Waste Treatment in Physical Input-Output Analysis

Erik Dietzenbacher

Abstract:

When compared to monetary input-output tables (MIOTs), a distinctive feature of physical input-output tables (PIOTs) is that they include the generation of waste as part of a consistent accounting framework. As a consequence, however, physical input-output analysis thus requires that the treatment of waste is explicitly taken into account, because otherwise the results will be grossly underestimated. The treatment of waste has recently led to an interesting methodological debate. This paper reviews the discussion and introduces a new alternative. This alternative reconciles the existing methods and enables us to obtain additional information that cannot be derived from the other methods.

Keywords: Waste, Physical input-output tables, Land use multipliers, Input-output analysis

JEL Classification Codes: D57, Q15

Address:

Erik Dietzenbacher, Faculty of Economics, University of Groningen, PO Box 800,

9700 AV Groningen, The Netherlands.

e-mail: e.dietzenbacher@eco.rug.nl.

tel: +31 50 363 3813, fax: +31 50 363 7337

Acknowledgements:

1. Introduction

Recent years have witnessed the first publications of physical input-output tables (Kratterl and Kratena, 1990; Kratena *et al.*, 1992; Konijn *et al.*, 1997; Stahmer *et al.*, 1997; Pedersen, 1999; Mäenpää, 2002; Hoekstra, 2003) and analyses thereof (Konijn *et al*., 1997; Stahmer, 2000; Strassert, 2001). Whereas monetary input-output tables (MIOTs) record all transactions in money terms, such as billion dollars, physical input-output tables (PIOTs) measure all deliveries in physical units, such as million tons. PIOTs seem to become an important tool in input-output analysis, in particular in fields where material flows and their links are paramount, such as in environmental, resource and energy economics.

In general, there is no simple conversion between an MIOT and a PIOT, even if full information with respect to prices were available. The reason is that the sectors that are distinguished in an input-output table consist themselves of many subsectors. Suppose that the deliveries from sector i to sector j consist to a large extent of deliveries by subsector i_1. The price of the deliveries from i to j will then be determined primarily by the price of product i_1. If, in contrast, the deliveries from sector i to sector h largely consist of deliveries by subsector i_2, the price will be close to the price of product i_2. So, converting the monetary deliveries from i to j into physical units would require a different price than converting the monetary deliveries from i to h. The simple conversion as based on the average price in sector i is therefore inappropriate. Typically, information for subsectors is not readily available. As a consequence, the production structure in physical terms may be radically different from the structure in monetary terms. For example, sector h might use (per billion dollars of its output) a large amount (in billion dollars) of product i, whereas it uses only a small physical amount of product i per million tons of its output, and for sector j this might well be exactly the other way. If we are interested in the material flows involved in the national production processes, it thus seems that PIOTs are more appropriate datasets than MIOTs are.

Another distinctive feature of PIOTs is that they provide detailed information on the generation of waste and they do so in a consistent accounting framework. Appending all sorts of extraneous information (such as labor inputs, CO_2 emissions, land use, and also waste) to MIOTs has a long tradition. In PIOTs, however, waste is not appended but is a central part of the accounting framework. This clearly is an advantage of a PIOT as a dataset. At the same time, this implies that in a physical input-output *analysis*, the treatment of waste becomes an important aspect. Very recently, this has led to an interesting methodological debate triggered by Hubacek and Giljum (2003), with a comment by Suh (2003) and a reply by Giljum *et al.* (2003), and a follow-up by Giljum and Hubacek (2004).

The purpose of this note is to review the discussion, to provide some additional insights, and to reconcile the suggested methods. This leads me to propose an alternative approach. The plan of the paper is as follows. The next section presents the method for waste treatment that was originally proposed in Hubacek and Giljum (HG, 2003) and the adapted version in Giljum and Hubacek (GH, 2004). In Section 3, I will focus on issues of production in input-output analysis and will arrive at the approach as proposed by Suh (2003). Section 4 deals with the reply by Giljum *et al.* (GHS, 2003). Section 5 shows how the approaches of GH and Suh can be reconciled and presents the alternative method. Section 6 concludes the paper with an evaluation.

2. Waste treatment in Hubacek and Giljum (2003) and in Giljum and Hubacek (2004)

The starting point is the PIOT as given in HG. In what follows I will adopt their notation and also their empirical case for Germany. The PIOTs are given in Tables 1 and 2. The $n \times n$ matrix $\mathbf{Z}$ denotes the intermediate deliveries of secondary inputs, $\mathbf{d}$ the vector of domestic final demand, $\mathbf{e}$ the vector of foreign final demand, $\mathbf{w}$ the vector of waste, and $\mathbf{x}$ the vector of (gross) output.[1] The row vector of primary material inputs is given by $\mathbf{r}'$. All entries are given in million tons (mt). Often the input-output table is appended by

[1] Matrices are indicated by bold, upright capital letters; vectors by bold, upright lower case letters, and scalars by italicized lower case letters. Vectors are columns by definition, so that row vectors are obtained by transposition, indicated by a prime (e.g. $\mathbf{x}'$). A diagonal matrix with the elements of vector $\mathbf{x}$ on its main diagonal and all other entries equal to zero is indicated by a circumflex (e.g. $\hat{\mathbf{x}}$).

additional information, such as the input of labor (in worker years or in money terms) per sector, the emission of CO_2, the expenses for R&D, or the appropriation of land (as in HG). The matrix of input coefficients is obtained as

$$\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} \tag{1}$$

where its typical element $a_{ij} = z_{ij} / x_j$ indicates the input (in mt) from sector *i* per mt of output in sector *j*. In the same way, the primary material input coefficients are given by $\mathbf{b}' = \mathbf{r}'\hat{\mathbf{x}}^{-1}$ with $b_j = r_j / x_j$ indicating the input of primary material per mt of output in sector *j*. Using $\mathbf{s}$ for the vector of land appropriation (in hectares), we have for the land appropriation coefficients

$$\mathbf{c}' = \mathbf{s}'\hat{\mathbf{x}}^{-1} \tag{2}$$

INSERT TABLES 1 AND 2

From Table 1, it is easily seen that $\mathbf{x} = \mathbf{Z1} + \mathbf{d} + \mathbf{e} + \mathbf{w} = \mathbf{Ax} + \mathbf{d} + \mathbf{e} + \mathbf{w}$, where $\mathbf{1}$ indicates the summation vector consisting of ones and where we have used $\mathbf{Z} = \mathbf{A}\hat{\mathbf{x}}$. Its solution is given by $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}(\mathbf{d} + \mathbf{e} + \mathbf{w}) = \mathbf{M}(\mathbf{d} + \mathbf{e} + \mathbf{w})$. Here,

$$\mathbf{M} = (\mathbf{I} - \mathbf{A})^{-1} \tag{3}$$

denotes the multiplier matrix. Its typical element m_{ij} gives the extra output in sector *i* that is (directly and indirectly) required to generate one mt of final demand or waste in sector *j*. Using these multipliers we are able to determine, for example, how much primary materials or land is used by the manufacturing sector to satisfy the domestic final demand for services.[2] The typical element (*i*, *j*) of the matrix $\hat{\mathbf{c}}\mathbf{M}$ gives the use of land in sector *i*, that is required to generate one mt of final demand or waste in sector *j*. Hence, the typical

