A Model to Analyse Financial Fragility*†

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Abstract

Our purpose in this paper is to produce a tractable model which illuminates problems relating to individual bank behaviour and risk-taking, to possible contagious interrelationships between banks, and to the appropriate design of prudential requirements and incentives to limit 'excessive' risk-taking. Our model is rich enough to include heterogenous agents (commercial banks and investors), endogenous default, and multiple commodity, and credit and deposit markets. Yet, it is simple enough to be effectively computable. Financial fragility emerges naturally as an equilibrium phenomenon.

In our model a version of the liquidity trap can occur. Moreover, the Modigliani-Miller proposition fails either through frictions in the (nominal) financial system or through incentives, arising from the imposed capital requirements, for differential investment behaviour because of capital requirements. In addition, a non-trivial quantity theory of money is derived, liquidity and default premia co-determine interest rates, and both regulatory and monetary policies have non-neutral effects.

The model also indicates how monetary policy may affect financial fragility, thus highlighting the trade-off between financial stability and economic efficiency.

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1 Introduction

It is a truism that the structure of a model needs to reflect the practical purposes which drive the research in the first place. In our case, we work for the Financial Stability wing of the Bank of England; our aim is to construct a model which illuminates problems relating to individual bank behaviour and risk-taking, to possible contagious inter-relationships between banks, and to the appropriate design of prudential requirements and incentives to limit 'excessive' risk-taking.

In order to reduce a model of the aggregate economy to manageable proportions, a common simplification is to assume that each sector has agents which behave identically, so that they can be presented in 'representative agent' format. So, in most models the banking system is represented by a single agent, which can either be viewed as a set of perfectly competitive identical banks, or, on occasions, as a single, monopolistic bank.

While the representative agent approach has many uses and advantages, applying it to the banking system inevitably obscures many of the economic and behavioural relationships, notably between banks, in which a regulatory authority is closely interested. For example, with a single 'representative' bank, there can be no interbank market. Again, either the whole banking system, as represented by the one agent, fails, or the whole banking system survives in face of some assumed shock. Typically in reality individual banks have differing portfolios, often reflecting differing risk/return preferences. So, typically, failures occur with the greatest probability amongst the riskiest banks. Such failures in turn generate interactions in the system more widely that may threaten the survival of other banks, a process of contagion. This may have several channels, both in interbank relationships more directly, and via changes in asset market flows and prices that may involve other sectors, e.g. persons and companies. Such interactions can hardly be studied in a model with a single representative bank, since many of these interactions, e.g. the interbank market, are ruled out by definition.

So the main innovation in our model is to incorporate a number of commercial banks. Each bank is distinguished by a unique risk/return preference. Since each bank is, and is roughly perceived as being, different, it follows that there is not a single market either for bank loans or bank deposits.[1] Instead, we assume that there is a separate market, with differing interest rates, in each case. We also allow individual non-bank agents to differ, with differing utility functions, and, hence differing attitudes towards potential bankruptcy; we also model in their case the incentives for avoiding bankruptcy, as we do in the case of the banks. Again we assume that the banks can observe the agents' differing riskiness with noise. This means that each borrower faces a different credit market. If each bank had its own individual, idiosyncratic information on each borrower, then if there were H borrowers and B banks, there would be $\frac{H!B!}{(H-1)!(B-1)!}$ bilateral markets for borrowing. If we assume, instead, that each bank has the same information on each borrower, an implausible assumption, then competition between banks would mean that there would be H separate markets. Alternatively, we can assume that borrowers have been pre-allocated at time t=0 to a particular bank, and that that allocation provides each respective bank with specialised information; such additional information allows them, for asymmetric information standard reasons, to lend cheaper than any other bank. Consequently there are B separate credit markets between each bank and a subset of borrowers that were initially randomly allocated.

This means that, instead of a single market for deposits/loans, we have multiple markets

[1] There would be such a single market if there was 100% complete deposit insurance in place, since then all deposits would be similarly riskless in any bank. We could adjust the market to take account of deposit insurance, with varying coverage, in future extensions.

for deposits (by separate bank) and for loans (by borrower and bank). Given the optimising conditions for the individual banks, after assuming an initial allocation of capital, the open market operations of the Central Bank, etc., etc., deposits may not be sufficient in each individual bank (plus capital), to finance that bank's asset portfolio (of cash, loans and Central Bank (public sector) debt), although within the banking sector as a whole outside liabilities must equal outside assets, and interest rates and/or cash flows adjust until that happens. So, deficit (surplus) banks borrow (lend) on the interbank market. In reality the interbank market is also segmented with banks of differing riskiness either borrowing at different rates, or facing limited 'caps' on such borrowing. At this stage in our exercise, however, we shall assume a single, undifferentiated interbank market with a common interest rate.

Since our focus is on financial fragility, the governance (public sector) institutions which we introduce are a financial regulator and a Central Bank; these two may, or may not, be the same institution, but will be assumed to cooperate where necessary. We abstract from fiscal policy. The financial regulator sets the penalties/incentives on bankruptcy in both the banking and the non-banking private sector, and also the required (minimum) capital adequacy ratios.

The Central Bank is established at time $t = 0$ with an allocation of (public sector, safe, fixed nominal value) debt as its assets. Against this it has as its liabilities cash, commercial banks reserve deposits and Central Bank debt. Reserve deposits and Central Bank debt are held by the commercial banks only, in an initial allocation, against an equivalent initial allocation of capital. Central Bank open market operations exchange its own (interest-bearing) debt for (non-interest bearing) deposits. Moreover, the Central Bank can lend, or borrow, in the interbank market.

In principle the non-bank public can insist on converting its commercial bank deposits into currency or into Central Bank deposits. It is this convertibility commitment that forces commercial banks to hold Central Bank deposits. Again we assume an initial allocation of Central Bank cash to the public, and model the public's choice between (safe) cash, which is non-interest-bearing, and deposits, which are risky but interest-bearing, and can be used for expenditures, and other risky (non-liquid) aspects.

The present model is based on the model introduced by Tsomocos (2003a) and (2003b) which introduced a commercial banking sector and capital requirements in a general equilibrium model with incomplete markets, money and default. However, we depart by introducing the possibility of capital requirements' violation and consequent penalties and a secondary market for the banks' equity. Moreover, we introduce limited access to consumer credit markets, thus allowing for different interest rates across the commercial banking sector. Finally, we simplify the model by removing the intratemporal loan markets and allow only for intertemporal borrowing and lending.

The closest precursor to this approach is the work of Shapley and Shubik (1977), Shubik (1973) and Shubik (1999) who introduced a central bank in a strategic market game. Shubik (1973) also emphasised the virtues of explicitly modeling each transaction (see also Grandmont (1983), Grandmont and Laroque (1973), Grandmont and Younes (1972 and 1973) who introduced a banking sector into general equilibrium with overlapping generations). The commercial banking sector follows closely Shubik and Tsomocos (1992). The modeling of money in an incomplete markets framework is akin to a series of models developed by Dubey and Geanakoplos (1992, 2003a, and 2003b) and by Drèze and Polemarchakis (2000) and Drèze, Bloise, and Polemarchakis (2002). Finally, default is modelled as in Dubey, Geanakoplos and Shubik (2000), Shubik (1973), and Shubik and Wilson (1977), namely by subtracting a linear term from the objective function of the defaulter proportional to the debt outstanding.[2]

For the rest we have tried to keep this model as simple, standardised and parsimonious as we can. The structure of the paper is as follows. In section 2, we set out the basic form of the model. In sections 3 and 4, we formally define the budget sets of the non-bank public, commercial banks and *Monetary Equilibrium with Commercial Banks and Default* (MECBD). In section 5, we show under which conditions a MECBD is achieved. We provide conditions such that trade would be beneficial to traders even under the presence of positive interest rates, and asset markets will be active irrespective of the possibility of positive default in equilibrium. Thus in a MECBD positive default and financial fragility are compatible with the orderly functioning of markets. So, given default and financial vulnerability, there is room for economic policy to improve upon the ensuing inefficiencies.

A formal definition of financial fragility is proposed in section 6, borrowed from Tsomocos (2003a and 2003b). Also, a Keynesian liquidity trap holds in equilibrium in which commodity prices stay bounded whereas the volume of trade in the asset markets tends to infinity whenever monetary policy is loosened. Also, whenever financial fragility is present in the economy, the role of economic policy is justified. Regulatory policy is shown to be non-neutral. Moreover, we address formally the Modigliani-Miller proposition and establish the conditions that cause its failure. In section 7, we note that Hicksian elements of the demand for money are active in equilibrium. We establish monetary non-neutrality that characterise the lack of the classical dichotomy between the real and nominal sectors of the economy. We also show that a non-trivial quantity theory of money holds and the liquidity structure of interest rates depends on aggregate liquidity and default in the economy.

Using the principles derived, we proceed in section 8 to analyse a concrete comparative statics change in computable general equilibrium models. We rationalise the outcomes by tracing the new equilibrium. The results are based on the prior principles that must hold simultaneously in equilibrium. For example, a change in monetary policy must satisfy contemporaneously the quantity theory of money, the liquidity structure of interest rates and the Fisher relation. Finally, we conclude in section 9 and all the proofs are relegated in the appendix.

This framework is more elaborate, but we believe that it also offers new insights into the analysis of financial fragility and systemic risk. We doubt whether contemporaneous models, without heterogenous agents, can adequately handle analyses relating to liquidity, default and contagion. After we gain experience with this model through parametric examples, we may possibly be able to derive our comparative statics results in a more general context.

2 The Model

2.1 The Economy

Consider the standard general equilibrium model with incomplete markets (GEI) in which time extends over two time periods (i.e. an endowment economy without production). The first period consists of a single initial state and the second period consists of S possible states. At $t = 0$, non-bank private sector (NBPS), commercial banks and the authorities take their decisions expecting (rationally) the realisation of any one of the S possible future scenarios to occur. At $t = 1$ one of the S states occurs and then again the economic actors take the

[2] Dubey and Geanakoplos (1992) have studied more general default specifications and also introduced the gains-from-trade hypothesis that guarantees the positive value of fiat money in finite horizon.

appropriate decisions. A detailed explanation of the sequence of events is contained in section 2.3 and figure 1. NBPS and commercial banks transact, maximising their respective objective functions, whereas the Central Bank and the regulator are modelled as 'strategic dummies' (i.e. their choices are exogenously fixed and are common knowledge to economic agents). NBPS trade in commodities, financial assets, consumer loans, deposits and shares of commercial banks. Commercial banks lend to the consumers and take deposits, they borrow and lend in the interbank credit market, invest in the asset market and issue equity in the primary market. The Central Bank operates in the interbank market via OMOs. The regulator fixes the bankruptcy code for households and commercial banks exogenously and sets the capital-adequacy requirements for commercial banks.

Formally, the notation that will be used henceforth is as follows:

$t \in T = \{0, 1\}$ = time periods,
$s \in S = \{1, ..., S\}$ = set of states at $t = 1$,
$S^* = \{0\} \cup \{S\}$ = set of all states,
$h \in H = \{1, ..., H\}$ = set of economic agents (households/investors),
$b \in B = \{1, ..., B\}$ = set of commercial banks,
$l \in L = \{1, ..., L\}$ = set of commodities,
$R_+^L \times R_+^{SL}$ = commodity space indexed by $\{1, ..., S\} \times \{1, ..., L\}$
$e^h \in R_+^L \times R_+^{SL}$ = endowments of households
$m^h \in R_+^{S^*}$ = monetary endowments of households
$e^b \in R_+^{S^*}$= capital endowments of commercial banks
$u^h : R_+^L \times R_+^{SL} \to R$ = utility function of agent $h \in H$,
χ_{sl}^h = consumption of commodity l in state s by $h \in H$.

