Accounting for Global Production of Exports: A Unified Framework

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ABSTRACT: The contributions of participants in the global production of exports are important concerns of trade analysts and policy makers. This paper aims to construct a unified framework with sound theoretical foundation for measuring global production of exports. In addition to harmonizing export decomposition at all levels, the framework embraces a broad range of global-value-chain indicators (such as trade in value-added measures and production position measures). World export decomposition analyses show that the supply-side diversification of China's export production structure is lower than the demand-side. The suppliers mainly concentrate in east Asia area, with a shift from Japan and Chinese Taipei to Korea. The purchasers spread over relatively broad regions, with a shift from the United States and Korea to Mexico and Vietnam.

KEYWORDS: Global production; gross export decomposition; global value chains; double-counting

1. Introduction

In the age of global value chains, goods are produced in stages performed in different countries. This is also true for export production. The value of gross export is therefore not fully contributed by the exporting country. First, gross export contains foreign value-added, because the production of vertical specialization exports requires imported inputs (Hummels et al., 2001). Second, gross export suffers from the double-counting issue and thus over measures the net output of global production for exports.

Tracing value-added content in gross export is therefore crucial to measure the net output of global production for exports and the contribution of each participant. Since the seminal work done by Koopman et al. (2014), great efforts have been devoted to decomposing value-added in gross exports at different levels (Johnson, 2018; Los and Timmer, 2018; Wang et al., 2018; Arto et al., 2019; Borin and Mancini, 2019; Miroudot and Ye, 2021). However, due to a lack of theoretical guarantee, there are still ongoing debates on two related issues. One is on the nature of double-counting and the other is on the additivity of decompositions. Although double-counting in an economy's aggregate export could be rather minor, it would become considerable in gross world export and in gross inter-regional export (e.g. gross trade among European Union members) and therefore covers the net output of export production.

First, on the nature of double-counting. The current definition of double-counting is based on the number of border-crossings. The value-added crossing "border" more than once is defined as double counted value-added in gross export. However, the border-crossing based definition is indefinite. It varies with the reference "border" selected from different perspectives. Miroudot and Ye (2021) classifies them into world perspective (the borders of all countries in the world), country perspective (the border of the exporting country), and bilateral perspective (the border between the exporting country and its partner). Different definitions of double-counting lead to different types of decompositions. This introduces additional troubles in model selection for trade analysts. Section 2.3 shows that an invalid selection would lead to mislabeled double-counting and therefore mismeasures the contributions of participants in global production. Therefore, a unified definition for double-counting and a unified analytical framework with sound theoretical foundation are required for understanding the global production of exports.

Second, on the additivity of decompositions. For bilateral export decompositions, there are two strands of studies. One strand of studies assumes that separate decompositions for a country's bilateral exports should add up to the corresponding decomposition for this country's aggregate export (see e.g. Wang et al., 2018). Another strand of studies believe that bilateral decompositions are not additive, because new double-counting would show up when summing bilateral decompositions (see e.g. Los and Timmer, 2018). Apparently, the debate on the additivity of decompositions is essentially related to the debate on the nature of double-counting. If double-counting is rigorously defined with clear economic interpretation, whether bilateral decompositions should satisfy the property of additivity would be self-evident.

This paper aims to provide a unified theoretical economic model for measuring the contributions of participants in the global production of exports. It harmonizes trade decompositions at all levels under the global production system and provide useful insights for resolving the important debates in this field. To this end, a unified definition for double-counting in gross export with clear economic meaning is proposed. From the perspective of accounting, the nature of double-counting is that identical value is counted twice or more in a gross measure. Export production activities are inter-connected in the global production system. Some exports are used up to produce other exports along global production chains. This "export for producing export" phenomenon leads to identical value-added appearing in two or more individual exports and therefore causes double-counting issues in gross trade measures. Given a bundle of exports, it can be divided into two mutually exclusive but inter-connected groups. One group is used up by the global production of the other group. The latter group is called “final export” of the given export bundle. It is the net output of global production for the given export bundle. The former group is called “joint export” associated to the “final export”. It is double counted component in the given export bundle, because its value is also counted in the value of “final export”.

By using inter-country input-output structures, “final export” and its associated “joint export” can be disentangled from any given bundle of exports. The decomposition outcome is a unique closed-form solution from the global Leontief production system and thus satisfies unicity. A unified analytical framework to account for global production of exports is constructed. In addition, this unified framework implies that export decompositions at different levels do not satisfy additivity. This is because production of bilateral exports is independent. A country’s export to one partner could be “joint export” associated to its export to another partner. Therefore, the aggregation of bilateral exports would yield new double-counting (Los and Timmer, 2018). Besides, a broad range of popular global value chain (GVC) measures can also be derived from this unified framework, such as trade in value-added measure (Johnson and Noguera, 2012), GVC income measure (Timmer et al., 2014), and production position measure (e.g., the average propagation length indicator proposed by Dietzenbacher et al., 2005; the upstreamness measure proposed by Antràs et al., 2012; see Antràs and Chor 2018 for a review and Chor et al., 2021 for a recent application).

Another contribution of this analytical framework is that it distinguishes “final export” and its associated “joint export”. This new feature provides useful information on inter-export linkages in the global production system. It can be used to measure the diversification of an economy’s export production structure from both supply-side and demand-side. Diversification of production and export structure is an important determinant of the abilities of economies to cope with shocks. Studies have shown that trade diversification increases an economy’s economic resilience to shocks (Caselli et al., 2020; World Trade Organization, 2021). To show this new application, China’s role in world export production chain is investigated based on the decomposition of world export of goods. Empirical results show that China experienced significant structural changes in its inter-export production linkages with world economies in the period 1995-2018. Both supply-side and demand-side diversification of China’s export production structure increased, but the supply-side diversification is lower than the demand-side. The suppliers mainly concentrate in east Asia area, with a shift from Japan and Chinese Taipei to Korea. The purchasers spread over relatively broad regions, with a shift from the United States and Korea to Mexico and Vietnam.

The reminder of the paper is organized as follows. Sections 2 introduced the accounting theory with a simplified example and compared the new definition of double-counting with traditional border-crossing based definitions. Section 3 developed a unified theoretical economic model for identifying value-added content in internal trade flows under the global Leontief production system and generalized to other popular GVC measures. As a new application, Section 4 measured the structure of global production for world export of goods based on the proposed framework. Section 5 concludes.

2. Accounting theory and illustration

2.1 Definitions

International fragmentation of production implies that some exports would be processed abroad and further imported to produce the other exports. For instance, China could export steel to Japan to produce engines and impots the engines back to produce cars exporting to US. In this example, China’s export of steel is used to produce its export of cars in the global production system. This “export for producing export” phenomenon leads to identical value-added appearing in two or more individual exports and therefore causes double-counting issues in gross export. To measure the inter-export linkages in global production, we have the following definitions for “final export” and “joint export” in gross export.

Definition 1 Final Export and Joint Export. *Given a bundle of exports, some exports are used in global production system to produce the other exports. We call the latter final export and the former joint export associated to the final export.*

According to Definition 1, given a bundle of exports, the gross export can be divided into two mutually exclusive parts, final export and joint export. The joint exports are used up to produce the final exports. Final exports can be further used as final products for consumption or as intermediate products for producing products outside the export bundle. Therefore, it should be stressed that final exports in a given export bundle are not necessarily export of final products, and export of intermediate products could be final exports of an export bundle.

As joint export is used to produce final export, its value (as a part of cost) is also counted in the value of final export. Adding joint export to its associated final export to yield gross export would lead to double-counting. Essentially, the value of joint export is double counted value in gross export and final export is the net output of global production for gross export (Definition 2).

Definition 2 Double-counting and Net Output of Global Production for Gross Export. *Final export is the net output of global production for gross export. The value of joint export associated to the final export is double counted value in gross export.*

The definition of double-counting in gross export implies: (a) An individual export does not contain double counted value, since it cannot be used to produce itself. (b) Export of final products does not contain double counted value, since they leave production system and cannot be used to produce other products. (c) Double counted value increases with the expansion of export bundle, because some exports in the export bundle before expansion could be used to produce new introduced exports in the expanded export bundle.

Use an example shown in Figure 1 for further explanation. Suppose that country *A* exports \$10 million coal (to country *B*) for producing \$50 million steel and imports the \$50 million steel back further to produce \$100 million metal products; country *A* exports the \$100 million metal products (to country *B* again) for producing \$1000 million engines and imports the \$1000 million engines back to produce \$2000 million cars; Finally, country *A* exports the \$2000 million cars to country *C* for household consumption.

<INSERT FIGURE 1>

(1) Country *A*'s bilateral export. Country *A*'s gross export to country *B* (e^{AB}) and country *C* (e^{AC}) are $e^{AB} = e_{\text{coal}} + e_{\text{metal}}$=\$110 million and $e^{AC} = e_{\text{car}}$=\$2000 million. In e^{AB}, e_{coal} is used to produce e_{metal}, hence the final export is e_{metal} and its joint export is e_{coal}. As joint export, the value of e_{coal} (\$10 million) is double counted in e^{AB}, because this value also appears in e_{metal}. As final export, e_{metal} is the net output of production for e^{AB}. It is available for the downstream production of cars. In summary, country *A*'s gross export to country *B* satisfies

\$110 million = \$100 million (net output) + \$10 million (double counted value) (1)

In e^{AC}, no exports are used to produce the other exports. All exports are final exports. Therefore, country *A*'s gross export to country *C* does not contain double counted value. It satisfies the following decomposition

\$2000 million=\$2000 million (net output) + \$0 million (double counted value) (2)

(2) Country *A*'s aggregate export[1]: $e_{\text{coal}} + e_{\text{metal}} + e_{\text{car}}$=\$2110 million. In this export bundle, e_{coal} and e_{metal} are used to produce e_{car}, so e_{car} is final export and e_{coal} and e_{metal} are joint exports. The net output of production for country *A*'s aggregate export is e_{car}=\$2000 million. As the value of e_{coal} and e_{metal} also appears in the value of e_{car}, $e_{\text{coal}} + e_{\text{metal}}$= \$110 million are double counted in

[1] We use the term "aggregate export" suggested by Los and Timmer (2018) to indicate a country's all exports to the world.

country A's aggregate export. In summary, we have the following decomposition for country A's aggregate export.

\$2110 million=\$2000 million (net output) +\$110 million (double counted value) (3)

If measure country A's bilateral exports separately, we only find \$10 million double counted value in country A's gross export to country B, indicated by equations (1) and (2). If measure country A's aggregate export (the sum of country A's bilateral exports), however, we find \$110 million double counted value, indicated by equation (3). Adding up equations (1) and (2) cannot yield equation (3). This is because productions for bilateral exports are interdependent in global production system. The production of country A's export of cars to country C (e_{car}) uses country A's export of metal products to country B (e_{metal}). The value of e_{metal}, that is not double counted in country A's bilateral export to country B, is also counted in e_{car}, and therefore double counted in country A's aggregate export. Due to the interdependence between export production, gross export decompositions do not satisfy additivity.

(3) World export: $e_{\text{coal}} + e_{\text{metal}} + e_{\text{car}} + m_{\text{steel}} + m_{\text{engine}}$=\$3160 million. In gross world export, e_{coal}, e_{metal}, m_{steel} and m_{engine} are joint exports associated to final export e_{car}. The net output of global production for world export therefore is e_{car} (\$2000 million), and the double counted value is $e_{\text{coal}} + e_{\text{metal}} + m_{\text{steel}} + m_{\text{engine}}$=\$1160 million. Gross world export satisfies the following decomposition

\$3160 million=\$2000 million (net output) +\$1160 million (double counted value) (4)

2.2 Value-added content in gross export

This section further investigates the value-added content in gross export. It measures the contributions and gains of participants in global production of exports. Use country A's aggregate export shown in Figure 1 as an example. The extensions to bilateral export and world export are straightforward.

