A Dynamic International Trade Model Using Overlapping Generation and General Equilibrium Concept

Mossa. Anisa Khatun* and Hajime Inamura**

Abstract

This paper develops a dynamic international trade model considering overlapping generation (OLG) and dynamic computable general equilibrium (CGE) concept. Most of the previous works used two period OLG model where only investment policy is analyzed. But in this paper, seven period OLG model is used where investment as well as reinvestment policy is analyzed for consumers and firms. This model is based on intertemporal dynamic optimization behaviors of consumers and firms. The effect of demography, interest rate and foreign exchange rate are also considered to analyze the dynamic trade pattern. The international input-output (IO) transaction table is used for analyzing Japan-US economy in details.

Keywords: Overlapping generation, computable general equilibrium, input-output, Leontief production-function, trade model.

1 Introduction

Airports and harbors are very important social capital since they work as international gateways where trades are supported. In international trade, a substantial share of domestic output of a country is exported to foreign countries and a large part of total expenditure is imported from foreign countries. So, it can be said that the trade prediction is an indispensable element and most important information for the port plan. This paper deals with dynamic trade analysis using OLG model and dynamic CGE concept.

There are several previous IO and CGE models which analyze supply and demand balance of goods. Leontief et. al. (1977) extended IO model to the trade model among many countries based on general equilibrium theory. Multiregional Input-Output (MRIO) models developed by Isard (1951), Moses (1955), Leontief and Shrout (1963) are capable of describing the interregional trade flows and the interindustry transactions in regional and industrial details. C. K. Liew and C. J. Liew (1985) introduced a model to measure the development impact of a transportation system considering regional input-output coefficients, trade coefficients and trade flows. Whalley (1985) developed two models (four-region and seven region model) for analyzing the impact of changes in trade policies among developed countries. Dirk Willenbockel (1999) analyzed a dynamic two-country model with international capital mobility and intertemporal optimizing agents. Jin Shui Zhang (2001) extended the linear dynamic input-output model to the computable non-linear dynamic input-output model.

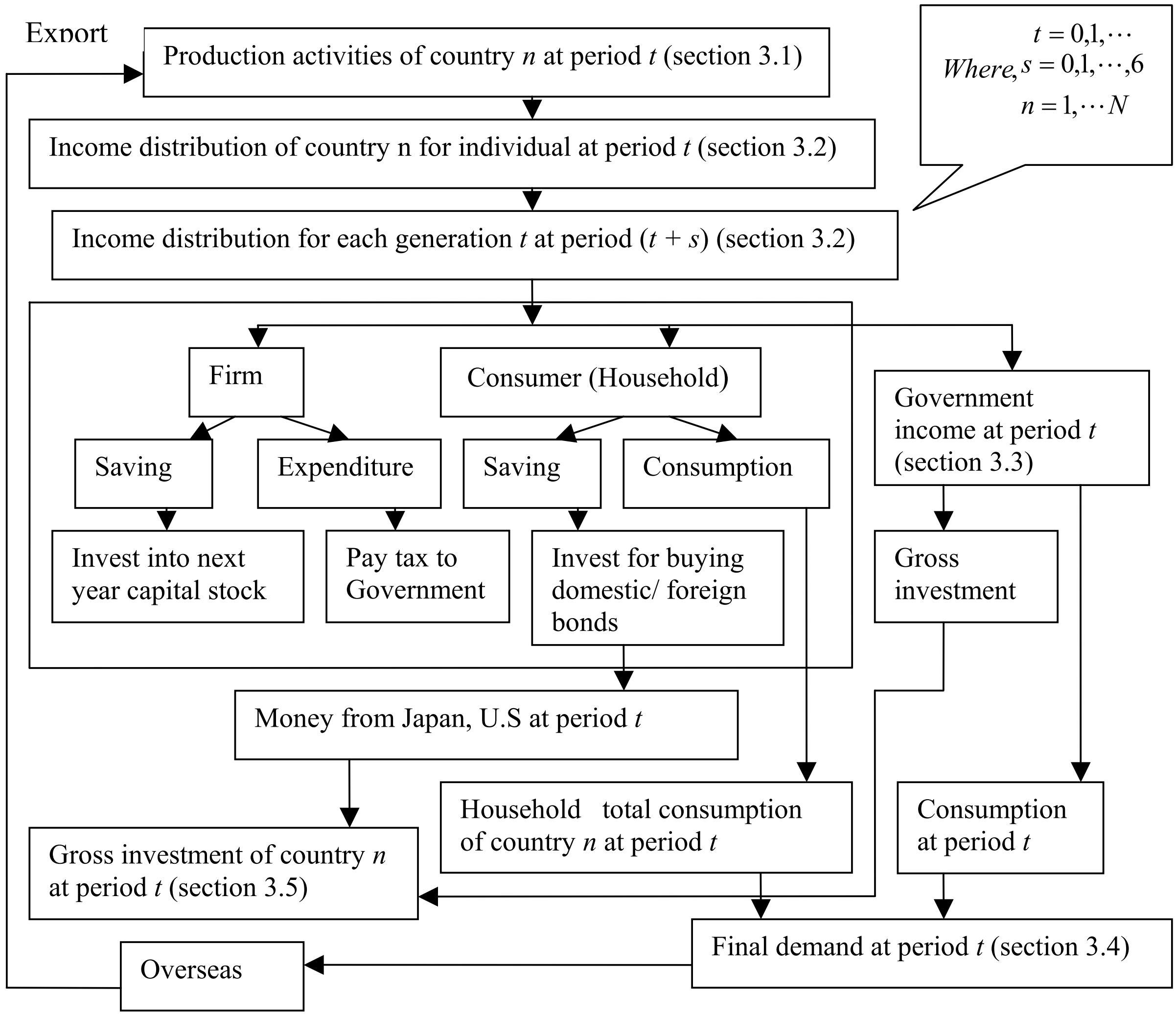
All of the above works on trade have been done within a static framework. Because of the static nature, these works can neither explain the changes in the variables over time nor organize the future trade markets. To overcome these problems, a dynamic trade model is developed in this paper considering seven period overlapping generation model. In OLG model, it is assumed that consumers receive disposable income for consumption and savings in lifetime budget condition. Income expenditure and savings of firms are also considered. Furthermore, it is assumed that consumers invest all savings for buying domestic/foreign bonds, and firms invest all savings into the next period capital stock. Several other papers have considered two period OLG model where only investment policy is analyzed. However, this paper considers seven period OLG model for analyzing the investment as well as reinvestment policy for consumers and firms. The same technique like OLG model is applied for developing a dynamic international trade model using dynamic CGE concept. The effect of demography, interest rate and foreign exchange rate are also considered to analyze the trade pattern.

