot{P}^L_{r,t}) + \\ \qquad\qquad \eta^{E,M}\ \varphi^M_{r,t-1}\ (\dot{P}^E_{r,t} - \dot{P}^M_{r,t}) + \mu^E_t + \mu^E_r + \epsilon^E_{r,t} \\ \dot{M}_{r,t} - \dot{Y}_{r,t} = \alpha^M + \eta^{M,K}\ \varphi^K_{r,t-1}\ (\dot{P}^M_{r,t} - \dot{P}^K_{r,t}) + \eta^{M,L}\ \varphi^L_{r,t-1}\ (\dot{P}^M_{r,t} - \dot{P}^L_{r,t}) + \\ \qquad\qquad \eta^{M,E}\ \varphi^E_{r,t-1}\ (\dot{P}^M_{r,t} - \dot{P}^E_{r,t}) + \mu^M_t + \mu^M_r + \epsilon^M_{r,t} \end{cases} \tag{19}$$

The system of equation (19) is solved for each sector s based on 2408 observations. The parameter α^i is the constant, η_{ij} are the elasticities of substitution between input i and j. μ^M_r and μ^M_t are respectively the country and the time fixed-effect terms, and $\epsilon_{i,t}$ is the error term. The input shares that intervene with a time lag in the system (19) to avoid endogeneity bias are computed according to equation (8):

$$\varphi_{r,t}^{X} = \frac{P_{r,t}^{X}\, X_{r,t}}{\sum_j P_{r,t}^{X}\, X_{r,t}} \tag{20}$$

As a generalization of the CES function, the VOE-CD also encompasses nested CES structures. We can therefore use it to test if the ES estimation is consistent with a nested CES structure. To do so, we adopt a standard three-level nested structure of the type $[[[X_2; X_3]X_4]X_1]$ as shown on Figure 1)

Figure 1: Nesting structure of a four inputs CES production function



Writing $\varphi_{k'}^{k}$ the share of the input k into the output k', (k' being either the final output Y at the first level of the nest or a composite input of production for lower levels)[17] , the model can be reformulated as follows :

$$\begin{cases} \dot{X}_1 = \dot{Y} + \rho^{X_1,X_{234}}\ \varphi_Y^{X_{234}}\ (\dot{P}^{X_1} - \dot{P}^{X_{234}}) \\ \dot{X}_{234} = \dot{Y} + \rho^{X_{234},X_1}\ \varphi_Y^{X_1}\ (\dot{P}^{X_{234}} - \dot{P}^{X_1}) \\ \dot{X}_2 = \dot{X}_{23} + \rho^{X_2,X_3}\ \varphi_{X_{23}}^{X_3}\ (\dot{P}^{X_2} - \dot{P}^{X_3}) \\ \dot{X}_3 = \dot{X}_{23} + \rho^{X_3,X_2}\ \varphi_{X_{23}}^{X_2}\ (\dot{P}^{X_3} - \dot{P}^{X_2}) \\ \dot{X}_4 = \dot{X}_{234} + \rho^{X_4,X_{234}}\ \varphi_{X_{234}}^{X_{23}}\ (\dot{P}^{X_4} - \dot{P}^{X_{23}}) \\ \dot{X}_{23} = \dot{X}_{234} + \rho^{X_4,X_{234}}\ \varphi_{X_{234}}^{X_4}\ (\dot{P}^{X_{23}} - \dot{P}^{X_4}) \end{cases} \tag{21}$$

We confront the two most used nesting structures of the production function in the literature. The first nesting is of the form $[[[[KL]E]M]$, which considers the value-added as a meaningful economic variable in relation to the intermediate inputs. The alternative case $[[[KE]L]M]$ sees The alternative case [[[KE]L]M] sees the Capital-Energy relation as grounding since it is based on engineering observations of a productive capital functioning (physical capital being run with an energy influx). The $[[[KL]E]M]$ nesting has been adopted in several articles (Okagawa and Ban, 2008; Koesler and Schymura, 2015; Antoszewski, 2019) whereas the $[[[KE]L]M]$ nesting has been preferred by others[18] (Prywes, 1986; Chang, 1994).

The system of equations we estimate is defined as follows:

$$\begin{cases} \dot{X}_1 - \dot{Y} = \alpha^{X_1} + \rho^{X_1,X_{234}}\ \varphi_Y^{X_{234}}\ (\dot{P}^{X_1} - \dot{P}^{X_{234}}) + \mu_t^{X_1} + \mu_r^{X_1} + \epsilon_{r,t}^{X_1} \\ \dot{X}_{234} - \dot{Y} = \alpha^{X_{234}} + \rho^{X_1,X_{234}}\ \varphi_Y^{X_1}\ (\dot{P}^{X_{234}} - \dot{P}^{X_1}) + \mu_t^{X_{234}} + \mu_r^{X_{234}} + \epsilon_{r,t}^{X_{234}} \\ \dot{X}_2 - \dot{X}_{234} = \alpha^{X_2} + \rho^{X_2,X_{34}}\ \varphi_{X_{234}}^{X_{34}}\ (\dot{P}^{X_2} - \dot{P}^{X_{34}}) + \mu_t^{X_2} + \mu_r^{X_2} + \epsilon_{r,t}^{X_2} \\ \dot{X}_{34} - \dot{X}_{234} = \alpha^{X_{34}} + \rho^{X_{34},X_2}\ \varphi_{X_{234}}^{X_2}\ (\dot{P}^{X_{34}} - \dot{P}^{X_2}) + \mu_t^{X_{34}} + \mu_r^{X_{34}} + \epsilon_{r,t}^{X_{34}} \\ \dot{X}_3 - \dot{X}_{34} = \alpha^{X_3} + \rho^{X_3,X_4}\ \varphi_{X_{34}}^{X_4}\ (\dot{P}^{X_3} - \dot{P}^{X_4}) + \mu_t^{X_3} + \mu_r^{X_3} + \epsilon_{r,t}^{X_3} \\ \dot{X}_4 - \dot{X}_{34} = \alpha^{X_4} + \rho^{X_4,X_3}\ \varphi_{X_{34}}^{X_3}\ (\dot{P}^{X_4} - \dot{P}^{X_3}) + \mu_t^{X_4} + \mu_r^{X_4} + \epsilon_{r,t}^{X_4} \end{cases} \tag{22}$$

Regarding the system of equations (22), this leads to $(X_1 = M; X_2 = E; X_3 = L; X_4 = K)$ in the first case $(((KL)E)M)$ and to $(X_1 = M; X_2 = L; X_3 = E; X_4 = K)$ in the second one $[[[KE]L]M]$.

By developing the system (21), we can derive the explicit production factors demand as in

[17] For the second level of the nested production function, the share of the composite good X_{23} into to the output X_{234} is $\varphi_{X_{234}}^{X_{23}} = (1 - \varphi_{X_{234}}^{X_4})$.

[18] In a three inputs-case $(K; L; E)$, we notice that the preferences are more oriented towards the $[[KE]L]$ form (Feng and Zhang, 2018; Kemfert, 1998).

the system (19). We can also write the ES between each pair of inputs implicitly defined by the system (21), η being a function of the ES ρ estimated in the nested specification of the production function. Extending the system of equation (21) by replacing the composite inputs leads to the explicit formulation of each input as in (9). This yields, after simplifying the equations[19], the relation between the ES of the form:

$$\begin{cases} \eta_{1,2} = \eta_{1,3} = \eta_{1,4} = \rho_{1,234} \\ \qquad\qquad\quad \eta_{2,3} = \dfrac{\rho_{2,3}}{1-\varphi_1-\varphi_4} - \dfrac{\rho_{1,234}\ \varphi_1}{1-\varphi_1} - \dfrac{\rho_{23,4}\ \varphi_4}{(1-\varphi_1)(1-\varphi_1-\varphi_4)} \\ \qquad\quad \eta_{2,4} = \eta_{3,4} = \dfrac{\rho_{23,4} - \rho_{1,234}\ \varphi_1}{1-\varphi_1} \end{cases} \tag{23}$$

It is to be noted that in this case, we still have three constrained values ($\eta_{1,3}$, $\eta_{1,4}$ and $\eta_{3,4}$) to the VOE-CD general case. We also consider for their computation the average shares φ on the whole period covered by the data panel.

[19] For a full demonstration see Reynès (2019).

5 Results

In this section, we expose the econometric results for three production function structure cases: two constrained cases and the unconstrained cases. Then we use them to calibrate a CGE model and simulate the impact of a carbon tax policy depending on the estimated production structure.

5.1 Estimation results

In order to facilitate the reading of the results due to the large number of sectors, we present them in a graphic form (see Figure 2). The detailed estimation tables are provided in 6.2 for the three cases.

The nesting structure has important implications since it leads to two opposite diagnostics regarding the substitution between Capital and Energy. In the $[[[KL]E]M]$ case, where Energy is a direct substitute for the Value-added component in the production function, the econometric estimation finds that the ES between capital and Energy is positive in a majority of sectors (thirty out of fifty-four). It indicates a strong complementary between these two inputs (see Prywes (1986)). In the $[[[KL]E]M]$ case, capital and Energy are, on the contrary, diagnosed as strong substitutes: forty-three sectoral estimations out of fifty-four have a negative ES, and among them, thirty-three with an absolute value greater than 1. Regarding at the average of sectors, the ES between Capital and Energy is 0,20 in the $[[[KL]E]M]$ case against -1,75 in the $[[[KL]E]M]$ case. It highlights a clear contrast with the Labor-Capital ES estimation where the results are consistent across the specifications (the average Labor-Capital ES is -0.31 for the case $[[[KL]E]M]$ and - 0,27 for the case $[[[KL]E]M]$). It indicates a specificity of the capital-energy relationship. When considered direct substitutes, they are strongly substitutable, and when integrated into a composite input, they are strongly complementary.