Along global production chains, the value of country A's exports can be expressed in value-added terms.

$$e_{\text{coal}} = v_1^A$$

\$10 million =\$10 million

$$e_{\text{metal}} = m_{\text{steel}} + v_2^A$$

$$= e_{\text{coal}} + v_1^B + v_2^A$$

$$= v_1^A + v_1^B + v_2^A$$

\$100 million =\$10 million +\$40 million +\$50 million

$$e_{\text{car}} = m_{\text{engine}} + v_3^A$$

$$= e_{\text{metal}} + v_2^B + v_3^A$$

$$= v_1^A + v_1^B + v_2^A + +v_2^B + v_3^A$$

\$2000 million =\$10 million +\$40 million +\$50 million +\$900 million+\$1000 million

Summing up all decompositions yields the decomposition for country *A*'s aggregate export.

$$e_{\text{coal}} + e_{\text{metal}} + e_{\text{car}} = v_1^A$$

$$+v_1^A + v_1^B + v_2^A$$

$$+v_1^A + v_1^B + v_2^A + v_2^B + v_3^A$$

\$2110 million

= \$10 million

+ \$10 million +\$40 million +\$50 million

+ \$10 million +\$40 million +\$50 million +\$900 million+\$1000 million

(5)

Equation (5) clearly shows that in country *A*'s aggregate export: value-added created by country *A*'s coal sector (v_1^A=\$10 million) is counted thrice, because it appears in three individual exports (coal, metal product, and car); Value-added created by country *A*'s metal product sector (v_2^A=\$50 million) and by country *B*'s steel sector (v_1^B=\$40 million) are both counted twice, because they appear in two individual exports (metal product and car). so, \$10 million*2+\$50 million=\$70 million value-added created by country *A* and \$40 million value-added created by country *B* are double counted in country A's aggregate export. In total, \$70 million+\$40 million=\$110 million value-added crated in the world is double counted, exactly the value of joint exports (e_{coal} and e_{metal}) in country A's aggregate export. In the global production of country A's aggregate export, $v_1^A + v_2^A + v_3^A$ =\$1060 million value-added of country A (domestic value-added) and $v_1^B + v_2^B$ =\$940 million value-added of country B (foreign value-added) are created. Total value-added created in the world is therefore \$2000 million, exactly the value of final export (e_{car}) in country A's aggregate export (Figure 2).

<INSERT FIGURE 2>

In summary, given a bundle of exports, the gross export can be decomposed into final export and joint export. Final export is the net output of global production for the given bundle of exports. The value of final export is comprised of the value-added created by each participant in the global production. Joint export is used up to produce final export. It is double counted content in gross export. The value of joint export is comprised of value-added double counted in gross export.

2.3 A comparison with border-crossing based double-counting

By using the example in Figure 1, the joint-export based double-counting proposed in this paper is compared with the border-crossing based double-counting used in main literature (see a comprehensive review in Miroudot and Ye, 2021). As the reference "border" can be selected from three perspectives (world perspective, country perspective, and bilateral perspective), the definition of border-crossing based double-counting has three versions.

Take country A's bilateral export to country C (e^{AC}) as an example to interpret the comparison results shown in Table 1. In this case, the export bundle (country A's export of cars to country C) does not include any joint export, so the joint-export based double-counting implies that there is no domestic double-counting (DDC) and foreign double-counting (FDC) in e^{AC}. The value-added created in the global production of e^{AC} are $v_1^A + v_2^A + v_3^A$=\$1060 million in country *A* (domestic value-added, DVA) and $v_1^B + v_2^B$=\$940 million in country *B* (foreign value-added, FVA). They are contributions of country *A* and country *B* in the global production of e^{AC} and add up to e^{AC} (\$2000 million). In other words, country *A* and country *B* gains \$1060 million and \$940 million in the global production of e^{AC}.

<INSERT TABLE 1>

The world perspective defines double-counting as the value-added crossing international borders more than once. International borders include the borders of all exporting countries in the world. From the world perspective, $v_1^A = 10$, $v_2^A = 50$, $v_1^B = 40$, and $v_2^B = 900$ all crosses international borders more than once in the global production of e^{AC} and therefore labeled as double-counting. $v_3^A = 1000$ crosses international borders once and is therefore not double counted value. So, e^{AC} contains \$60 million (=\$10 million + \$50 million) DDC and \$940 million (= \$40 million + \$900 million) FDC. The DVA and FVA are \$1000 million and \$0 million in e^{AC}, respectively. Compared with the joint-export based double-counting, the border-crossing based double-counting with world perspective over-measures the double counted value in e^{AC} and therefore under-measures the contribution of participants in the global production of e^{AC}.

The country perspective defines double-counting as the value-added crossing the border of exporting country more than once. From the country perspective, $v_1^A = 10$, $v_2^A = 50$, and $v_1^B = 40$ all cross the border of country *A* more than once in the global production of e^{AC}. $v_3^A = 1000$ and $v_2^B = 900$ both cross the border of country *A* once. The country perspective implies that e^{AC} contains \$60 million (=\$10 million + \$50 million) DDC, \$40 million FDC, \$1000 million DVA, and \$900 million FVA. Apparently, the border-crossing based double-counting with country perspective also over-measures the double counted value in e^{AC}.

The bilateral perspective defines double-counting as the value-added crossing the bilateral border of the exporting country more than once. For e^{AC}, the corresponding bilateral border is the border between country A and country C ($A \rightarrow C$). From the bilateral perspective, all value-added cross the bilateral border ($A \rightarrow C$) only once, so there is no domestic double-counting (DDC) and foreign double-counting (FDC) in e^{AC}. This is consistent with the joint-export based double-counting.

Table 1 also reveals the relationship between joint-export based double-counting and border-crossing based double-counting with different perspectives. The joint-export based double-counting in bilateral export (of a country), aggregate export (of a country), and world export are consistent with the border-crossing based double-counting with bilateral perspective, country perspective, and world perspective, respectively. An invalid selection of border-crossing based definitions from these three perspectives would lead to mislabeled double-counting and therefore mismeasures the contributions of participants in the global production of exports.

2.4 Position of suppliers in trade flows

Double-counting contains useful information on the position of suppliers in trade flows. For suppliers located upstream, their value-added also appears in the trade flows of downstream suppliers via inter-export production linkages. If sum up all trade flows to arrive at a gross trade measure, the value-added of upstream suppliers would be double counted in a larger degree than that of downstream suppliers. Hence, the double-counting ratio defined below can measure the relative position of suppliers in trade flows.

$$r_i = d_i / v_i^*$$

Given a bundle of trade flows, v_i^* is the value-added created by supplier i in the global production of the gross trade and d_i is the double counted value-added of supplier i in the value of gross trade. r_i measures the numbers of double-counting for the value-added of supplier i in gross trade. A higher ratio indicates a more upstream position. Section 3 further shows that if focus on all trade flows (both domestic trade flows and international trade flows) in the world, the double-counting ratio is essentially the "upstreamness" measure proposed by Antràs et al. (2012).

Continue using the example in Figure 1 to interpret the double-counting ratio as a position measure. For international trade flows in the world $e_{\text{coal}} + e_{\text{metal}} + e_{\text{car}} + m_{\text{steel}} + m_{\text{engine}}$=\$3160 million, the value-added content and double-counting ratios of suppliers are listed in Table 2. Double-counting ratios in Table 2 show that the value-added of coal industry is double-counted four times, because it visits other four suppliers. It is the largest ratio, indicating the coal industry of country *A* located the most upstream. The relative position of suppliers in international trade flows, from upstream to downstream, are coal industry, steel industry, metal product industry, engine industry, car industry, respectively. This outcome well fits the example in Figure 1.

<INSERT TABLE 2>

For country *A*'s export flows (e_{coal}, e_{metal}, e_{car}), there are three suppliers (i.e. country *A*'s coal industry, metal product industry, and car industry). According to their double-counting ratios in Table 3, the relative position of suppliers in country *A*'s trade flows, from upstream to downstream, are coal industry, metal product industry, and car industry, respectively. The outcome also fits the example well.

<INSERT TABLE 3>

3. Analytical framework

According to the accounting theory introduced in Section 2, the essential question for measuring global production of exports is to disentangle final exports and its associated joint exports from a given export bundle. This requires dividing the export bundle into two mutually exclusive but inter-connected sub-bundles. The first sub-bundle of exports (joint exports) is used to produce the second one (final exports) in global production system. Value-added in joint exports is double counted value-added in gross export. The value of final exports consists of the value-added created by each participant in the global production of exports.

Identifying final exports and joint exports from gross export is not a straightforward procedure, because the inter-export production relationship could be indirect. It requires the information on inter-country-industry linkages. This section aims to provide an identification framework based on input-output structures of global Leontief production system. We first propose a general framework for identifying double-counting and net output in any bundle of output flows in the Leontief input-output system and then focus on the identification for international trade flows.

3.1 The input-output table

Table 4 gives an input-output table of the global economy with n industries. $\mathbf{Z} = (z_{ij})_{n\times n}$ is the intermediate delivery matrix. z_{ij} is industry i's output used by industry j's production. $\mathbf{f} = (f_i)_{n\times 1}$ is the final use vector. f_i is industry i's output sold to final users for consumption and investment. $\mathbf{v}' = (v_j)_{1\times n}$ is the value-added vector. v_j is the value-added created by industry j.[2] $\mathbf{x} = (x_i)_{n\times 1}$ is the output vector. x_i is industry i's output.

<INSERT TABLE 4>

Define the input coefficient $a_{ij} = z_{ij}/x_j$ and the matrix form $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$, where $\hat{\mathbf{x}}$ is a diagonal matrix generated from vector $\mathbf{x}$. a_{ij} gives the output of industry i directly used to produce per unit industry j's output. The standard Leontief input-output model satisfies

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{L}\mathbf{f} \tag{6}$$

In which, $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief inverse matrix. Its element l_{ij} denotes the

[2] Use prime (') to represent a transpose operation on a column vector **v**.

output of industry i directly and indirectly required to produce per unit output of industry j (Miller and Blair, 2009).

3.2 Identify double-counting and net output in input-output flows

First focus on the identification for a bundle of output flows including z_{ij}, z_{ks}, f_i, and f_k (see Table 4). They add up to the gross value $\bar{x}$. The identification will be generalized to any combination of output flows afterwards. In the global production system, some output in the flows could be used to produce the other output. For instance, some output of industry i is used to produce the output of industry j. Some output of industry j is further used to produce the output of industry k for final use. In this scenario, a part of z_{ij} is used to produce f_k. Therefore, $\bar{x}$ can be divided into two mutually exclusive bundles of output based on production linkages. The first bundle of output is used to produce the output in the second bundle. Namely, the first bundle is double counted content and the second bundle is net output of $\bar{x}$.

For the second bundle, further use y_{ij} and y_{ks} to represent the net output from z_{ij} and z_{ks}, respectively. f_i and f_k are output used for final uses and therefore are all net output. For the first bundle, use $z^*_{ij}(ij)$, $z^*_{ij}(ks)$, $z^*_{ij}(f_i)$, $z^*_{ij}(f_k)$ to represent the output in z_{ij} used to produce y_{ij}, y_{ks}, f_i, f_k, respectively. Use $z^*_{ks}(ij)$, $z^*_{ks}(ks)$, $z^*_{ks}(f_i)$, $z^*_{ks}(f_k)$ to represent the output in z_{ks} used to produce y_{ij}, y_{ks}, f_i, f_k, respectively. When output flows z_{ij}, z_{ks}, f_i, and f_k add up to $\bar{x}$, the value $z^*_{ij}(ij)$, $z^*_{ij}(ks)$, $z^*_{ij}(f_i)$, and $z^*_{ij}(f_k)$ in z_{ij} as well as the value $z^*_{ks}(ij)$, $z^*_{ks}(ks)$, $z^*_{ks}(f_i)$, and $z^*_{ks}(f_k)$ in z_{ks} become double counted content in $\bar{x}$.