[2] Answering this question has recently led to some confusion, see Bicknell *et al*. (1998) and Ferng (2001).

element (i, j) of the matrix $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{d}}$ gives the use of land in sector i, that is required to generate the actual domestic final demand in sector j (i.e. d_j). Using this matrix, two frequently posed questions may be readily answered. First, how much land use takes place in each sector and, second, how much land use can be imputed to each sectoral domestic final demand. The jth element of the row vector of column sums (i.e. $\mathbf{1}'\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{d}} = \mathbf{c}'\mathbf{M}\hat{\mathbf{d}}$) gives the total land use (i.e. in all sectors) required to satisfy domestic final demand d_j. The ith element of the column vector of row sums (i.e. $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{d}}\mathbf{1} = \hat{\mathbf{c}}\mathbf{M}\mathbf{d}$) gives the land use in sector i required to satisfy all domestic final demands.

The sectoral land use necessary to satisfy the final demands (both domestic and foreign) therefore equals $\hat{\mathbf{c}}\mathbf{M}(\mathbf{d}+\mathbf{e})$. HG then argue that this answer yields a serious underestimation of the "true" land use that should be imputed to the final demands. Note that $\hat{\mathbf{c}}\mathbf{M}(\mathbf{d}+\mathbf{e}+\mathbf{w}) = \hat{\mathbf{c}}\mathbf{x} = \hat{\mathbf{s}}\hat{\mathbf{x}}^{-1}\mathbf{x} = \mathbf{s}$, i.e. the actual land appropriation (given in Table 1 as row vector). So, the actual land appropriation can be divided (by imputation) over domestic final demand **d**, exports **e**, and waste **w**. It is thus obvious that the final demands only, do not require all the actually appropriated land (because a substantial part can be imputed to waste generation). Unlike satisfying domestic final demands or exports, the generation of waste is not an aim of the production process. It is a consequence of production and thus of satisfying final demands. Therefore it seems reasonable to distribute the waste over domestic and foreign final demands. So, in imputing land use to domestic final demand (or to exports) part of the land use involved in generating waste should be included.

The distribution that HG propose is such that the total waste is distributed over domestic and foreign final demand. First, they define the ratio of total waste to total primary material inputs as $\rho = (\mathbf{1}'\mathbf{w}/\mathbf{r}'\mathbf{1})$. Next the primary material inputs that end up as waste are given by the vector $\bar{\mathbf{r}}' = \rho\mathbf{r}'$. Finally this vector is distributed over the two final demand categories, according to their shares. This yields

$$d_i^{ext} = d_i + \frac{d_i}{d_i + e_i}\bar{r}_i \quad \text{and} \quad e_i^{ext} = e_i + \frac{e_i}{d_i + e_i}\bar{r}_i \qquad (4)$$

We thus obtain the following procedure for waste treatment.

Approach of HG: Starting from Table 1, using the definitions in (1) – (3), let the extended final demand vectors $\mathbf{d}^{ext}$ and $\mathbf{e}^{ext}$ be given by (4). The land use in sector *i* imputed to all exports is given by the *i*th row sum of the matrix $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{e}}^{ext}$. The total land use imputed to the exports of sector *j* are given by the *j*th column sum of $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{e}}^{ext}$.

Unfortunately, this approach yields serious inconsistencies. Note that if we take the sum of the vectors we have $\mathbf{d}^{ext} + \mathbf{e}^{ext} = \mathbf{d} + \mathbf{e} + \rho\mathbf{r}$, which – in general – will be different from **d** + **e** + **w**. Only the *total* waste is distributed over domestic and foreign final demand, as follows from $\mathbf{1}'(\mathbf{d}^{ext} + \mathbf{e}^{ext}) = \mathbf{1}'(\mathbf{d} + \mathbf{e} + \rho\mathbf{r}) = \mathbf{1}'(\mathbf{d} + \mathbf{e} + \mathbf{w})$, because $\rho\mathbf{1}'\mathbf{r} = (\mathbf{1}'\mathbf{w}/\mathbf{r}'\mathbf{1})\mathbf{1}'\mathbf{r} = \mathbf{1}'\mathbf{w}$. Consider now $\hat{\mathbf{c}}\mathbf{M}(\mathbf{d} + \mathbf{e} + \mathbf{w}) = \hat{\mathbf{c}}\mathbf{x} = \mathbf{s}$, which is the actual land appropriation as recorded in the Tables 1 and 2. Because $\mathbf{d}^{ext} + \mathbf{e}^{ext} \neq \mathbf{d} + \mathbf{e} + \mathbf{w}$, it is also likely that $\hat{\mathbf{c}}\mathbf{M}(\mathbf{d}^{ext} + \mathbf{e}^{ext}) \neq \hat{\mathbf{c}}\mathbf{M}(\mathbf{d} + \mathbf{e} + \mathbf{w}) = \mathbf{s}$. Therefore, applying the HG approach, generally yields answers for the sectoral land use that do not sum to the actually recorded land use. Even the total land use found by imputation may be different from the actual total land use.

The results for the HG approach are given in Table 3. If the domestic final demands and the exports are added to the distributed waste in columns (1) and (2), we arrive at Table 11 in HG.[3] Note that 24,901,392 million hectares of land are appropriated according to the total in column (12), whereas in reality 24,484,142 million hectares are used (the sum of land appropriation in Table 2). This is caused by the fact that the distribution of waste in column (3) differs from the actual disposal to nature in Table 2, only their totals (4,251.8) are the same.

INSERT TABLE 3

[3] Note that small differences occur, because the PIOT was slightly adapted to make it internally consistent (see also Suh, 2003).

In GH, this inconsistency has been removed. The waste in sector *i* is distributed to domestic and foreign final demand proportional to the size of each of the two categories. The new vectors of extended domestic final demand ($\mathbf{d}^{ext}$) and exports ($\mathbf{e}^{ext}$) are given by

$$d_i^{ext} = d_i + \frac{d_i}{d_i + e_i} w_i \quad \text{and} \quad e_i^{ext} = e_i + \frac{e_i}{d_i + e_i} w_i \tag{5}$$

The approach of GH may be summarized as follows.

Approach of GH: Starting from Table 1, using the definitions in (1) – (3), let the extended final demand vectors $\mathbf{d}^{ext}$ and $\mathbf{e}^{ext}$ be given by (5). The land use in sector *i* imputed to all exports is given by the *i*th row sum of the matrix $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{e}}^{ext}$. The total land use imputed to the exports of sector *j* are given by the *j*th column sum of $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{e}}^{ext}$.