The standard assumptions hold:

(A1) $\forall s \in S^*$ and $l \in L$, $\sum\limits_{h \in H} e_{sl}^h > 0$,

(i.e. every commodity is present in the economy).

(A2) $\forall s \in S^*$ and h $(b) \in H$ (B), $e_{sl}^h > 0$ $(e_{sl}^b > 0)$ for some $l \in L$, $s \in S^*$,

(i.e. no household (commercial bank) has the null endowment of commodities (capital) in every state of the world).

(A3) Let A be the maximum amount of any commodity sl that exists and let $\mathbf{1}$ denote the unit vector in R^{SL}. Then $\exists Q > 0 \ni u^h(0, ..., Q, ..., 0) > u^h(A\mathbf{1})$ for Q in an arbitrary component

(i.e. strict monotonicity in every component).[3] Also, continuity and concavity are assumed.

Let $u^b(\pi_0^b, \pi_1^b, ..., \pi_s^b) : R_+^{S^*} \to R$ = objective function of commercial banks.
π_s^b = monetary holdings of b at $s \in S^*$.

A straightforward assumption is imposed.

(A4) Let A_m be the maximum amount of money present in the economy and let $\mathbf{1}$ denote the unit vector in R^{S^*}. Then $\exists Q > 0 \ni u^b(0, ..., Q, ..., 0) > u(A_m\mathbf{1})$ for Q in an arbitrary component.

[3]The results remain unaffected if, instead of the previous condition, we assume smoothness of u^h.

(i.e. strict monotonicity in every component).[4] Also, continuity and concavity are assumed.

2.2 Central Bank and the Regulator

The Central Bank conducts open market operations in the interbank credit market (though it could also do so by buying, or selling, its own debt instruments).[5]

Formally, the following vector gives the Central Bank's action

$$(M^{CB}, \mu^{CB}, m^{CB}) \equiv (M^{CB}, \mu^{CB}, (m^{CB}_{bs})_{b\in B, s\in S^*})$$

where,
M^{CB} = OMOs on behalf of the Central Bank,
μ^{CB} = bond sales by the Central Bank,
m^{CB} = money financed Emergency Liquidity Assistance to commercial banks.

Note that the Central Bank is not required to spend less than it borrows; the existence of equilibrium is compatible with the Central Bank printing money to finance its expenditures. All the results hold for both cases (i.e. with or without money financing) except where otherwise stated. Also, the Central Bank may fix the interbank interest rate ρ and then accommodate the ensuing money demand.

Similarly, the following vector gives the regulator's actions

$$(k, \lambda, \omega) \equiv ((\bar{k}_t)_{t\in T}, (\lambda^h_{sz})_{h\in H\cup B, s\in S, n\in N}; (\omega_{tj})_{t\in T, j\in Z})$$

where
k_t = time-dependent capital requirements $\forall b \in B$,
λ^h_{sz} = bankruptcy (or capital requirements violation) penalties imposed upon $h \in H \cup B$ when contractual obligations are abrogated
$z \in Z = \{N\} \cup \{J\} \cup \{k_1, k_2\} = \{0^*, 0^1, ..., 0^B\} \cup \{\overline{0}^1, ..., \overline{0}^B\} \cup \{1, ..., J\} \cup \{k_1, k_2\}$
Z is the set of all credit and deposit markets (i.e. loan markets $(0^1, ..., 0^B)$, deposit markets $(\overline{0}^1, ..., \overline{0}^B)$ and interbank markets (0^*)), secondary asset markets and time-dependent capital requirements. (see section 2.4-2.6)
ω_{tj} = risk-weights $\forall b \in B, \forall t \in T = \{0, 1\}$ and $j \in \{0, 1, ..., J\}$.

The risk weights may be functions of other macroeconomic variables such as aggregate default levels as in Caterineu-Rabell, Jackson and Tsomocos (2003), interest rates, volumes of trade, prices, etc. Consumer loans are bank specific (see section 2.5) whereas the interbank credit market is an aggregate market where the Central Bank and all the commercial banks participate. Finally, since our focus is on formulating a framework for financial stability and not monetary policy, we have collapsed the interbank and the repo markets into one.

2.3 The Time Structure of Markets

At $t = 0$, the commodity, asset, equity, credit, deposit and interbank markets meet. At the end of the first period consumption and settlement (including any bankruptcy and capital requirements' violation penalties) take places.

[4] The results remain unaltered if, instead of the previous condition, we assume smoothness of u^b.

[5] LOLR assistance to commercial banks could also be modelled in this context, a subject for future analysis. For more see Goodhart (1989) and Goodhart and Huang (1999).

At $t = 1$, commodity and equity markets meet again, loans, deposits and assets are delivered. At the end of the second period consumption and settlement for default and second period capital requirements' violations take place. Also, commercial banks are liquidated. Figure 2 makes the time line of the model explicit.

2.4 Asset Markets

The set of assets is $J = \{1, ..., J\}$. Assets are promises sold by the seller in exchange for a price paid by the buyer today. They are traded at $t = 0$ and the contractual obligations are delivered at $t = 1$ for a particular state $s \in S$. An asset $j \in J$ is denoted by a vector $A^j \in R_+^{S(L+1)}$ indicating the collection of goods deliverable and the money at any future state $s \in S$. Therefore, the asset market is summarised by an $((L + 1)S) \times J$ matrix $\mathbf{A}$.

All the deliveries are made in money (outside cash or inside deposits/loans). When the assets promise commodities the seller delivers the money equivalent of the value of the agreed commodities at their spot prices in the relevant state. Whenever rank $|J|$ = rank $|S|$ the capital markets are said to be complete whereas when rank $|J| <$ rank $|S|$ the markets are said to be incomplete.

Furthermore, we assume

(A5) $A^j \neq 0, \forall j \in J$

(i.e. no asset makes zero promises).

(A6) $A^j \geq 0, \forall j \in J$

(i.e. asset payoffs are non-negative).

Finally, note that agents do not hold positive endowments of assets and thus all sales of assets are effectively short sales.

Individuals are price takers in asset markets, where θ_j represent asset prices. Let $b_j^h \equiv$ amount of money sent by $h \in H \cup B$ in the market of asset j. Also, let $q_j^h \equiv$ promises sold of asset j by agent $h \in H \cup B$. In equilibrium, at positive levels of trade, $0 < \theta_j < \infty$,

$$\theta_j = \frac{\sum\limits_{h \in H} b_j^h + \sum\limits_{b \in B} b_j^b}{\sum\limits_{h \in H} q_j^h + \sum\limits_{b \in B} q_j^b}$$

for $j \in J, h \in H, b \in B$. All the asset markets meet contemporaneously; hence cash obtained from the sale of asset j cannot be used for the purchase of another asset $j' \neq j$. Thus, the volume of trade in the asset market is affected by the overall liquidity of the economy. In this way monetary policy interacts with asset markets and influences asset prices (i.e. asset price inflation channel).

2.5 Money, Credit, and Deposit Markets

Money is the stipulated means of exchange. All commodities can be traded for money, and (as noted) all asset deliveries are exclusively in money. Money can be either *inside* or *outside* fiat (see Gurley and Shaw (1960)). At the outset some individuals and banks hold *net* monetary assets-outside money-(which includes Central Bank liabilities). Inside money is credit created

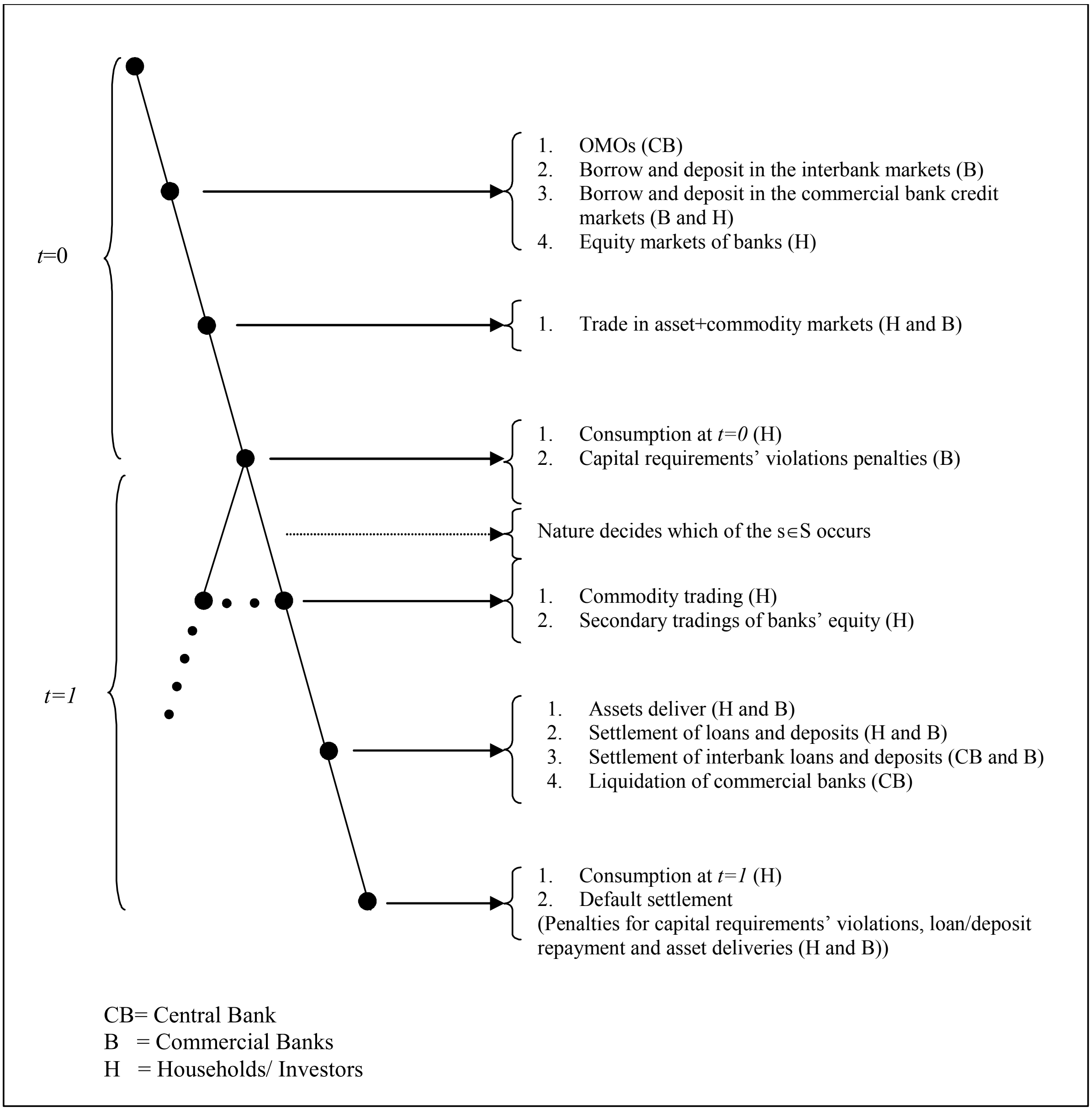

Figure 1: The Time Structure of the Model

by the banking sector through the credit markets in period 0, in part depending on current monetary policy, and is matched by the individual borrowers' debt obligation to the banks. When the Central Bank undertakes expansionary OMOs, in the interbank market, the commercial banks gain cash reserve assets matched by an interbank deposit liability to the Central Bank. In turn, commercial banks lend to borrowers and accept deposits. This represents an asset of the commercial bank and thus a liability to investors. The *net* assets of the private sector as a whole remain unchanged. Cash-in-advance is required for any purchase.

A market involves a symmetric exchange between two instruments. Just as agents cannot sell money which they do not have in a market, so in the model agents cannot sell commodities they do not have. The only exceptions are assets, credit and deposit markets, where we allow agents to write their own promises (bonds).