In summary, each output flow in $\bar{x}$ can be decomposed into net output and double counted content as follows

$$z_{ij} = y_{ij} + z^*_{ij}(ij) + z^*_{ij}(ks) + z^*_{ij}(f_i) + z^*_{ij}(f_k)$$

$$z_{ks} = y_{ks} + z^*_{ks}(ij) + z^*_{ks}(ks) + z^*_{ks}(f_i) + z^*_{ks}(f_k)$$

$$f_i = f_i$$

$$f_k = f_k$$

(7)

The relationship between double-counting components and net-output components satisfies

$$z_{ij}^*(ij) = a_{ij}l_{ji}y_{ij},\ z_{ij}^*(ks) = a_{ij}l_{jk}y_{ks},\ z_{ij}^*(f_i) = a_{ij}l_{ji}f_i,\ z_{ij}^*(f_k) = a_{ij}l_{jk}f_k$$

$$z_{ks}^*(ij) = a_{ks}l_{si}y_{ij},\ z_{ks}^*(ks) = a_{ks}l_{sk}y_{ks},\ z_{ks}^*(f_i) = a_{ks}l_{si}f_i,\ z_{ks}^*(f_k) = a_{ks}l_{sk}f_k$$

(8)

Combining equations (7-8), further have

$$\begin{cases} (1 + a_{ij}l_{ji})(y_{ij} + f_i) + a_{ij}l_{jk}(y_{ks} + f_k) = z_{ij} + f_i \\ a_{ks}l_{si}(y_{ij} + f_i) + (1 + a_{ks}l_{sk})(y_{ks} + f_k) = z_{ks} + f_k \end{cases} \quad (9)$$

Use $\bar{x}_i = z_{ij} + f_i$ and $\bar{x}_k = z_{ks} + f_k$ to denote the output in $\bar{x}$ produced by industry *i* and industry *k*, respectively; Use $\bar{y}_i = y_{ij} + f_i$ and $\bar{y}_k = y_{ks} + f_k$ to denote the net output produced by industry *i* and industry *k*, respectively. Equation group (9) are further rewritten as

$$\begin{cases} (1 + a_{ij}l_{ji})\bar{y}_i + a_{ij}l_{jk}\bar{y}_k = \bar{x}_i \\ a_{ks}l_{si}\bar{y}_i + (1 + a_{ks}l_{sk})\bar{y}_k = \bar{x}_k \end{cases} \quad (10)$$

In matrix form,

$$\begin{bmatrix} 1 + a_{ij}l_{ji} & a_{ij}l_{jk} \\ a_{ks}l_{si} & 1 + a_{ks}l_{sk} \end{bmatrix} \begin{bmatrix} \bar{y}_i \\ \bar{y}_k \end{bmatrix} = \begin{bmatrix} \bar{x}_i \\ \bar{x}_k \end{bmatrix}$$

In $\bar{x}$, the net output produced by industry *i* and industry *j* is therefore solved as

$$\begin{bmatrix} \bar{y}_i \\ \bar{y}_k \end{bmatrix} = \begin{bmatrix} 1 + a_{ij}l_{ji} & a_{ij}l_{jk} \\ a_{ks}l_{si} & 1 + a_{ks}l_{sk} \end{bmatrix}^{-1} \begin{bmatrix} \bar{x}_i \\ \bar{x}_k \end{bmatrix}$$

Further extend Equation group (10) to take all *n* industries in the input-output table into account by setting zeros for industries that have no output included in the gross value $\bar{x}$. In the present case, these industries are industries $1, 2, \dots, (i-1), (i+1), \dots (k-1), (k+1), \dots n$. The following *n* equations hold.

$$0 \times \bar{y}_1 + \cdots + 0 \times \bar{y}_n = \bar{x}_1 = 0$$
$$\cdots$$
$$0 \times \bar{y}_1 + \cdots + 0 \times \bar{y}_n = \bar{x}_{i-1} = 0$$
$$0 \times \bar{y}_1 + \cdots + 0 \times \bar{y}_{i-1} + \left(1 + a_{ij}l_{ji}\right)\bar{y}_i + 0 \times \bar{y}_{i+1} + \cdots + 0 \times \bar{y}_{k-1} + a_{ij}l_{jk}\bar{y}_k$$
$$+0 \times \bar{y}_{k+1} + \cdots + 0 \times \bar{y}_n = \bar{x}_i$$
$$0 \times \bar{y}_1 + 0 \times \bar{y}_2 + \cdots + 0 \times \bar{y}_n = \bar{x}_{i+1} = 0$$
$$\cdots$$
$$0 \times \bar{y}_1 + 0 \times \bar{y}_2 + \cdots + 0 \times \bar{y}_n = \bar{x}_{k-1} = 0$$
$$0 \times \bar{y}_1 + \cdots + 0 \times \bar{y}_{i-1} + a_{ks}l_{si}\bar{y}_i + 0 \times \bar{y}_{i+1} + \cdots + 0 \times \bar{y}_{k-1} + (1 + a_{ks}l_{sk})\bar{y}_k$$
$$+0 \times \bar{y}_{k+1} + \cdots + 0 \times \bar{y}_n = \bar{x}_k$$
$$0 \times \bar{y}_1 + 0 \times \bar{y}_2 + \cdots + 0 \times \bar{y}_n = \bar{x}_{k+1} = 0$$
$$\cdots$$
$$0 \times \bar{y}_1 + 0 \times \bar{y}_2 + \cdots + 0 \times \bar{y}_n = \bar{x}_n = 0$$

By using conformable extension shown above, a generalized model for disentangling net output and double counted content from any combination of output flows can be developed. The relation between the gross value of a given bundle of output flows $\bar{\mathbf{x}}$ and its net output $\bar{\mathbf{y}}$ satisfies the following matrix equation

$$\left(\mathbf{I} + \bar{\mathbf{A}}\,\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}\right)\bar{\mathbf{y}} = \bar{\mathbf{x}} \qquad (11)$$

In which:

$\mathbf{I}$ is an identity matrix (n by n);

$\bar{\mathbf{A}}$ is a partial input coefficient matrix (n by n) generated based on the input coefficient matrix $\mathbf{A}$ by keeping some coefficients unchanged and setting the other coefficients zeros. The location of the unchanged coefficients in $\mathbf{A}$ is the same as the location of the intermediate use flows of the given output flow bundle in $\mathbf{Z}$;

$\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}$ is a partial Leontief inverse matrix (n by n) generated based on the Leontief inverse matrix $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-\mathbf{1}}$ by keeping some coefficients unchanged and setting the other coefficients zeros. The location of the unchanged coefficients in $\mathbf{L}$ is determined by the row-column index set $[\boldsymbol{\beta},\boldsymbol{\alpha}]$. $\boldsymbol{\alpha}$ represents the row index set and $\boldsymbol{\beta}$ represents the column index set covered by the location of given output flows in the input-output table. The operation $[\boldsymbol{\beta},\boldsymbol{\alpha}]$ yields the row-column index set generated by index sets $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$. For example, in the previous case, the output flows are z_{ij}, z_{ks}, f_i and f_k, so the row index set is $\boldsymbol{\alpha} = \{i,k\}$ and the column index set is $\boldsymbol{\beta} = \{j,s\}$.[3] $[\boldsymbol{\beta},\boldsymbol{\alpha}] = [\{j,s\},\{i,k\}] = \{(j,i),(j,k),(s,i),(s,k)\}$.

Further introduce how set up equation (11) by using an input-output table with 3 industries. Suppose that we are interested in a bundle of output flows including intermediate use flows z_{12}, z_{13}, z_{22}, z_{23}, and final use flows f_1, f_2. The intermediate use flows are located at the 1st and 2nd rows as well as the 2nd and 3rd columns, so we have

$$\boldsymbol{\alpha} = \{1,2\},\ \boldsymbol{\beta} = \{2,3\}$$

$$[\boldsymbol{\beta},\boldsymbol{\alpha}] = [\{2,3\},\{1,2\}] = \{(2,1),(2,2),(3,1),(3,2)\}$$

The parameters and variables in equation (11) are set as follows

$$\bar{\mathbf{A}} = \begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix},\ \bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]} = \begin{bmatrix} 0 & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & 0 \end{bmatrix},\ \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\mathbf{x}} = \begin{bmatrix} z_{12} + z_{13} + f_1 \\ z_{22} + z_{23} + f_2 \\ 0 \end{bmatrix}$$

In Equation (11), $\bar{\mathbf{y}}$ and $\bar{\mathbf{A}}\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}\bar{\mathbf{y}}$ (denoted by $\bar{\mathbf{z}}$ afterwards) are the net output and double counted content in the gross value of $\bar{\mathbf{x}}$. Solving Equation (11), these two bundles can be disentangled from $\bar{\mathbf{x}}$.

$$\bar{\mathbf{y}} = \left(\mathbf{I} + \bar{\mathbf{A}}\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}\right)^{-1}\bar{\mathbf{x}} \tag{12}$$

$$\bar{\mathbf{z}} = \bar{\mathbf{A}}\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}\bar{\mathbf{y}} = \bar{\mathbf{A}}\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}\left(\mathbf{I} + \bar{\mathbf{A}}\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}\right)^{-1}\bar{\mathbf{x}} \quad \text{or} \quad \bar{\mathbf{z}} = \bar{\mathbf{x}} - \bar{\mathbf{y}} \tag{13}$$

Equations (12-13) are the key models for identifying net output and double counted content in the gross value of a bundle of output flows.

When output flows include all intermediate use flows and final use flows of the whole economy, $\bar{\mathbf{x}} = \mathbf{x}$, $\bar{\mathbf{A}} = \mathbf{A}$, $\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]} = \mathbf{L}$. Equation (11) becomes $(\mathbf{I} + \mathbf{AL})\bar{\mathbf{y}} = \mathbf{x}$. As $\mathbf{I} + \mathbf{AL} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \cdots = \mathbf{L}$, so $\mathbf{L}\bar{\mathbf{y}} = \mathbf{x}$. Hence,

$$\bar{\mathbf{y}} = \mathbf{L}^{-1}\mathbf{x} = (\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{f} \tag{14}$$

$$\bar{\mathbf{z}} = \mathbf{AL}\bar{\mathbf{y}} = \mathbf{ALf} = \mathbf{Ax}. \tag{15}$$

This special case of the general model implies that net output and double counted

[3] The intermediate use flows have both row index and column index. The final use flows only have row index.

content in gross output of the whole economy are final product $\mathbf{f}$ and intermediate product $\mathbf{Ax}$, respectively. This is fully consistent with the rule of GDP accounting.

3.3 Identify final exports and joint exports in gross export

This section focuses on the identification of final exports and its joint exports in a bundle of exports based on the global Leontief production system. It is a specific application of Equations (12-13) to international trade flows in an inter-country input-output table. For simplification, assume that the inter-country input-output table has three economies (economy *r*, economy *s* and economy *t*) and each economy has *n* industries. The structure of the inter-country input-output table is shown in Table 5.