The more general unconstrained VOE case rather indicates substitutability between capital and Energy. Out of the 31 sectors providing significant results, 29 sectors have a negative elasticity[20]. For results significant at a 99% level [21], we find an average ES between Capital and Energy of -0,76 (resp. -0.5 for the median ES), of -0,83 (resp. -0,85) between Capital and Materials of -0,48 (resp. -0,42) between Labor and Energy, of -1,65 (resp. -1,31) between Labor and Materials of -0,80 (resp.

[20]Out of the three sectors showing complementarity J61 = Telecommunications; C27 = Manufacture of electrical equipment and R_S = Other services activities), two of them are related to electrical equipment, suggesting a sectoral feature in capital and energy use.

[21]After excluding an outlier: the values estimated for the sector E37-E39 (Sewerage; waste collection, treatment; materials recovery, and other waste management services) are in absolute terms higher than 10 for two ES, suggesting misspecification of a data issue.

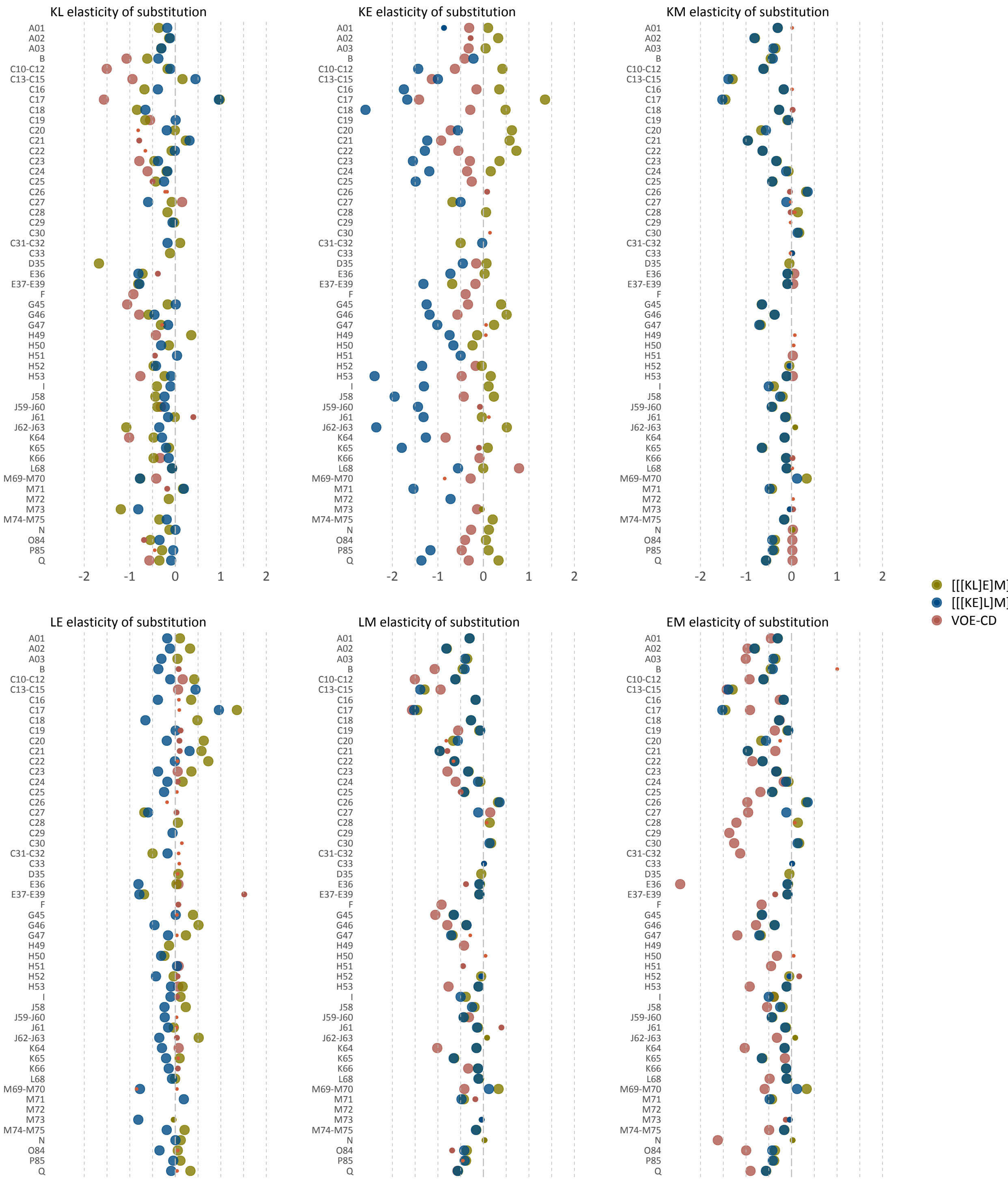
-0,65) and of -2,37 (resp. -1,72) between Energy and Materials. The results confirm recent findings from the Literature. Based on an empirical analysis of the production function using a Translog specification as a benchmark, Lagomarsino and Turner (2017) conclude that a [[[KE]L]M] nested structure is the most appropriate form.

Another point to raise is the differences in estimation that brings the VOE-CD specification concerning the nested specifications. Indeed due to the restrictions imposed by the constraints on the ES estimations (see equation 23), non-energy-inputs (M) are considered less substitutable with the other inputs than in the VOE case[22].

Without further statistical tests, it remains tedious to assess the superiority of a specification to another from an empirical point of view. However, imposing a nesting structure necessarily induces more constraints on the estimation. In the case of the KM, LM, and EM ES, it seems that these restrictions can even be misleading. If it matches pretty well the estimations from the VOE for the KM substitutability, the findings suggesting a complementarity between capital and Energy for most of the sectors remains questionable.

[22] In the alternative cases, Materials are substitute to the composite input $[[KL]E]$ or $[[KE]L]$.

Figure 2: Estimation of the elasticities



Note: The size of the points indicates the level of significance of the estimations (big = 1%; medium = 5%; small = 10%).

5.2 Simulations

As stated in the introduction, results from simulations conducted on CGE are sensitive to the distribution of the exogenous parameters, including the elasticities of substitution. In this part, we will mobilize the CGE model ThreeME to conduct a sensitivity analysis regarding the distribution of these parameters on the aggregate and sectoral results. The model ThreeME is a dynamic CGE model characterized by neo-Keynesian features. It allows for sub-optimal equilibria and transition phases before reaching a long-term steady-state equilibrium (see details in Appendix C). We take as the baseline a 17 sectors version of the model[23], calibrated on the NAF nomenclature, compatible with the NACE Rev2.1 EU nomenclature (and therefore WIOD), which has been used to estimate the impact of the COVID restrictions on the french economy (Malliet et al., 2020).

Our ES estimations are based on the WIOD sectoral disaggregation. They must be adjusted to match the NAF nomenclature sector disaggregation used in ThreeME. For each sector of the NAF nomenclature, its ES are calculated as the weighted average of the estimated ES from WIOD data using the production weight from WIOD on the related sectors. The distribution is provided in the Figure 3.

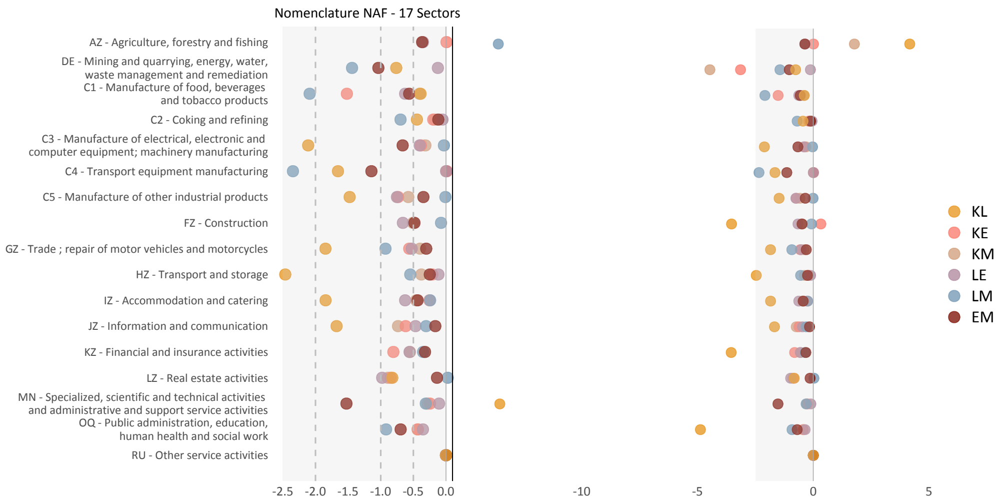
We consider a neutral carbon tax scenario with no monetary transfers between households and firms: proceeds of the carbon tax paid by households are redistributed to them, while each sector receives a share of the carbon tax paid by the private sector proportional to its share of total employment. This mode of allocation is favorable to labor-intensive sectors. Following the Quinet commission report (Quinet, 2019), we assume a constant increasing carbon tax trajectory is reaching 250 EUR in 2030, 500 EUR in 2040, and 775 EUR in 2050.

We compare four simulations of this scenario where only the value of the elasticities of substitution is altered. The first one with an aggregate elasticity of substitution calibrated to -0.5 represents a relative inelastic case, and the second one to -1 corresponds to a Cobb-Douglas production function specification. The third one with -2 states an elastic version of the production function, and in the last one, we report the results obtained from the econometric regression and calibrated on the 17 sectors. The results are reported in Figure 4 in relative deviation to the baseline scenario (where no carbon tax policy is implemented).