Money enters the economy in three ways. First, it may be present at the outset ($t = 0$) in the private endowments of agents and commercial banks. Agent $h \in H$ has an endowment m^h of money, $\forall s \in S^*$ and commercial banks have initial capital endowment e_s^b, $\forall s \in S^*$, part of which may be held in cash or deposits at the Central Bank. Second, when the Central Bank lends on the interbank market, or purchases bonds with currency or the government engages in money financed fiscal transfers, then the money stock increases. Third, previously issued Central Bank bonds, or interbank loans, are repaid; money then exits the system via redemptions of debt from investors and from commercial banks.

Agents are permitted to borrow from, and deposit with, each particular commercial bank. However, borrowers are distributed initially to a particular bank from which they borrow. This may be a result of relationship banking, or some other informational advantage that a commercial bank has with particular borrowers. This restricted participation assumption generates different interest rates charged by commercial banks, but could be relaxed in more complicated versions of the model.

So, let us partition $H = \{1, ..., H\}$ to $\{h_1^\alpha, ..., h_k^\alpha\} \cup \{h_1^b, ..., h_l^b\} \cup ... \cup \{h_1^B, ..., h_m^B\}$ disjoint sets whose union is H. Let the generic element of a subset H^b be h^b indicating the particular agent who can borrow from $b \in B$. Note that $h \in H$ can deposit in any commercial bank she wishes.

(A7) Each $h^b \in H^b$ can only borrow from $b \in B$ (i.e. restricted participation in lending markets)

Let μ^{h^b} be the amount of fiat money agent $h^b \in H^b$ chooses to owe on the loan market of bank b. If all agents repay exactly what they owe, then $\forall b \in B$ we must have that,

$$1 + r^b = \frac{\sum\limits_{h^b \in H^b} \mu^{h^b}}{\overline{m}^b}$$

where $\overline{m}^b$ is the amount of credit that commercial banks extend which is also subject to their capital requirements set by the regulator (see section 2.6).

Thus, the ratio of nominal value of loans over loans supply (i.e. commercial banks credit extension) determines the gross *nominal* interest rate. We add that r^b is the *ex ante* nominal interest rate that incorporates both the liquidity and default premium of loans since default is permitted in equilibrium. The effective (*ex post*) interest is suitably adjusted to account for default on loans.

Similarly, there exist B deposit markets, one for each commercial bank, in which investors may deposit funds. Note, that we do not impose the limited participation assumption on the different deposit markets.

Let μ_d^b be the demand of deposits by each $b \in B$ and d_b^h the amount of deposits that each $h \in H$ deposits with each $b \in B$. If all deposits are repaid fully by commercial banks, the $\forall b \in B$ we must have that

$$1 + r_d^b = \frac{\mu_d^b}{\sum\limits_{h \in H} d_b^h}$$

The deposit rates need not be the same as the lending rate since the parties involved in each market will, in principle, manifest different default patterns in equilibrium. Moreover, even though we have removed the limited participation assumption, deposit rates would in general be different since different banks may make different choices as to their respective repayment rates. Thus, as with lending rates so the deposit rates would also incorporate default and liquidity premia in equilibrium.

These financial assets, bank loans, bank deposits, and bank equities (but *not* commodities) can be inventoried; they are the only stores of value in our model.

2.6 Capital Requirements

As already mentioned in section 2.1, the regulator sets the banks' minimum capital requirements. Given that the assets of commercial banks consist of loans (including interest rate payments), investments in marketable assets and some initial distribution of government bonds, the capital requirements constraint becomes,

$$\overline{k}_t \leq \frac{e_s^b + \sum\limits_{h \in H} \psi_b^h}{\overline{\omega}_t(\eta, \sigma)\overline{R}_s^b \overline{m}^b (1 + r^b) + \sum\limits_{j \in J} \omega_{tj}(\eta, \sigma) R_{sj}(p_s A_s^j)\left(\frac{b_j^b}{\theta_j}\right) + \omega_t(\eta, \sigma) R_s^b d^b (1 + \rho)}, \quad \forall s \in S^*, b \in B$$

The variables are defined as follows:

$\eta \equiv$ set of macro variables,
$\sigma \equiv$ choice variables of the investors and commercial banks,
$\overline{\omega}_t(\cdot) \equiv$ risk-weights for loans,
$\omega_{tj}(\cdot) \equiv$ risk-weights for marketable assets,
$\omega_t(\cdot) \equiv$ risk weights for interbank market deposits,
$\psi_b^h \equiv$ equity of commercial banks,
$p_s \equiv$ commodity prices, $\forall l \in L, s \in S$,
$\rho \equiv$ interbank interest rate
$e_s^b \equiv$ commercial banks' initial capital endowment $\forall s \in S^*$,
$d^b \equiv$ amount of money that $b \in B$ deposits in the interbank market
R's $\equiv$ expected rates of delivery of various instruments (see also section 2.7)

Capital requirements are set by the regulators at each $t \in T, \forall b \in B$. However, evaluation of the capital and the risk-weighted assets occur at each state's, $s \in S$, prices, delivery rates, initial capital endowments and capital adjustments. Banks may not necessarily hold the same capital since precautionary capital over and above the regulatory minimum can vary across banks. In addition, as we will describe in the next section, banks are allowed to violate the capital requirements constraints, subject to a penalty payment.

Note that credit requirements in the first period are calculated with respect to the realised asset deliveries in the *ex post* equilibrium and not to the expected *ex ante* ones as in $t = 0$. The impact of regulatory policy, since it affects credit extension and banks' portfolio composition, is akin to the workings of monetary policy.

2.7 Default

Default can be either strategic or due to ill fortune. Lenders cannot distinguish whether default (or equivalently capital requirements constraints' violation) occurs because the debtors are unable to honour their contractual obligations or they choose not to do so even though they have the necessary resources. The default (or capital requirements' violation) penalties are proportional to the level of default (or violation). Their purpose is to induce debtors to honour their obligation when they are able to do so and to refrain from making promises that they know they will not honour in the future.

Let us define $D^h_{sz} = (1 - v^h_{sz})\mu^{h^b}$ ($D^b_{sz} = (1 - v^b_{sz})\mu^b$) where v^h_{sz} (v^b_{sz}) is the rate of repayment by households (banks). D^h_{sz} is the nominal value of debt under default in the credit markets (analogously in the asset market, or on deposit and interest rate obligations). In practice, default penalties and the bankruptcy code depend normally on the *nominal* values of debt and are only adjusted at discrete intervals as the general level of prices increases. In the model, nominal values are deflated so that penalties are real. Finally, note that households are not allowed to default on their obligations either in the primary or secondary equity markets of commercial banks.

Let the parameters λ^h_{sz} (λ^b_{sz}) represent the marginal disutility of defaulting for each 'real' dollar on liabilities in state s.[6] Therefore, the payoffs to investors and commercial banks will be respectively $\forall s \in S^*$,

$$\Pi^h_s(\chi^h_s, (D^h_{sz})_{z\in Z}, p_s) = u^h_s(\chi^h_s) - \frac{\sum\limits_{z\in Z} \lambda^h_{sz}[D^h_{sz}]^+}{p_s g_s}$$

and

$$\Pi^b_s(\pi^b_s, (D^b_{sz})_{z\in Z}, p_s) = u^b_s(\pi^b_s) - \frac{\sum\limits_{z\in Z} \lambda^b_{sz}[D^b_{sz}]^+}{p_s g_s}$$

where g_s is the base basket of goods which serves as a price deflator with respect to which the bankruptcy penalty is measured and

$$[c]^+ \equiv \max[0, c]$$

Also, since we allow commercial banks to violate their capital requirement constraints, $\lambda^b_{k_t}$ represents the marginal disutility of violating their capital requirement constraint for each 'real' dollar. Thus, we need to subtract from the payoff of commercial banks the additional term:

$$\frac{\lambda^b_{k_t} \max[0, \overline{k}_t - k^b_s]}{p_s g_s}$$

[6] We consider real penalties (i.e. non-pecuniary) to avoid interpersonal comparison of welfare. However, most of the arguments hold with nominal penalties as well.

Note that, since we allow for capital requirements' violations, these do not appear in the budget set of banks, but only in the objective function (see section 3.2). Since capital requirements constraints do not enter the commercial banks' optimisation problem as constraints, it is only the possibility of incurring the penalty for any violation that provides an incentive to banks to fulfil their capital requirements. If, for example, $\lambda^b_{k_t} = +\infty$, $\forall b \in B$, then commercial banks would never violate their capital adequacy ratios.

This specification of default captures the idea (first introduced by Shubik and Wilson (1997)) that utility decreases monotonically in the level of default. In equilibrium, agents equalise the marginal utility of defaulting with the marginal disutility of the bankruptcy penalty. Thus, the expected rates of delivery of interbank, loans, assets and deposits $R = (R^b_s, \overline{R}^b_s, R_{sj}, \overline{R}^b_s)$ $\forall s \in S, j \in J$ and $b \in B$ are equal to actual rates of delivery in equilibrium. This is a crucial ingredient of this model. It allows us to establish default as an equilibrium phenomenon and produces different interest rates in each credit market. Different interest rates are produced because different risk attitudes and initial capital endowments of both individual borrowers and commercial banks induce different default levels that in turn generate different default risk premia in each credit market.

2.8 Commodity Markets

Commodity prices p_{sl} are taken as exogenously given by the agents. Let $b^h_{sl} \equiv$ amount of fiat money spent by $h \in H$ to trade in the market of commodity $sl \in L$. In addition, let $q^h_{sl} \equiv$ amount of good $l \in L$ offered for sales at state $s \in S^*$ by $h \in H$. Agents cannot sell commodities they do not own, so $q^h_{sl} \leq e^h_{sl}$. In equilibrium, at positive levels of trade $0 < p_{sl} < \infty$,

$$p_{sl} = \frac{\sum\limits_{h \in H} b^h_{sl}}{\sum\limits_{h \in H} q^h_{sl}}$$

All markets meet simultaneously; hence cash obtained from the sale of commodity l at state s cannot be used for the purchase of another commodity $l \in L$ at some $s \in S^*$. This institutional arrangement is a fundamental feature of a model that captures the importance of liquidity constraints and the transaction demand for cash. Cash-in-advance constraints should be viewed as liquidity constraints that distinguish commodities from liquid wealth. Without loss of generality one could extend the present model to accommodate different liquidity characteristics of commodities, or of other assets, by introducing liquidity parameters for each commodity that would be determined in equilibrium.[7]

2.9 Commercial Banks

Commercial banks enter the model because of their importance both for enabling agents to smooth consumption, for the transmission of monetary policy, and for contagion of financial crises during periods of financial fragility.

Let $b \in B = \{1, ..., B\}$ be the set of commercial banks. We assume:

(A8) perfectly competitive banking sector (i.e. commercial banks take interest rates and asset prices as exogenously given)

[7] See section 3.3 for further discussion.

Although each bank has a group of borrowers allocated to it in this model, we could think of each bank as a set of similar competitive banks of the same type.

The balance sheet of commercial banks is as follows:

A	L
Loans to individual agents	Deposits from individual agents
Interbank deposits	Interbank borrowing
Asset investments	Equity

Similarly, the balance sheet of the Central Bank,

A	L
Interbank loans	Currency (Fiat money)
Government bonds	
Money financed emergency liquidity assistance	
Residual[8]	

The modelling of banking behaviour here is akin to the portfolio balance approach of the banking firm introduced by Tobin (1963 and 1982).