<INSERT TABLE 5>

The global intermediate delivery matrix $\mathbf{Z}$ (3n-by-3n), final demand matrix $\mathbf{F}$ (3n-by-3), gross output vector $\mathbf{x}$ (3n-by-1) and value-added vector $\mathbf{v}'$ (1-by-3n) are given by

$$\mathbf{Z}=\begin{bmatrix}\mathbf{Z}^{rr} & \mathbf{Z}^{rs} & \mathbf{Z}^{rt}\\ \mathbf{Z}^{sr} & \mathbf{Z}^{ss} & \mathbf{Z}^{st}\\ \mathbf{Z}^{tr} & \mathbf{Z}^{ts} & \mathbf{Z}^{tt}\end{bmatrix},\ \mathbf{F}=\begin{bmatrix}\mathbf{f}^{rr} & \mathbf{f}^{rs} & \mathbf{f}^{rt}\\ \mathbf{f}^{sr} & \mathbf{f}^{ss} & \mathbf{f}^{st}\\ \mathbf{f}^{tr} & \mathbf{f}^{ts} & \mathbf{f}^{tt}\end{bmatrix},\ \mathbf{x}=\begin{bmatrix}\mathbf{x}^{r}\\ \mathbf{x}^{s}\\ \mathbf{x}^{t}\end{bmatrix},\ \mathbf{v}'=\begin{bmatrix}\mathbf{v}^{r\prime} & \mathbf{v}^{s\prime} & \mathbf{v}^{t\prime}\end{bmatrix}$$

Furthermore, the global input coefficient matrix $\mathbf{A}=\mathbf{Z}\hat{\mathbf{x}}^{-1}$, value-added coefficient vector $\mathbf{w}'=\mathbf{v}'\hat{\mathbf{x}}^{-1}=\begin{bmatrix}\mathbf{w}^{r\prime} & \mathbf{w}^{s\prime} & \mathbf{w}^{t\prime}\end{bmatrix}$ and Leontief inverse matrix $\mathbf{L}=(\mathbf{I}-\mathbf{A})^{-1}$ are given by

$$\mathbf{A}=\begin{bmatrix}\mathbf{A}^{rr} & \mathbf{A}^{rs} & \mathbf{A}^{rt}\\ \mathbf{A}^{sr} & \mathbf{A}^{ss} & \mathbf{A}^{st}\\ \mathbf{A}^{tr} & \mathbf{A}^{ts} & \mathbf{A}^{tt}\end{bmatrix},\ \widetilde{\mathbf{W}}=\begin{bmatrix}\mathbf{w}^{r\prime} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{w}^{s\prime} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{w}^{t\prime}\end{bmatrix},\ \mathbf{L}=\begin{bmatrix}\mathbf{L}^{rr} & \mathbf{L}^{rs} & \mathbf{L}^{rt}\\ \mathbf{L}^{sr} & \mathbf{L}^{ss} & \mathbf{L}^{st}\\ \mathbf{L}^{tr} & \mathbf{L}^{ts} & \mathbf{L}^{tt}\end{bmatrix}$$

Parameters in Equations (12-13) vary with the exports to be measured and are determined by the rule introduced in Section 3.2. The detailed settings for an economy's bilateral export, aggregate export and gross world export are summarized in Table 6.

<INSERT TABLE 6>

3.4 Measuring value-added content in exports

As an industry's production requires intermediate input from other industries, the value of an industry's output is created by the whole production system. Almost all industries directly and indirectly participate in the production of an output by using their labor and capital and therefore add value to the output. The value of an output equals the sum of value-added created by industries participating in the production of this output (Arto et al., 2019). Therefore, according to the accounting theory in Section 2.2, value-added of each economy created in global production of a given export bundle $\bar{\mathbf{x}}$ equals the value-added in final exports $\bar{\mathbf{y}}$ and satisfies $\mathbf{v}^* = \widetilde{\mathbf{W}}\mathbf{L}\bar{\mathbf{y}}$. It accounts for the contribution and gains of each economy in the global production of exports. Double counted value-added of each economy in the gross value of $\bar{\mathbf{x}}$ equals the value-added in joint exports $\bar{\mathbf{z}}$ and satisfies $\mathbf{d} = \widetilde{\mathbf{W}}\mathbf{L}\bar{\mathbf{z}}$. It accounts for the double counted content in traditional gross trade statistics and contains useful information for measuring position of suppliers in international trade flows.

As $\bar{\mathbf{y}}$ and $\bar{\mathbf{z}}$ are unique solutions from our analytical framework, users can avoid the trouble of selecting appropriate decomposition perspectives (bilateral perspective, country perspective and world perspective) before carrying out the decomposition. As discussed in Section 2.3, the solutions for bilateral export, aggregate export and gross world export are consistent with the bilateral perspective, country perspective and world perspective, respectively. We further investigated the relationship between domestic value-added in export solved from our framework with those proposed in main literature. Appendix 1 proved that our solution is consistent with Los and Timmer (2018) for bilateral export decomposition and is consistent with Koopman et al. (2014) and Los et al. (2016) for aggregate export decomposition.

For value-added content in gross world export (the gross export of all economies in the world), Theorem 1 shows that value-added created by each economy in global production of gross world export and of its own aggregate export are the same. Namely, an economy cannot gain more value-added from gross world export than from its own aggregate export. One intuitive interpretation is as follows. Economy *r* exports intermediate product to other economies for export production and thus gains value-added from other economies' exports. This value-added, however, has been counted in economy *r*'s aggregate export. Therefore, expanding the trade flows from economy *r*'s aggregate export to gross world export would increase the double counted value-added of economy *r* but would not change the value-added created by economy *r*.

Theorem 1: *The value-added of an economy created in global production of gross world export (the gross export of all economies in the world) and in global production of aggregate export of this economy are identical.* See Appendix 2 for a proof.

Theorem 1 provides important guidance for measuring an economy's performance in world trade by looking at the economy's capability of adding value to world export production. Theorem 1 implies that computing domestic value-added in the economy's aggregate export is sufficient for answering the question. This can be done based on national input-output tables or inter-country input-output tables and the outcomes are identical (Los et al., 2016). The advantage of computing based on inter-country input-output tables is that all countries' value-added in gross world export can be computed simultaneously in a harmonized framework and therefore comparable. However, it should be stressed that Theorem 1 holds only when all exports are studied. For a sub-group of exports (for instance, exports of goods), Theorem 1 does not hold. An economy's value-added created in global production of its exports of goods does not equal its value-added created in global production of gross world export of goods.

A broad range of popular GVC indictors can be derived from our analytical framework, such as trade in value-added indicator (Johnson and Noguera, 2012) and GVC income indicator (Timmer et al., 2014). Trade in value-added indicator measures the value-added of an economy finally consumed by another economy. This requires identifying the value-added content in a bundle of world final products consumed by an economy. Take the value-added import of economy *r* for an example. In this context, the output flow bundle $\bar{\mathbf{x}}$ to be studied is the world final products consumed by economy *r* ($\mathbf{f}^r$). GVC income indicator measures the value-added created by each economy in global production of a bundle of world final products. If the GVC income of manufactures is studied, it requires identifying the value-added content in a bundle of world manufacturing final products. In this context, the output flow bundle $\bar{\mathbf{x}}$ to be studied is all manufacturing final products in the world ($\mathbf{f}^m$). Final products leave production system and cannot be used to produce each other, so a bundle of final products is net output and contains no double-counting. Therefore, we have $\bar{\mathbf{y}} = \mathbf{f}^r$ (or $\mathbf{f}^m$) and $\bar{\mathbf{z}} = \mathbf{0}$. The value-added of each economy imported by economy *r* is $\mathbf{v}^{*r} = \widetilde{\mathbf{W}}\mathbf{L}\bar{\mathbf{y}} = \widetilde{\mathbf{W}}\mathbf{L}\mathbf{f}^r$ and the GVC income of manufactures obtained by each economy is $\mathbf{v}^m = \widetilde{\mathbf{W}}\mathbf{L}\bar{\mathbf{y}} = \widetilde{\mathbf{W}}\mathbf{L}\mathbf{f}^m$. They are exactly the formulae for computing trade in value-added indicator (Johnson and Noguera, 2012) and GVC income indicator (Timmer et al., 2014).

3.5 Measuring position of suppliers in international trade flows

Section 2.3 shows that double-counting ratio measures the relative position of suppliers in trade flows. Suppliers with higher double-counting ratios are located more upstream. Given a bundle of exports $\bar{\mathbf{x}}$, the double counting ratio by economy $\mathbf{r}$ is the ratio of double counted value-added $\mathbf{d}$ to created value-added $\mathbf{v}^*$ and satisfies

$$\mathbf{r} = \frac{\mathbf{d}}{\mathbf{v}^*} = \frac{\widetilde{\mathbf{W}}\mathbf{L}\bar{\mathbf{z}}}{\widetilde{\mathbf{W}}\mathbf{L}\bar{\mathbf{y}}}$$

In which, $\bar{\mathbf{y}}$ and $\bar{\mathbf{z}}$ are final exports and joint exports disentangled from the export bundle $\bar{\mathbf{x}}$ and are determined by Equations (12-13). The vector division in this paper is defined as element-wise division.

If the position of suppliers at industry level in trade flows of the whole world (both domestic trade and international trade) is studied, our position measure is essentially the upstreamness measure proposed by Antràs et al. (2012). In this special case, the gross trade flows $\bar{\mathbf{x}}$ is exactly the world gross output $\mathbf{x}$. so, $\bar{\mathbf{y}} = \mathbf{f}$ and $\bar{\mathbf{z}} = \mathbf{ALf}$ (see Equations 14 and 15). The created value-added ($\mathbf{v}^*$) and double counted value-added ($\mathbf{d}$) at the industry level are $\mathbf{v}^* = \widehat{\mathbf{W}}\mathbf{L}\bar{\mathbf{y}} = \widehat{\mathbf{W}}\mathbf{L}\mathbf{f}$ and $\mathbf{d} = \widehat{\mathbf{W}}\mathbf{L}\bar{\mathbf{z}} = \widehat{\mathbf{W}}\mathbf{L}\mathbf{A}\mathbf{L}\mathbf{f} = \widehat{\mathbf{W}}\mathbf{L}(\mathbf{L}-\mathbf{I})\mathbf{f}$. In which, $\widehat{\mathbf{W}}$ is a diagonal matrix generated from the value-added vector $\mathbf{w}'$. The double-counting ratios by industry ($\mathbf{r}$) is

$$\mathbf{r} = \frac{\mathbf{d}}{\mathbf{v}^*} = \frac{\widehat{\mathbf{W}}\mathbf{L}(\mathbf{L}-\mathbf{I})\mathbf{f}}{\widehat{\mathbf{W}}\mathbf{L}\mathbf{f}} = \frac{\mathbf{L}(\mathbf{L}-\mathbf{I})\mathbf{f}}{\mathbf{L}\mathbf{f}} \tag{16}$$

Formula (16) is slightly different from the upstreamness indicator (Antràs et al., 2012).The upstreamness indicator measures the weighted average distance of each industry's output to the final market. The distance counts from "1". Final product has the nearest distance to final market, so its distance is "1". If the distance counts from "0" (the distance of final product to final market is "0"), the upstreamness indicator becomes

$$\frac{(0\times\mathbf{f}+1\times\mathbf{A}\mathbf{f}+2\times\mathbf{A}^2\mathbf{f}+3\times\mathbf{A}^3\mathbf{f}+\cdots)}{\mathbf{x}} = \frac{\mathbf{L}(\mathbf{L}-\mathbf{I})\mathbf{f}}{\mathbf{L}\mathbf{f}}.$$

It is exactly Formula (16).

In summary, the analytical framework proposed in the present paper has a sound theoretical foundation and embraces a broad range of GVC indicators, including value-added content in trade, trade in value-added, GVC income, position in trade flows etc. It provides a unified framework for analyzing the structure of global production.