The results for the GH approach are given in Table 4. Note that columns (1) - (3) show that the waste in each sector is distributed over domestic and foreign final demand. Column (3) is the sum of columns (1) and (2) and equals the column with disposals to nature in Table 2. As a consequence, the results are consistent now. That is, the sum – in column (12) – of land used in sector *i* and imputed to domestic final demand – i.e. column (7) – and land used in sector *i* and imputed to exports – i.e. column (11) – equals the land appropriation as given in Table 2

INSERT TABLE 4

3. Waste treatment in Suh (2003)

Input-output tables were developed to describe the production structure of an economy. The SNA (see CEC/IMF/OECD/UN/WB, 1993) gives a very explicit definition of production: “All goods and services produced as outputs must be such that they can be sold on markets or at least be capable of being provided by one unit to another, with or without charge. The System includes within the production boundary all production actually destined for the market, whether for sale or barter.” (p. 4, par. 1.20). So generating waste clearly does *not* belong to production, it is a consequence of production. Therefore, waste is not an output, it is merely an outflow of the production process and linked to productive activities. Doubling the production in a sector would double the generation of waste.

Production aims at making usable outputs that are sold to other sectors to be used in their production processes or that are sold to be used for final demand purposes (either domestically or abroad). Waste is not a part of the usable output and should be treated as a necessary “input” for production. Just like the production of a sector requires per unit of output a certain amount of labor and land, it also requires that a certain amount of waste is generated (in the same way as it requires that a certain amount of, for example, CO_2 is emitted).

Table 1 is entirely correct from a bookkeeping perspective. Each of the first *n* rows describes how many mt come out of this sector. Part of this are the usable outputs, which are sold to production sectors and sold for final demand purposes. The other part is the outflow of waste, which is not accounted as production. For an *analysis* of production, Table 1 is therefore somewhat misleading because only usable outputs belong to production, waste is only a consequence of production. The possibility of generating waste is thus a requirement for production. Input-output tables typically record the output (i.e. production in the SNA sense) of each sector as the total of its deliveries. Tables 5 and 6 are obtained from Tables 1 and 2, by recording the sectoral waste as a negative input instead of as an output. They thus seem to better reflect the production principle and to be more appropriate for input-output analysis. The vector of usable output is denoted by $\bar{\mathbf{x}}$. The material balance in each sector was given in Tables 1 and 2 by the rows, in Tables 5 and 6 it is readily obtained from the columns. For example, in sector 3 (Services) it is seen that the production of usable outputs amounts to 160.8 mt, while 1000.4 mt of waste are generated due to the production activities in this sector. The total amount of materials emanating from sector 3 is thus 1161.2 mt. Of course, $\mathbf{x} = \bar{\mathbf{x}} + \mathbf{w}$.[4]

[4] It should be stressed that the two alternative ways of recording waste is in one very important aspect different from the two ways of recording imports in input-output tables (in contrast to what Giljum *et al.*,

INSERT TABLES 5 AND 6

Suh's approach is the input-output analysis as based on Tables 5 and 6. Define

$$\overline{\mathbf{A}} = \mathbf{Z}\hat{\mathbf{x}}^{-1}, \; \overline{\mathbf{M}} = (\mathbf{I} - \overline{\mathbf{A}})^{-1}, \text{ and } \overline{\mathbf{c}}' = \mathbf{s}'\hat{\mathbf{x}}^{-1} \tag{6}$$

Suh's approach: Starting from Table 5, using the definitions in (6), the land use in sector i imputed to all exports is given by the ith row sum of the matrix $\hat{\overline{\mathbf{c}}}\overline{\mathbf{M}}\hat{\mathbf{e}}$. The total land use imputed to the exports of sector j are given by the jth column sum of $\hat{\overline{\mathbf{c}}}\overline{\mathbf{M}}\hat{\mathbf{e}}$.

The results for Suh's approach are given in Table 7. Note that just like GH's approach (and unlike HG's approach), Suh's approach is consistent. That is, the total land use in each sector as given in column (9) equals the land appropriation in Table 2. When Suh's approach is compared to GH's approach (neglecting HG's approach because of its inconsistency), the differences are striking. Very little land use is imputed to the final demands of Agriculture in Suh's approach (less than 10% of the imputed land use in GH's approach). In contrast, much more (more than twice as much) land use is imputed to the final demands of Manufacturing by Suh than by GH. I will come back to this later.

INSERT TABLE 7

4. The reply by Giljum *et al.* (2003)

In this section, I will discuss the reply by Giljum *et al.* (GHS, 2003). First, GHS observe that in Suh's approach some elements in the matrix $\overline{\mathbf{A}}$ of input coefficients are larger than one. By no means, however, does this imply that the matrix $\overline{\mathbf{A}}$ would be inappropriate for an input-output analysis. A simple example may illustrate this. Consider

---

2003, suggest). The alternatives in the case of waste do affect the output, whereas the alternatives in the an arbitrary input-output table and suppose that the units of measurement are changed in sector 1. That is, suppose that the outputs of sector 1 are measured in thousand tons instead of in mt (or, for an MIOT, that they are measured in thousand dollars instead of million dollars). As a consequence, the entire first (and only the first) row of the input-output table is multiplied by 1000. The output, all intermediate deliveries and the final demands in sector 1 become 1000 times as large as they were before the change. Note that a simple summation within the columns of the input-output table no longer makes sense, because the units of measurement are no longer the same this would imply adding apples and oranges. The effects of this change on the input matrix are as follows. Element (1, 1) remains the same, but elements (1, 2), …, (1, *n*) all become 1000 times larger, and elements (2, 1), …, (*n*, 1) become 1000 times smaller. It may be expected that several elements in the first row are now (much) larger than one. Still, we are dealing with exactly the same economy. Really nothing has changed in the production structure, except that – for one reason or another – we decided to measure the deliveries of product 1 in thousands of tons instead of in mt. Although this particular example of changing the unit of measurement may be a silly exercise, it certainly is a valid exercise. The outcomes of an input-output analysis remain the same (except that the findings for sector 1 will be 1000 times as large).

Although the misconception is not uncommon, it is *not* necessary that all input coefficients are smaller than one for an input-output analysis to be viable. The only requirements are that the model yields a non-negative output vector as a solution for any given non-negative final demand vector. This is also known as the existence problem (see, for example, Takayama, 1985, Chapter 4). Consider a numerically given input-output table (no matter whether a PIOT or MIOT), as in Table 8. Here $\mathbf{Z}_0$ is the matrix of intermediate deliveries, $\mathbf{f}_0$ the vector of final demands (i.e. **d**+**e**+**w** in case of Table 1 and **d**+**e** in case of Table 5), $\mathbf{v}'_0$ the row vector of primary inputs (i.e. $\mathbf{r}'$ for Table 1 and $\mathbf{r}'-\mathbf{w}'$ for Table 5), and $\mathbf{x}_0$ the output vector (i.e. **x** in Table 1 and $\bar{\mathbf{x}}$ in Table 5). The accounting equations yield $\mathbf{x}_0 = \mathbf{Z}_0\mathbf{1} + \mathbf{f}_0$ and $\mathbf{x}'_0 = \mathbf{1}'\mathbf{Z}_0 + \mathbf{v}'_0$. The matrix of input coefficients is given by $\mathbf{A}_0 = \mathbf{Z}_0\hat{\mathbf{x}}_0^{-1}$. The first accounting equation then yields

case of imports yield the same output.