From a macroeconomic point of view, the scenario with the VOE-CD estimation does not appear as an outlier. It evolves in the same range as the ad-hoc elasticities scenarios with a long-term effect between the $ES : -2$ and $ES : -0.5$ scenarios. Regarding the GDP, we observe a positive increase by 2050 of 0.24% (the amplitude of the deviation is slightly under the Cobb-Douglas scenario for

[23] The source code can be retrieved from the Github repository: `https://github.com/fosem/ThreeME_V3-open`.

Figure 3: Elasticities for the NAF nomenclature



Note: The gray area corresponds to a zoom of the distribution of the elasticities between −2.5 and 0 values/ The dashed lines corresponds to the assumptions made for the alternative scenarios for the values of the elasticities of substitution.

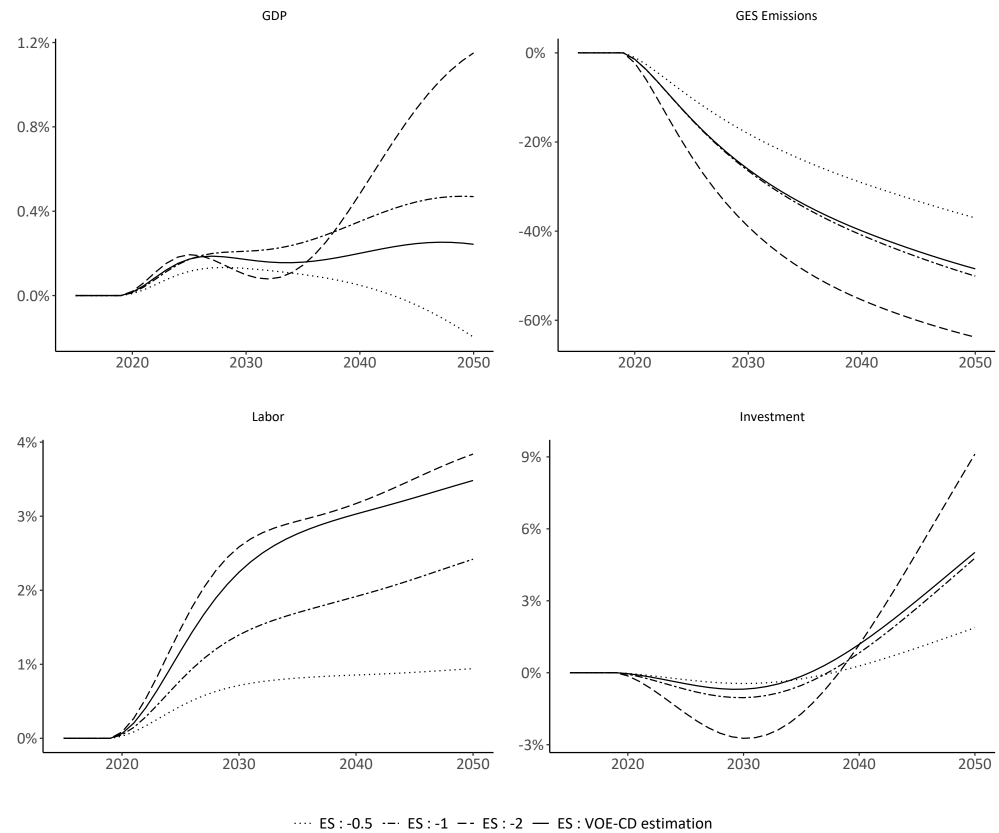
which the impact is 0.47%). From a general point of view, we can see that the results are strongly related to the ES assumption since the results cover a broad amplitude. At the end year of the simulation, 2050, we find a range from −0.20% for the scenario $ES : -0.5$ to +1.15% for the scenario $E : -2$).

For the more elastic case, we can see a downturn in the GDP trajectory from 2025 to 2032, which corresponds to a similar shrinking of investments in the same period before catching up and reaching 2050, a 1.15% increase with respect to the baseline. In the most inelastic case (ES:−0.5), the GDP % deviation remains small compared to the others and leads to a long-run negative impact with a 0.2% deviation by 2050.

The dynamics induced in the labor market are pretty straightforwards as well—the more substitutable the inputs, the larger the impact on employment. It reaches by 2050 a positive deviation of 0.9% for the scenario $ES : -0.5$, 2.4% for the intermediate case $ES : -1$, and 3.8% for the scenario $ES : -2$. The estimated elasticities scenario follows the same dynamic as the latest, with a long-term impact of 3.5%. It should be noted that the recycling scheme plays a central role in the direction of the results. Other recycling schemes would not necessarily lead to a positive effect on employment.

Finally, the overall impact on emissions ranges from −63% to −37% by 2050, the lowest reduction

Figure 4: Simulations of a carbon tax policy for each distribution of parameters



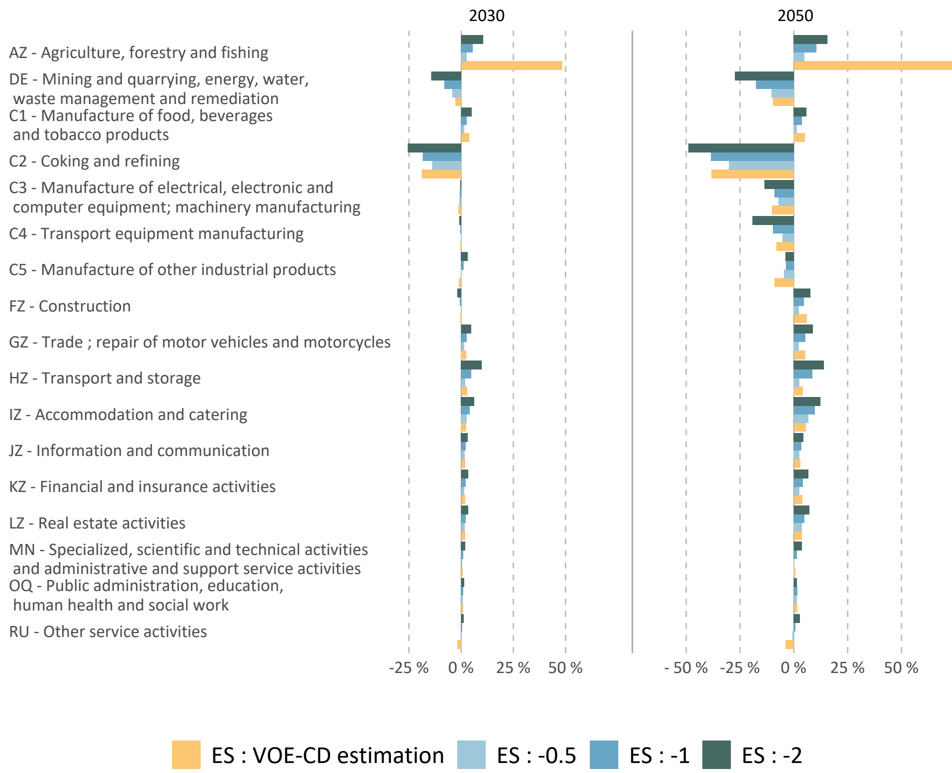
Note: Simulations conducted with the model ThreeME

being associated with the scenario $ES : -0.5$ and the highest to the scenario $ES : -2$. The more substitutable the inputs, the lower the fossil fuel energy demand. The scenario with the estimated ES leads to a similar reduction of emissions than in the $ES : -1$ case, with a relative deviation by 2050 equals to -48.5% (resp. -50.1%). It can be seen as the direct consequence of the values for ES between Capital and Energy, which for some sectors are closer to -2 than to -1[24]

Breaking down at the sectoral level (see Figure 5 and looking at the value-added variable, the results from the estimated elasticities deliver the same conclusions as for the aggregate indicators. The effects dwell within the same range as the macroeconomic indicators, except for the agricultural sector, where the variation of labor use is much larger. It is the direct consequence of the estimated

[24]The estimations for the ES between Energy and Labor though are not as much elastic since the results are distributed between 0 and -1.

Figure 5: Value added in % deviation wrt baseline for two selected years



Note: Simulations conducted with the model ThreeME

value of its elasticity between materials and labor, which is equal to -15.2 for this sector. Such an outlying value raises questions about the data quality for this sector since such a value is an outlier. It leads to a long-term variation of value added and employment of more than 60%, much greater than for other scenarios.

6 Conclusion

We contribute to the empirical literature on substitutions between production factors by proposing the first econometric estimation of the VOE-CD specification. Moreover, we constructed an original panel dataset derived from the WIOD database, one of the most used sources for CGE analysis. We then estimate the ES between KLEM inputs for 54 economic sectors. We evaluate and compare three specifications of the production function, among which two main forms of nested CES production function, namely [[[KL]E]M] and [[[KE]L]M]. We obtain highly significant estimation results for most of the sectors. A comparison of the different specifications allows for deriving three main conclusions:

- By imposing constraints on the estimations, the form of the nest has important implications on the estimated results.
- The Capital-Energy substitution behavior is especially highly dependent on the nest structure since it leads to opposing conclusions: either substitution or complementarity depending on the nest structure's choice.
- The VOE specification supports substitutability between these two factors of production, suggesting that the [[[KE]L]M] nest may be closer to reality.

The VOE-CD specification appears as a relevant, flexible, functional form of the production function. It has the advantage of linear tractability while relaxing the constraint imposed by the CES production function. It is, therefore, a relevant alternative for CGE models. When applied to energy and carbon policy evaluations, the VOE-CD function shows that the nest's choice affects the results critically.