** Professor Hajime Inamura, Dept of Human Social Information Sciences, Tohoku University,
(Aobayama, Sendai -980, Japan, e-mail: inamura@plan.civil.tohoku.ac.jp, TEL +81-22-217-7494, FAX +81-22-217-7494)
* Graduate student, dept of human Social Information Sciences, Tohoku University,
((Aobayama, Sendai -980, Japan, e-mail:anisa@plan.civil.tohoku.ac.jp, TEL +81-22-217-7497)

The paper is organized as follows. Section 2 presents the framework of the model. In section 3, the model is formulized for analyzing the production activities, income distribution for household, firm and government, final demand for household and government and dynamic international trade pattern. Finally section 4 summarizes the conclusion.

2 Framework of Model

This model describes the economic activities of a country based on four calculation steps: analysis of production activities, income distribution, final demand, and accumulation of capital. The model is built for two or more countries considering the export and import of goods and households investment among the countries. The framework of the model is given bellow.



Consumer goods import (section 3.6)

3 Formulization of Model

3.1 Analysis of production activities and equilibrium price

It is assumed that the following assumptions (a)-(c) from Armington are also hold for this model.

(a) Market equilibrium is considered for all goods and excess demand is eliminated.

(b) Production factors, labor and capital movement between countries are impossible but movement between domestic industries are possible.
(c) Commodities are distinguished by place of production.

All countries except the candidate countries (*m, n=1* to *N* from here after) are treated as Rest of the World (ROW). Although the activities of ROW are not taken into consideration but it has been taken only for balancing of the market. Furthermore, the classification is not considered about the commodities from ROW because all commodities from ROW are regarded as a single commodity.

Production function: In this paper, the production function of country *n* industry *j* is assumed as the Leontief type production function of concerning intermediate inputs and production factors which is expressed by the following relation.

$$X_{jt}^{n} = \min\left(\cdots, \frac{x_{ijt}^{mn}}{a_{ijt}^{mn}}, \cdots, \frac{x_{jt}^{ROWn}}{a_{jt}^{ROWn}}, \cdots \frac{Y_{jt}^{n}}{a_{Y_j t}^{n}}\right) \tag{1}$$

where

X_{jt}^{n} = quantity of production of country *n* industry *j* at period *t,*

x_{ijt}^{mn} = the intermediate input to country *n* industry *j* imported commodity *i* from country *m* at period *t,*

x_{jt}^{ROWn} = the intermediate input to country *n* industry *j* imported commodity *i* from ROW at period *t,*

Y_{jt}^{n} = production capacity of country *n* industry at period *t,*

a_{ijt}^{mn}, a_{jt}^{ROWn} = input coefficient at period *t,* and

$a_{Y_j t}^{n}$ = a ratio of production capacity per unit quantity of production in country *n* at period *t.*

Production capacity: In above equation (1), a Cobb-Douglas type production capacity function Y_{jt}^{n} is assumed for production factors. The function can be expressed as follows.

$$Y_{jt}^{n} = \theta_{jt}^{n}\left(L_{jt}^{n}\right)^{\alpha_{L_j t}^{n}}\left(K_{jt}^{n}\right)^{\alpha_{K_j t}^{n}} \tag{2}$$

where

$$\left(\alpha_{L_j t}^{n} + \alpha_{K_j t}^{n}\right) = 1,$$

L_{jt}^{n}, K_{jt}^{n} = labor and capital of country *n* which are supplied in industry *j* at period *t,* and