$\mathbf{x}_0 = \mathbf{A}_0\mathbf{x}_0 + \mathbf{f}_0$. The input-output model assumes fixed coefficients and asks whether for any *arbitrary* final demand vector $\mathbf{f}$ a solution $\mathbf{x}$ exists such that $\mathbf{x} = \mathbf{A}_0\mathbf{x} + \mathbf{f}$? Mathematically the answer is affirmative, provided that the matrix $(\mathbf{I} - \mathbf{A}_0)$ is non-singular. In that case the solution is given by $\mathbf{x} = (\mathbf{I} - \mathbf{A}_0)^{-1}\mathbf{f} = \mathbf{M}_0\mathbf{f}$. However, the solution must also be economically meaningful, which yields the existence problem. That is, for any non-negative vector $\mathbf{f} \geq 0$, is there a non-negative vector $\mathbf{x} \geq 0$ such that $\mathbf{x} = \mathbf{A}_0\mathbf{x} + \mathbf{f}$? The answer to this existence problem is positive if $\mathbf{M}_0$ exists and is positive.

INSERT TABLE 8

The existence problem and its solution have received a lot of attention in the early days of input-output analysis when computing power was still very limited (see Takayama, 1985, for an excellent overview of the mathematical aspects of input-output analysis). I will not go into the details of this discussion and only present a sufficient condition that is easily checked in practical cases. It turns out that if the analysis uses a numerically given input-output table as its starting-point, existence can usually be guaranteed.

Theorem 1. Consider Table 8 and suppose that $\mathbf{Z}_0$ is positive. If $\mathbf{v}_0'$ is non-negative and at least one of its elements is positive, then $\mathbf{M}_0$ exists and is positive. Also, if $\mathbf{f}_0$ is non-negative and at least one of its elements is positive, then $\mathbf{M}_0$ exists and is positive.

Proof. See the Appendix.

The first statement in this theorem is well known and says that for no sector j its intermediate inputs should exceed its output while for some sector it should be less. The second statement is much less known and expresses that in no sector i its intermediate outputs should exceed its total output while for some sector is should be less. Although the conditions in Theorem 1 are only sufficient conditions, it should be clear that they are easily met in practical cases (i.e. working with real-life input-output tables).

Let us now return to the reply of GHS on Suh's approach, namely that some input coefficients are larger than one. Looking at Tables 5 and 6, it is obvious that the first condition in Theorem 1 may well be violated. That is, $\mathbf{v}_0' = \mathbf{r}' - \mathbf{w}'$ is likely to contain at least some negative elements. The second condition, however, will be met because $\mathbf{f}_0 = \mathbf{d} + \mathbf{e}$ may be expected to be non-negative and to include some positive elements.

The second point of reply in GHS is that the interpretation of the input coefficients is problematic. In my view, however, the interpretation can remain as it was. Let us consider the production in Services (sector 3, see Table 6). In order to produce 160.8 mt of usable output, sector 3 requires 336.2 mt inputs from sector 1, 206.2 mt from sector 2, 50.9 mt from itself, and 567.9 mt of primary material inputs. In addition it is required that 1000.4 mt of waste is generated. So, per mt of usable output in sector 3, the requirements are 2.09 mt from sector 1, 1.28 mt from sector 2, 0.32 mt from sector 3 itself, 3.53 mt of primary materials and 6.22 mt of waste is generated. Because so much waste is generated per mt of usable output in sector 3, it is not surprising that huge amounts of inputs are required, otherwise the material balance would be violated. The production in this sector is such that a large part of the inputs is transformed into waste and only a minor part into usable output.

The third point in GHS's reply to Suh is that in a more detailed (i.e. with more sectors) PIOT for Germany, it was found that some sectors had usable outputs close to zero or even negative. Looking at the rows in Table 5, it is clear that a negative value can occur only if the total final demand (i.e. domestic plus foreign) in such a sector is negative, which would be extremely difficult to interpret in physical units. From the columns in Table 5, it follows that the generation of waste in this sector must be larger than the sum of all its inputs. In Stahmer's (2000) PIOT with twelve sectors, negative values for the usable outputs do not occur. There is, however, a single sector for which the usable output is close to zero. This sector ("Environmental protection services") has a usable output of 13.2 mt, generates 4442.8 mt of waste, and uses 4456 mt of inputs (of which 9.8 mt primary materials and 0.2 imports). Of the output of this sector, 11.1 mt are delivered to other sectors and 2.1 is exported. It turns out that the environmental protection services merely transform the waste of the other sectors (which are given as inputs into this sector) into another type of waste that is less damaging for nature. In principle, it is well possible that for this sector of environmental protection services the usable output becomes zero. In that case, however, the sector can easily be removed from the system without affecting the results.

5. A reconciliation of approaches

The approach in GH distributes the waste over the two final demand categories, using each category's share in total final demand as weight. In this section, I suggest to adapt the weighting scheme. If the generation of waste depends on production, then it seems more appropriate to distribute the waste according to how much waste is generated in the production necessary to satisfy domestic final demands, respectively exports. Using the model in Section 3 and viewing the generation of waste as a necessary requirement for producing usable outputs yields the following. First define the waste input coefficients as

$$\bar{\mathbf{q}}' = \mathbf{w}'\hat{\bar{\mathbf{x}}}^{-1} \tag{7}$$

Then, the waste generation required directly and indirectly to produce domestic final demand and exports is given by the matrices

$$\mathbf{W}^{d} = \hat{\bar{\mathbf{q}}}\bar{\mathbf{M}}\hat{\mathbf{d}} \quad \text{and} \quad \mathbf{W}^{e} = \hat{\bar{\mathbf{q}}}\bar{\mathbf{M}}\hat{\mathbf{e}} \tag{8}$$

Element (i, j) of matrix $\mathbf{W}^{e}$, for example, indicates the amount of waste generated *in* sector i and required for the production of the exports in sector j. So, if we are interested in the amount of land use imputed to the exports j, we should include the land use necessary for the waste that is attributed to the exports of sector j. That is, the jth column of $\mathbf{W}^{e}$. Note that the matrices $\mathbf{W}^{d}$ and $\mathbf{W}^{e}$ distribute all waste over the two final demand categories. Adding the ith row sums of both matrices exactly yields the waste in sector i. That is, $\mathbf{W}^d\mathbf{1}+\mathbf{W}^e\mathbf{1}=\hat{\mathbf{q}}\overline{\mathbf{M}}(\mathbf{d}+\mathbf{e})=\hat{\mathbf{q}}\overline{\mathbf{x}}=\mathbf{w}$ as follows from $\overline{\mathbf{x}}=\overline{\mathbf{A}}\overline{\mathbf{x}}+\mathbf{d}+\mathbf{e}=\overline{\mathbf{M}}(\mathbf{d}+\mathbf{e})$ and from the definition in equation (7).