4. The structure of global production for world export of goods

As an empirical study, this section measured the structure of global production for world export of goods based on the 2021 release of OECD inter-country input-output (ICIO) tables (OECD, 2021). The OECD ICIO tables cover 45 industries and 67 economies (including an economy called "rest-of-the-world" or "ROW") from 1995 to 2018. Industries in the ICIO table that supply export of goods include agriculture, mining, and manufacture industries (from industry 1 to industry 22).

4.1 Value-added content and position of suppliers

Table 7 accounts for the global production of world export of goods in 2018. It shows that \$14120789 million goods were produced in the world for export in 2018. Of which, the final export is $11172676 million, accounting for 79.1% (=11172676/14120789). It is net output of global export production and was available for foreign final demand and for non-export of goods production. The joint export of goods associated with the final export of goods is $2948113 million (accounting for 20.9%), which was used in global production to produce above-mentioned final export. A large share of export of goods were supplied by EU 27 (28.0%), China (14.4%) and the United States (8.0%). China's share in world final export of goods (15.1%) is 3.4 percentage points higher than its share in world joint export of goods (11.7%). It indicates China specializes in supplying final export of goods in global production of export. This is also true for Mexico. On the contrary, the United States, Korea and Russia specialize in supplying joint export of goods for final export production.

<INSERT TABLE 7>

$11172676 million value-added was created in the global production of world export of goods, exactly the value of world final export of goods. Of which, EU 27, China, and the United States are the top three economies of creating value-added in world export production. The gains of United States in value-added term (9.8%) are obviously higher than that in gross term (8.0%). On the contrary, the gains of ASEAN economies decrease from 7.4% in gross term to 6.0% in value-added term.

The ranking of double-counting ratio indicates the relative position of suppliers in international trade flows of goods in 2018. Mexico, Canada and China have quite low ratios (0.16, 0.19 and 0.21), indicating that these economies locate at the downstream of international trade flows and specialize in final production stages of world export production. Russia and Korea locate at the upstream with high ratios (0.34 and 0.30). These economies specialize in supplying raw materials and parts for world export production.

Figure 3 and Figure 4 further compare the value-added share and double-counting ratio by economy in 2018 against their values in 1995. As in Figure 3, the observations of EU 27, Japan, and the United States were below and obviously off the 45-degree line, indicating that the share of these economies in the total value-added created in world export production decreased substantially in the 1995-2018 period. Of which, the share of EU 27 decreased from 34.8% to 25.4%, with a shrinkage of 9.5 percentage points. Japan and the United States decreased 5.8 and 3.6 percentage points, respectively. On the contrast, the share of China increased from 2.3% to 14.9%, with an expansion of 12.6 percentage points. The contribution of China to world export production becomes increasingly important.

<INSERT FIGURE 3>

As in Figure 4, the observations of all economies are above the 45-degree line, indicating that the double-counting ratios of all economies increased in the 1995-2018 period. This reveals the world export production in 2018 was more fragmented than that in 1995. The value-added of each economy needed to visit more export suppliers before the global production of export ends. The global production chain of world export was lengthened. The relative position in international trade flows basically unchanged for most economies, except for the United States and Korea. This can be identified by comparing the position of each point in Figure 4 along the horizontal axis and along the vertical axis, respectively. The United States approached from upstream to mid-stream and Korea moved to a more upstream position.

<INSERT FIGURE 4>

4.2 Inter-export linkages in global production: the role of China

This section investigated the inter-export linkages between China and other economies in global export production from both supply and demand perspectives. To do this, joint exports associated to China’s final export are further decomposed by economies based on our framework. This can identify the distribution of exports of goods across economies directly and indirectly required by the global production of China's final export. It therefore can measure the dependency of China's export production on its upstream suppliers as well as the diversification of its supply side. In addition, the inter-export linkages can also be measured from the downstream demand side, by attributing China's joint exports to each economy's final export production. This can measure the dependency of China's export production on its downstream users as well as the diversification of its demand side.

Figure 5 shows the dependency of China's export production measured from the supply side. It gives top four suppliers for China's export production in 2018. Around 38% of the joint exports required by the production of China's export of goods were supplied by Korea, Chinese Taipei, Japan, and the United States. Most of the economies were concentrated in east Asia area. The decreasing shares of Chinese Taipei, Japan, and the United States indicate that China's export production dependency on these economies was weakened over 1995-2018. The dependency on Korea, however, was strengthened, with the share increased from 11.3% to 14.4%. The share of China itself also increased over time, indicating that more Chinese goods were exported for producing its other exports. China's self-dependency was strengthened in world export production.

<INSERT FIGURE 5>

<INSERT FIGURE 6>

Figure 6 shows the dependency of China's export production measured from the demand side. It attributes China's joint exports to each economy who uses China's export for its final export production. In 2018, around 49% of China's joint exports were induced by the final export production in Mexico, Vietnam, Korea, the United States, Chinese Taipei, Japan, and Germany. These economies spread over relatively broad regions, including north America, southeast Asia, east Asia, and Europe. The demand-side diversification of China's export production is much higher than the supply-side (see Figure 7), but the diversification of its supply side increased substantially after China's access to WTO. Although the demand-side diversification increased slightly, it experienced significant structural changes over 1995-2018. Figure 6 shows that Mexico and Vietnam replaced the United States and Korea to become top two economies that drive China's joint export. China's downstream dependency on Japan and Chinese Taipei were weakened as well in recent years. The strengthened self-dependency of China was also captured from the downstream demand perspective. An increasing share of China's export of goods were driven endogenously by its own export.

<INSERT FIGURE 7>

5. Conclusion

Export production activities are inter-connected in the global production system. Some exports are required to produce other exports along global production chains. This joint export phenomenon, on the one hand, indicates that the value of export is a mixture of value-added created in the world. On the other hand, it leads to identical value-added appearing in two or more individual exports and therefore causes double-counting issues in gross trade measures.

This paper found that two mutually exclusive but inter-connected groups of exports can be disentangled from a given bundle of exports by using inter-country input-output structures. One of the groups is used to produce the other in global production. The latter is the net output of global production for the given export bundle, called final export. The former is the joint export associated to the final export. It is double counted content in the given export bundle.

Based on the inter-export linkages in global Leontief production system, this paper developed a unified framework for accounting for global production of exports. Like previous literature, this framework can attribute gross value of exports to the value-added created by each economy as well as double counted content. It harmonizes export decompositions at all levels in a unified framework and embraces popular GVC indicators, such as trade in value-added, GVC income, and upstreamness indicators. This framework also provides additional information on final exports and joint exports and their linkages across economies.

The framework provides important implications and guidance for trade accounting. First, the border-crossing based definition for double-counting is not a rigorous one. It varies with the reference "border" selected from different perspectives. An invalid selection would lead to mislabeled double-counting and therefore mismeasure the contributions of participants in the global production of exports. Second, export decompositions do not satisfy additivity. This is because export production activities are inter-dependent in global production system. For instance, the production of an economy's exports to one partner would indirectly use its exports to another partner. Aggregation would cause extra double-counting. Third, an economy gains the same amount of value-added from gross world export (the exports of all economies in the world) and from its own aggregate export. If one would like to measure an economy's performance in world trade by looking at the economy's capability of adding value to the global production of world exports, computing domestic value-added in the economy's aggregate export is sufficient and can be done based on national input-output tables. The advantage of using inter-country input-output tables is that all countries' value-added in gross world export can be computed simultaneously in a harmonized framework and therefore comparable.

The global production of world exports of goods in the 1995-2018 period was analyzed under our framework. China's contribution to world export production became increasingly important. Its share in total value-added created in the global production of world exports of goods increased from 2.3% in 1995 to 14.9% in 2018. Mexico, Canada, and China located at the downstream of international trade flows, specializing in final production stages of world export production. Russia and Korea located at the upstream, supplying raw materials and parts for world export production in recent years. Significant structural changes in China's inter-export production linkages with world economies were identified. Both supply-side and demand-side diversification of China's export production structure increased in the period 1995-2018, but the supply-side diversification is much lower than the demand-side. The underlying suppliers for China's export production concentrated in east Asia, with a shift from Japan and Chinese Taipei to Korea. The demand side of China's export in world export production spreads over relatively broad regions. Its main demand side shifted from the United States and Korea to Mexico and Vietnam in recent years.

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TABLE 1. Double-counting in international trade flows (unit: $million)

Trade flows	Gross export	Joint-export based double-counting				Boder-crossing based double-counting											
						World perspective				Country perspective				Bilateral perspective			
		DVA	FVA	DDC	FDC	DVA	FVA	DDC	FDC	DVA	FVA	DDC	FDC	DVA	FVA	DDC	FDC
(1) e^{AB}	110	60	40	10	0	60	0	10	40	60	40	10	0	60	40	10	0
(2) e^{AC}	2000	1060	940	0	0	1000	0	60	940	1000	900	60	40	1060	940	0	0
(3) e^{A}	2110	1060	940	70	40	1060	0	70	980	1060	940	70	40	-	-	-	-
(4) e^{*}	3160	1060	940	140	1020	1060	940	140	1020	-	-	-	-	-	-	-	-

Note: 1. e^{AB}=country *A*'s bilateral export to country *B*; e^{AC}=country *A*'s bilateral export to country *C*; e^{A}=country *A*'s aggregate export; e^{*}=gross world export; DVA=domestic value-added; FVA=foreign value-added; DDC=domestic double-counting; FDC=foreign double-counting.

2. In this table, domestic value-added refers to the value-added created by country *A*; Foreign value-added refers to the value-added created by country *B*; Country *C* does not create value-added.

TABLE 2. Double-counting ratios for suppliers in international trade flows

Suppliers of international trade flows	Created value-added (unit: $million)	Double counted value-added (unit: $million)	Double-counting ratio
Coal (Country *A*)	10	40	4
Steel (Country *B*)	40	120	3
Metal products (Country *A*)	50	100	2
Engines (Country *B*)	900	900	1
Cars (Country *A*)	1000	0	0

TABLE 3. Double-counting ratios for suppliers in country *A*'s export flows

Suppliers of country *A*'s export flows	Created value-added (unit: $million)	Double counted value-added (unit: $million)	Double-counting ratio
Coal (Country *A*)	10	20	2
Metal products (Country *A*)	50	50	1
Cars (Country *A*)	1000	0	0

TABLE 4. An input-output table of the global economy with *n* industries

<table>
<tr><td colspan="2" rowspan="2"></td><td>Intermediate use</td><td rowspan="2">Final use</td><td rowspan="2">Output</td></tr>
<tr><td>1 2 … n</td></tr>
<tr><td>Intermediate input</td><td>1
2
…
n</td><td>$\mathbf{Z} = (z_{ij})_{n\times n}$</td><td>$\mathbf{f} = (f_i)_{n\times 1}$</td><td>$\mathbf{x} = (x_i)_{n\times 1}$</td></tr>
<tr><td colspan="2">Value added</td><td>$\mathbf{v}' = (v_j)_{1\times n}$</td><td colspan="2" rowspan="2"></td></tr>
<tr><td colspan="2">Output</td><td>$\mathbf{x}' = (x_j)_{1\times n}$</td></tr>
</table>