θ_{jt}^{n} = manufacturing technique parameter in country *n* at period *t.*

Equilibrium price: In the following equation (3), it is assumed that the producer price of commodity is the sum of payments to primary factors and intermediate inputs. For calculating the payments to primary factors (labor and capital), it is necessary to consider the labor and capital demand which can be found by solving the cost minimization problem. At the market equilibrium condition, an equilibrium price can be obtained from the following equation (3) which is considered as the prices of commodity of a country. A price equilibrium equation is formulized as the following equation.

$$P_{jt}^{n} = \sum_{m}\sum_{i}\left(q_{it}^{mn} a_{ijt}^{mn}\right) + q_{t}^{ROWn} a_{jt}^{ROWn} + a_{Y_j t}^{n}\left(\omega_{t}^{n} DL_{jt}^{n} + r_{t}^{n} DK_{jt}^{n}\right) \tag{3}$$

where

P^{n}_{jt} = the producer price of country *n* industry *j* at period *t,*

q^{mn}_{it} = the consumer price in country *n* of imported commodity *i* from country *m* at period *t,*

q^{ROWn}_{t} = the price of commodities in ROW at period *t,*

ω^{n}_{t} = the rate of wages in country *n* at period *t,*

r^{n}_{t} = capital service cost in country *n* at period *t,*

DL^{n}_{jt} = labor demand per unit production capacity in country *n* at period *t,* and

DK^{n}_{jt} = capital demand per unit production capacity in country *n* at period *t.*

Cost minimization problem: Labor demand and capital demand per unit production capacity are obtained by solving the following cost minimization problem.

$$\min_{DL^{n}_{jt}, DK^{n}_{jt}} \omega^{n}_{t} DL^{n}_{jt} + r^{n}_{jt} DK^{n}_{jt}$$

$$s.t\, \theta^{n}_{jt} \left(DL^{n}_{jt}\right)^{\alpha^{n}_{L_j t}} \left(DK^{n}_{jt}\right)^{\alpha^{n}_{K_j t}} = 1 \tag{4}$$

Therefore solving the above equation (4), labor demand and capital demand per unit production capacity is found as follows.

$$DL^{n}_{jt} = \frac{1}{\theta^{n}_{jt}} \left(\frac{\alpha^{n}_{L_j t} r^{n}_{t}}{\alpha^{n}_{K_j t} \omega^{n}_{t}} \right)^{\alpha^{n}_{K_j t}} \; and \; DK^{n}_{jt} = \frac{1}{\theta^{n}_{jt}} \left(\frac{\alpha^{n}_{K_j t} \omega^{n}_{t}}{\alpha^{n}_{L_j t} r^{n}_{t}} \right)^{\alpha^{n}_{L_j t}} \tag{5}$$

Producer price and consumer price: The relation between consumer price and producer price of a production country with respect to domestic goods demand and overseas goods demand can be expressed as follows.

$$q^{nn}_{it} = p^{n}_{it}, q^{mn}_{it} = \frac{p^{m}_{it}}{\varepsilon^{mn}} \tag{6}$$

where

ε^{mn} = exchange rate.

3.2 Income distribution for consumer and firm

In this section two types of income distribution, primary income distribution and secondary income distribution are considered for consumer and firm. A labor and capital income is assumed as the primary income for a consumer and a firm. For secondary income distribution, it is assumed that a household receives disposable income after paying the corporation tax to the government from labor income, and a firm receives the capital income after paying corporation tax to the government from operating surplus.

3.2.1 Primary income distribution

A labor income is distributed as employee compensation for a consumer (household). In this model, at first, a labor income is formulized at period *t* for considering OLG technique as the following equation.

$$ce^{n}_{t} = \sum_{j} \left(\omega^{n}_{t} dl^{n}_{jt} a^{n}_{Y_j t} X^{n}_{jt} \right) \tag{7}$$

where

$$dl^n_{jt} = \frac{DL^n_{jt}}{N^n_t}, \qquad N^n_t = L^n_t,$$

$ce^n_t =$ employee compensation for individual of country n at period t,

$N^n_t =$ total working population of country n at period t,

$L^n_t =$ total labor supply of country n at period t, and

$dl^n_{jt} =$ individual labor demand of country n for industry j at period t.

Capital income per worker at period t is served as fixed capital depreciation and operating surplus which is shown bellow.

$$fk^n_t = \sum_j \left(\delta^n_{jt} P^n_{K_jt} dk^n_{jt} a^n_{Y_jt} X^n_{jt}\right) \quad and \tag{8}$$

$$os^n_t = \sum_j \left[\left(r^n_t - \delta^n_{jt} P^n_{K_jt}\right) dk^n_{jt} a^n_{Y_jt} X^n_{jt}\right] \tag{9}$$

where

$$dk^n_{jt} = \frac{DK^n_{jt}}{N^n_t},$$

$fk^n_t =$ fixed capital depreciation amount per worker of country n at period t,

$\delta^n_{jt} =$ the rate of depreciation of capital stock at period t,

$os^n_t =$ operating surplus per worker of country n at period t,

$P^n_{K_jt} =$ the price of capital stock at period t, and

$dk^n_{jt} =$ individual capital demand of country n for industry j at period t.