Recall that GH used the extended export vector as given by (5) after which the land use imputed to the exports of sector j is given by the jth column sum of the matrix $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{e}}^{ext}$. The distribution of the waste that I suggest is to use matrices $\mathbf{E}^{ext}$ and $\mathbf{D}^{ext}$ instead of the diagonal matrices $\hat{\mathbf{e}}^{ext}$ and $\hat{\mathbf{d}}^{ext}$, with

$$\mathbf{D}^{ext}=\hat{\mathbf{d}}+\mathbf{W}^d \quad \text{and} \quad \mathbf{E}^{ext}=\hat{\mathbf{e}}+\mathbf{W}^e \tag{9}$$

Note that the jth column of matrix $\mathbf{E}^{ext}$ gives the exports in sector j and the waste (which is generated in each of the sectors) imputed to the exports in sector j. This yields the following approach.

Approach 4. Using the definitions in (7) – (9), let the diagonal matrices $\hat{\mathbf{e}}^{ext}$ and $\hat{\mathbf{d}}^{ext}$ with the extended final demand vectors in GH's approach be replaced by the matrices $\mathbf{E}^{ext}$ and $\mathbf{D}^{ext}$. The land use in sector i imputed to all exports is given by the ith row sum of the matrix $\hat{\mathbf{c}}\mathbf{M}\mathbf{E}^{ext}$. The total land use imputed to the exports of sector j are given by the jth column sum of $\hat{\mathbf{c}}\mathbf{M}\mathbf{E}^{ext}$.

The Appendix shows that Approach 4, which is an adapted form of the approach of GH, yields the same result as Suh's approach.

At first sight it may seem as if Approach 4 differs largely from GH's procedure. Closer inspection, however, shows that if we are interested only in the land use in each sector (i.e. the row sums of $\hat{\mathbf{c}}\mathbf{M}\mathbf{E}^{ext}$) both approaches are fairly similar. In that case we can use GH's formula based on $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{e}}^{ext}$, which gives $\hat{\mathbf{c}}\mathbf{M}\mathbf{e}^{ext}$ for its row sums. The only difference with Approach 4, is that the latter uses $\mathbf{e}^{ext}=\mathbf{e}+\mathbf{w}^e$ (with $\mathbf{w}^e=\mathbf{W}^e\mathbf{1}$, the row sums of $\mathbf{W}^e$) instead of (5).

Table 9 gives the results for the waste distribution in Approach 4. The most striking outcome is that satisfying the final demands (both domestic and foreign) of

Manufacturing generates huge amounts of waste *in* Agriculture. This explains why Suh's approach (which yields the same results as Approach 4 for the land use) finds that so much land use in Agriculture must be attributed to the final demands of Manufacturing. Table 9 indicates that a large part of this is due to the waste in Agriculture that is imputed to the final demands of Manufacturing. At the same time this explains why GH's approach finds that much agricultural land use should be attributed to the final demands of Agriculture. In distributing the waste, all waste in the sector Agriculture is in GH divided between (and therefore implicitly attributed to) the two final demand categories of the *same* sector. In Approach 4, however, most of the waste *in* Agriculture is attributed to the final demands of Manufacturing.

INSERT TABLE 9

Further details on land use are given in Table 10. Note that if the waste part is added to the non-waste part, Table 7 for Suh's approach is obtained. Observe that more than 80% of the overall land use falls in the waste part. So, the largest part of the land use is attributed to the waste that is generated in order to satisfy the final demands. Therefore it is of crucial importance that waste is properly treated in analyses of this type. It turns out that no less than 45% of all the appropriated land is used in Agriculture and can be attributed to the waste that is necessary for the domestic final demand in Manufacturing. This is in sharp contrast to the findings from GH's approach. This is because, in GH, the waste generated in Agriculture is attributed to the final demands (domestic and foreign) of Agriculture, and not to the final demands of Manufcaturing.

INSERT TABLE 10

6. Evaluation

In this paper I have introduced a fourth method for the treatment of waste in a physical input-output analysis. This Approach 4 was in its construction similar to the approach of

GH (namely by distributing the waste), whereas its results were exactly the same as those of Suh's approach. Should the major conclusion now be that I have shown my own proposal to be redundant? The answer is negative, because in my view Approach 4 enables us to derive additional information that cannot be obtained from Suh's method.

Recall that the motivation for GH (and its predecessor HG) to come up with their suggestion was that the conventional way of imputation led to misleading results. The land use multiplier matrices $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{d}}$ and $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{e}}$, cover only a part of the story, because also waste has to be taken into account. Clearly, this applies to an analysis of a PIOT, which explicitly includes the generation of waste into its accounting framework. So, in order to find the "true" amounts of land imputed to exports, for example, one has to somehow deal with waste. GH's approach was to distribute the waste over domestic and foreign final demand. Suh's approach was to consider only usable outputs and therefore did not have to treat waste explicitly.

In my view it is also relevant to know explicitly how much land can be attributed to the waste that is imputed to for example the exports. Of the four approaches described in this paper, only Approach 4 allows us to do so. As an example, suppose that we would like to know the effects on land use if the exports of services were to expand by 1 mt (which is 5% of the exports in 1990). Define the vector $\Delta\mathbf{e} = (0, 0, 1)'$. The waste involved in producing this extra final demand equals $\hat{\mathbf{q}}\overline{\mathbf{M}}(\Delta\mathbf{e})$, which – in this case – is the third column of the matrix $\hat{\mathbf{q}}\overline{\mathbf{M}}(\Delta\hat{\mathbf{e}})$. For the land use required to satisfy the extra exports, two parts may be distinguished. First, the land use in each sector that is attributed strictly to the exports (i.e. the non-waste part) amounts to $\hat{\mathbf{c}}\mathbf{M}(\Delta\mathbf{e})$, the third column of $\hat{\mathbf{c}}\mathbf{M}(\Delta\hat{\mathbf{e}})$.[5] Second, the land use in each sector attributed to the waste that was imputed to the exports, which equals $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{q}}\overline{\mathbf{M}}(\Delta\mathbf{e})$.

Many exercises of the type above can be carried out. For example, if the export change in sector 3 had been 2.2 instead of 1, the results should simply be multiplied by 2.2. Exercises that involve unit changes in exports of sector 1 (or 2) are based on the first (respectively second) column of the matrices $\hat{\mathbf{c}}\mathbf{M}$ and $\hat{\mathbf{c}}\mathbf{M}\hat{\mathbf{q}}\overline{\mathbf{M}}$. So, any exogenously

[5] Note that this equals the third column of matrix $\hat{\mathbf{c}}\mathbf{M}$, because the exports have changed by one unit.

specified change in domestic or foreign final demands can be easily dealt with by using these multiplier matrices. The matrix $\hat{\mathbf{c}}\mathbf{M}$ reflects the non-waste part and its element (i, j) gives the (extra) land use *in* sector i attributed to one mt of (extra) final demand (either domestic or foreign) in sector j. Similarly, the waste part is given by matrix $\hat{\mathbf{c}}\mathbf{M}\hat{\bar{\mathbf{q}}}\overline{\mathbf{M}}$, whose element ($i$, j) gives the (extra) land use in sector i attributed to the waste involved in satisfying one mt of (extra) final demand in sector j.