These results shed some light on the Capital-Energy controversy initiated by opposite estimations of the value of the ES between these two inputs: On the one hand, Berndt and Christensen (1973) found complementarity; on the other hand, Griffin and Gregory (1976) found substitutability.

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The estimated values we obtained from the econometric regressions compared with a Cobb-Douglas production function specification (i.e., with an ES equal to -1) indicate relatively lower substitutability between energy and capital, leading to fewer emissions reduction.

To investigate these estimations' implications on a CGE model's simulation results, we perform a sensitivity analysis regarding the level of ES. Including the values estimated econometrically, we compare them to 3 standard cases of ES. Our results confirm the crucial role of the distribution of the ES parameters on the results of CGE conducted simulations. The implications of the simulation results are paramount. It sketches a more labor-intensive substitution effect from the carbon tax policy than what could be expected in the Cobb-Douglas case but associated with an equivalent reduction in emissions. More specifically, the elasticities between labor and energy in most sectors are lower than -1. These results could be further investigated in several directions. A first lead would be to compare them with those estimated from another flexible production function, such as the Translog, which would disentangle the data's respective role and the estimated results specification. Another possible investigation is the indicators' impact on the estimated elasticity level. For instance, the definition of the capital stock used may impact the results. Investigation of the original dataset should also be carried on since they result from a necessary transformation process of raw data from statistical institutes that can be a source of estimation bias, especially at the sector level. Nonetheless, the implications of the calibration of the elasticities are critical in terms of effect and cannot be ignored or neglected. The data stringency argument raised in the past to justify ad-hoc values appears no longer valid, and the development of econometric studies for CGE modeling should be more systematized.

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6.1 Appendix A: Econometric results

Table 1: Estimation results for the VOE-CD production function

Sectors	η^{KL}	η^{KE}	η^{KM}	η^{LE}	η^{LM}	η^{EM}
A01	-0.389^{***}	-0.283^{***}	-0.294^{***}	0.908	-0.165^{***}	-2.875^{**}
	(0.053)	(0.464)	(0.035)	(4.47)	(0.133)	(2.063)
A02	-0.563^{**}	-3.32^{***}	-1.053^{***}	7.606	-0.71^{***}	12.087
	(0.081)	(1.961)	(0.158)	(9.618)	(0.129)	(6.17)
A03	-0.635^{***}	-1.407	-0.475^{***}	3.548	-0.6^{***}	2.904^{**}
	(0.092)	(0.748)	(0.092)	(4.065)	(0.196)	(2.569)
B	-0.428^{***}	-0.465	-0.668^{***}	-0.492^{***}	-0.216^{***}	-0.911^{***}
	(0.067)	(0.067)	(0.083)	(0.16)	(0.274)	(0.172)
C10-C12	-0.459^{***}	-0.287^{***}	-0.572^{***}	-5.753	-0.808^{***}	-0.456^{**}
	(0.053)	(0.423)	(0.064)	(2.076)	(0.101)	(1.201)
C13-C15	-0.433^{***}	-0.769^{***}	-0.61^{***}	-0.724^{***}	-0.793^{***}	-1.074
	(0.064)	(0.162)	(0.091)	(0.714)	(0.126)	(0.352)
C16	-0.712^{***}	-0.497^{***}	-0.331^{***}	1.08	-0.772^{***}	-0.844^{**}
	(0.098)	(0.535)	(0.048)	(2.516)	(0.129)	(0.681)
C17	-0.474^{***}	0.165^{***}	-0.572^{***}	-1.466	-1.084^{***}	-2.654
	(0.075)	(0.189)	(0.053)	(1.468)	(0.113)	(0.361)
C18	-0.733^{***}	-0.125^{***}	-0.52^{***}	-0.046	-1.078^{***}	-1.047^{*}
	(0.088)	(0.366)	(0.067)	(1.717)	(0.121)	(1.041)
C19	-1.022^{***}	0.224^{*}	-0.234^{***}	0.103	-0.108^{**}	0.163^{***}
	(0.135)	(0.207)	(0.076)	(0.457)	(0.159)	(0.074)
C20	-1.488^{**}	-0.478^{**}	-0.348^{***}	0.554	-0.119^{***}	-1.146
	(0.209)	(0.255)	(0.07)	(1.662)	(0.299)	(0.432)
C21	-0.393^{***}	-0.21^{*}	-0.142^{***}	-1.4	-1.161	-2.47
	(0.092)	(0.406)	(0.129)	(1.45)	(0.136)	(0.548)
C22	-0.392^{***}	-0.875^{***}	-0.588^{***}	-0.903^{***}	-1.027^{***}	-1.621
	(0.098)	(0.254)	(0.056)	(1.043)	(0.106)	(0.415)
C23	-0.457^{***}	-1.062^{***}	-0.453^{***}	1.514^{***}	-0.796^{***}	-1.524
	(0.087)	(0.309)	(0.048)	(1.602)	(0.127)	(0.552)
C24	-0.483^{***}	-0.313^{***}	-0.465^{***}	-6.222	-0.579^{***}	-1.085
	(0.102)	(0.243)	(0.05)	(1.914)	(0.092)	(0.387)
C25	-0.692^{***}	-0.321	-0.569^{***}	1.159	-0.831^{***}	-1.576^{*}
	(0.077)	(0.354)	(0.054)	(1.237)	(0.099)	(0.336)
C26	0.129^{**}	0.008	0.356	-0.535	-1.102	-0.876
	(0.073)	(0.034)	(0.21)	(0.607)	(0.148)	(0.486)
C27	1.023^{*}	0.203	4.612^{***}	-0.043	-1.447^{***}	-0.09
	(0.134)	(0.123)	(0.45)	(0.743)	(0.178)	(0.488)
C28	0^{*}	-0.008	-0.001	-0.751	-1.033	-2.257
	(0.072)	(0.03)	(0.067)	(0.638)	(0.144)	(0.672)
C29	1.798^{*}	0.455	5.127^{***}	-1.88^{**}	-2.732^{***}	-0.657
	(0.203)	(0.155)	(0.499)	(0.916)	(0.286)	(0.953)
C30	4.666^{***}	-0.062	5.551^{***}	-5.147	-3.528^{***}	2.161
	(0.469)	(0.393)	(0.55)	(3.096)	(0.755)	(4.053)
C31-C32	0.019	-0.108	0.032	-5.21^{*}	-1.39	-0.081
	(0.049)	(0.05)	(0.072)	(0.464)	(0.164)	(0.398)
C33	-0.569^{*}	-0.718^{*}	-0.636^{**}	1.67	-0.565^{***}	-1.563

	(0.197)	(1.067)	(0.144)	(2.663)	(0.319)	(0.959)
D35	−0.457***	0.162***	−0.166***	−1.253***	−0.551***	0.128***
	(0.058)	(0.027)	(0.037)	(0.165)	(0.349)	(0.066)
E36	−1.714***	−0.238	−0.09***	3.514	−1.913	−5.929***
	(0.254)	(0.423)	(0.102)	(5.291)	(1.083)	(2.982)
E37-E39	−0.608***	0.204	−0.161***	−0.721	−0.909	−4.024***
	(0.094)	(0.35)	(0.108)	(1.009)	(0.146)	(1.314)
F	−0.238***	−1.139***	−0.618***	−0.825**	−0.537***	−0.892**
	(0.066)	(0.366)	(0.087)	(0.408)	(0.082)	(0.381)
G45	−0.627***	−0.395***	−0.733***	−0.788	−0.661***	−1.431
	(0.067)	(0.295)	(0.059)	(0.39)	(0.081)	(0.388)
G46	−0.748***	−0.361***	−0.766***	−0.175	−0.514***	−1.971*
	(0.057)	(0.311)	(0.062)	(0.441)	(0.086)	(0.44)
G47	0.317***	0.047	1.139***	−1.717	−1.678***	−3.518
	(0.026)	(0.062)	(0.061)	(0.906)	(0.156)	(0.588)
H49	−0.165***	−0.559	0.006	−2.591***	−1.472	−2.551***
	(0.104)	(0.158)	(0.048)	(1.288)	(0.235)	(0.53)
H50	−0.236***	−1.102***	−1.086***	−0.017*	−1.011***	−0.052*
	(0.065)	(0.451)	(0.198)	(1.231)	(0.172)	(1.206)
H51	−0.497***	−0.745***	−0.266***	2.122***	−0.964***	−1.431**
	(0.056)	(0.211)	(0.053)	(1.267)	(0.143)	(0.524)
H52	−0.416***	−1.014***	−0.343***	−1.263*	−0.843***	−0.076
	(0.101)	(0.477)	(0.101)	(0.837)	(0.112)	(0.531)
H53	−0.702***	−0.385***	−0.721***	−1.156*	−1.263***	−1.737***
	(0.062)	(0.184)	(0.082)	(0.495)	(0.193)	(0.882)
I	−0.478***	−0.315***	−0.623***	2.623	−1.036***	−2.109*
	(0.072)	(0.479)	(0.09)	(1.066)	(0.122)	(0.691)
J58	−0.316	−42.303	−2.269	67.411	−1.287**	226.966
	(0.449)	(76.828)	(0.689)	(116.917)	(0.929)	(369.362)
J59-J60	−0.161***	−0.613***	−0.432***	−0.209*	−0.323***	−2.956
	(0.038)	(0.258)	(0.055)	(0.665)	(0.112)	(0.819)
J61	5.31***	0.108**	13.352***	−0.509	−3.814***	−1.532
	(0.371)	(0.186)	(0.802)	(0.61)	(0.334)	(0.372)
J62-J63	−0.617***	−0.505**	−0.366***	−1.968*	−0.594***	−1.022
	(0.107)	(0.204)	(0.071)	(0.569)	(0.086)	(0.202)
K64	−0.733***	0.31*	−0.441***	−1.197	−0.619***	−1.602**
	(0.136)	(0.27)	(0.113)	(0.628)	(0.172)	(0.325)
K65	−1.534***	−2.938	−0.444*	1.697	−0.633*	−1.665
	(0.669)	(1.494)	(0.188)	(1.051)	(0.14)	(0.426)
K66	−0.311***	−3.623**	−0.212***	2.293	−2.847**	6.318
	(0.07)	(2.207)	(0.067)	(53.546)	(1.399)	(11.484)
L68	−0.027	0.221	0.371	−0.987	−0.736***	−2.249***
	(0.025)	(0.203)	(0.111)	(0.816)	(0.066)	(1.123)
M69-M70	−0.457	−0.592***	−1.259***	0.088	−0.26***	−2.175**
	(0.084)	(0.518)	(0.168)	(0.2)	(0.078)	(0.612)
M71	−0.51	−0.833***	−0.438***	−1.034***	−0.826**	−0.512**
	(0.041)	(0.189)	(0.145)	(0.295)	(0.116)	(0.134)
M72	−0.897***	0.717*	−1.095***	1.414	−1.075***	−1.177**
	(0.265)	(1.044)	(0.141)	(1.143)	(0.16)	(0.263)
M73	−1.138***	0.25***	−0.786***	0.444	−0.646***	−1.809***