Next, the primary income distribution is formulized for each generation t at period $t+s$. Because, in this paper, seven period OLG model is considered for income distribution. For simplicity, it is assumed that consumers live seven periods in infinite horizon, so that seven generations coexist in every period t. Furthermore, it is assumed that consumers in each generation start to work for saving and consumption after reaching age 30 and retire at age 60. All activities for the remaining lifetime until death are considered similar and included with the retirement period. So, all consumers belonging to the same generation have the same income for every period as described by the following equations. In all equations from (10) to equation (26), the superscript denotes the year of birth of each generation t of country n and subscript denotes the date of consumption.

Employee compensation per worker (household) ce^{nt}_{t+s} for each generation t at period ($t+s$) is expressed as the following equations.

$$ce^{nt}_{t+s} = \sum_j \left(\omega^{nt}_{t+s} dl^{nt}_{j,t+s} a^n_{Y_jt} X^n_{jt}\right) \tag{10}$$

where,

$$dl^{nt}_{j,t+s} = \frac{1}{\theta^n_{j,t+s}} \left(\frac{\alpha^n_{L_jt} r^{nt}_{t+s}}{\alpha^n_{K_jt} \omega^{nt}_{t+s}}\right)^{\alpha^n_{K_jt}} \times \frac{1}{N^n_t},$$

$dl^{nt}_{j,t+s} =$ individual labor demand of industry j for each generation t at period $t+s$,
$t = 0,1,\cdots, \quad and \quad s = 0,1,2,\cdots,6$ from here after.

Capital income distribution per worker for each generation t at period ($t+s$) is formulized by the following equations.

$$fk^{nt}_{t+s} = \sum_j \left(\delta^n_{j,t} P^n_{K_j,t+s} dk^{nt}_{j,t+s} a^n_{Y_j t} X^n_{jt}\right) \quad and \tag{11}$$

$$os^{nt}_{t+s} = \sum_j \left[\left(r^{nt}_{t+s} - \delta^n_{j,t} P^n_{K_j,t+s}\right) dk^{nt}_{j,t+s} a^n_{Y_j t} X^n_{jt}\right] \tag{12}$$

where

$$dk^{nt}_{j,t+s} = \frac{1}{\theta^{nt}_{j,t+s}} \left(\frac{\alpha^n_{K_j t}\,\omega^{nt}_{t+s}}{\alpha^n_{L_j t}\, r^{nt}_{t+s}}\right)^{\alpha^n_{L_j t}} \times \frac{1}{N^n_t},$$

$fk^{nt}_{t+s} =$ fixed capital depreciation per worker of country n for each generation t at every period $t+s$,
$os^{nt}_{t+s} =$ operating surplus per worker of country n for each *generation* t at every period $t+s$, and
$dk^{nt}_{t+s} =$ individual capital demand of country n for industry j for each generation t at every period $t+s$.

From first order zero profit maximization problem, the labor wage rate and capital service rate can be calculated for each generation at his lifetime by the following equations.

$$\omega^{nt}_{t+s} = \theta^{nt}_{t+s} \alpha^n_{Lt} \left(\frac{K^{nt}_{t+s}}{L^n_t}\right)^{\alpha^n_{Kt}} \tag{13}$$

$$r^{nt}_{t+s} = \theta^{nt}_{t+s} \alpha^n_{Kt} \left(\frac{L^n_t}{K^{nt}_{t+s}}\right)^{\alpha^n_{Lt}} \tag{14}$$

3.2.2 Secondary income distribution

A household receives disposable income after paying income tax to the government from labor income. He also receives transfer income from government for pension, social security, health, transportation etc. which is included with disposable income. The relation can be expressed as

$$di^{nt}_{t+s} = ce^{nt}_{t+s} - dt^{nt}_{h,t+s} + trg^{nt}_{t+s} \tag{15}$$

where

$$dt^{nt}_{h,t+s} = \tau^{nt}_{h,t} ce^{nt}_{t+s},$$

$di^{nt}_{t+s} =$ household disposable income for each generation t at period $t+s$ of country n,
$dt^{nt}_{h,t+s} =$ income tax for each generation t at period $t+s$ of country n,
$trg^{nt}_{t+s} =$ transfer income for each generation t of country n, and
$\tau^{nt}_{h,t} =$ income tax rate which is same for all over the model.