Shares of ownership of commercial banks are determined on a prorated manner as follows:

$$s_b^h = \frac{\psi_b^h}{\sum\limits_{h\in H} \psi_b^h}, \forall b \in B$$

where $\psi_b^h \equiv$ amount of money offered by h for ownership shares of banks $b \in B$, and

$s_b^h \equiv$ percentage of shares of ownership by $h \in H$ acquired at $t = 0$ at the initial public offering of $b \in B$

As can be seen from the time structure of the model, at $t = 1$, $\forall s \in S$, retrading occurs and the secondary bank equity market clears as follows:

$$\theta_s^b = \frac{\sum\limits_{h\in H} b_{sb}^h}{\sum\limits_{h\in H} s_{sb}^h V_s^b}, \forall s \in S, b \in B$$

where $b_{sb}^h \equiv$ amount of fiat money offered by $h \in H$ at $s \in S$ for shares of ownership at the secondary bank equity market of $b \in B$,

$s_{sb}^h \equiv$ percentage shares of ownership by $h \in H$ sold at $t = 1$ at the secondary bank equity market of $b \in B$, and finally $V_s^b \equiv e_s^b + \triangle(2^b)$ (i.e. retained profits at $s = 0$, see section 3.2)

Also, $s_{sb}^h \leq s_b^h$ (i.e. no short sales of bank equity, for the sake of simplicity). Note that dividends are not distributed at the end of $t = 0$, since our focus does not lie only on the capital structure of commercial banks. At $t = 1$, the profits (if any) of commercial banks are liquidated and distributed back to the individual owners according to their ownership shares, (and for default penalties the same rule applies). This way we close the model. We also remark that, because of the capital requirements' violation penalty, banks will never go bankrupt and

[8] The residual entry in the balance sheet of the Central Bank accounts for the endogenous default in the interbank market.

therefore bank equity prices will be non-zero. The formal existence argument is presented in section 5.

Finally, as will be discussed in section 6, we analyse not only the default channel and liquidity trap for the banking system, but also the effect on financial fragility of a collapse in bank equity values.

2.10 Interbank Credit Market

The Central Bank conducts its monetary policy through OMOs in the interbank market, (though other routes for OMOs are also possible in practice). Also, interbank lending and borrowing occurs in this market. Alternatively, the Central Bank could set the interbank interest rate and accommodate the ensuing excess demand (or supply) of liquidity.

The interbank interest rate is established in equilibrium at positive levels of trade,

$$(1+\rho) = \frac{\sum\limits_{b\in B} \mu^b + \mu^{CB}}{\sum\limits_{b\in B} d^b + M^{CB}}$$

where $\mu^b \equiv$ amount of zero coupon bonds issued by $b \in B$, or equivalently the amount of money b chooses to owe in the interbank credit market, d^b = amount of money that b deposits. Similarly, $\mu^{CB} \equiv$ amount of zero-coupon bonds issued by the Central Bank and $M^{CB} \equiv$ Central Bank money supply.

Note that monetary policy is *not* symmetric since default can lead to varying responses to the Central Bank's OMOs actions. Also, the Central Bank could determine the interest rate instead and let borrowing and lending equilibrate the market, as the current practice of implementing monetary policy is nowadays.

3 The Budget Set

It is assumed that commodities are perishable, lasting only one period, and that each market meets once in each period. In order to ensure that agents have the necessary liquidity before they spend the order in which markets meet should be carefully chosen. Accordingly, the interbank market meets first to enable commercial banks to acquire funds to supply in the credit markets which in turn meet before commodity markets meet to allow investors to borrow, if necessary, for their expenditures. However, if the time horizon is extended to a large T the order does not very much matter as long as receipts from sales cannot be used contemporaneously for the purchase of commodities.

As in Tsomocos (2003a, and 2003b), we assume that asset markets (as well as the banks' equity market) clear automatically via a giant clearing house. Thus,we attempt to capture the fact that financial markets clear faster than commodity markets.

3.1 Investors

Macro variables ($\eta = (p, \rho, r, \theta, s, R)$) are determined in equilibrium and every agent takes them as given. Agents are perfect competitors and therefore are price takers. The choice of investors, $h \in H$, are determined by $\sigma^h \in \sum^h(\eta)$ where,

$\sigma^h = (\chi^h, \mu^{h^b}, d^h_b, b^h, q^h, \psi^h, b^h_{sb}, s^h_{sb}, v^h) \in R^{LS^*}_+ \times R_+ \times R_+ \times R^{LS^*+J}_+ \times R^{LS^*+J}_+ \times R^B_+ \times R^{SB}_+ \times R^{SB}_+ \times R^{S+J}_+$ is the vector of all of investors' decisions.

$B^h(\eta) = \{\sigma^h \in \Sigma^h(\eta) : (1^h) - (7^h) \text{ below}\}$ is the budget set where $\triangle(i)$ represents the difference between RHS and LHS of inequality (i).

For $t = 0$,

$$\sum_{j\in J} b_j^h + \sum_{b\in B} \psi_b^h + \sum_{l\in L} b_{0l}^h + \sum_{b\in B} d_b^h \leq \left(\frac{\mu^{h^b}}{1+r^b}\right) + \sum_{j\in J} \theta_j q_j^h + m_0^h \tag{1^h}$$

(i.e. expenditures for assets, banks' equity in the primary market, and commodities + bank deposits $\leq$ borrowed money at the credit markets + receipts from sales of assets + initial private monetary endowments)

$$q_{0l}^h \leq e_{0l}^h, \ \forall l \in L \tag{2^h}$$

(i.e. sales of commodities $\leq$ endowments of commodities)

$$\chi_{0l}^h \leq e_{0l}^h - q_{0l}^h + \frac{b_{0l}^h}{p_{0l}}, \quad \forall l \in L \tag{3^h}$$

(i.e. consumption $\leq$ initial endowment - sales + purchases)

$\forall s \in S$,

$$\sum_{l\in L} b_{sl}^h + \sum_{b\in B} b_{sb}^h \leq \triangle(1^h) + \sum_{l\in L} p_{0l} q_{0l}^h + \sum_{b\in B} s_{sb}^h \theta_s^b V^b + m_s^h \tag{4^h}$$

(i.e. expenditures for commodities and banks' equity in the secondary market $\leq$ money at hand + receipts from sales of commodities + receipts from sales of banks' equity in the secondary market + initial private monetary endowment in states s)

$$\begin{aligned}\sum_{j\in J}(v_{sj}^h p_s A_s^j) q_j^h + \sum_{b\in B} v_{sb}^h \mu^{h^b} \leq \triangle(4^h) + \sum_{l\in L} p_{sl} q_{sl} + \sum_{b\in B}\left(\frac{b_{sb}^h}{\theta_s^b V_s^b} + s_b^h - s_{sb}^h\right)\pi_s^b \\ + \sum_{b\in B} \overline{R}_{sd}^b d_b^h (1 + r_d^b) + \sum_{j\in J} (R_{sj} p_s A_s^j)\left(\frac{b_j^h}{\theta_j}\right)\end{aligned} \tag{5^h}$$

(i.e. asset and loan deliveries $\leq$ money at hand + receipts from commodities sales + distribution of commercial banks' profits + deposit and interest payment + asset deliveries)

$$q_{sl}^h \leq e_{sl}^h, \ \forall l \in L, \forall s \in S \tag{6^h}$$

(i.e. sales of commodities $\leq$ endowments of commodities)

$$\chi_{sl}^h \leq e_{sl}^h - q_{sl}^h + \frac{b_{sl}^h}{p_{sl}}, \quad \forall l \in L, \forall s \in S \tag{7^h}$$

(i.e. consumption $\leq$ initial endowment - sales + purchases)

3.2 Commercial banks

Denote the choices of commercial banks $b \in B, \sigma^b \in \Sigma^b(\eta)$ where $\sigma^b = (\mu^b, d^b, \overline{m}^b, \mu_d^b, b^b, q^b, v^b, \pi^b) \in R_+ \times R_+ \times R_+ \times R_+ \times R_+^J \times R_+^J \times R_+^{J+2} \times R_+^{S^*}$ is the vector of all of their choices.

$B^b(\eta) = \{\sigma^b \in \Sigma^b(\eta) : (1^b) - (3^b) \text{ below}\}$ is the budget set where $\triangle(i)$ represents the difference between RHS and LHS of inequality (i).

For $t = 0$,

$$d^b \leq \sum_{h \in H} \psi_b^h + e_0^b \tag{1^b}$$

(i.e. deposits in the interbank market $\leq$ receipts from banks' primary equity market + initial capital endowment)

$$\overline{m}^b + \sum_{j \in J} b_j^b \leq \triangle(1^b) + \frac{\mu^b}{(1+\rho)} + \sum_{j \in J} \theta_j q_j^b + \sum_{h \in H} d_b^h \tag{2^b}$$

(i.e. credit extension + expenditures for assets $\leq$ money at hand + interbank loans + receipts from asset sales + consumer deposits)

$\forall s \in S$,

$$\sum_{h \in H} \overline{v}_s^b (1+r_d^b) d_b^h + \widetilde{v}_s^b \mu^b + \sum_{j \in J} v_{sj}^b p_s A_j^j q_j^b \leq \triangle(2^b) + \sum_{h \in H} \overline{R}_s \mu^{h^b} + \sum_{j \in J} (R_{sj} p_s A_s^j)(\frac{b_j^b}{\theta_j}) + \overline{R}_{ds} d^b (1+\rho) \Psi + e_s^b \tag{3^b}$$

(i.e. deposits and interest repayment + interbank loan repayment + expenditures for asset deliveries $\leq$ money at hand + loan repayments + money received from asset payoffs + interbank deposits and interest repayment + initial capital endowment in state s)

where

$\Psi = d^b / (\sum_{b \in B} d^b + M^{CB})$

$\pi_0^b \equiv \triangle(2^b)$ and $\pi_s^b \equiv \triangle(3^b)$

Note that since the interbank market is perfectly competitive, in cases where there has been default, deposit repayments are made proportional to the deposits made by each commercial bank relative to the aggregate supply of credit by the entire commercial banking sector and the Central Bank.

3.3 A Remark on Cash-In-Advance

A common criticism of the cash-in-advance (C-I-A)[9] models is that these constraints are *ad hoc* and do not adequately capture liquidity. Our view is that C-I-A constraints are the simplest form of liquidity constraints and can be straightforwardly generalised to model more complicated liquidity constraints. The main intuition of these constraints is that the different instruments and commodities of the economy are not equally liquid. Put differently, not all receipts from sales can be contemporaneously used for other purchases. As long as there exist some liquidity parameters for the commodity endowment which are less than 1 (otherwise, the

[9] C-I-A constraint can be traced back at least in Clower (1967).

budget constraints collapse to the standard Arrow-Debreu constraints), money (or liquidity or credit) demand is positive in order to bridge the gap between expenditures and receipts. Indeed, Grandmont and Younes (1972) have used these liquidity parameters. However, the pure C-I-A constraint offers accounting clarity and ease of exposition. (see section 7)

4 Equilibrium

We say that[10] $(\eta, (\sigma^h)_{h\in H}, (\sigma^b)_{b\in B})$ is a Monetary Equilibrium with Commercial Banks and Default (MECBD) for the economy

$$E\{(u^h, e^h, m^h)_{h\in H}; (u^b, e^b)_{b\in B}; A, M^{CB}, \mu^{CB}, m^{CB}, k, \lambda, \omega\}$$

iff:

(i) $p_{sl} = \dfrac{\sum\limits_{h\in H} b^h_{sl}}{\sum\limits_{h\in H} q^h_{sl}}, \ \forall s \in S^*, l \in L;$

Condition (i) shows that all commodity markets clear (or equivalently that price expectations are rational).

(ii) $1 + \rho = \dfrac{\sum\limits_{b\in B} \mu^b + \mu^{CB}}{\sum\limits_{b\in B} d^b + M^{CB}}$

Condition (ii) shows that the interbank credit market clears (or equivalently that interbank interest rate forecasts are rational).

(iii) $1 + r^b = \dfrac{\sum\limits_{h^b\in H^b} \mu^{h^b}}{\overline{m}^b}, \ \forall b \in B, h^b \in H^b;$

Condition (iii) shows that the long-term credit markets clear (or equivalently that prediction of the long-term interest rate is rational).