	(0.168)	(0.824)	(0.109)	(0.65)	(0.08)	(0.252)
M74-M75	−0.002	−0.027	−0.002	−2.183	−0.604	−2.37**
	(0.009)	(0.398)	(0.031)	(3.69)	(0.084)	(1.331)
N	−0.639***	0.097	−0.334***	−2.232	−1.619*	−4.493***
	(0.041)	(0.619)	(0.151)	(0.998)	(0.448)	(17.978)
O84	−0.432***	−0.8	−0.384***	−1.178***	−1.478**	7.859***
	(0.044)	(0.201)	(0.142)	(0.724)	(0.461)	(5.661)
P85	−0.635***	−0.461	−0.514***	−0.625	−0.902**	−4.957***
	(0.046)	(0.336)	(0.163)	(1.076)	(0.375)	(6.271)
Q	−0.888***	−0.353***	−0.405***	0.633*	−0.414***	−2.779***
	(0.061)	(0.164)	(0.082)	(0.602)	(0.205)	(0.683)

Table 2: Estimation results for the [[[KL]E]M] nested production function

Sectors	$\rho^{K.L}$	$\rho^{E.KL}$	$\rho^{M.KLE}$
A01	-0.348	0.007^{***}	-0.303^{***}
	(0.016)	(0.001)	(0.03)
A02	-0.318	0.013^{***}	-0.803^{***}
	(0.016)	(0.003)	(0.082)
A03	-0.32	-0.051^{***}	-0.355^{***}
	(0.021)	(0.003)	(0.074)
B	-0.306	-0.003^{***}	-0.457^{***}
	(0.033)	(0.005)	(0.033)
C10-C12	-0.341	0.014^{***}	-0.611^{***}
	(0.016)	(0.002)	(0.04)
C13-C15	-0.395^{**}	0.017^{***}	-1.294^{***}
	(0.015)	(0.015)	(0.081)
C16	-0.397	0.066^{***}	-0.173^{***}
	(0.018)	(0.007)	(0.022)
C17	-0.171	0.022^{***}	-1.453^{***}
	(0.011)	(0.005)	(0.064)
C18	-0.559	0.122^{***}	-0.276^{***}
	(0.024)	(0.02)	(0.031)
C19	-0.133	0.013^{***}	-0.091^{***}
	(0.024)	(0.02)	(0.029)
C20	-0.269^{*}	0.053^{***}	-0.665^{***}
	(0.022)	(0.008)	(0.048)
C21	-0.201	0.01^{***}	-0.957^{***}
	(0.013)	(0.002)	(0.071)
C22	-0.362^{*}	0.014^{***}	-0.637^{***}
	(0.016)	(0.005)	(0.038)
C23	-0.374^{*}	0.03^{***}	-0.324^{***}
	(0.015)	(0.005)	(0.034)
C24	-0.123	0.051^{***}	-0.067^{***}
	(0.022)	(0.007)	(0.025)
C25	-0.413	0.005^{***}	-0.418^{***}
	(0.017)	(0.005)	(0.037)
C26	-0.014	-0.022^{***}	0.321
	(0.007)	(0.033)	(0.05)
C27	-0.047	-0.432^{***}	-0.057^{***}
	(0.01)	(0.05)	(0.038)
C28	-0.053	0.085^{***}	0.138^{***}
	(0.011)	(0.018)	(0.042)
C29	-0.003	-0.024^{***}	0.023^{**}
	(0.001)	(0.038)	(0.039)
C30	0.007	0.152^{***}	0.168
	(0.007)	(0.078)	(0.055)
C31-C32	0.085	-0.32^{***}	0.029^{***}
	(0.01)	(0.052)	(0.048)
C33	-0.044^{*}	0.005^{***}	0.015^{**}

	(0.015)	(0.005)	(0.006)
D35	−1.319***	0.049***	−0.047***
	(0.164)	(0.007)	(0.016)
E36	−0.64***	0.015***	−0.077***
	(0.029)	(0.003)	(0.023)
E37-E39	−0.645	−0.555***	−0.092***
	(0.023)	(0.03)	(0.006)
F	−0.003	−0.016***	−0.004
	(0.003)	(0.038)	(0.009)
G45	−0.309	0.023***	−0.656***
	(0.017)	(0.008)	(0.039)
G46	−0.461	0.153***	−0.375***
	(0.019)	(0.008)	(0.03)
G47	−0.377	0.056***	−0.677***
	(0.017)	(0.009)	(0.065)
H49	0.288	−0.109***	−0.005***
	(0.04)	(0.036)	(0.013)
H50	−0.054	−0.169***	0.044*
	(0.023)	(0.023)	(0.024)
H51	−0.005	0.001***	0.007
	(0.008)	(0.002)	(0.032)
H52	−0.351	−0.04***	−0.05***
	(0.02)	(0.009)	(0.018)
H53	−0.171***	0.06***	−0.106***
	(0.021)	(0.022)	(0.015)
I	−0.391	0.025***	−0.39***
	(0.022)	(0.004)	(0.059)
J58	−0.333	0.068***	−0.196***
	(0.016)	(0.008)	(0.045)
J59-J60	−0.398*	0.01***	−0.423***
	(0.021)	(0.007)	(0.055)
J61	−0.035	−0.055***	−0.121***
	(0.007)	(0.016)	(0.025)
J62-J63	−0.606	0.348***	0.081***
	(0.025)	(0.02)	(0.039)
K64	−0.296	0.01***	−0.151***
	(0.018)	(0.017)	(0.024)
K65	−0.303	−0.159***	−0.641***
	(0.027)	(0.013)	(0.053)
K66	−0.288	0.047***	−0.121***
	(0.028)	(0.065)	(0.02)
L68	−0.07***	−0.006***	−0.096***
	(0.008)	(0)	(0.023)
M69-M70	−0.217	−0.291***	0.333***
	(0.025)	(0.16)	(0.033)
M71	−0.089**	−0.058***	−0.429***
	(0.009)	(0.048)	(0.06)
M72	−0.079	0.002***	−0.004***
	(0.019)	(0.003)	(0.004)
M73	−0.444	−0.037***	−0.038***

	(0.03)	(0.018)	(0.015)
M74-M75	−0.23	0.022***	−0.156***
	(0.019)	(0.008)	(0.018)
N	−0.056	0.083***	0.024**
	(0.018)	(0.026)	(0.012)
O84	−0.541	0.044***	−0.365***
	(0.014)	(0.003)	(0.072)
P85	−0.293	0.09***	−0.381***
	(0.022)	(0.011)	(0.047)
Q	−0.357	0.299***	−0.557***
	(0.02)	(0.009)	(0.061)
R-S	−0.632	0.053***	−0.419***
	(0.018)	(0.004)	(0.053)

Table 3: Estimation results for the [[[KE]L]M] nested production function

Sectors	$\rho^{K.E}$	$\rho^{L.KE}$	$\rho^{M.KLE}$
A01	-0.677	-0.208^{***}	-0.305^{*}
	(0.269)	(0.012)	(0.03)
A02	-0.632	-0.31^{***}	-0.815
	(0.474)	(0.015)	(0.083)
A03	-13.241	-0.328^{***}	-0.401^{***}
	(0.528)	(0.023)	(0.074)
B	-0.158^{***}	-0.376^{***}	-0.413^{***}
	(0.022)	(0.119)	(0.03)
C10-C12	-0.844	-0.31^{***}	-0.619^{***}
	(0.152)	(0.018)	(0.039)
C13-C15	-0.847	-0.258^{***}	-1.388^{***}
	(0.094)	(0.014)	(0.082)
C16	-0.541	-0.268^{***}	-0.171^{***}
	(0.105)	(0.017)	(0.022)
C17	-1.38	-0.218^{***}	-1.521^{***}
	(0.157)	(0.01)	(0.067)
C18	-1.109^{*}	-0.473^{***}	-0.272^{***}
	(0.176)	(0.024)	(0.031)
C19	0.004	-0.031^{***}	-0.07
	(0.02)	(0.01)	(0.023)
C20	-0.408	-0.354^{***}	-0.563^{***}
	(0.038)	(0.029)	(0.047)
C21	-0.956	-0.158^{***}	-0.963^{***}
	(0.122)	(0.012)	(0.071)
C22	-0.609	-0.337^{***}	-0.636^{***}
	(0.088)	(0.018)	(0.038)
C23	-0.682	-0.359^{***}	-0.336^{***}
	(0.058)	(0.018)	(0.034)
C24	-0.508	-0.147^{***}	-0.118^{***}
	(0.064)	(0.023)	(0.026)
C25	-0.58^{*}	-0.333^{***}	-0.427^{***}
	(0.058)	(0.017)	(0.037)
C26	0.016	0.019^{***}	0.354
	(0.011)	(0.009)	(0.047)
C27	0.037	-0.404^{***}	-0.114^{***}
	(0.009)	(0.038)	(0.037)
C28	-0.007	-0.007^{***}	0.073
	(0.016)	(0.011)	(0.043)
C29	0	-0.022^{***}	0.037
	(0.002)	(0.009)	(0.035)
C30	0.006	-0.032^{***}	0.134
	(0.007)	(0.036)	(0.051)
C31-C32	0.528	-0.093^{***}	0.056^{***}
	(0.059)	(0.01)	(0.048)
C33	-0.051	-0.001^{***}	0.013
	(0.067)	(0.003)	(0.006)
D35	-0.36^{***}	-3.347^{***}	-0.027^{***}
	(0.036)	(0.276)	(0.017)