Using Suh's approach, the distinction between the non-waste and the waste part cannot be made, because only their sums are obtained. In GH's approach, such a distinction would be possible, but would be based on the assumption that a 10% increase in the exports in sector j, for example, would raise e_j^{ext} - which includes the distribution of waste to sector j according to (5) – by 10%. In the previous section, however, it was shown that increasing the exports of sector j also increases the waste generation in any other sector i.

Table 11 reports the land use multipliers. These can be readily used to analyze the effects of exogenously specified changes in the final demand components. Note that also Table 10 may be obtained straightforwardly from Table 11. Multiplying the column A (either in the non-waste or in the waste part) in Table 11 by the domestic final demand of 46.8 (see Table 2) gives the figures in the columns A in the upper part of Table 10. Multiplying by the agricultural exports value of 36.7, yields the outcomes in the lower part of Table 10. In the same way, the columns M should be multiplied by 552.5 (domestic final demand) and by 155.9 (exports), to obtain columns M in the upper, respectively lower, part of Table 10. For columns S we have 16.3 and 20.0.

INSERT TABLE 11

References

Bicknell, K.B., R.J. Ball, R. Cullen and H.R. Bigsby (1998) New Methodology for the Ecological Footprint with an Application to the New Zealand Economy, *Ecological Economics*, 27, pp. 149-160.

CEC/IMF/OECD/UN/WB (1993) *System of National Accounts 1993* (Brussels, New York, Paris, Washington, DC, CEC/IMF/OECD/UN/WB).

Ferng, J.-J. (2001) Using Composition of Land Multiplier to Estimate Ecological Fotprints Associated with Production Activity, *Ecological Economics*, 37, pp. 159-172.

Giljum, S. and K. Hubacek (2004) Alternative Approaches of Physical Input-Output Analysis to Estimate Primary Material Inputs of Production and Consumption Activities, *Economic Systems Research*, 16, forthcoming.

Giljum, S., K. Hubacek and L. Sun (2003) Beyond the Simple Material Balance: A Reply to Sangwon Suh's Note on Physical Input-Output Analysis, *Ecological Economics*, forthcoming.

Hoekstra, R. (2003) *Structural Change of the Physical Economy – Decomposition Analysis of Physical and Hybrid Input-Output Tables* (Free University of Amsterdam, Tinbergen Insitute Research Series, no. 315, PhD thesis).

Hubacek, K. and S. Giljum (2003) Applying Physical Input-Output Analysis to Estimate Land Appropriation (Ecological Footprints) of International Trade Activities, *Ecological Economics*, 44, pp. 137-151.

Konijn, P.J.A., S. de Boer and J. van Dalen (1997) Input-Output Analysis of Material Flows with Application to Iron, Steel and Zinc, *Structural Change and Economic Dynamics*, 8, pp. 31-45.

Kratena, K., A. Chovanec and R. Konechy (1992) *Eine ökologische volkswirtschaftliche Gesammtrechnung für Österreich. Die Umwelt Input Output Tabelle 1983* (Wien, Institut für sozial-, wirtschafts- und umweltpolitische Forschung).

Kratterl, A. and K. Kratena (1990) *Reale Input-Output Tabelle und ökologischer Kreislauf* (Heidelberg, Physica-Verlag).

Mäenpää, I. (2002) Physical Input-Output Tables of Finland 1995 – Solutions to Some Basic Methodological Problems, *Paper* presented at the Fourteenth International Conference on Input-Output Techniques, Montréal.

Pedersen, O.G. (1999) *Physical Input-Output Tables for Denmark. Products and Materials 1990, Air Emissions 1990-92* (Copenhagen, Statistics Denmark).

Stahmer, C. (2000) The Magic Triangle of Input-Output Tables, in: S. Simon and J. Proops (eds) *Greening the Accounts* (Cheltenham, Edward Elgar), pp. 123-154.

Stahmer, C., M. Kuhn and N. Braun (1997) *Physische Input-Output Tabellen, Beiträge zu den Umweltökonomischen Gesamtrechnungen*, Band 1 (Stuttgart, Metzler-Poeschel Verlag).

Strassert, G. (2001) Interindustry Linkages: The Flow Network of a Physical Input-Output Table (PIOT): Theory and Application for Germany, in: M.L. Lahr and E. Dietzenbacher (eds) *Input-Output Analysis : Frontiers and Extensions* (Basingstoke, Palgrave), pp. 35-53.

Suh, S. (2003) A Note on the Calculus for Physical Input-Output Analysis and its Application to Land Apropriation of International Trade Activities, *Ecological Economics*, forthcoming.

Takayama, A. (1985) *Mathematical Economics,* second edition (Cambridge, Cambridge University Press).

Appendix

Proof of Theorem 1

Note that, because $\mathbf{Z}_0$ is positive, also the outputs $\mathbf{x}_0$ are positive. This immediately follows from $\mathbf{x}_0' = \mathbf{1}'\mathbf{Z}_0 + \mathbf{v}_0'$, with $\mathbf{v}_0'$ non-negative. Therefore also $\mathbf{A}_0 = \mathbf{Z}_0\hat{\mathbf{x}}_0^{-1}$ is positive. Using that $\mathbf{v}_0'$ is non-negative and at least one of its elements is positive, yields that no column sum of $\mathbf{A}_0$ is larger than one while at least one is smaller then one. This implies that matrix $\mathbf{A}_0$ satisfies the so-called Brauer-Solow condition. As a consequence, the matrix $(\mathbf{I} - \mathbf{A}_0)$ is non-singular and its inverse $\mathbf{M}_0$ is positive. This proves the first statement of the theorem.

