

## **A Montgomery Additive Decomposition with disaggregate factors within the Leontief Inverse.**

Topic: Input-Output Theory and Methodology - VI

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Structural Decomposition Analysis (SDA) of input-output tables assists in identifying the leading drivers in changes throughout the time of a given variable. There are multiple ways of performing a given decomposition, each having its own advantages and drawbacks. In the case of Additive SDA, de Boer and Rodrigues (2020) recommend the Montgomery Decomposition or the Bennet Indicator. On the one hand, since the Montgomery method is simpler than the Bennet Indicator, demanding only one decomposition form, it should be a better choice for the general practice of SDA, except when 'changes-in-sign robustness' is required. On the other hand, identifying the contributions of nested factors inside a Leontief Inverse or different kinds of inverse matrices—such as the Miyazawa Matrix—is, until now, bounded to the Bennet method. Despite its relevance to the input-output field, the decomposition of 'nested factors' did not receive the same highlights as other themes of SDA, with little development. This paper continues the discussion provided by Muradov (2021), in which different methods to decompose the Leontief Decompose are compared through their results. We evaluate his considerations in parallel with those of Rose and Casler (1996), specifically the Multiplicative and Additive Identity Splitting methods. We connect both works by demonstrating how the Additive Identity Splitting method can be a 'complete' decomposition, without any residual term, as displayed in Muradov (2021). We then introduce a new decomposition method that allows us to consider the contributions of the nested factors inside the Leontief Inverse while using the Montgomery Decomposition. This new method first calculates the weights of the contributions to the changes of each cell of the Inverse Matrix by those of its nested factors. Then these weights are used at the Montgomery decomposition for decomposing the contributions of the Inverse Matrix to the changes of the decomposed variable. The new method is compatible with the Additive and Multiplicative Identity Splitting methods or all of the shortcuts tested by Muradov (2021). The proposed method requires a smaller number of decomposition forms than other complete methods, thus alleviating the computational burden of the SDA. The limitations of this new method are that: (1) it is not change-in-sign robust, as any other Montgomery Decomposition; (2) it requires that if the change of a cell of the inverse matrix is zero, all of its nested factors must also have contributions equal to 0 to that cell; which is an improbable case. Moreover, we illustrate how, given the aforementioned limitations, the traditional hierarchical decompositions, using Bennet method, can also be seen as a distribution of the nested factors proportionally to their contributions to their nest matrix. Therefore, we demonstrate that the values from the traditional hierarchical decomposition and those of our proposed method differ because of the difference between the Bennet and Montgomery methods at the first-level decompositions.