E36	−0.625	−0.73***	−0.09***
	(0.082)	(0.033)	(0.024)
E37-E39	−1.005***	−0.634***	−0.088***
	(0.181)	(0.027)	(0.005)
F	−0.006	−0.008***	−0.003
	(0.01)	(0.01)	(0.009)
G45	−0.32	−0.221***	−0.649***
	(0.072)	(0.015)	(0.039)
G46	−0.283	−0.423***	−0.371***
	(0.042)	(0.024)	(0.028)
G47	−0.493	−0.266***	−0.707***
	(0.062)	(0.017)	(0.064)
H49	−0.411	−0.003***	−0.01***
	(0.054)	(0.009)	(0.013)
H50	−0.386*	−0.223***	0.047***
	(0.038)	(0.047)	(0.026)
H51	−0.223	0.02***	−0.044***
	(0.041)	(0.009)	(0.028)
H52	−0.847	−0.322***	−0.046***
	(0.076)	(0.022)	(0.018)
H53	−1.138	−0.1***	−0.108***
	(0.195)	(0.022)	(0.017)
I	−0.805	−0.176***	−0.501***
	(0.096)	(0.016)	(0.061)
J58	−0.793	−0.242***	−0.25***
	(0.081)	(0.016)	(0.046)
J59-J60	−1.267	−0.256***	−0.437***
	(0.349)	(0.017)	(0.055)
J61	−0.904*	−0.153***	−0.14***
	(0.057)	(0.011)	(0.025)
J62-J63	−0.436***	−0.222***	−0.013***
	(0.054)	(0.019)	(0.041)
K64	−0.427	−0.234***	−0.153***
	(0.046)	(0.019)	(0.023)
K65	−0.985	−0.359***	−0.654***
	(0.08)	(0.033)	(0.053)
K66	−0.571	−0.132***	−0.118***
	(0.101)	(0.023)	(0.02)
L68	−0.541	−0.074***	−0.11***
	(0.065)	(0.007)	(0.023)
M69-M70	0.053	−0.351***	0.121
	(0.038)	(0.028)	(0.024)
M71	−0.086**	−0.137***	−0.482***
	(0.02)	(0.018)	(0.06)
M72	−0.072	0***	−0.005***
	(0.021)	(0)	(0.004)
M73	−0.673**	−0.343***	−0.039***
	(0.068)	(0.029)	(0.015)
M74-M75	−0.631	−0.176***	−0.165***
	(0.071)	(0.017)	(0.018)

N	-0.006	0.002^{***}	0.001
	(0.03)	(0.001)	(0.009)
O84	-0.157	-0.349^{***}	-0.422
	(0.227)	(0.015)	(0.071)
P85	-0.663	-0.062^{***}	-0.412^{***}
	(0.152)	(0.011)	(0.048)
Q	-0.737^{*}	-0.109^{***}	-0.563^{***}
	(0.117)	(0.012)	(0.061)
R-S	-0.811	-0.44^{***}	-0.467^{***}
	(0.065)	(0.017)	(0.053)

6.2 Appendix B: Nomenclature description

Table 4: Country codes and names

iso3 code	countries names
AUS	Australia
AUT	Austria
BEL	Belgium
BGR	Bulgaria
BRA	Brazil
CAN	Canada
CHE	Switzerland
CHN	China
CYP	Cyprus
CZE	Czechia
DEU	Germany
DNK	Denmark
ESP	Spain
EST	Estonia
FIN	Finland
FRA	France
GBR	United Kingdom
GRC	Greece
HRV	Croatia
HUN	Hungary
IDN	Indonesia
IND	India
IRL	Ireland
ITA	Italy
JPN	Japan
KOR	South Korea
LTU	Lithuania
LUX	Luxembourg
LVA	Latvia
MEX	Mexico
MLT	Malta
NLD	Netherlands
NOR	Norway
POL	Poland
PRT	Portugal
ROU	Romania
RUS	Russia
SVK	Slovakia
SVN	Slovenia
SWE	Sweden
TUR	Turkey
TWN	Taiwan
USA	United States

Table 5: Sectors labels and nomenclatures correspondence

code sectors WIOD	WIOD sectors names	code NAF17	NAF17 sectors names
A01	Crop and animal production, hunting and related service activities	AZ	Agriculture
A02	Forestry and logging	AZ	Agriculture
A03	Fishing and aquaculture	AZ	Agriculture
B	Mining and quarrying	DE	Energy, water, waste
C10-C12	Manufacture of food products, beverages and tobacco products	C1	Agro-food industries
C13-C15	Manufacture of textiles, wearing apparel and leather products	C5	Other industrial branches
C16	Manufacture of wood and of products of wood and cork	C5	Other industrial branches
C17	Manufacture of paper and paper products	C5	Other industrial branches
C18	Printing and reproduction of recorded media	C3	Capital goods
C19	Manufacture of coke and refined petroleum products	C2	Coking and refining
C20	Manufacture of chemicals and chemical products	C5	Other industrial branches
C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations	C5	Other industrial branches
C22	Manufacture of rubber and plastic products	C5	Other industrial branches
C23	Manufacture of other non-metallic mineral products	C5	Other industrial branches
C24	Manufacture of basic metals	C5	Other industrial branches
C25	Manufacture of fabricated metal products, except machinery and equipment	C5	Other industrial branches
C26	Manufacture of computer, electronic and optical products	C3	Capital goods
C27	Manufacture of electrical equipment	C3	Capital goods
C28	Manufacture of machinery and equipment n.e.c.	C3	Capital goods
C29	Manufacture of motor vehicles, trailers and semi-trailers	C4	Transport equipment
C30	Manufacture of other transport equipment	C4	Transport equipment
C31_C32	Manufacture of furniture; other manufacturing	C5	Other industrial branches
C33	Repair and installation of machinery and equipment	C5	Other industrial branches
D35	Electricity, gas, steam and air conditioning supply	DE	Energy, water, waste
E36	Water collection, treatment and supply	DE	Energy, water, waste
E37-E39	Sewerage; waste collection, treatment and disposal activities	DE	Energy, water, waste
F	Construction	FZ	Construction
G45	Wholesale and retail trade and repair of motor vehicles and motorcycles	GZ	Trade
G46	Wholesale trade, except of motor vehicles and motorcycles	GZ	Trade
G47	Retail trade, except of motor vehicles and motorcycles	GZ	Trade
H49	Land transport and transport via pipelines	HZ	Transport
H50	Water transport	HZ	Transport
H51	Air transport	HZ	Transport
H52	Warehousing and support activities for transportation	HZ	Transport
H53	Postal and courier activities	HZ	Transport
I	Accommodation and food service activities	IZ	Accommodation and food service
J58	Publishing activities	JZ	Information and communication
J59_J60	Motion picture, video, audio and television programme production	JZ	Information and communication
J61	Telecommunications	JZ	Information and communication
J62_J63	Computer programming, consultancy and related activities; information service activities	JZ	Information and communication
K64	Financial service activities, except insurance and pension funding	KZ	Financial services
K65	Insurance, reinsurance and pension funding, except compulsory social security	KZ	Financial services
K66	Activities auxiliary to financial services and insurance activities	KZ	Financial services
L68	Real estate activities	LZ	Real estate services
M69_M70	Legal and accounting activities; activities of head offices; management consultancy activities	MN	Business services
M71	Architectural and engineering activities; technical testing and analysis	MN	Business services
M72	Scientific research and development	MN	Business services
M73	Advertising and market research	MN	Business services
M74_M75	Other professional, scientific and technical activities; veterinary activities	MN	Business services
N	Administrative and support service activities	MN	Business services
O84	Public administration and defence; compulsory social security	OQ	Non-market services
P85	Education	OQ	Non-market services
Q	Human health and social work activities	OQ	Non-market services
R_S	Other service activities	RU	Household services
T	Activities of households as employers	RU	Household services
U	Activities of extraterritorial organizations and bodies	RU	Household services

7 Appendix C: Description of the main equation of ThreeME

7.1 Specification of adjustment mechanisms

Unlike Walrasian models that assume that equality between supply and demand is achieved through a perfect flexibility of prices and quantities, ThreeME represents more realistically the functioning of the economy by taking into account explicitly the slow adjustment of prices and quantities (factors of production, consumption). In this Keynesian framework, permanent or transitory underemployment equilibria are possible and supply is determined by demand. ThreeME assumes that the actual levels of prices and quantities gradually adjust to their notional level. The notional level corresponds to the optimal (desired or target) level that the economic agent in question (the company for prices and the demand for production factors, the household for consumption, the Central bank for the interest rate, etc.) would choose in the absence of adjustment constraints. These constraints mainly come from adjustment costs, physical or temporal boundaries and uncertainties. Formally, we assume that the adjustment process and expectations for prices and quantities are represented by the following equations:

$$\log F_t = \lambda_0^F \ \log F_t^n + \left(1 - \lambda_t^{0,F}\right) \ (\log F_{t-1} + \mathit{\Delta} \left(\log F_t^e\right)) \tag{24}$$

$$\mathit{\Delta} \left(\log F_t^e\right) = \lambda_t^{1,F} \ \mathit{\Delta} \left(\log F_{f,s,t-1}^e\right) + \lambda_t^{2,F} \ \mathit{\Delta} \left(\log F_{f,s,t-1}\right) + \lambda_t^{3,F} \ \mathit{\Delta} \left(\log F_t^n\right) \tag{25}$$

Where F_t is the actual value of a given variable (e.g. the production price, labor, capital, etc.), F_t^n is its notional level, F_t^e its anticipated value at period t and α_i^F are the adjustments parameters (with $\alpha^{1,F} + \alpha^{2,F} + \alpha^{3,F} = 1$).