Define a new matrix of so-called allocation coefficients as $\mathbf{B}_0 = \hat{\mathbf{x}}_0^{-1}\mathbf{Z}_0$. Its elements $b_{ij}^0 = z_{ij}^0 / x_i^0$ indicate the percentage of the output in sector *i* that is delivered to sector *j*. The accounting equation $\mathbf{x}_0 = \mathbf{Z}_0\mathbf{1} + \mathbf{f}_0$ shows that the outputs are positive because $\mathbf{Z}_0$ is positive and $\mathbf{f}_0$ is non-negative. Using that $\mathbf{f}_0$ is non-negative and at least one of its elements is positive, yields that no row sum of $\mathbf{B}_0$ is larger than one while at least one is smaller then one. This implies that matrix $\mathbf{B}_0$ satisfies the Brauer-Solow condition. As a consequence, the matrix $(\mathbf{I} - \mathbf{B}_0)$ is non-singular and its inverse $(\mathbf{I} - \mathbf{B}_0)^{-1}$ is positive. Next, note that $\mathbf{Z}_0 = \mathbf{A}_0\hat{\mathbf{x}}_0 = \hat{\mathbf{x}}_0\mathbf{B}_0$ so that $\mathbf{A}_0 = \hat{\mathbf{x}}_0\mathbf{B}_0\hat{\mathbf{x}}_0^{-1}$. Hence, $\mathbf{M}_0 = (\mathbf{I} - \mathbf{A}_0)^{-1} = (\mathbf{I} - \hat{\mathbf{x}}_0\mathbf{B}_0\hat{\mathbf{x}}_0^{-1})^{-1} = (\hat{\mathbf{x}}_0\hat{\mathbf{x}}_0^{-1} - \hat{\mathbf{x}}_0\mathbf{B}_0\hat{\mathbf{x}}_0^{-1})^{-1} = \hat{\mathbf{x}}_0(\mathbf{I} - \mathbf{B}_0)^{-1}\hat{\mathbf{x}}_0^{-1}$, which is positive because $(\mathbf{I} - \mathbf{B}_0)^{-1}$ and $\mathbf{x}_0$ are positive. This proves the second statement in the theorem.[6]

[6] I have tried to keep the mathematical details as simple as possible. As a consequence, the results in Theorem 1 can be further strengthened. For example, it is not necessary that all intermediate deliveries in $\mathbf{Z}_0$ are positive, many of them may even be zero. Such refinements are easily obtained using the mathematical exposition in Takayama (1985).

Proof of equivalence of Approach 4 and Suh's approach

Take Approach 4 as a starting point and consider $\hat{\mathbf{c}}\mathbf{M}\mathbf{E}^{ext}$ with $\mathbf{E}^{ext} = \hat{\mathbf{e}} + \hat{\bar{\mathbf{q}}}\overline{\mathbf{M}}\hat{\mathbf{e}}$. Using the definitions of $\mathbf{c}$, $\mathbf{M}$, $\bar{\mathbf{q}}$ and $\overline{\mathbf{M}}$ we have

$$\hat{\mathbf{c}}\mathbf{M}\mathbf{E}^{ext} = \hat{\mathbf{s}}\hat{\mathbf{x}}^{-1}(\mathbf{I}-\mathbf{A})^{-1}[\hat{\mathbf{e}} + \hat{\mathbf{w}}\hat{\bar{\mathbf{x}}}^{-1}(\mathbf{I}-\overline{\mathbf{A}})^{-1}\hat{\mathbf{e}}] \tag{A1}$$

Note that $\hat{\mathbf{x}}^{-1}(\mathbf{I}-\mathbf{A})^{-1} = [(\mathbf{I}-\mathbf{A})\hat{\mathbf{x}}]^{-1} = (\hat{\mathbf{x}}-\mathbf{Z})^{-1}$ and, similarly, $\hat{\bar{\mathbf{x}}}^{-1}(\mathbf{I}-\overline{\mathbf{A}})^{-1} = (\hat{\bar{\mathbf{x}}}-\mathbf{Z})^{-1}$. So, (A1) equals

$$\hat{\mathbf{s}}(\hat{\mathbf{x}}-\mathbf{Z})^{-1}[\hat{\mathbf{e}} + \hat{\mathbf{w}}(\hat{\bar{\mathbf{x}}}-\mathbf{Z})^{-1}\hat{\mathbf{e}}] = \hat{\mathbf{s}}(\hat{\bar{\mathbf{x}}}+\hat{\mathbf{w}}-\mathbf{Z})^{-1}[\hat{\mathbf{e}} + \hat{\mathbf{w}}(\hat{\bar{\mathbf{x}}}-\mathbf{Z})^{-1}\hat{\mathbf{e}}] \tag{A2}$$

where $\mathbf{x} = \bar{\mathbf{x}} + \mathbf{w}$ was used. Writing $(\hat{\bar{\mathbf{x}}}-\mathbf{Z})(\hat{\bar{\mathbf{x}}}-\mathbf{Z})^{-1}\hat{\mathbf{e}}$ for $\hat{\mathbf{e}}$ gives

$$\begin{aligned}&\hat{\mathbf{s}}(\hat{\bar{\mathbf{x}}}+\hat{\mathbf{w}}-\mathbf{Z})^{-1}[(\hat{\bar{\mathbf{x}}}-\mathbf{Z})(\hat{\bar{\mathbf{x}}}-\mathbf{Z})^{-1}\hat{\mathbf{e}} + \hat{\mathbf{w}}(\hat{\bar{\mathbf{x}}}-\mathbf{Z})^{-1}\hat{\mathbf{e}}] = \\ &\hat{\mathbf{s}}(\hat{\bar{\mathbf{x}}}+\hat{\mathbf{w}}-\mathbf{Z})^{-1}[(\hat{\bar{\mathbf{x}}}-\mathbf{Z})+\hat{\mathbf{w}}](\hat{\bar{\mathbf{x}}}-\mathbf{Z})^{-1}\hat{\mathbf{e}} = \hat{\mathbf{s}}(\hat{\bar{\mathbf{x}}}-\mathbf{Z})^{-1}\hat{\mathbf{e}}\end{aligned} \tag{A3}$$

Writing $(\hat{\bar{\mathbf{x}}}-\mathbf{Z})^{-1} = \hat{\bar{\mathbf{x}}}^{-1}(\mathbf{I}-\overline{\mathbf{A}})^{-1} = \hat{\bar{\mathbf{x}}}^{-1}\overline{\mathbf{M}}$ yields for (A3)

$$\hat{\mathbf{s}}\hat{\bar{\mathbf{x}}}^{-1}\overline{\mathbf{M}}\hat{\mathbf{e}} = \hat{\bar{\mathbf{c}}}\overline{\mathbf{M}}\hat{\mathbf{e}} \tag{A4}$$

which is the result in Suh's approach.

Table 1. A simplified PIOT

Supply	Use				
	Sectors (1,…,n)	Final demand		Disposal to nature	Total output
		Domestic	Exports		
Sectors (1,…,n)	$\mathbf{Z}$	$\mathbf{d}$	$\mathbf{e}$	$\mathbf{w}$	$\mathbf{x}$
Primary material inputs (domestic extraction and imports)	$\mathbf{r}'$				
Total input	$\mathbf{x}'$				
Land appropriation	$\mathbf{s}'$				

Table 2. Three-sector PIOT for Germany 1990 (million tons)

Supply	Use						
	Agriculture	Manufacturing	Services	Final demand		Disposal to nature	Total output
				Domestic	Exports		
Agriculture	2,247.7	1,442.2	336.2	46.8	36.7	2,404.8	6,514.4
Manufacturing	27.4	1,045.4	206.2	552.5	155.9	846.6	2,834.0
Services	5.1	68.5	50.9	16.3	20.0	1,000.4	1,161.2
Primary material inputs (domestic extraction and imports)	4,234.2	277.9	567.9				
Total input	6,514.4	2,834.0	1,161.2				
Land appropriation (in hectares)	21,019,662	1,912,694	1,551,786				24,484,142