TABLE 5. An inter-country input-output table with three economies

	Intermediate use			Final use			Output
	Economy *r*	Economy *s*	Economy *t*	Economy *r*	Economy *s*	Economy *t*	
Economy *r*	$\mathbf{Z}^{rr}$	$\mathbf{Z}^{rs}$	$\mathbf{Z}^{rt}$	$\mathbf{f}^{rr}$	$\mathbf{f}^{rs}$	$\mathbf{f}^{rt}$	$\mathbf{x}^{r}$
Economy *s*	$\mathbf{Z}^{sr}$	$\mathbf{Z}^{ss}$	$\mathbf{Z}^{st}$	$\mathbf{f}^{sr}$	$\mathbf{f}^{ss}$	$\mathbf{f}^{st}$	$\mathbf{x}^{s}$
Economy *t*	$\mathbf{Z}^{tr}$	$\mathbf{Z}^{ts}$	$\mathbf{Z}^{tt}$	$\mathbf{f}^{tr}$	$\mathbf{f}^{ts}$	$\mathbf{f}^{tt}$	$\mathbf{x}^{t}$
Value added	$(\mathbf{v}^{r})'$	$(\mathbf{v}^{s})'$	$(\mathbf{v}^{t})'$				
Output	$(\mathbf{x}^{r})'$	$(\mathbf{x}^{s})'$	$(\mathbf{x}^{t})'$				

TABLE 6. Parameter settings for identifying final exports and joint exports

Trade flows	$\bar{\mathbf{x}}$	$\bar{\mathbf{A}}$	$\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}$
Economy *r*'s bilateral export to economy *s*	$\begin{bmatrix}\mathbf{Z}^{rs}\mathbf{i}+\mathbf{f}^{rs}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}$	$\begin{bmatrix}\mathbf{0} & \mathbf{A}^{rs} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}$	$\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{L}^{sr} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}$
Economy *r*'s aggregate export	$\begin{bmatrix}(\mathbf{Z}^{rs}+\mathbf{Z}^{rt})\mathbf{i}+\mathbf{f}^{rs}+\mathbf{f}^{rt}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}$	$\begin{bmatrix}\mathbf{0} & \mathbf{A}^{rs} & \mathbf{A}^{rt}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}$	$\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{L}^{sr} & \mathbf{0} & \mathbf{0}\\ \mathbf{L}^{tr} & \mathbf{0} & \mathbf{0}\end{bmatrix}$
Gross world export	$\begin{bmatrix}(\mathbf{Z}^{rs}+\mathbf{Z}^{rt})\mathbf{i}+\mathbf{f}^{rs}+\mathbf{f}^{rt}\\ (\mathbf{Z}^{sr}+\mathbf{Z}^{st})\mathbf{i}+\mathbf{f}^{sr}+\mathbf{f}^{st}\\ (\mathbf{Z}^{tr}+\mathbf{Z}^{ts})\mathbf{i}+\mathbf{f}^{tr}+\mathbf{f}^{ts}\end{bmatrix}$	$\begin{bmatrix}\mathbf{0} & \mathbf{A}^{rs} & \mathbf{A}^{rt}\\ \mathbf{A}^{sr} & \mathbf{0} & \mathbf{A}^{st}\\ \mathbf{A}^{tr} & \mathbf{A}^{ts} & \mathbf{0}\end{bmatrix}$	$\begin{bmatrix}\mathbf{L}^{rr} & \mathbf{L}^{rs} & \mathbf{L}^{rt}\\ \mathbf{L}^{sr} & \mathbf{L}^{ss} & \mathbf{L}^{st}\\ \mathbf{L}^{tr} & \mathbf{L}^{ts} & \mathbf{L}^{tt}\end{bmatrix}$

TABLE 7. Accounting for the world export of goods (2018) Unit: million USD

Economies	Export of goods	Final export	Joint export	Created value-added	Double counted value-added	Double counting ratio
World	14,120,789	11,172,676	2,948,113	11,172,676	2,948,113	0.26
ASEAN	7.4%	7.5%	7.0%	6.0%	5.6%	0.25
CAN	2.5%	2.7%	1.8%	2.3%	1.7%	0.19
CHN	14.4%	15.1%	11.7%	14.9%	11.9%	0.21
EU27	28.0%	27.9%	28.2%	25.4%	25.3%	0.26
JPN	4.5%	4.5%	4.6%	4.9%	4.8%	0.26
KOR	4.2%	4.0%	4.9%	3.6%	4.1%	0.30
MEX	2.8%	3.1%	1.5%	2.1%	1.2%	0.16
RUS	2.7%	2.5%	3.4%	3.3%	4.2%	0.34
SCA	2.9%	2.9%	2.9%	3.3%	3.3%	0.26
USA	8.0%	7.8%	8.6%	9.8%	9.9%	0.27
OTH	22.6%	21.9%	25.6%	24.5%	28.0%	0.30

Note: Outcomes for 67 economies are further aggregated to 11 groups. See Appendix 3 for economy codes.

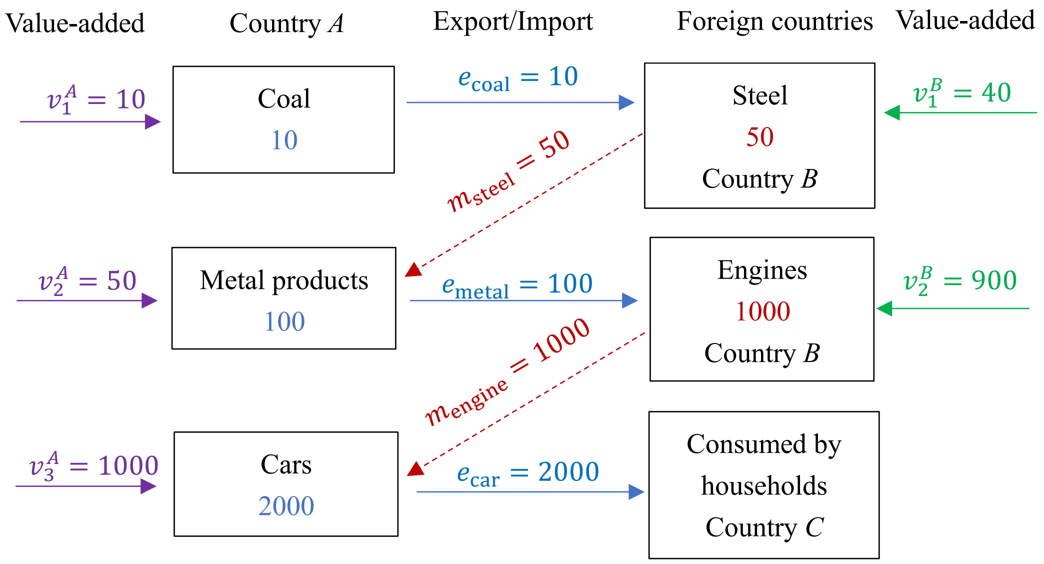

FIGURE 1 Country *A*'s international trade (unit: $million)

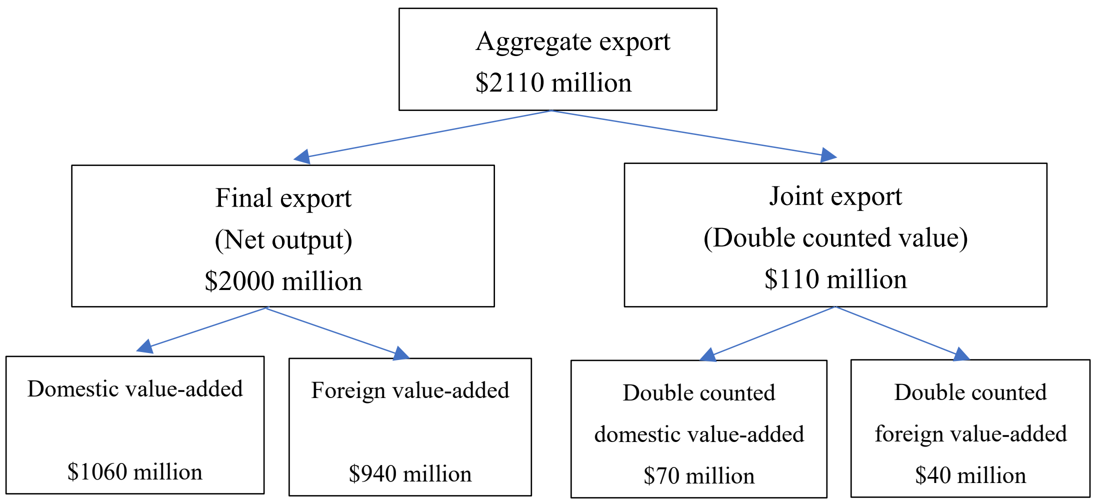

FIGURE 2 Value-added content in country *A*'s aggregate export

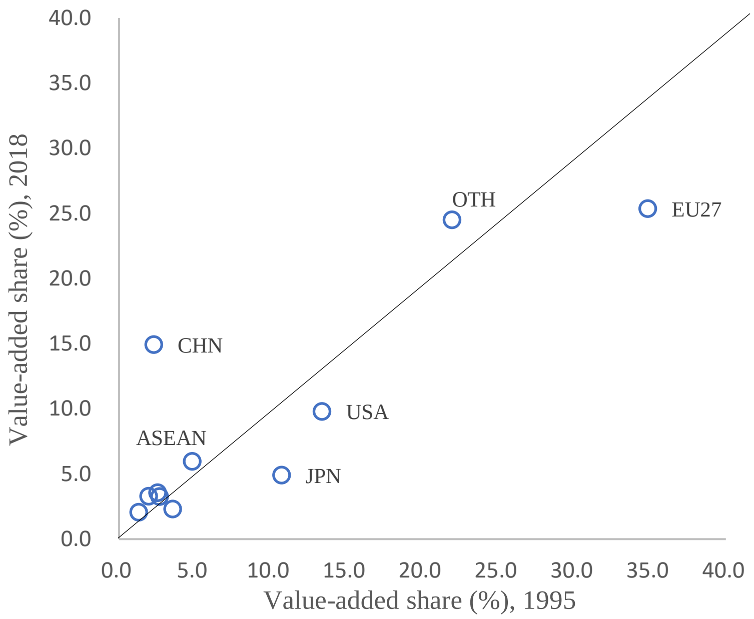

FIGURE 3 Value-added share by economy (1995 *vs*. 2018)

Note: The straight line is the 45-degree line.

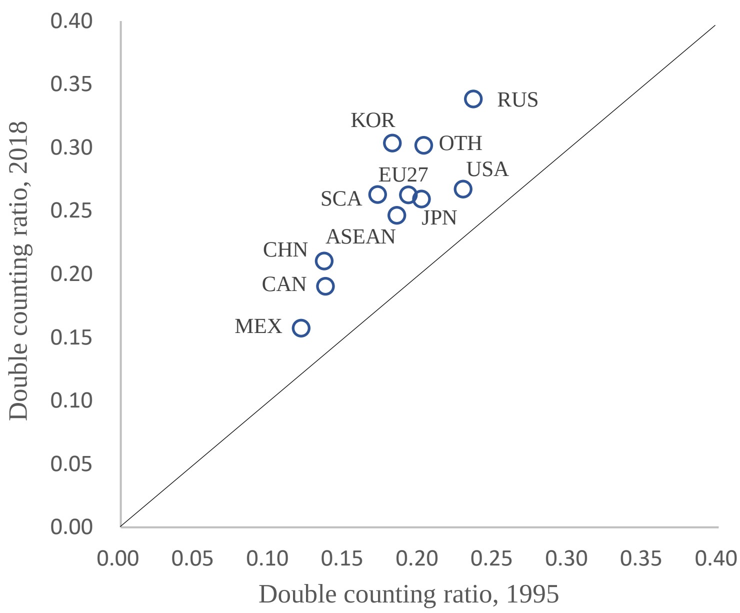

FIGURE 4 Double-counting ratio by economy (1995 *vs*. 2018)

Note: The straight line is the 45-degree line.