In this paper, it is assumed that a household spends his disposable income for saving and consumption. All savings are invested/reinvested for buying domestic/foreign bonds in each generation t at his lifetime. The household lifetime budget constraint is formulized by the following equation.

$$s_{h,t+s}^{nt} = di_{t+s}^{nt} + s_{h,t-1+s}^{nt}\left(1 + R_s\right) - c_{h,t+s}^{nt} \tag{16}$$

where

$s_{h,t+s}^{nt}$ = household saving of country n for each generation t at period $t+s$, and

$c_{h,t+s}^{nt}$ = household consumption of country n for each generation t at period $t+s$,

Since bequests are not considered, initial return is zero that is,

$s = 0, s_{h,t-1+s}^{nt}\left(1 + R_s\right) = 0$

Household saving and disposable income are also zero at the retirement period that is,

$s = 6, s_{h,t+s}^{nt} = 0, di_{t+s}^{nt} = 0$

The household budget constraint is split for understanding the model clearly which is illustrated below.

Age 30 Age 35 ------------- Age 60 to 80

$$c_t^{nt} = di_t^{nt} - s_{h,t}^{nt},\ c_{t+1}^{nt} = di_{t+1}^{nt} + s_{h,t}^{nt}\left(1 + R_1\right) - s_{h,t+1}^{nt}, \dots,\ c_{t+6}^{nt} = s_{h,t+5}^{nt}\left(1 + R_6\right)$$

Invest Return Reinvest Return

For buying domestic/ foreign bond

For buying domestic/ foreign bond

Household consumption demand is obtained by solving the following Utility maximization problem subject to household lifetime budget constraint for each generation t of country n which is formulized as follows.

$$\max u\left(c^{nt}\right) = \sum_{s=0}^{6} \beta^s u\left(c_{t+s}^{nt}\right)$$

$$s.t.\ c_t^{nt} + \frac{c_{t+1}^{nt}}{(1 + R_1)} + \cdots + \frac{c_{t+6}^{nt} = \sum_{s=6}^{10} c_{t+s}^{nt}}{(1 + R_1)\cdots(1 + R_6)} = DI^{nt} \tag{17}$$

where

$$DI^{nt} = di_t^{nt} + \frac{di_{t+1}^{nt}}{(1 + R_1)} + \cdots + \frac{di_{t+5}^{nt}}{(1 + R_1)\cdots(1 + R_5)},$$

DI^{nt} = lifetime disposable income for each generation t of country n,

R = interest rate, and

β = discount factor.

The household consumption demand for each generation t at every period t can be derived from the above equation (17) by the Lagrange multipliers method as follows.

$$c_{t}^{nt} = DI^{nt} \frac{1}{(1+\beta+\cdots+\beta^{6})} \tag{18}$$

$$c_{t+1}^{nt} = (1+R_1) DI^{nt} \frac{\beta}{(1+\beta+\cdots+\beta^{6})} \tag{19}$$

$$\vdots$$

$$c_{t+6}^{nt} = (1+R_1)\cdots(1+R_6) DI^{nt} \frac{\beta^{6}}{(1+\beta+\cdots+\beta^{6})} \tag{20}$$

Firm receives the money after paying corporation tax to the government from operating surplus that is considered as savings of firm. All savings of firm are invested/ reinvested into the next year capital stock of country n. Firm's saving and capital stock for each generation t at period $(t+s)$ are formulized by the following equations.

$$s_{f,t+s}^{nt} = os_{t+s}^{nt} - dt_{f,t+s}^{nt}, \tag{21}$$

$$dt_{f,t+s}^{nt} = \tau_{f,t}^{nt} os_{t+s}^{nt}, \tag{22}$$

$$s_{f,t+s}^{nt} = I_{f,t+s}^{nt} \quad and \tag{23}$$

$$k_{t+s}^{nt} = k_{t+s-1}^{nt}\left(1-\delta_{t}^{n}\right) + I_{f,t+s-1}^{nt}, \quad when, s=0, \quad k_{t}^{n} = k_{t}^{nt} \tag{24}$$

where

$s_{f,t+s}^{nt}$ = firm savings for each generation t at period $t+s$ of country n,

$dt_{f,t+s}^{nt}$ = corporation tax for each generation t at period $t+s$ of country n,

$\tau_{f,t}^{nt}$ = corporation tax rate which is same for all over the model.

k_{t+s}^{nt} = per worker capital stock for each generation t at period $t+s$ of country n

$I_{f,t+s}^{nt}$ = investment amount of firm for each generation t at period $t+s$ of country n.

In this model, seven generations coexist in every period t. For calculating the economic activities of country n in period t, it is necessary to calculate the economic activities of country n for each generation t at period $t+s$. For this reason, all above equations from (10) to (24) are formulized for each generation t at period $t+s$ of country n. Now, considering these equations, the following activities for consumers and firms of country n can be easily formulized in every period t.