Table 3. Results for the HG approach

	Distribution of waste			Imputation of land use to								
				Extended domestic final demand				Extended exports				Total
	to domestic final demand	to exports	Total	A	M	S	Total	A	M	S	Total	Sum
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Agriculture	1986	1558	3544	10,074,628	2,995,914	518,877	13,589,419	7,900,403	845,363	636,659	9,382,425	22,971,844
Manufacturing	181	51	233	14,633	794,798	46,706	856,137	11,475	224,270	57,308	293,053	1,149,190
Services	213	262	475	4,149	40,799	323,595	368,543	3,253	11,512	397,049	411,815	780,358
Total	2381	1871	4252	10,093,410	3,831,511	889,178	14,814,099	7,915,132	1,081,145	1,091,016	10,087,292	24,901,392

Notes: (3) = (1) + (2), (7) = (4) + (5) + (6), (11) = (8) + (9) + (10), (12) = (7) + (11)

Table 4. Results for the GH approach

	Distribution of waste			Imputation of land use to								
				Extended domestic final demand				Extended exports				Total
	to domestic final demand	to exports	Total	A	M	S	Total	A	M	S	Total	Sum
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Agriculture	1348	1057	2405	6,910,942	4,950,774	1,051,416	12,913,133	5,419,478	1,396,970	1,290,081	8,106,529	21,019,662
Manufacturing	660	186	847	10,038	1,313,411	94,641	1,418,090	7,872	370,608	116,124	494,604	1,912,694
Services	449	551	1000	2,846	67,420	655,711	725,977	2,232	19,024	804,553	825,809	1,551,786
Total	2457	1794	4252	6,923,827	6,331,605	1,801,768	15,057,200	5,429,582	1,786,601	2,210,759	9,426,942	24,484,142

Notes: (3) = (1) + (2), (7) = (4) + (5) + (6), (11) = (8) + (9) + (10), (12) = (7) + (11)

Table 5. A PIOT with waste as negative "input"

Supply	Use			
	Sectors (1,…,*n*)	Final demand		Total output
		Domestic	Exports	
Sectors (1,…,*n*)	$\mathbf{Z}$	$\mathbf{d}$	$\mathbf{e}$	$\bar{\mathbf{x}}$
Primary material inputs (domestic extraction and imports)	$\mathbf{r}'$			
Disposal to nature	$-\mathbf{w}'$			
Total input	$\bar{\mathbf{x}}'$			
Land appropriation	$\mathbf{s}'$			

Table 6. Three-sector PIOT for Germany 1990 (million tons) with waste as negative "input"

Supply	Use					
	Agriculture	Manufacturing	Services	Final demand		Total output
				Domestic	Exports	
Agriculture	2,247.7	1,442.2	336.2	46.8	36.7	4,109.6
Manufacturing	27.4	1,045.4	206.2	552.5	155.9	1,987.4
Services	5.1	68.5	50.9	16.3	20.0	160.8
Primary material inputs (domestic extraction and imports)	4,234.2	277.9	567.9			
Disposal to nature	-2,404.8	-846.6	-1,000.4			
Total input	4,109.6	1,987.4	160.8			
Land appropriation (in hectares)	21,019,662	1,912,694	1,551,786			24,484,142

Table 7. Results for Suh's approach

	Imputation of land use to								
	Extended domestic final demand				Extended exports				Total
	A	M	S	Total	A	M	S	Total	Sum
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Agriculture	555,387	13,313,176	1,328,682	15,197,245	435,528	3,756,605	1,630,285	5,822,417	21,019,662
Manufacturing	2,297	1,354,125	77,402	1,433,824	1801	382,096	94,973	478,870	1,912,694
Services	3,064	730,369	273,849	1,007,282	2403	206,090	336,011	544,504	1,551,786
Total	560,748	15,397,670	1,679,934	17,638,351	439,732	4,344,790	2,061,268	6,845,791	24,484,142

Notes: (4) = (1) + (2) + (3), (8) = (5) + (6) + (7), (9) = (4) + (8)

Table 8. A simplified input-output table

Supply	Use		
	Sectors (1,…,*n*)	Final demand	Total output
Sectors (1,…,*n*)	$\mathbf{Z}_0$	$\mathbf{f}_0$	$\mathbf{x}_0$
Primary inputs	$\mathbf{v}_0'$		
Total input	$\mathbf{x}_0'$		

Table 9. The distribution of waste in Approach 4

	Waste imputed to								
	domestic final demands				exports				Total
	A	M	S	Total	A	M	S	Total	Sum
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Agriculture	63.54	1,523.12	152.01	1,738.67	49.83	429.78	186.52	666.13	2,404.8
Manufacturing	1.02	599.37	34.26	634.64	0.80	169.12	42.04	211.96	846.6
Services	1.8	470.85	176.54	649.37	1.55	132.86	216.62	351.03	1,000.4
Total	66.53	2,593.34	362.81	3,022.69	52.17	731.77	445.17	1,229.11	4,251.8

Notes: (4) = (1) + (2) + (3), (8) = (5) + (6) + (7), (9) = (4) + (8)

Table 10. Land use in Approach 4

	Land use imputed to domestic final demands								
	Non-waste part				Waste part				Total
	A	M	S	Total	A	M	S	Total	Sum
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Agriculture	231,911	2,255,388	36,815	2,524,114	323,476	11,057,788	1,291,867	12,673,131	15,197,245
Manufacturing	337	598,341	3,314	601,992	1,960	755,784	74,089	831,832	1,433,824
Services	96	30,714	22,960	53,769	2,969	699,655	250,889	953,513	1,007,282
Total	232,343	2,884,443	63,089	3,179,875	328,405	12,513,227	1,616,845	14,458,476	17,638,351
	Land use imputed to exports								
	Non-waste part				Waste part				Total
Agriculture	181,862	636,407	45,172	863,441	253,666	3,120,198	1,585,112	4,958,976	5,822,417
Manufacturing	264	168,835	4,066	173,165	1,537	213,261	90,907	305,705	478,870
Services	75	8,667	28,171	36,913	2,328	197,423	307,840	507,591	544,504
Total	182,201	813,908	77,410	1,073,519	257,531	3,530,882	1,983,859	5,772,271	6,845,791

Notes: (4) = (1) + (2) + (3), (8) = (5) + (6) + (7), (9) = (4) + (8)

Table 11. Land use multipliers in Approach 4

	Land use imputed to one unit of final demand					
	Non-waste part			Waste part		
	A	M	S	A	M	S
Agriculture	4,955.36	4,082.15	2,258.61	6,911.88	20,014.10	79,255.62
Manufacturing	7.20	1,082.97	203.30	41.88	1,367.93	4,545.34
Services	2.04	55.59	1,408.57	63.43	1,266.34	15,391.99
Total	4,964.60	5,220.71	3,870.48	7,017.19	22,648.37	99,192.94