Equation (24) assumes a geometric adjustment process. Taking into account the anticipations guaranties that the actual variables converge to their notional levels in the long run. Equation (25) assumes that the anticipations are adaptive (*"backward-looking"*). One can see that Equation (24) and Equation (25) can be reformulated into an Error Correction Model used in the econometric estimations to take into account the non-stationary propriety of some variables:

$$\Delta \log(X_{t-1}) = \alpha_1 \ \Delta \log(X_{t-1} + \alpha_2 \ \Delta \log(X_{t-1}^n) - \alpha_3 \ \log(X_{t-1})/(X_{t-1}^n))$$

For this, the following constraints must hold: $\lambda_0^X = \alpha_3, \lambda_1^X = 0, \lambda_2^X = \alpha_1/(1-\alpha_3), \lambda_3^X = (\alpha_2 - \alpha_3)/(1-\alpha_3)$

We also assume that the substitution effects ($SUBST\_X$) adjust slowly to the notional substitution effects ($SUBST\_X^n$):

$$SUBST\_X_t = \lambda_4^X * SUBST\_X_t^n + (1 - \lambda_4^X) * SUBST\_X_{t-1} \tag{26}$$

The three equations above allow a rich set of adjustment as they integrate different types of rigidity (on prices and quantities, on expectations and on substitution mechanisms). For illustrative purposes, we present the full specification of the demand for labor (L). For simplicity, the sector index is omitted. The notional labor demand (L^n is derived by minimizing production costs. It depends positively on the level of the output (Y), negatively on the labor productivity ($PROG_L$) and on an element gathering all the substitution phenomena with the other production factors ($SUBST\_L$):

$$\Delta \log(L_t^n) = \Delta \log(Y_{t-1})\Delta \log(PROG\_L_t) + \Delta SUBST\_L_t \tag{27}$$

We introduce a distinction between the actual and notional substitution effects to account for the fact that labor demand generally responds more quickly to changes in the level of production than to substitution phenomena: while it is *physically* necessary to increase employment to meet rising production, substitutions involve changes to the structure of production whose implementation takes longer. The actual substitution therefore adjusts gradually to the notional substitution ($SUBST\_L^n$) which depends on the relative prices between the production factors:

$$\Delta SUBST\_L_t^n = -\eta^{LK}\varphi_{t-1}^K \Delta \log(C_t^L/C_t^K) - \eta^{LE}\varphi_(t-1)^E \Delta \log(C_t^L/C_t^E) - \eta^L \varphi_{t-1}^M \Delta \log(C_t^L/C_t^M) \tag{28}$$

Where η^{LK}, η^{LE}, η^{LM} are the elasticities of substitution between labor and the other production factors respectively capital, energy, material (i.e. non-energy intermediate consumption). φ^K, φ^E, φ^M are respectively the capital, energy and materials shares in the production costs. C^K, C^L, C^E, C^M are respectively the unitary costs of production of capital, labor, energy and material. The next section provides more information on the derivation of factors demands. Finally, the adjustment mechanisms being defined according to the equations (1), (2) and (3), the three following relationships are used:

$$\log(L_t) = \lambda_0^L \ \log(L_t^n) + (1 - \lambda_0^L) \ (\log(L_{t-1}) + \Delta \log(L_t^e))$$

$$\Delta \log(L_t^e) = \lambda_1^L \ \Delta \log(L_{t-1}^e) + \lambda_2^L \ \Delta \log L_{t-1}) + \lambda_3^L \ \Delta \log(L_t^n) \tag{29}$$

$$SUBST\_L_t = \lambda_4^L \; SUBST\_L_t^n + (1 - \lambda_4^L) \; SUBST\_L_{t-1}$$

7.2 The production function and the production factors demand

The production structure is decomposed into three levels (see Figure 6). The first one assumes a production function with 4 inputs (or production factors), often referred as KLEM (capital, labor, energy and materials). The first level has a fifth element: the transport and commercial margins. *Stricto sensu*, they cannot be considered as production factors since they intervene after the production process. Thus they are not substitutable with the production factors. But they are closely related to the level of production since once a good has been processed, it has to be transported and commercialized. At the second level, the investment, energy, material and margins aggregates are further decomposed by type of commodities (e.g. energy sources). At the third level, the demand for each factor or margin is either imported or produced domestically. The demands for production factors are derived from the minimization of the firm's production costs. We assume a production function with constant returns-to-scale more general than the CES (Constant Elasticity of Substitution) insofar as substitution elasticities may differ between different inputs pair (Reynès, 2019). The production costs minimization program leads to the following equations for the notional factors demand. This holds for every economic activity, but for algebraic simplicity the sector index is omitted here:

$$\Delta \log(FP_{j,t}^n) = \Delta \log(Y_t) - \Delta \log(PROG_F P_{j,t}) + \Delta SUBST\_FP_{j,t} \tag{30}$$

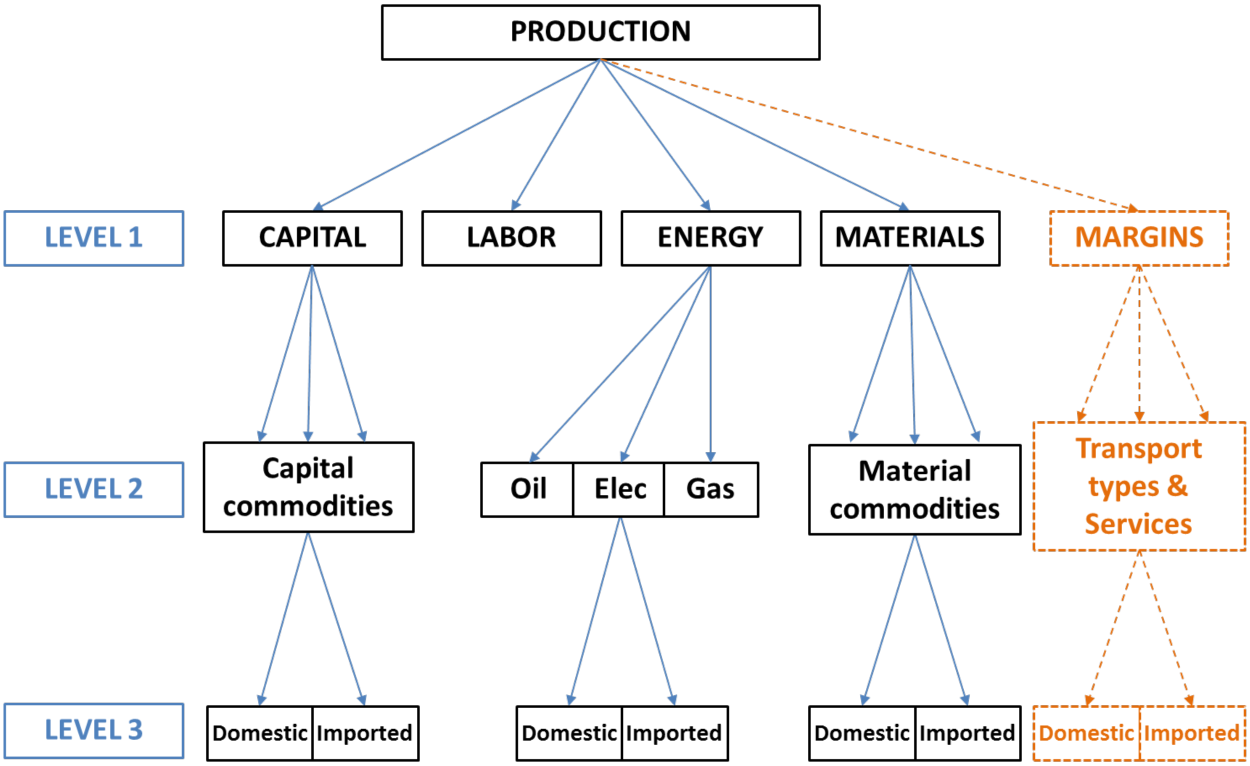
$$\Delta SUBST\_FP_{j,t}^n = - \sum_{\substack{j'=1 \\ j \neq j'}} \eta_{j,j'} \; \varphi_{t-1}^{j'} \; \Delta \log(C_(j', t)^{FP} / C_{j,t}^{FP})) \tag{31}$$

with $\varphi_{j,t-1} = (C_{j,t}^{FP} \; FP_{j,t-1}) / (\sum_j C_{j,t}^{FP} \; FP_{j,t-1})$ and $j = \{K, L, E, M\}$

Where FP_j^n is the notional demand of input j (KLEM), $\eta_{j,j'}$ the elasticity of substitution between the pairs of inputs j and j', $PROG\_FP_{j,t}$ the technical progress related to input j, C_j^{FP} the cost/price of input j and Y the level of production of the sector under consideration.