(iv) $1 + r^b_d = \dfrac{\mu^b_d}{\sum\limits_{h\in H} d^h_b}, \ \forall b \in B, h \in H;$

Condition (iv) shows that the deposit markets for each bank clear (or equivalently that prediction of the deposit rates is rational).

(v) $\theta_j = \dfrac{\sum\limits_{h\in H} b^h_j + \sum\limits_{b\in B} b^b_j}{\sum\limits_{h\in H} q^h_j + \sum\limits_{b\in B} q^b_j}, \forall j \in J$

Condition (v) shows that every asset market clears (or equivalently, asset price expectations are rational).

(vi) $\sum\limits_{h\in H} s^h_b = 1, \forall b \in B;$

Condition (vi) shows that the primary equity market for the bank ownership clears (or equivalently bank equity shareholding expectation are rational).

[10]Recall that by assumption p, ρ, r^b, θ, R are different from 0 and ∞ in each component.

(vii) $\theta_s^b = \dfrac{\sum\limits_{h\in H} b_{sb}^h}{\sum\limits_{h\in H} s_{sb}^h V^b}, \forall b \in B, s \in S;$

Condition (vii) shows that the secondary equity market of commercial banks clears (or equivalently secondary market bank equity price expectations are rational.)

$$\text{(viii) } R_{sj} = \left\{ \begin{array}{c} \dfrac{\sum\limits_{h\in H\cup B}\left(v_{sj}^h p_s q_j^h A^j\right)}{\sum\limits_{h\in H\cup B}\left(p_j q_j^h A^j\right)}, \text{ if} \sum\limits_{h\in H\cup B}\left(p_j q_j^h A^j\right) > 0 \\ \text{arbitrary, if} \sum\limits_{h\in H\cup B}\left(p_j q_j^h A^j\right) = 0 \end{array} \right\}$$

Condition (viii) shows that each asset buyer is correct in his expectations about the fraction of assets that will be delivered to him.

$$\text{(ix)-(xii) } \overline{R}_s(R_{sd}, \widetilde{R}_{sd}, \widetilde{R}_s) = \left\{ \begin{array}{c} \dfrac{\sum\limits_{h\in H\cup B}\left(\overline{v}_s \mu^{h^b}(d^{h^b}(1+r^b), d^b(1+\rho), \mu^b)\right)}{\sum\limits_{h\in H\cup B}\left(\mu^{h^b}(d^{h^b}(1+r^b), d^b(1+\rho), \mu^b)\right)}, \\ \text{if} \sum\limits_{h\in H\cup B}\left(\mu^{h^b}(d^{h^b}(1+r^b), d^b(1+\rho), \mu^b)\right) > 0, \forall s \in S \\ \text{arbitrary, if} \sum\limits_{h\in H\cup B}\left(\mu^{h^b}(d^{h^b}(1+r^b), d^b(1+\rho), \mu^b)\right) = 0 \end{array} \right\}$$

Conditions (ix)-(xii) show that the Central Bank and commercial banks are correct in their expectations about the fraction of loans that will be delivered to them. Similarly, investors and commercial banks are correct in their expectations about the fraction of deposits and interest rate payment that will be delivered to them.

(xiii) (a) $\sigma^h \in \underset{\sigma^h \in B^h(\eta)}{\text{Argmax}} \Pi^h(\chi^h)$

(b) $\sigma^b \in \underset{\sigma^b \in B^b(\eta)}{\text{Argmax}} \Pi^b(\pi^b)$

Condition (xii) shows that all agents optimise.

In sum, all markets clear and agents optimise given their budget sets. These are the defining properties of a competitive equilibrium.

5 Orderly Functioning of Markets: Existence of a Monetary Equilibrium with Commercial Banks and Default

If a MECBD exists, then default and financial instability manifest themselves as equilibrium phenomena entirely consistent with the proper-functioning of markets. Thus, if any of these phenomena are deemed detrimental for the economy and for the welfare of the society, then regulatory intervention may be justified. Moreover, active crisis management and prevention can become necessary.

As can be seen from conditions (viii)-(xii) of section 4, expected deliveries of assets, loans and deposits are equal to realised deliveries in equilibrium. However, the specification of expectations for inactive markets is 'arbitrary'. Thus, we need a hypothesis to rule out trivial equilibrium (in which trade in the corresponding markets collapses). Following Tsomocos (2003a) we impose the Inactive Market Hypothesis.

Inactive Market Hypothesis (IMH): Whenever credit or asset markets are inactive the corresponding rates of delivery are set equal to 1.

This hypothesis follows closely Dubey, Geanakoplos, and Shubik (2000), and Dubey and Shubik (1978) that allow an external agent to be added in these markets that always supplies an ε amount and never abrogates his contractual obligations. It may be thought as the FDIC or an analogous institution.

Economic agents in our model are not required to trade and they always have the option to consume their own endowment. This situation arises when interest rates are prohibitively high and thus there is no demand for credit. Then, it also becomes uncertain whether the interbank market will be active as well. This happens whenever the marginal cost of borrowing (i.e. interest rate payments) is higher than the marginal benefit of the extra consumption.

We are thus naturally led to adopt a condition that guarantees sufficient gains from trade. For an extensive discussion on this issue see Dubey and Geanakoplos (1992), (2003a). Geanakoplos and Tsomocos (2002) extend the gains from trade condition to a model related to the present one. The crucial insight of this condition is that, even if transaction costs are high, utility from extra consumption would still make such transaction attractive. We use the definition of gains from trade from Tsomocos (2003a) which is given as follows:

Definition:

Let $(\chi^h, \pi^b) \in R_+^{S^* \times (L+1)} \quad \forall h \in H,\ b \in B$. $\forall \delta > 0$, we will say that $(\chi^1, ..., \chi^h; \pi^1, ..., \pi^b) \in (R_+^{S^* \times L})^H \times R_+^{S^* \times B}$ permits at least $\delta-$*gains-from-trades* $\tau^1, ..., \tau^H; \tau^1, ..., \tau^B$ in R^{L+1} such that

1. $\sum\limits_{h \in H} \tau^h + \sum\limits_{b \in B} \tau^b = 0$
2. (a) $\chi_s^h + \tau^h \in R_+^L$, $\forall h \in H$
   (b) $\pi_s^b + \tau^h \in R$, $\forall b \in B$
3. (a) $u^h(\overline{\chi}^h) > u^h(\chi^h)$, $\forall h \in H$
   (b) $u^b(\overline{\pi}^b) > u^b(\pi^b)$, $\forall b \in B$

where,

$$\overline{\chi}_{tl}^h = \left\{ \begin{array}{c} \chi_{tl}^h,\ t \in S^* \backslash \{s\} \\ \chi_{tl}^h + \min\{\tau_l^h, \tau_l^h/(1+\delta)\} \ \text{ for } l \in L \text{ and } t = s \end{array} \right\}$$

$$\overline{\pi}_t^b = \left\{ \begin{array}{c} \pi_t^b,\ t \in S^* \backslash \{s\} \\ \pi_t^b + \min\{\tau^b, \tau^b/(1+\delta)\} \ \text{ for } t = s \end{array} \right\}$$

Note that when $\delta > 0$, $\overline{\chi}_l^h < \chi_l^h + \tau_l^h$, if $\tau_l^h > 0$ and $\overline{\chi}_{tl}^h = \chi_{tl}^h + \tau_l^h$ if $\tau_l^h \leq 0$. Also, $\overline{\pi}_t^b < \pi_t^b + \tau^b$, if $\tau^b > 0$ and $\overline{\pi}_t^b = \pi_t^b + \tau^b$, if $\tau^b \leq 0$

Formally, the hypothesis that we impose on the economy for sufficient gains from trades is:

G from T:

$\forall s \in S$, the initial endowment $(e^h, e^b)_{h \in H \cup B}$ permits at least δ_s-gains to trade in state s, where

$$\delta_s = \frac{\sum\limits_{h \in H} m_0^h + \sum\limits_{h \in H} m_s^h + \sum\limits_{b \in B} e_0^b + \sum\limits_{b \in B} e_s^b}{M^{CB}}$$

We are now ready to state the existence theorem that establishes default and financial fragility compatible with equilibrium and the orderly functioning of markets. Note that the existence theorem also resolves the 'Hahn paradox' (1965) whereby money has no-value in finite horizon.

Theorem:
If in the economy $E = \{(u^h, e^h, m^h)_{h\in H}; (u^b, e^b)_{b\in B}; A, M^{CB}, \mu^{CB}, m^{CB}, \lambda, \omega\}$
1. G from T and IMH hold,
2. $M^{CB} > 0$,
3. $\forall s \in S^*$, $\sum_{h\in H} m^h_s + \sum_{b\in B} e^b_s > 0$ and
4. $\lambda >> 0, \forall h \in H, b \in B$

then a MECBD exists.[11]

6 Financial Fragility, Default and the Liquidity Trap

6.1 Financial Fragility and Contagion: Concepts and Definitions

We adopt a definition of financial fragility introduced in Tsomocos (2003a), where a MECBD is financially fragile whenever a substantial 'number' of households and commercial banks default on some of their obligations (i.e. a liquidity 'crisis'), without necessarily becoming bankrupt, and the aggregate profitability of the banking sector decreases significantly (i.e. a banking 'crisis').

The formal definition of financial fragility is as follows;

Definition: A MECBD $(\eta, (\sigma^h)_{h\in H}, (\sigma^b)_{b\in B})$ is financially fragile at s whenever $D^{h^*}_{sz}$, $D^{b^*}_{sz} \geq \overline{D}$, $\sum_{b\in B} \pi^b_s \leq \overline{\Pi}$, for $|H^*| + |B^*| \geq \overline{Z}$, and $s \in S^*$ where $\overline{Z} \in (0, |H| + |B|)$ and $\overline{\Pi}, \overline{D} \in R_{++}$.

This definition requires both increased default *and* reduced aggregate profitability. Increased default by itself might indicate excessive risk taking without necessarily engendering a serious strain on the financial sector of the economy, whereas a decrease in profitability by itself might indicate the onset of a recession in the real economy and not of financial vulnerability. Also, with heterogenous agents, the welfare of society depends not only on aggregate outcomes, but also on their distribution over agents.

The interaction of investors and commercial banks in the various markets of this model allows us to precisely trace the different channels of contagion given an adverse shock. The first channel of contagion is the one generated by increased default in a specific sector of the economy. For example, if a specific bank charges exobitantly high interest rates on its clients then their subsequent default impacts upon the rest of the economy. Commercial banks reduce their repayment rates in the interbank market and investors and/or commercial banks abrogate their obligation in the asset markets. Alternatively, the commodity markets may be affected either through reduced supply (or demand) which in turn affects expected income of the household sector (or the supplier). The upshot of this chain of contagion is that reduced liquidity hurts the lenders whose income (or equivalently their expenditures) is reduced, thus decreasing their consumption and welfare. We note that this chain may be broken, for example, with emergency liquidity assistance that neutralises the reduced loan repayment rate to the

[11] For an extensive discussion of the theorem in such a model see Tsomocos (2003a). See also the proof of theorem in the appendix where the modification from the arguments of Tsomocos (2003a and 2003b) is presented.

initial commercial bank. The same reasoning applies for contagion through the interbank market's increased default.

Second, contagion may commence through the collapse of the banking sector's equity value in the secondary market. Since the distribution of profits to investors is determined by the shares of ownership as they are specified in the secondary banks' equity market, weakness of the banking sector is translated to investors' income. Reduced expected profitability of the banking sector will be reflected in a reduced value of the shares of ownership of banks' equity and thus the reduced income will lower such agents' repayment rates of loans and asset deliveries. For example, if bank b's equity drops in value then its investors will increase their default in the rest of the economy which will adversely affect other agents' welfare as well, who transact with them in the asset market. Finally, the last channel of contagion which will be discussed in section 6.2 is generated by a possible ineffectiveness of monetary policy. As monetary policy eases without affecting the real side of the economy (i.e. we enter a liquidity trap), the extra liquidity inflates activity in some asset markets. This in turn leads commercial banks to violate excessively their capital requirements which adversely affects their profitability and subsequently their equity value. Through the investor sector's ownership of bank shares contagion spreads outside the banking sector and may reduce welfare in the rest of the economy.