Total employee compensation CE_{t}^{n}, total disposable income DI_{t}^{n}, total income tax $DT_{h,t}^{n}$, total consumption C_{t}^{n} and total savings $S_{h,t}^{n}$ of households of country n in period t are formulized as follows:

$$CE_{t}^{n} = \sum_{s=0}^{5} N^{t-s} ce_{t}^{t-s}, \quad DI_{t}^{n} = \sum_{s=0}^{5} N^{t-s} di_{t}^{t-s}, \quad DT_{h,t}^{n} = \sum_{s=0}^{5} N^{t-s} dt_{h,t}^{t-s},$$

$$C_t^n = \sum_{s=0}^{6} N^{t-s} c_t^{n,t-s} \; and \qquad S_{h,t}^n = \sum_{s=0}^{5} N^{t-s} s_{h,t}^{n,t-s} \tag{25}$$

where

$$N^t = (1+\eta) N^{t-1}, \quad N_t^n = \sum_{s=0}^{6} N^{t-s}$$

$\eta =$ working population growth rate for generation wise.

Total fixed capital depreciation FK_t^n, total operating surplus OS_t^n, total corporation tax (expenditure of firm) $DT_{f,t}^n$ and total savings $S_{f,t}^n$ of firms of country *n* in period *t* are formulized as follows:

$$FK_t^n = \sum_{s=0}^{6} N^{t-s} fk_t^{n-s}, OS_t^n = \sum_{s=0}^{5} N^{t-s} os_t^{t-s}, DT_{f,t}^n = \sum_{s=0}^{5} N^{t-s} dt_{f,t}^{t-s} \; and \quad S_{f,t}^n = \sum_{s=0}^{5} N^{t-s} s_{f,t}^{t-s} \tag{26}$$

3.3 Income expenditure for government

Although the government behavior is not considered for each generation *t* at period *t+s* like other agents but it is analyzed in every period *t*. Government receives income after paying transfer income and returning on bonds issued to households from his revenues. Government revenues are obtained by tax collection from household and firm and bonds issuing for household. Government spends his income for consumption and gross investment as described bellow.

$$G_t^n = DT_{h,t}^n + DT_{f,t}^n + \sum_{m=Japan,U.S} Bn_t^{nm} - \sum_{m=Japan,U.S} Bn_{t-1}^{nm} - TRG_t^n \tag{27}$$

$$GI_{g,t}^n = sav_{g,t}^n G_t^n, \; GC_t^n = \left(1 - sav_{g,t}^n\right) G_t^n \tag{28}$$

where

$G_t^n =$ government income of country *n* at period *t,*

$GI_{g,t}^n =$ government gross investment of country *n* at period *t,*

$GC_t^n =$ government consumption of country *n* at period *t,*

$\sum_m Bn_t^{nm} =$ amount of bonds issued by government of country *n* at period *,*

$\sum_m Bn_{t-1}^{nm} =$ amount of bond issued by government at last period of country *n,* and

$sav_{g,t}^n =$ gross investment rate of government of country *n* at period *t.*

3.4 Final demand

Final demand is an economic activity performed by households and the government consumption demand. If utility maximization problem subject to consumption restrictions is assumed Cobb-Douglas type, then the household consumption demand for commodity can be formulized as follows.

$$\max U_t^n = \left[\prod_m \prod_i \left(c_{it}^{mn}\right)^{\beta_{it}^{mn}}\right]\left(c_t^{ROWn}\right)^{\beta_t^{ROWn}}$$

$$s.t\left(\sum_m \sum_i q_{it}^{mn} c_{it}^{mn}\right) + q_t^{ROWn} c_t^{ROWn} = \sum_i C_{it}^n \tag{29}$$

where $$C_t^n = \sum_i C_{it}^n$$

The household consumption demand for commodity can be derived from the above equation (29) by the Lagrange multipliers method as follows.

$$c_{it}^{mn} = \frac{\beta_{it}^{mn} C_t^n}{q_{it}^{mn}}, \qquad c_t^{ROWn} = \frac{\beta_t^{ROWn} C_t^n}{q_t^{ROWn}} \tag{30}$$

where

$\left(\sum_m \sum_i \beta_{it}^{mn}\right) + \beta_t^{ROWn} = 1,$

c_{it}^{mn} = the household consumption demand of country *n* at period *t* for commodity *i* imported from country *m,*

$\beta_{it}^{mn}, \beta_t^{ROWn}$ = preference parameter at period *t*, and

c_t^{ROWn} = the household consumption demand of country *n* at period *t* for commodities imported from *ROW.*

Similarly, the government consumption demand function also can be formulized as follows.

$$g_{it}^{mn} = \frac{\zeta_{it}^{mn} GC_t^n}{q_{it}^{mn}}, \qquad g_t^{ROWn} = \frac{\zeta_t^{ROWn} GC_t^n}{q_t^{ROWn}} \tag{31}$$

where

g_{it}^{mn} = the government consumption demand of country *n* for commodity *i* imported from country *m* ,and

$\zeta_{it}^{mn}, \zeta_t^{ROWn}$ = government preference parameter at period *t*.