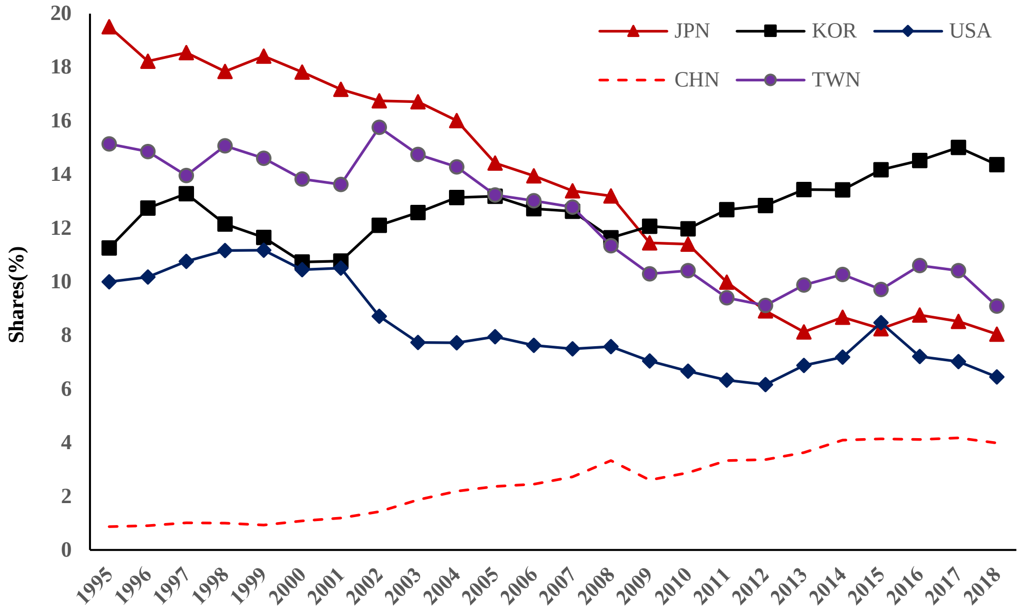

FIGURE 5 Shares by economy in world joint export associated to China's final export

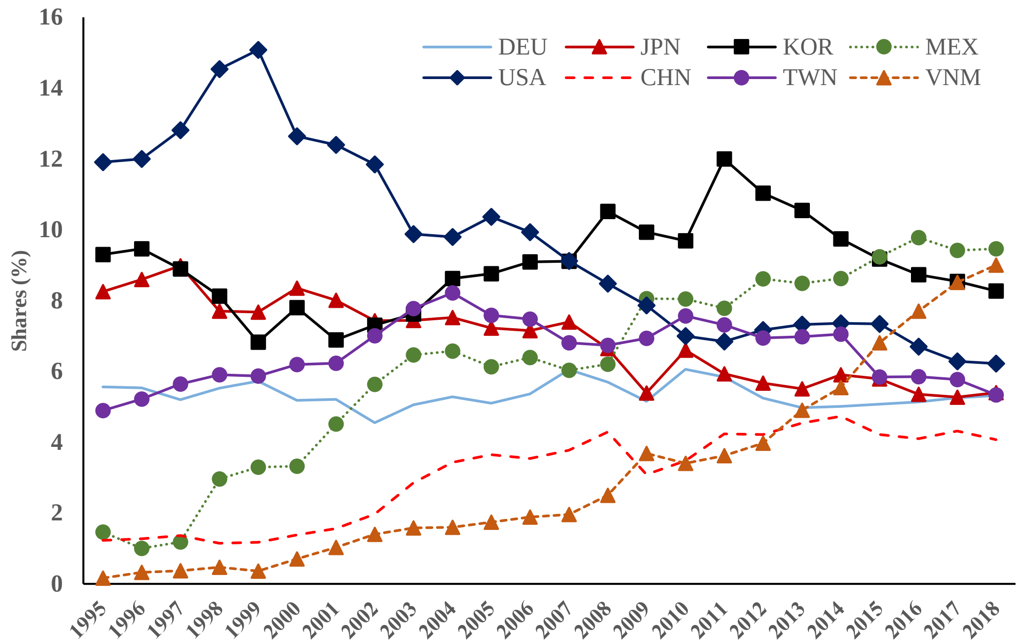

FIGURE 6 Attribution of China's joint exports to each economy's final export

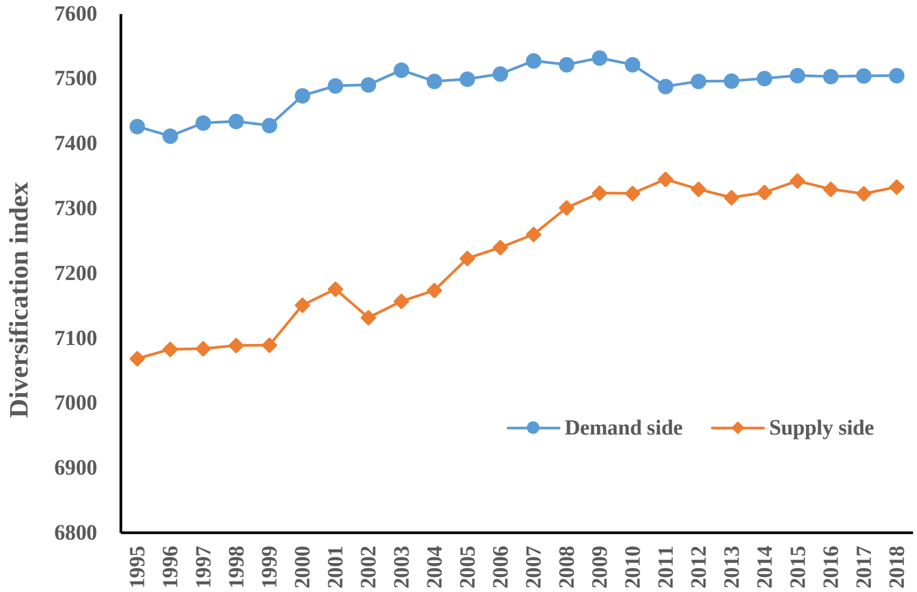

FIGURE 7 China's export production diversification

Note: The diversification index is computed based on the Herfindahl-Hirschman (HH) index. HH index $= \sum_{i=1}^{n} s_i^2$, where s_i is the share of each economy (%) contributing to China's export production (supply-side or demand-side). The diversification index is computed as 10000-HH.

APPENDIX 1.

Theorem A1: *For computing domestic value-added in bilateral export, the hypothetical extraction method is a mathematical equivalence to the method proposed in this paper.*

Proof:

First introduce the hypothetical extraction method for computing domestic value-added in bilateral export. If we are interested in economy *r*'s domestic value added in its gross export to economy *s*, we can recalculate economy *r*'s GDP by assuming that economy *r* does not export to economy *s*. The difference between economy *r*'s original GDP and the hypothetical GDP is economy *r*'s domestic value-added in its gross export to economy *s*. The assumption of economy *r* exporting nothing to economy *s* can be modeled by setting zeros for $\mathbf{A}^{\mathrm{rs}}$ and $\mathbf{f}^{\mathrm{rs}}$ in the input-output model. The hypothetical input coefficient matrix $\mathbf{A}^{*(\mathrm{rs})}$, Leontief inverse matrix $\mathbf{L}^{*(\mathrm{rs})}$ and final demand matrix $\mathbf{F}^{*(\mathrm{rs})}$ without economy *r*'s gross export to economy *s* are given by

$$\mathbf{A}^{*(rs)} = \begin{bmatrix} \mathbf{A}^{rr} & \mathbf{0} & \mathbf{A}^{rt} \\ \mathbf{A}^{sr} & \mathbf{A}^{ss} & \mathbf{A}^{st} \\ \mathbf{A}^{tr} & \mathbf{A}^{ts} & \mathbf{A}^{tt} \end{bmatrix}, \mathbf{L}^{*(rs)} = \left(\mathbf{I} - \mathbf{A}^{*(rs)}\right)^{-1}, \ \mathbf{F}^{*(rs)} = \begin{bmatrix} \mathbf{f}^{rr} & \mathbf{0} & \mathbf{f}^{rt} \\ \mathbf{f}^{sr} & \mathbf{f}^{ss} & \mathbf{f}^{st} \\ \mathbf{f}^{tr} & \mathbf{f}^{ts} & \mathbf{f}^{tt} \end{bmatrix}$$

The hypothetical GDP of economy *r* is

$$g_r^{*(rs)} = \bar{\mathbf{w}}^{r\prime}\mathbf{L}^{*(rs)}\mathbf{F}^{*(rs)}\mathbf{i}$$

In which, $\bar{\mathbf{w}}^{r\prime} = [\mathbf{w}^{r\prime} \quad \mathbf{0} \quad \mathbf{0}]$; $\mathbf{i}$ is a conformable column vector of ones.

The original GDP of economy *r* is

$$g_r = \bar{\mathbf{w}}^{r\prime}\mathbf{LFi}$$

Economy *r*'s domestic value-added in its gross export to economy *s* is given by

$$VAXD_r^{rs} = g_r - g_r^{*(rs)} = \bar{\mathbf{w}}^{\mathrm{r}\prime}\mathbf{LFi} - \bar{\mathbf{w}}^{\mathrm{r}\prime}\mathbf{L}^{*(\mathrm{rs})}\mathbf{F}^{*(\mathrm{rs})} \qquad \text{(A1)}$$

Without loss of generality, this proof focuses on economy *r*'s domestic value-added in its gross export to economy *s*.

(1) The hypothetical extraction method

According to equation (A1), economy *r*'s domestic value-added in its gross export to economy *s* is

$$VAXD_r^{rs} = g_r - g_r^{*(rs)} = \bar{\mathbf{w}}^{r'}\mathbf{LFi} - \bar{\mathbf{w}}^{r\prime}\mathbf{L}^{*(rs)}\mathbf{F}^{*(rs)} \quad \text{(A2)}$$

In addition, we have the following input-output identity

$$\begin{bmatrix}\mathbf{x}^r\\ \mathbf{x}^s\\ \mathbf{x}^t\end{bmatrix} = \begin{bmatrix}\mathbf{A}^{rr} & \mathbf{0} & \mathbf{A}^{rt}\\ \mathbf{A}^{sr} & \mathbf{A}^{ss} & \mathbf{A}^{st}\\ \mathbf{A}^{tr} & \mathbf{A}^{ts} & \mathbf{A}^{tt}\end{bmatrix}\begin{bmatrix}\mathbf{x}^r\\ \mathbf{x}^s\\ \mathbf{x}^t\end{bmatrix} + \begin{bmatrix}\mathbf{f}^{rr} & \mathbf{0} & \mathbf{f}^{rt}\\ \mathbf{f}^{sr} & \mathbf{f}^{ss} & \mathbf{f}^{st}\\ \mathbf{f}^{tr} & \mathbf{f}^{ts} & \mathbf{f}^{tt}\end{bmatrix}\mathbf{i} + \begin{bmatrix}\mathbf{e}^{rs}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}$$

i.e. $\mathbf{x} = \mathbf{A}^{*(rs)}\mathbf{x} + \mathbf{F}^{*(rs)}\mathbf{i} + \bar{\mathbf{e}}^{rs}$

Solving the equation, we have

$$\mathbf{x} = \mathbf{L}^{*(rs)}\mathbf{F}^{*(rs)}\mathbf{i} + \mathbf{L}^{*(rs)}\bar{\mathbf{e}}^{rs} \quad \text{(A3)}$$

We also have the standard input-output model

$$\mathbf{x} = \mathbf{LFi} \quad \text{(A4)}$$

Combining equations (A3) and (A4), further have

$$\mathbf{LFi} = \mathbf{L}^{*(rs)}\mathbf{F}^{*(rs)}\mathbf{i} + \mathbf{L}^{*(rs)}\bar{\mathbf{e}}^{rs}$$