Finally, due to limited liability and active default in equilibrium, the Modigliani-Miller irrelevance proposition does not hold in our model for the commercial banking sector. Equity is default free, whereas debt (either through interbank or credit market loans) is defaultable.

6.2 Liquidity Trap

The Keynesian liquidity trap describes a situation in which monetary policy would not affect real expenditures in the economy. If interest rates are sufficiently low and investors expect them to rise in the future, then they do not invest into assets like bonds whose value is expected to fall. Thus, they hold the extra money balances due to expansionary monetary policy for speculative purposes without affecting commodity prices. Various authors provide models that allow for the occurrence of a liquidity trap (e.g. Tobin (1963, 1982), Grandmont and Laroque (1973), and Hool (1976)).

Dubey and Geanakoplos (2003) provide a novel interpretation of this phenomenon. They argue that in a monetary *GEI* model, as monetary policy eases, then commodity prices remain unaffected whereas the extra liquidity is channeled into asset market(s) where trading activity becomes large. However, in the aggregate there is almost no new *net* trading activity in such asset markets. This possibility is non-generic, and occurs only in an equilibrium where the corresponding real *GEI* economy possesses no equilibrium (i.e. the case of the Hart (1975) counterexample).

In the present model, the same phenomenon reappears, (coupled with financial instability), only when capital requirements are non-binding. Otherwise, such increased trading activity would lead commercial banks to increase their risk-weighted assets and thus violate their capital requirements even more so. Moreover, the liquidity trap originates from the interbank market and may propagate to the asset markets via investments of the banks only. However, note that this is a non-generic case and occurs only when equilibrium fails to exist in the underlying real *GEI* economy. Of course, the assets most commonly bought by commercial banks in a liquidity trap are government bonds, and these usually bear a zero risk-weight. This analysis therefore provides a rationale for imposing some positive risk-weighting on these assets as well.

However, when capital requirements are binding, the liquidity trap may still be present via excessive trading activity in equity markets. Banks now switch to credit extension and consumers spend in the primary bank equity market (thus helping to satisfy the capital requirements of banks). Alternatively, in anticipation of a higher liquidation value of commercial banks, consumers restructure their portfolio of banks' equity in the secondary market. The next proposition summarises this intuition.

Proposition 1

Suppose that the economy has a riskless asset $A^j_{sm} = (1, ..., 1)$ (i.e. monetary payoffs in every state are equal to one) and $A^j_{sl} = 0, \forall s \in S$ and $l \in L$ for $k^b_t = 0$. Also, consider the case in which the underlying economy has no *GEI*. Then as $M^{CB} \to \infty$, then

(i) $\overline{\omega}(\eta,\delta)\overline{R}^b_s\overline{m}^b(1+r^b) + \sum_{j\in J}\omega_{ij}(\eta,\delta)R_{sj}(p_sA^j_s)\left(\frac{b^b_j}{\theta_j}\right) + \omega(\eta,\delta)R^b_sd^b \to \infty, \forall s \in S^*$, from some $b \in B$, $\frac{M^{CB}}{\|p_{0l}\|} \to \infty$ and $(\sum_{h\in H} q^h_j + \sum_{b\in B} q^b_j) \to \infty$.

(ii) There exists $\overline{D}, \overline{\Pi}$ such that $D^{h^*}_{sz}, D^{b^*}_{sz} > \overline{D} > 0$ for some $b^* \in B, h^* \in H$, and $\sum_{b\in B}\pi^b_s \leq \overline{\Pi}$.

(iii) Suppose now that $\lambda^b_{k_t} = +\infty, \forall b \in B$ and $k^b_t > 0$. Then, $\frac{M^{CB}}{\|p_{0l}\|} \to \infty$ and $(\sum_{h\in H} q^h_j + \sum_{b\in B} q^b_j) < K$, for some $K \in (0, +\infty)$, but $\sum_{h\in H} s^h_{sb}V^b \to \infty$.

6.3 Regulatory Policy and Default

Since both default and capital requirements' violations incur a cost, consumers and banks weigh the marginal costs and benefits of abrogating their contractual or regulatory obligation. Thus, for sufficiently high penalties, default and capital requirements' violations vanish in equilibrium. We therefore observe the importance of capital requirements for financial stability. For example, whenever credit is fully collateralised, the regulator guarantees future financial stability. This, however, has an opportunity cost since the resulting higher interest rates due to stricter capital requirements would reduce efficient trade.

In sum, we note that at least one aspect of the well-known trade-off between financial stability and efficiency is present and this indicates the interconnectedness of monetary and regulatory policies.

Proposition 2

There exist $\overline{\lambda}^b_{k_t}$ and $\overline{\lambda}^h_{sz}, \overline{\lambda}^b_{sz}$ such that $D^h_{sz} = D^b_{sz} = 0$ and $\overline{k}_t - k^b_t = 0, \forall b \in B, h \in H$

Since agents may opt to default or violate their capital requirements in equilibrium, changes in regulatory practice affect their marginal rates of substitution among various choices and thus produce equilibria with different allocations, as the following proposition indicates. The model is liquidity based with well-defined transaction technology and settlement processes; therefore both real changes as well as changes of the nominal constraints have a necessarily non-neutral effect.

We define a MECBD to be *bank-indecomposable* if for any $s \in S^*$ and any partition of assets into disjoint sets Γ_1 and Γ_2 there is some $b \in B$ who transacts in at least one asset from each set in $s \in S^*$.

Proposition 3

Suppose that u^h, u^b are differentiable and $m^h_s, e^b_s > 0$ or $\lambda^b < \overline{\lambda}^b_{sz}$ and $\lambda^h < \overline{\lambda}^h_{sz}$ and $k^b_t > 0$ for all $h \in H, b \in B$ and $s \in S^*$. Then at a *bank-indecomposable* MECBD any change by the regulator of λ, k or ω results in a different MECBD in which for some $b \in B$ the payoff is different.

6.4 Modigliani-Miller Proposition

The modeling of commercial banks that have diverse financing and investment opportunities sheds light on the Modigliani-Miller proposition. The traditional argument for the validity of the irrelevance of financing rests on perfect and frictionless capital markets. Various arguments such as limited liability, bankruptcy costs and differential taxation between debt and equity have been offered to invalidate this proposition.

In the present model, only when (i) markets are complete, (ii) limited participation does not produce different borrowing behaviour, and (iii) banks' risk taking behaviour, and capital requirements are identical, then financing does not matter. Put differently, only when we remove all the frictions of the model and also impose homogeneity across banks, do we recover the Modigliani-Miller proposition. When one models active banks with diversified portfolios, not only lack of frictions but also identical investment guarantees the validity of the irrelevance proposition.

As our next proposition shows, any deviation from these principles destroys the symmetry of debt-equity financing. First, limited participation, even for banks with identical financing, creates different returns from credit extension, and thus changes the value of banks in $t = 1$. Second, even if all other variables remain the same across banks, different risk-appetites lead them to form different portfolios. Given differential returns amongst the various investments (i.e. credit extension, investment in the interbank and asset markets), banks' preferences towards risk generate different terminal values for banks' portfolios[12] Third, different capital requirements and/or risk-weights provide different incentives to banks when forming their portfolios provided that the capital requirements' violation penalties are transferable to banks' shareholders. Otherwise, it may very well be the case that the penalty is internalised by banks management who still form identical portfolios and thus generate equal profits. Thus, the forces of demand and supply will typically equilibrate the markets so that banks' equity will trade at different prices in the secondary equity market. This may be relevant for analysing the impact of the New Basel Accord.

Finally, even in the absence of the previous frictions but with incomplete markets, i.e. $|J| < |S|$, different financing alters the space of marketed assets and therefore produces different equilibria and consequently different values for banks. In other words, this is akin to the distinction of comparing *within* an equilibrium two different financing structures that produce identical payoffs and *across* different equilibria of a bank that changes its financing. In the first instance, value is not affected, whereas in the second it is. If two different financing schemes have the same payoffs, in an equilibrium, then the corresponding values of the banks will be the same by a standard no-arbitrage argument. However, if one bank changes its financing then the monetary payoffs of its assets in the new equilibrium will typically be different, given incomplete markets and limited liability. This distinction holds only when markets are incomplete; otherwise since the space of marketed assets does not change, the two cases are equivalent.

[12] This point has also been discussed extensively by King (1977).

Proposition 4

Let $b_1, b_2 \in B$ with $\mu^{b_1} = \mu^{b_2}$ and $\sum_{h \in H} \psi^h_{b_1} = \sum_{h \in H} \psi^h_{b_2}$. Moreover, suppose $\overline{\lambda}^{b_1}_{k_t} = \overline{\lambda}^{b_2}_{k_t}, \lambda^{b_1}_{sz} = \lambda^{b_2}_{sz}$ and $e^{b_1}_s = e^{b_2}_s$, $\forall s \in S^*$ (i.e., identical financing, default and initial capital endowments)

(i) *Limited participation*: If $u^{b_1} = u^{b_2}$ but $\exists h \in H_1$ and $\widehat{h} \in H_2$, $h \neq \widehat{h}$ with respect to their endowments or preferences such that either $\psi^{\widehat{h}}_{b_1}, \psi^{\widehat{h}}_{b_2} > 0$ or $b^{\widehat{h}}_{sb_1}, s^{\widehat{h}}_{sb_1}$ and $b^{\widehat{h}}_{sb_2}, s^{\widehat{h}}_{sb_2} > 0$, then $\theta^{b_1}_s \neq \theta^{b_2}_s$ for some $s \in S$.

(ii) *Risk Preferences*: If $h = \widehat{h}$ for all $h \in H_1$ and $\widehat{h} \in H_2$ with respect to their endowments and preferences but $u^{b_1} \neq u^{b_2}$ then $\theta^{b_1}_s \neq \theta^{b_2}_s$.

(iii) *Regulation*: Let either $[0, \overline{k}_t - k^{b_1}_t]^+$ and/or $[0, \overline{k}_t - k^{b_2}_t]^+ > 0$ but $[0, \overline{k}_t - k^{b_1}_t]^+ \neq [0, \overline{k}_t - k^{b_2}_t]^+$. Also, let $\frac{\lambda^b_{k_t}[0, \overline{k}_t - k^b_t]^+}{p_s g_s}$ be transferable to banks' shareholders. If both $u^{b_1} = u^{b_2}, h = \widehat{h}$ $\forall h \in H_1$ and $\widehat{h} \in H_2$ but either $k^{b_1}_t \neq k^{b_2}_t$ or $\omega^{b_1}_{tj} \neq \omega^{b_2}_{tj}$ for some $j \in J$ or $t \in T$ then $\theta^{b_1}_s \neq \theta^{b_2}_s$.

(iv) *Incomplete markets*: Suppose $|J| < |S|, u^{b_1} = u^{b_2}, h = \widehat{h}$ $\forall h \in H_1$ and $\widehat{h} \in H_2$, $k^{b_1}_t = k^{b_2}_t$ or $\omega^{b_1}_{tj} = \omega^{b_2}_{tj}$, $\forall j \in J$ and limited liability. Now let $\widehat{\mu}^{b_1} \neq \mu^{b_2}$ but $\sum_{h \in H_1} \hat{\psi}^h_{b_1} + \widehat{\mu}^{b_1} = \sum_{h \in H_1} \psi^h_{b_2} + \mu^{b_2}$. Then $\theta^{b_1}_s \neq \theta^{b_2}_s$ for some $s \in S$ in the new MECBD.