3.5 Capital accumulation

In this paper, it is assumed that households invest their savings for buying domestic/foreign bonds. The investment to ROW is not considered. So, the gross investment of a country is estimated by the amount of money from domestic country (household investment amount and government gross investment) and foreign countries (household investment amount) as formulized by the following equations.

$$S_{h,t}^n = I_{h,t}^n = \sum_{m=Japan,U.S} I_t^{nm} = I_{Db,t}^{nn} + I_{Fb,t}^{nm} \tag{32}$$

$$GI_t^n = I_{Db,t}^{nn} + I_{Fb,t}^{mn} + GI_{g,t}^n \tag{33}$$

$$\sum_m Bn_t^{nm} = I_{Db,t}^{nn} + I_{Fb,t}^{mn} \tag{34}$$

where

$I_{Db,t}^{nn}$ = the amount of money of country *n* that is invested for buying domestic bonds at period *t,*

$I_{Fb,t}^{nm}$ = the amount of money of country *n* that is invested for buying foreign bonds at period *t,*

GI_t^n = gross investment of country *n* at period *t,* and

$I_{Fb,t}^{mn}$ = the amount of money of foreign country that is invested for buying bonds of country *n* at period *t*.

3.6 Dynamic gross domestic product and trade analysis

In international trade, a substantial share of domestic output of a country is exported to foreign countries and a large part of total expenditure is imported from foreign countries. So, it can be said that the analysis of dynamic gross domestic product is important to analyze the dynamic trade pattern of a country.

Dynamic gross domestic product: The gross domestic product GDP_t^n for each country at each period is the sum of operating surplus, employee compensation, and fixed capital depreciation of that country. The next period GDP_{t+1}^n is analyzed for each country using the OLG technique as formulized by the following equations.

$$GDP_t^n = OS_t^n + CE_t^n + FK_t^n \quad and \tag{35}$$

$$GDP_{t+1}^n = OS_{t+1}^n + CE_{t+1}^n + FK_{t+1}^n \tag{36}$$

where

$$OS_{t+1}^n = \left(\sum_j \left[\left(r_{t+1}^n - \delta_{j,t}^n P_{K_j,t+1}^n \right) dk_{j,t+1}^n a_{Y_j t}^n X_{j,t+1}^n \right] \right) N_{t+1}^n,$$

$$CE_{t+1}^n = \left(\sum_j \left(\omega_{t+1}^n dl_{j,t+1}^n a_{Y_j t}^n X_{j,t+1}^n \right) \right) N_{t+1}^n,$$

$$FK_{t+1}^n = \left(\sum_j \left(\delta_{j,t}^n P_{K_j,t+1}^n dk_{j,t+1}^n a_{Y_j t}^n X_{j,t+1}^n \right) \right) N_{t+1}^n.$$

Dynamic trade pattern: Since the volume of import for each country *n* at period *t* and *t+1* is the sum of the intermediate demand and consumption demand for households and government of commodities from overseas, it is expressed by the following formulas.

$$m_{it}^{mn} = \sum_j \left(a_{ijt}^{mn} X_{jt}^n \right) + c_{it}^{mn} + g_{it}^{mn}, \tag{37}$$

$$m_{i,t+1}^{mn} = \sum_j \left(a_{ijt}^{mn} X_{j,t+1}^n \right) + c_{i,t+1}^{mn} + g_{i,t+1}^{mn}, \tag{38}$$

$$m_{t+1}^{ROWn} = \sum_j \left(a_{jt}^{ROWn} X_{j,t+1}^n \right) + c_{t+1}^{ROWn} + g_{t+1}^{ROWn}, \tag{39}$$

$$M_t^n = \sum_i q_{it}^{mn} m_{it}^{mn} + q_t^{ROWn} m_t^{ROWn}, and \tag{40}$$

$$M_{t+1}^n = \sum_i q_{i,t+1}^{mn} m_{i,t+1}^{mn} + q_{t+1}^{ROWn} m_{t+1}^{ROWn}. \tag{41}$$

where

m_{it}^{mn} = the volume of import of commodity *i* to country *n* from country *m* at period *t,*

M_{t+1}^n = total import to country *n* from country *m* and ROW at period *t+1,* and

q_{it}^{mn} = the consumer price in country *n* of imported commodity *i* from country *m* at period *t.*

Total export E_t^n and E_{t+1}^n from country *n* to country *m* and ROW at period *t* and *t+1* is formulized bellow.

$$E_t^n = \sum_i q_{it}^{nm} m_{it}^{nm} + \sum_i p_{it}^n e_{it}^{nROW} \quad and \tag{42}$$

$$E^{n}_{t+1} = \sum_i q^{nm}_{i,t+1} m^{nm}_{i,t+1} + \sum_i p^{n}_{i,t+1} e^{nROW}_{it} \tag{43}$$

For analyzing the dynamic trade of a country, it is necessary to consider the commodity consumption demand for household $c^{mn}_{i,t+1}$ and government $g^{mn}_{i,t+1}$ of each country at period *t+1* which is formulized using OLG technique as shown bellow.