According to national accounts data, ThreeME assumes that each commodity may be produced by more than one sector. For instance, electricity can be produced by several sectors such as nuclear or wind power. The production of each sector is defined by the following equations:

Figure 6: Structure of production in ThreeME



$$Y_{c,a} = \varphi_{c,a} YQ_c \tag{32}$$

$$Y_a = \sum_c Y_{a,c} \tag{33}$$

Where YQ_c is the aggregated domestic production of commodity c. It is determined by the demand (intermediate & final consumption, investment, public spending, exports and stock variation). $\varphi_{c,a}$ is then the share of commodity c produced by the sector a (with $\sum_a \varphi_(c,a) = 1$) and Y_a is the aggregated production of sector a.

7.3 Equations for investment & capital

Investment in ThreeME depends on the anticipated production, on its past dynamic, on substitution phenomena and on a correction mechanism, which guaranties that companies reach their level of long-term notional capital stock. The stock of capital is deducted from the investment according to the standard capital accumulation equation:

$$\Delta \log(IA_t) = \theta_1^I A \, \Delta \log(IA_{t-1}) + \theta_2^I A \, \Delta \log(Y_t^e) \; + \theta_3^I A \, (\log(K_{t-1}^n) - \log(K_{t-1})) + \Delta SUBST\_K_t \tag{34}$$

$$K_t = (1 - \delta^K) K_{t-1} + IA_t$$

Where IA is the investment, Y^e anticipated production, K and K^n the actual and notional stocks of capital, $SUBST\_K$ a variable gathering substitution phenomena between capital and the other inputs, and δ^K the depreciation rate of capital. Moreover, we impose the constraint $\theta_1^I A + \theta_2^I A = 1$ in order to guaranty the existence of the stationary equilibrium path. This specification is a compromise between the short-term dynamics empirically observed and the consistency of the model in the long run. Like the MESANGE econometric model (**?**), it is common to estimate an investment equation rather than capital stock equation for several reasons. Firstly, time series capital stock data are often unreliable. Secondly, this approach better represents the short-term dynamics of investment. In particular, it avoids capital destruction phenomena (negative investment) that are in practice unusual, since companies generally prefer to wait for the technical depreciation of their installed capital. Unlike MESANGE, we assume in addition that investment depends on the difference between the actual and notional capital stock. This element ensures that the effective capital stock converges over time towards its notional level. In the long-term, the model is then consistent with the production function theory that establishes a relationship between the levels of production and capital stock (and not with the flow).

7.4 Wage equation

Several studies have shown that the theoretical arguments and empirical estimates difficultly allow choosing between the two specifications. However, this difference of specification has important implications on the definition of the equilibrium unemployment rate (NAIRU) and thus on the inflationary dynamic and the long-term proprieties of a macroeconomic model (**?**). In ThreeME, we choose a general specification that includes the Phillips and WS curves. It assumes that the notional nominal wage (W_t^n) positively depends on the anticipated consumption price (P_t^e) and on the labor productivity ($PROG\_L_t$), and negatively on the unemployment rate (U_t):

$$\Delta \log(W_t^n) = \rho_1^W + \rho_2^W \, \Delta \log(P_t^e) + \rho_3^W \, \Delta \log(PROG\_L_t) - \rho_4^W U_t - \rho_5^W \Delta U_t \tag{35}$$

This relation can alternatively be identical, either to the Phillips curve, or to the WS curve depending on the value of the selected parameters (**??**). The Phillips curve corresponds to the case where $\rho_4^W > 0$ whereas the WS curve assumes $\rho_4^W = 0$. For the model to have a consistent steady-state in the long-run, the WS curve must also impose the constraints identified by **?**: a unit indexation of wages on prices and productivity: $(\rho_2^W = \rho_3^W = 1)$ and $\rho_1^W = 0$.

7.5 Equation of households' consumption

In the standard version of the model, consumption decisions are modeled through a Linear Expenditure System (LES) utility function generalized to the case of a non-unitary elasticity of substitution between the commodities **?**. Households' expenditures for each commodity evolve (more or less) proportionally to their income:

$$(EXP_c^n - NEXP_c) \, PEXP_c = \beta_c^{EXP} [(1 - MPS) \, DISPINC\_VAL - \sum_c PEXP_c \, NEXP_c] \tag{36}$$

With $\sum_c \beta_c^{EXP} = 1$

Where EXP_c^n corresponds to the volume of notional consumption (expenditures) in commodity c and $PEXP_c$ to its price. $NEXP_c$ is the incompressible volume of expenditures in commodity c, $DISPINC\_VAL$ is the households' disposable income and MPS their marginal propensity to save. In the case of no incompressible expenditures ($NEXP_c = 0$), households aim at allocating a share $\beta_c^E XP$ of their total expenditure (in value), $(1 - MPS) \, DISPINC\_VAL$, to commodity c. This share is constant if the elasticity of substitution between the commodities is equal to one (Cobb-Douglas assumption). In this case (Cobb-Douglas utility function without incompressible expenditures), commodity c expenditures stay exactly proportional to income. In the case of a CES function where the elasticity of substitution is η^{LES_CES}, the marginal propensity to spend varies depending on the relative prices according to the following specification:

$$\Delta \beta_{c,t}^{EXP} = (1 - \eta^{LES\_CES}) \, \Delta PEXP_{c,t} / (PEXP_t^{CES}) \tag{37}$$

$$PEXP_t^{CES} = (\sum_c \beta_{c,0}^{EXP} \, PEXP_{c,t}^{(1-\eta^{LES\_CES})})^{1/1-\eta^{LES\_CES}} \tag{38}$$

7.6 Equations of prices and of the mark-up rate

The production price for each sector is set at the lowest level by applying a mark-up over the unit cost of production (which includes labor, capital, energy and other intermediate consumption costs) :

$$PY_t^n = CU_t\ (1 + TMD_t) \tag{39}$$

$$\Delta \log(1 + TM_t^n) = \sigma^{TM}\ (\Delta \log(Y_t) - \Delta \log(Y_{t-1})) \tag{40}$$

$$TMD_t = \lambda^{TM}\ TM_t^n + (1 - \lambda^{TM})\ TMD_{t-1} \tag{41}$$

Where PY_t^n is the notional price, CU_t the unitary cost of production and Y_t the level of production. TMD_t and TM_t^n are respectively the desired and notional mark-up. The equation of notional price is a behavioral equation: by assuming that the addressed demand to a firm is a negative function of its price, one can easily demonstrate that the optimal price corresponds to a mark-up over the marginal cost of production. The mark-up equation reflects the fact that the returns-to-scale are decreasing in the short-term. Therefore, a non-expected increase in production results into a higher marginal cost of production and therefore into a higher notional price. The other prices are calculated according to their accounting definition and are therefore (directly or indirectly) a function of the producer price. The price of the domestically produced commodity c is a weighted average of the production prices of activities (indexed by a) producing that commodity. For example, the price of electricity is a weighted average of the production prices of the sectors producing electricity. The price paid by the final user (consumer, government, sector, rest of the world) integrates in addition the commercial and transportation margins, and the taxes net from subsidies. Combined with the price of imports, we get the average price for each commodity paid by each end user.

7.7 Equations of foreign trade

Exports are determined by the external demand addressed to domestic products and the ratio between the export and world prices:

$$\Delta \log(X_{c,t}) = \Delta \log(WD_{c,t}) + \Delta SUBST\_X_{c,t} \tag{42}$$

$$\Delta SUBST\_X^{n}_{c,t} = -\eta^{X}\ \Delta \log(P^{X}_{c,t}/P^{W}_{c,t}/TC_t)$$

Where $WD_{c,t}$ is the world demand, $P^{W}_{c,t}$ its price. $P^{X}_{c,t}$ is the export price that depends on the production costs and which reflects the price-competitiveness of the domestic products. TCt is the exchange rate; η^{X} is the price-elasticity (assumed constant). We assume imperfect substitution between domestic and imported goods (**?**). The demand for domestic and imported products is :

$$\Delta \log(A^{D}_{c,t}) = \Delta \log(A_{c,t}) + \Delta SUBST\_AD_{c,t}$$

$$\Delta SUBST\_AD^{n}_{c,t} = \eta^{A}_{c}\ \Delta \log(P^{AD}_{c,t}/P^{A}_{c,t}M)\ \frac{(P^{A}_{c,t-1}MA^{M}_{c,t-1})}{(P^{A}_{c,t-1}\ A_{c,t-1})} \tag{43}$$

$$A^{M}_{c,t} = A_{c,t} - A^{D}_{c,t}$$

Where $A_{c,t}$ represents the demand for each type of use (intermediary consumption, investment, consumption, public spending, exports, etc.), $P^{A}_{c,t}$ is its price. $A^{M}_{c,t}$ and $A^{D}_{c,t}$ are the imports and the domestic products demanded for each type of use A, $P^{A}_{c,t}M$ and $P^{A}_{c,t}D$ are their respective prices. The elasticity of substitution η^{A}_{c} by type of use A of a given commodity c can potentially be different, which allows a high degree of flexibility. The full description of the model can be found online at www.threeme.org