(v) *Complete markets*: If $|J| = |S|, u^{b_1} = u^{b_2}, h = \widehat{h}$ $\forall h \in H_1$ and $\widehat{h} \in H_2$, $k^{b_1}_t = k^{b_2}_t$ or $\omega^{b_1}_{tj} = \omega^{b_2}_{tj}$ then $\theta^{b_1}_s = \theta^{b_2}_s$.

In sum, the Modigliani-Miller principle is violated primarily from two sources. First, when structural frictions such as limited participation and market incompleteness are present. Second, when investment behaviour of *active* banks is affected by different incentives or different attitudes towards risk.

7 Money Demand, Interest Rates, and the Non-Neutrality of Monetary Policy

The monetary/financial sector of our simple specification of the economy, coupled with its transaction technology, produces the traditional motives for holding money. Thus, we observe that the standard Hicksian determinants of money demand are present in the model.

In particular, liquidity provision by banks and default by both banks and households produce an intricate relationship among interest rates. Since base money is fiat and the horizon is finite[13], in the end money exits the system. This means that both central bank money, M^{CB} (i.e. inside money) and money and liquidity present in the initial endowments of banks and households (i.e. outside money) would exit the system either via loan repayments to commercial banks or to the Central Bank by the commercial banks. Thus, the overall liquidity of the economy affects the determination of interest rates. Moreover, endogenous default is possible in equilibrium and inevitably affects interest rates as well. In sum, both a liquidity and default premium affects interest rates. However, further structural assumptions are needed to be able to disentangle these term premia.

[13] Had we used an infinite horizon model, as long as there is settlement and liquidation in regular time intervals, similar results would hold.

Liquidity Structure of Interest Rates Proposition:
In any MECBD, $\forall s \in S$

$$\sum_{b \in B} \overline{m}^b r^b = \sum_{h \in H} (m_0^h + m_s^h)(\frac{1}{v_{sb}^h}) + \sum_{b \in B} (e_0^b + e_s^b)(\frac{1}{\overline{v}_s^b})$$

In our multi-period setting, if $|B| > |S|+1$ then there are more interest rates than equations. Thus, they depend on the real data of the economy and are subject to policy intervention. The only exception is when $m_s^h = e_s^b = 0, \forall s \in S^*, h \in H, b \in B$ and $v_{sb}^h = \overline{v}_s^b = 1, \forall s \in S^*, h \in H, b \in B$. In such a case, all interest rates are zero (including ρ) and money is essentially a veil.

If all interest rates are positive, then all the available liquidity will be channeled in the commodity markets $\forall s \in S$. However, this is not the case at $s = 0$ because of uncertainty and incomplete markets, investors may opt to spend it in the asset markets or hold some precautionary reserves.

Quantity Theory of Money Proposition:
In any MECBD with $\rho > 0$,

$$\sum_{h \in H} \sum_{l \in L} p_{sl} q_{sl}^h = \sum_{b \in B} \overline{m}^b + \sum_{h \in HUB} \sum_{j \in J} v_{sj}^h p_s q_j^h A^j + \sum_{h \in H} \sum_{b \in B} s_{bs}^h \theta_s^b V^b + \Delta(1^h)$$

$\forall s \in S$.

For $s = 0$,

$$\sum_{h \in H} \sum_{l \in L} p_{0l} q_{0l}^h = M^{CB} + \sum_{b \in B} d^b - \sum_{h \in HUB} b_j^h - \sum_{h \in H} \sum_{b \in B} \psi_b^h - \Delta(1^h) - \sum_{b \in B} \pi_0^b$$

This is no 'crude' quantity theory of money. Velocity will always be less than or equal to 1 (one if all interest rates are positive). However, since quantities supplied in the markets are chosen by agents (unlike the representative agent model's *sell-all* assumption), the real velocity of money, that is how many real transactions can be moved by money per unit time, is endogenous.

The interest rates determined in equilibrium are in nominal terms. Thus, they depend not only on the intertemporal marginal rate of substitution, but also on the inflation rate of the economy. So, the well known Fisher relation holds, as is argued in the next proposition.

Fisher Effect Proposition:
Suppose that for some $h \in H^b, b_{0l}^h$ and $b_{sl}^h > 0$ for $l \in L$ and $s \in S$. Suppose further that $\Delta(4^h) > 0$. Then in a MECBD,

$$(1 + r^b) = \left(\left(\frac{\partial u^h(\chi)}{\partial \chi_{0l}} \right) / \left(\frac{\partial u^h(\chi)}{\partial \chi_{sl}} \right) \right) \left(\frac{p_{sl}}{p_{0l}} \right)$$

Taking the logarithm of both sides and interpreting loosely, this says that the nominal rate of interest is equal to the real rate of interest plus the (expected) rate of inflation.

As in the case with regulatory policy, monetary policy also has non-neutral effects. As it is proved in Tsomocos (2001), MECBD are finite with respect to both real allocations and nominal variables. Thus, any monetary change (except the one mentioned in the remark after proposition 5) affects interest rates and therefore economic agents' decisions. Finally, since

MECBD are typically constrained inefficient, policy changes do not necessarily affect welfare and financial stability monotonically.

We define a MECBD to be *investors-indecomposable* if for any $s \in S^*$ any partition of goods into disjoint sets L_1 and L_2 there is some $h \in H$ who transacts in at least one commodity from each set in $s \in S^*$.

Proposition 5

Suppose that all u^h, u^b are differentiable and $m_s^h, e_s^b > 0$ or $\lambda^b < \overline{\lambda}^b$ and $\lambda^h < \overline{\lambda}^h$ for all $h \in H, b \in B$ and $s \in S^*$. Suppose at an indecomposable MECBD at every $s \in S^*$ all $h \in H$ consume positive amounts of all goods $l \in L$ and that some $h \in H$ carries over money from period 0 to 1. Then any change by the Central Bank (except the one described in the remark) results in a different MECBD in which for some $h \in H$ consumption is different.

Remark: The 'no money illusion' property holds in the model. A proportional increase of all the nominal endowments of the consumers and commercial banks while k stays fixed and penalties are scaled down proportionally does not affect the real variables of MECBD.

8 Application

In this section we specialise the general model presented earlier to the case of three households and two banks, where the time horizon extends over two periods ($t \in \{0, 1\}$) and three possible states ($s \in \{1, 2, 3\}$) in the first period. One bank, bank γ, is relatively poor at $t = 0$ and therefore has to seek external funds to finance its loans to its *nature-selected* borrower, Mr. α. Bank γ can raise its funds either by borrowing from the interbank or the deposit markets. Bank γ's assets comprise only of its credit extension to the consumer loan market. This way we can focus on the effects of policies on banks that cannot quickly restructure their portfolios, perhaps due to inaccessibility of capital and asset markets, during periods of financial adversity. The other bank, bank δ, is a relatively rich commercial bank. In addition to its lending activities to its *nature-selected* borrower, Mr. β, its portfolio consists of deposits in the interbank market. Bank δ which is relatively richer compared to bank γ, has also alternative investment opportunities such as depositing in the interbank market. Its richer portfolio allows it to diversify quickly and more efficiently than bank γ. Mr. α and Mr. β are poor in terms of their commodity endowment at $t = 0$, and therefore have to borrow money from banks γ and δ, respectively, to buy commodities. As they are rich at $t = 1$, they sell commodities in the three states of period 1. On the other hand, Mr. ϕ is rich in $t = 0$ and poor in $t = 1$ (both in terms of commodity and monetary endowments).

We allow endogenous defaults in the interbank market, i.e. bank γ can default on its interbank loans in which it borrows from bank δ. Furthermore, Mr. ϕ has a choice to deposit his money with either bank. To distinguish between bank γ's and bank δ's deposits, we assume that the relatively more risky bank, bank γ, can default on its borrowing from the deposit market. Finally, we assume that the cost of default in the interbank and deposit market is quadratic. This in turn implies that the marginal cost of default in these markets increases as the size of borrowing becomes larger.

Formally, the agents' utility/profit functions can be described as follows;

$$
\begin{aligned}
U^{\alpha} &= [\chi_0^{\alpha} - c_0^{\alpha}(\chi_0^{\alpha})^2] + \sum_{s=1}^{3}[\chi_s^{\alpha} - c_s^{\alpha}(\chi_s^{\alpha})^2] - \sum_{s=1}^{3}\lambda_{s\gamma}^{\alpha}\max[0, \mu^{\alpha^{\gamma}} - v_{s\gamma}^{\alpha}\mu^{\alpha^{\gamma}}] \\
U^{\beta} &= [\chi_0^{\beta} - c_0^{\beta}(\chi_0^{\beta})^2] + \sum_{s=1}^{3}[\chi_s^{\beta} - c_s^{\beta}(\chi_s^{\beta})^2] - \sum_{s=1}^{3}\lambda_{s\delta}^{\beta}\max[0, \mu^{\beta^{\delta}} - v_{s\delta}^{\alpha}\mu^{\beta^{\delta}}] \\
U^{\phi} &= [\chi_0^{\phi} - c_0^{\phi}(\chi_0^{\phi})^2] + \sum_{s=1}^{3}[\chi_s^{\phi} - c_s^{\phi}(\chi_s^{\phi})^2] \\
U^{\gamma} &= \sum_{s=1}^{3}\pi_s^{\gamma} - \sum_{s=1}^{3}\left(\lambda_{ks}^{\gamma}\max[0, \overline{k} - k_s^{\gamma}] + \lambda_s^{\gamma}[\mu^{\gamma} - v_s^{\gamma}\mu^{\gamma}]^2 + \lambda_{s\phi}^{\gamma}[\mu_d^{\gamma} - v_s^{\gamma}\mu_d^{\gamma}]^2\right) \\
U^{\delta} &= \sum_{s=1}^{3}\pi_s^{\delta} - \sum_{s=1}^{3}\lambda_{ks}^{\delta}\max[0, \overline{k} - k_s^{\delta}]
\end{aligned}
$$

Given the chosen set of exogenous parameters/variables, we solve the agents' optimisation problems using a version of Newton's method and obtain the initial MECBD equilibrium. We then conduct a series of comparative statics by perturbing each of the exogenous parameters/variables. However, as the purpose of this section is merely to illustrate how the general model developed in this paper can be applied to analyse the issue of financial fragility in practice, we only report the result of one comparative statics below. The fully-analysed application of the general model which also involves a variation of specialisation of the general model can be found in the companion papers, Goodhart et al. (2003). Moreover, the full description of the agents' optimisation problems, the chosen value of exogenous parameters and the initial equilibrium used in this section can be found *op. cit.*

8.1 Expansionary Monetary Policy

Let the Central Bank engage in expansionary monetary policy by increasing the money supply in the interbank market (or equivalently lowering the interbank market rate). Due to the lower cost of funds in the interbank market, bank γ extends more credit to Mr. α and thus its lending rate decreases. Also, it restructures its liability side by reducing its deposit demand, thus lowering r_d^{γ}. In turn, Mr. ϕ increases his deposits with bank δ whose deposit rate now also falls. Consequently, bank δ expands its credit extension and deposits in the interbank market.

Given more liquidity in the economy, by the quantity theory of money proposition, all prices increase. So, the expected income of Mr. α and β increases and so do their repayment rates. This could also have been predicted by the liquidity structure of interest rate proposition since expansionary monetary policy has increased liquidity.

With respect to the welfare effects of expansionary monetary policy, as expected, the household sector benefits from lower borrowing cost. However, lower deposit rates adversely affect Mr. ϕ whose welfare decreases. Finally, banks' profits are lower since the higher repayment rates are compensated by the capital requirements' violation penalties they incur since their risk-weighted assets increase. Capital requirements worsen because the effect of higher credit extension and higher repayment rates dominate the beneficial effect of lower lending rates.