Replacing $\mathbf{LFi}$ in equation (A2) by $\mathbf{L}^{*(rs)}\mathbf{F}^{*(rs)}\mathbf{i} + \mathbf{L}^{*(rs)}\bar{\mathbf{e}}^{rs}$, the new expression for $VAXD_r^{rs}$ is given by

$$VAXD_r^{rs} = \bar{\mathbf{w}}^{r\prime}\mathbf{L}^{*(rs)}\bar{\mathbf{e}}^{rs} \quad \text{(A5)}$$

Next, we introduce the relation between changes in Leontief inverse matrix and changes in input coefficient matrix by using the Sherman-Morrison-Woodbury formula (Sherman and Morrison, 1950; Woodbury, 1950; Horn and Johnson, 2013). According to the Sherman-Morrison-Woodbury formula, if submatrix $\mathbf{A}^{rs}$ of the global input coefficient matrix $\mathbf{A}$ changes to $\mathbf{0}$, the global Leontief inverse matrix $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ changes to $\mathbf{L}^{*(rs)}$ and satisfies

$$\mathbf{L}^{*(rs)} = (\mathbf{I} - \mathbf{A} + \mathbf{U}\bar{\mathbf{A}})^{-1} = \mathbf{L} - \mathbf{LU}(\mathbf{I} + \bar{\mathbf{A}}\mathbf{LU})^{-1}\bar{\mathbf{A}}\mathbf{L}$$

In which,

$$\mathbf{U} = \begin{bmatrix}\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}$$

$$\bar{\mathbf{A}} = \begin{bmatrix}\mathbf{0} & \mathbf{A}^{rs} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix}\mathbf{L}^{rr} & \mathbf{L}^{rs} & \mathbf{L}^{rt}\\ \mathbf{L}^{sr} & \mathbf{L}^{ss} & \mathbf{L}^{st}\\ \mathbf{L}^{tr} & \mathbf{L}^{ts} & \mathbf{L}^{tt}\end{bmatrix}$$

Therefore, we have

$$\mathbf{U\bar{A}} = \begin{bmatrix} \mathbf{0} & \mathbf{A}^{rs} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{\bar{A}LU} = \mathbf{\bar{A}L}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]} = \begin{bmatrix} \mathbf{A}^{rs}\mathbf{L}^{sr} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$(\mathbf{I} + \mathbf{\bar{A}LU})^{-1} = \left(\mathbf{I} + \mathbf{\bar{A}L}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}\right)^{-1} = \begin{bmatrix} (\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \quad \text{(A6)}$$

By straightforward matrix operations, we have

$$\mathbf{L}^{*(rs)}\mathbf{\bar{e}}^{rs} = [\mathbf{L} - \mathbf{LU}(\mathbf{I} + \mathbf{\bar{A}LU})^{-\mathbf{1}}\mathbf{\bar{A}L}]\mathbf{\bar{e}}^{rs}$$
$$= \begin{bmatrix} \mathbf{L}^{rr}(\mathbf{I} - (\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{A}^{rs}\mathbf{L}^{sr})\mathbf{e}^{rs} \\ \mathbf{L}^{sr}(\mathbf{I} - (\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{A}^{rs}\mathbf{L}^{sr})\mathbf{e}^{rs} \\ \mathbf{L}^{tr}(\mathbf{I} - (\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{A}^{rs}\mathbf{L}^{sr})\mathbf{e}^{rs} \end{bmatrix}$$

As

$$\mathbf{I} - (\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{A}^{rs}\mathbf{L}^{sr} = (\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}(\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr}) - (\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{A}^{rs}\mathbf{L}^{sr}$$
$$= (\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}(\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr} - \mathbf{A}^{rs}\mathbf{L}^{sr})$$
$$= (\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}$$

So,

$$\mathbf{L}^{*(rs)}\mathbf{\bar{e}}^{rs} = \begin{bmatrix} \mathbf{L}^{rr}(\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{e}^{rs} \\ \mathbf{L}^{sr}(\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{e}^{rs} \\ \mathbf{L}^{tr}(\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{e}^{rs} \end{bmatrix}$$

Finally, according to equation (A5) we have

$$VAXD_r^{rs} = \mathbf{\bar{w}}^{r\prime}\mathbf{L}^{*(rs)}\mathbf{\bar{e}}^{rs}$$
$$= [\mathbf{w}^{r\prime} \quad \mathbf{0} \quad \mathbf{0}] \begin{bmatrix} \mathbf{L}^{rr}(\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{e}^{rs} \\ \mathbf{L}^{sr}(\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{e}^{rs} \\ \mathbf{L}^{tr}(\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{e}^{rs} \end{bmatrix}$$
$$= \mathbf{w}^{r\prime}\mathbf{L}^{rr}(\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{e}^{rs} \quad \text{(A7)}$$

(2) Method proposed in this paper

We have

$$VAXD_r^{rs} = \mathbf{\bar{w}}^{r\prime}\mathbf{L\bar{y}}^{rs} = \mathbf{\bar{w}}^{r\prime}\mathbf{L}\left(\mathbf{I} + \mathbf{\bar{A}L}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}\right)^{-1}\mathbf{\bar{e}}^{rs}$$

In which, $\mathbf{\bar{w}}^{r\prime} = [\mathbf{w}^{r\prime} \quad \mathbf{0} \quad \mathbf{0}]$.

Recall equation (A6),

$$VAXD_r^{rs} = [\mathbf{w}^{r\prime} \quad \mathbf{0} \quad \mathbf{0}] \begin{bmatrix} \mathbf{L}^{rr} & \mathbf{L}^{rs} & \mathbf{L}^{rt} \\ \mathbf{L}^{sr} & \mathbf{L}^{ss} & \mathbf{L}^{st} \\ \mathbf{L}^{tr} & \mathbf{L}^{ts} & \mathbf{L}^{tt} \end{bmatrix} \begin{bmatrix} (\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{e}^{rs} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$= \mathbf{w}^{r\prime}\mathbf{L}^{rr}(\mathbf{I} + \mathbf{A}^{rs}\mathbf{L}^{sr})^{-1}\mathbf{e}^{rs} \tag{A8}$$

Comparing with equation (A7), equation (A8) is the same expression given by the hypothetical extraction method. Therefore, the hypothetical extraction method is a mathematical equivalence to our method for computing the domestic value-added in bilateral export. ■

For the domestic value-added in an economy's aggregate, we find that Theorem A1 also holds. The proof resembles the proof of Theorem A1.

APPENDIX 2. The Proof for Theorem 1

Theorem 1: *The value-added of an economy created in global production of gross world export and in global production of aggregate export of this economy are identical.*

Proof:

Let $\mathbf{A}^D = \begin{bmatrix} \mathbf{A}^{rr} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}^{tt} \end{bmatrix}$ represent the domestic input coefficient matrix. It consists of the domestic input coefficient matrices of economies *r*, *s* and *t*. Then, in the setting of gross world export decomposition $\bar{\mathbf{A}} = \mathbf{A} - \mathbf{A}^D$.

In gross world export decomposition, we further have

$$\left(\mathbf{I} + \bar{\mathbf{A}}\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}\right)^{-1} = [\mathbf{I} + (\mathbf{A} - \mathbf{A}^D)\mathbf{L}]^{-1} = (\mathbf{I} + \mathbf{AL} - \mathbf{A}^D\mathbf{L})^{-\mathbf{1}}$$
$$= (\mathbf{L} - \mathbf{A}^D\mathbf{L})^{-\mathbf{1}} = [(\mathbf{I} - \mathbf{A}^D)\mathbf{L}]^{-\mathbf{1}}$$
$$= \mathbf{L}^{-\mathbf{1}}(\mathbf{I} - \mathbf{A}^D)^{-\mathbf{1}}$$

The value-added of each economy in gross world export satisfies

$$\mathbf{v_y} = \widetilde{\mathbf{W}}\mathbf{L}\left(\mathbf{I} + \bar{\mathbf{A}}\bar{\mathbf{L}}_{[\boldsymbol{\beta},\boldsymbol{\alpha}]}\right)^{-1}\bar{\mathbf{e}} = \widetilde{\mathbf{W}}\mathbf{L}\mathbf{L}^{-\mathbf{1}}(\mathbf{I} - \mathbf{A}^D)^{-\mathbf{1}}\bar{\mathbf{e}} = \widetilde{\mathbf{W}}(\mathbf{I} - \mathbf{A}^D)^{-\mathbf{1}}\bar{\mathbf{e}}$$
$$= \begin{bmatrix} \mathbf{w}^{r\prime} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{w}^{s\prime} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{w}^{t\prime} \end{bmatrix} \begin{bmatrix} (\mathbf{I} - \mathbf{A}^{rr})^{-\mathbf{1}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (\mathbf{I} - \mathbf{A}^{ss})^{-\mathbf{1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (\mathbf{I} - \mathbf{A}^{tt})^{-\mathbf{1}} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{e}}^r \\ \bar{\mathbf{e}}^s \\ \bar{\mathbf{e}}^t \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{w}^{r\prime}(\mathbf{I} - \mathbf{A}^{rr})^{-\mathbf{1}}\bar{\mathbf{e}}^r \\ \mathbf{w}^{s\prime}(\mathbf{I} - \mathbf{A}^{ss})^{-\mathbf{1}}\bar{\mathbf{e}}^s \\ \mathbf{w}^{t\prime}(\mathbf{I} - \mathbf{A}^{tt})^{-\mathbf{1}}\bar{\mathbf{e}}^t \end{bmatrix}$$

$\mathbf{w}^{r\prime}(\mathbf{I} - \mathbf{A}^{rr})^{-\mathbf{1}}\bar{\mathbf{e}}^r$, $\mathbf{w}^{s\prime}(\mathbf{I} - \mathbf{A}^{ss})^{-\mathbf{1}}\bar{\mathbf{e}}^s$ and $\mathbf{w}^{t\prime}(\mathbf{I} - \mathbf{A}^{tt})^{-\mathbf{1}}\bar{\mathbf{e}}^t$ are the value-added of economies *r*, *s*, and *t* in their aggregate exports computed based on national input-output tables (identical with the outcomes computed based on inter-country input-output tables, Los et al., 2016), respectively. Therefore, the value-added of an economy created in global production of gross world export and in global production of aggregate export of this economy are identical. ■

APPENDIX 3. Economy codes

	Economy
ASEAN	BRN (Brunei Darussalam); IDN (Indonesia); KHM (Hong Kong, China); LAO (Lao People's Democratic Republic); MYS (Malaysia); MMR (Myanmar); PHL (Philippines); SGP (Singapore); THA (Thailand); VNM (Viet Nam)
CAN	Canada
CHN	China
EU27	AUT (Austria); BEL (Belgium); CZE (Czech Republic); DNK (Denmark); EST (Estonia); FIN (Finland); FRA (France); DEU (Germany); GRC (Greece); HUN (Hungary); IRL (Ireland); ITA (Italy); LVA (Latvia); LTU (Lithuania); LUX (Luxembourg); NLD (Netherlands); POL (Poland); PRT (Portugal); SVK (Slovak Republic); SVN (Slovenia); ESP (Spain); SWE (Sweden); BGR (United Kingdom); CYP (Cyprus); HRV (Croatia); MLT (Malta); ROU (Romania)
JPN	Japan
KOR	Korea
MEX	Mexico
RUS	Russia
SCA	CHL (Chile); ARG (Argentina); BRA (Brazil); COL (Colombia); CRI (Costa Rica); PER (Peru)
USA	United States
OTH	AUS (Australia); ISR (Israel); NZL (New Zealand); TUR (Turkey); IND (India); KAZ (Kazakhstan); MAR (Morocco); SAU (Saudi Arabia); TUN (Tunisia); ZAF (South Africa); ROW