$$c^{mn}_{i,t+1} = \frac{\beta^{mn}_{it} C^{n}_{t+1}}{q^{mn}_{i,t+1}}, \quad c^{ROWn}_{t+1} = \frac{\beta^{ROWn}_{t} C^{n}_{t+1}}{q^{ROWn}_{t+1}}, \quad g^{mn}_{i,t+1} = \frac{\zeta^{mn}_{it} GC^{n}_{t+1}}{q^{mn}_{i,t+1}} \tag{44}$$

where

$$C^{n}_{h,t+1} = C^{n}_{t+1} = \sum_i C^{n}_{i,t+1} = \left(1 - sav^{n}_{h,t+1}\right) DI^{n}_{t+1},$$

$$DI^{n}_{t+1} = CE^{n}_{t+1} - DT^{n}_{h,t+1} + TRG^{n}_{t+1}$$

C^{n}_{t+1} = total consumption of each country *n* at period *t+1*,

DI^{n}_{t+1} = total household disposable income of each country *n* at period *t+1*, and

$sav^{n}_{h,t+1}$ = households saving rate at period *t+1.*

Similarly, the government consumption of each country for period *t+1* can be calculated.

3.7 Equilibrium conditions

The intertemporal general equilibrium requires that all markets clear in each country and in each period. Clearing is required on (1) the commodity market, (2) the labor market, and (3) the capital market as expressed by the following equations.

(1) Supply and demand balance of the commodity for each country at period *t* and *t+1* is shown bellow.

$$X^{m}_{it} = \sum_n \sum_j \left[\left(a^{mn}_{ijt} X^{n}_{jt} \right) + c^{mn}_{it} + g^{mn}_{it} \right] + e^{mROW}_{it} \tag{45}$$

$$X^{m}_{i,t+1} = \sum_n \sum_j \left[\left(a^{mn}_{ijt} X^{n}_{j,t+1} \right) + c^{mn}_{i,t+1} + g^{mn}_{i,t+1} \right] + e^{mROW}_{it} \tag{46}$$

where

e^{mROW}_{it} = Export to ROW from country *m* commodity *i* at period *t.*

(2) Supply and demand balance of labor for each country at period *t* and *t+1* is shown bellow.

$$L^{n}_{t} = \sum_j \left(DL^{n}_{jt} a^{n}_{y_j t} X^{n}_{jt} \right) \tag{47}$$

$$L^{n}_{t+1} = \sum_j \left(DL^{n}_{j,t+1} a^{n}_{y_j t} X^{n}_{j,t+1} \right) \tag{48}$$

where $$L^{n}_{t+1} = \left(1 + \eta\right) L^{n}_{t},$$

$\eta =$ periodic labor growth rate.

(3) Supply and demand balance of capital for each country at period *t* and *t+1* is shown bellow.

$$K^n_t = \sum_j \left(DK^n_{jt} a^n_{y_j t} X^n_{jt} \right) \tag{49}$$

$$K^n_{t+1} = \sum_j \left(DK^n_{j,t+1} a^n_{y_j t} X^n_{j,t+1} \right) \tag{50}$$

where $K^n_{t+1} = \left(1-\delta^n_t\right) K^n_t + I^n_{f,t}$

4 Conclusion

Labor and capital demand are considered for per worker to formulize the primary income distribution for consumers and firms. Next, the primary income distribution is formulized for each generation at seven periods for considering seven period OLG techniques. Several other papers have considered two period OLG model where only investment policy is analyzed. However, this work considers seven period OLG model for analyzing the investment as well as reinvestment policy for consumers and firms. In two period model, last period capital accumulation is not considered because first period capital will fully depreciate in the next period. The main limitation of seven period model is that a lot of capital is accumulated at last period for each generation. In the secondary income distribution, it is assumed that consumers receive disposable income for consumption and savings in seven period budget conditions. Income expenditure and savings of firms are also considered. Furthermore, it is assumed that consumers invest all savings for buying domestic/foreign bonds, and firms invest all savings into the next period capital stock. Households and firms investment policy are distinguished. A firm will invest all savings into the next period capital stock because of the dynamic nature and for calculating dynamic wage and capital rental price. In cost minimization problem, it is observed that household wage, capital rental price and production technology have an influence on labor and capital demand.

The OLG model allows the utility maximization problem subject to a household lifetime budget constraint for obtaining the household consumption. Not only household lifetime consumption but also the commodity consumption demand for households and the government at every period are obtained by solving the utility maximization problem. The main contribution is that the same technique like OLG model is applied for developing a dynamic international trade model using dynamic general equilibrium concept. The effect of demography, interest rate and foreign exchange rate are also considered to analyze the trade pattern.

Currently the work is in data analysis phase where tables seven periods (1965, 1970, 1975, 1980, 1985, 1990 and 1995) international input-output transaction table, national accounts statistics and time series data are being used by the model for analyzing Japan-U.S economy in details. In this paper, seven periods value-added part for different sectors are used for analyzing the model. For deficiency of data, the value-added part for different sectors for four periods (1965,1970,1975 and 1980) are manipulated based on actual total yearly GDP data and three periods (1985, 1990 and 1995) international I-O transaction tables. The remaining three periods (1985, 1990 and 1995) value-added data are used from actual I-O table. The complete international I-O table is used for only year 1995.

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