In sum, monetary policy has ambiguous welfare and profit effects depending on the state of the economic cycle. While the impact on borrowers is typically positive, the effect on investors and financial intermediaries depends on their portfolio and the regulatory regimes in which they operate.

8.2 Conclusion

In Goodhart et. al (2003), we explain in detail other relevant experiments, e.g. a change in household commodity and monetary endowment, a change in risk weight on loans etc. However, in this subsection we highlight some of the main lessons that can be drawn from them. First, in an adverse economic environment, expansionary monetary policy can aggravate financial fragility since the extra liquidity injected by the Central Bank may be used by certain banks to gamble for resurrection, worsening their capital position, and therefore the overall financial stability of the economy. Thus, a trade-off between efficiency and financial stability need not exist only for regulatory policies, but also for monetary policy.

Second, agents which have more investment opportunities can deal with negative shocks more effectively by using their flexibility in quickly restructuring their investment portfolios as a means of transferring 'negative externalities' to other agents with a more restricted set of investment opportunities. This result has various implications. Among others, banks which have no well-diversified portfolios tend to follow a countercyclical credit extension policy in face of a negative regulatory shock in the loan market (e.g. tighter loan risk weights). In contrast, banks which can quickly restructure their portfolio tend to reallocate their portfolio away from the loan market, thus following a procyclical credit extension policy. Moreover, regulatory policies which are selectively targeted at different groups of banks can produce very *non-symmetric* results. When the policy is aimed at banks which have more investment opportunities, much less contagion effect to the rest of the economy is observed since those banks simply restructure their portfolios between interbank and asset markets without greatly perturbing the credit market, and therefore interest rates and prices in the economy. On the contrary, when the same policy is targeted at banks which have relatively limited investment opportunities, they are forced to 'bite the bullet' by altering their credit extension. This produces changes in a series of interest rates, and therefore the cost of borrowing for agents. This in turn produces a contagion effect to the *real* sector in the economy.

Thirdly, an improvement such as a positive productivity shock, which is concentrated in one part of the economy, can worsen that for others. The key reason for this lies in the fact that our model has *heterogenous* agents and distributional effects therefore operate through various feedback channels among various sectors in the economy which all are active in equilibrium.

Finally, increasing the endowment of banks produces much the same result as increasing their corresponding capital violation penalties, particularly in regards to its contagion effects to the rest of the economy. Thus, a direct injection of emergency recapitalisation funds to banks is, to a certain extent, substitutable by increasing the banks' capital violation penalties.

9 Concluding Remarks

In reality, the economic system is both complex and heterogenous. In order to model it in a way that is both mathematically tractable, rigorous, and yet simple enough to be illuminating, economists have tended to assume homogeneity amongst agents in the sectors involved. Unfortunately that prevents analysis of certain key features of the real world, especially those relating to interbank interactions and financial fragility.

We have sought to focus on such inter-active channels. That has inevitably raised the complexity of our modelling; however we have tried to limit such complexity by adopting an endowment economy with just endowed consumers and banks, (no firms, no external sector, no other financial intermediaries, a black-box official sector).

We see this paper as the start of a major programme to use models such as this for the analysis of financial fragility. We shall stick to two main principles; first, heterogeneity is essential; second our approach to modelling default is the best available modelling strategy. Otherwise we hope to examine a wide range of alternative structures.

The main challenges ahead will be, first to represent a complex reality in a manner that is both illuminating and yet reflects that reality, and second to be able to draw general conclusions from an array of models that may, each individually, be sensitive to their individual particularities. This paper is the first step in our planned programme.

10 Appendix I: Proofs

10.1 Proof of Theorem

The proof follows *mutatis mutandis* from Tsomocos (2003). we will here provide on the parts that are different.

First, we need to extend the strategy spaces as follows:

$\Sigma_{\varepsilon}^{h}=\{(\chi^{h},\mu^{h^{b}},d_{b}^{h},b^{h},q^{h},\psi^{h},b_{sb}^{h},s_{sb}^{h},v^{h})\in R_{+}^{LS^{*}}\times R_{+}\times R_{+}\times R_{+}^{LS^{*}+J}\times R_{+}^{LS^{*}+J}\times R_{+}^{B}\times R_{+}^{SB}\times R_{+}^{SB}\times R_{+}^{S+J}:0\leq\chi^{h}\leq 2A1,\varepsilon m_{0}^{h}\leq\mu^{h^{b}}\leq\frac{1}{\varepsilon},\varepsilon m_{0}^{h}\leq d_{b}^{h}\leq M^{*},\ \varepsilon m_{s}^{h}\leq b_{sl}^{h}(b_{j}^{h})\leq\frac{1}{\varepsilon},\varepsilon\leq q_{sl}^{h}(q_{j}^{h})\leq\frac{1}{\varepsilon},\varepsilon m_{0}^{h}\leq\psi^{h}\leq\frac{1}{\varepsilon},\varepsilon m_{s}^{h}\leq b_{sb}^{h}\leq\frac{1}{\varepsilon},\varepsilon\leq s_{sb}^{h}\leq 1,\varepsilon\leq v^{h}\leq 1\}$

and

$\Sigma_{\varepsilon}^{b}=\{(\mu^{b},d^{b},\overline{m}^{b},\mu_{d}^{b},b^{b},q^{b},v^{b},\pi^{b})\in R_{+}\times R_{+}\times R_{+}\times R_{+}\times R_{+}^{J}\times R_{+}^{J}\times R_{+}^{J+2}\times R_{+}^{S^{*}}:\varepsilon e_{0}^{b}\leq\mu^{b}\leq\frac{1}{\varepsilon},0\leq d^{b}\leq M^{*},\varepsilon e_{0}^{b}\leq\overline{m}^{b}\leq\frac{1}{\varepsilon},\varepsilon e_{0}^{b}\leq\mu_{d}^{b}\leq\frac{1}{\varepsilon},\ \varepsilon e_{0}^{b}\leq b_{j}^{b}\leq\frac{1}{\varepsilon},\varepsilon\leq q_{j}^{b}\leq\frac{1}{\varepsilon},\varepsilon\leq v_{s}^{b}\leq 1,\varepsilon e_{s}^{b}\leq\pi_{s}^{b}\leq M^{*}\}$

which are both compact and convex. Also, $M^{*}\equiv M^{CB}+\sum\limits_{h\in H}\sum\limits_{s\in S^{*}}m_{s}^{h}+\sum\limits_{b\in B}\sum\limits_{s\in S^{*}}e_{s}^{b}$.

We need to bound $r_{d}^{b}(\varepsilon)$. $r_{d}^{b}(\varepsilon)\geq 0$ by Σ_{ε}^{h} and Σ_{ε}^{b}. Now let $r_{d}^{b}(\varepsilon)\to\infty$. Then b could reduce $\mu^{h^{b}}(\varepsilon)$ by $\triangle$ and improve its pay off, since $r_{d}^{b}(\varepsilon)\triangle\to\infty$, a contradiction.

Furthermore, we need to show that $0<\theta_{s}^{b}(\varepsilon)<\infty,\forall b\in B,s\in S$. First, let $\theta_{s}^{b}(\varepsilon)\to 0$ for some $s\in S,b\in B$. Then choose $h\in H$. He could have borrowed $\triangle$ more to buy $\triangle/\theta_{s}^{b}(\varepsilon)\to\infty$ of bank b. He could have then increased his income from the liquidation of b by $\triangle/\theta_{s}^{b}V_{s}^{b}\to\infty$ and use it to pay back his loan and reduce either his sales q_{sl}^{h} or d_{sb}^{h} or increase his repayment rates v_{sj}^{h},v_{sb}^{h} and thus improve his payoff, a contradiction.

Now let $\theta_{s}^{b}(\varepsilon)\to\infty$ for some $s\in S,b\in B$. Let $h^{b}\in H$ borrow a very large $\mu^{h^{b}}$ and use it to buy any sl. Then, h^{b} could sell $\triangle$ of bank b, acquire $\triangle\theta_{s}^{b}(\varepsilon)\to\infty$ to defray his loan and therefore improve his payoff, a contradiction. □

10.2 Proof of Proposition 1

(i) Let $M^{CB}\to\infty$ and assume that $(\sum\limits_{h\in H}q_{j}^{h}+\sum\limits_{b\in B}q_{j}^{b})\nrightarrow\infty$. Then by choosing subsequences and further subsequences select a subsequence along which all relative σ's and η's converge. Thus, the limit of the last subsequence coincides with a GEI, a contradiction. Also by feasibility, $M^{CB}/\left\|\theta_{j}\right\|\to\infty$ and therefore $\overline{\omega}(\eta,\sigma)\overline{R}_{s}^{b}\overline{m}^{b}(1+r^{b})+\sum\limits_{j\in J}\omega_{ij}(\eta,\sigma)R_{sj}(p_{s}A_{s}^{j})\left(b_{j}^{b}/\theta_{j}\right)+\omega(\eta,\sigma)R_{s}^{b}d^{b}\to\infty,\forall s\in S^{*},b\in B$. Finally, the relative boundedness of η's (by the proof of the Theorem 1) yields $M^{CB}/\left\|p_{0,l}\right\|\to\infty$.

(ii) $\exists \overline{z} \ni \nabla\Pi^{b^*}_s(\pi^{b^*}_s) > \lambda^{b^*}_s$ and $\nabla\Pi^{h^*}_s(\chi^{h^*}_s) > \lambda^{h^*}_s$ for some $b^* \in B$ and $h^* \in H$ by Tsomocos (2003a). Thus, $D^{h^*}_{sz}, D^{b^*}_{sz} > \overline{D} > 0$. Interiority of the maximum implies bounded aggregate profits and consumption.

(iii) Assume now that $\sum_{h\in H} \psi^h_b < +\infty$. Then again by choosing subsequences and further subsequences select a subsequence along which all relative σ's and η's converge. Again the limit of this subsequence yields a MECBD that coincides with a GEI, a contradiction. Thus, $\sum_{h\in H} \psi^h_b \to +\infty$. Moreover, since $k^b_t > 0$ and $\lambda^b_{k_t} = +\infty$ the argument of (i) does not obtain and so $\forall b \in B$, $\overline{m}^b, \mu^b \to \infty$ thus we get $\sum_{h\in H} s^h_{sb} V^b_s \to \infty$ by the boundedness of s^h_{sb}. □

10.3 Proof of Proposition 2

From the theorem, $p_{sl}, \theta_j > c$, $\forall s \in S^*$, $l \in L$. Let

$$\overline{Q} = 1 + \max\{[\nabla\Pi^b_s(\pi^b_s)/\nabla\Pi^b_{\acute{s}}(\pi^b_{\acute{s}}) :\ s, s' \in S^*,\ b \in B,\ \pi^b_s \in \diamondsuit], [\nabla\Pi^h_s(\chi^h_s)/\nabla\Pi^h_{\acute{s}}(\chi^h_{\acute{s}}) :\ s, \acute{s} \in S^*,\ h \in$$

$$\text{where } \diamondsuit = \{\ \pi^b_s \in IR^{s+1}_t :\ \pi^b_s \leq 1 + \max_{0\leq s\leq S} \sum_{b\in B} e^b_s + \max_{0\leq s\leq S} \sum_{h\in H} m^h_s\}$$

Let $\widehat{\lambda}^h_{sz} = (\overline{Q}/c)\max\{\nabla\Pi^h_l(\chi^h_l), h \in H,\ l \in L,\ \chi^h_l < A\}$ and $\overline{\lambda}_{sz} > \widehat{\lambda}^h_{sz}$,
$\widehat{\lambda}^b_{sz} = (\overline{Q}/c)\max\{\nabla\Pi^b_s(\pi^b_s),\ s \in S,\ b \in B,\ \pi^b_s \in \diamondsuit\}$ and $\overline{\lambda}_{sz} > \widehat{\lambda}^b_{sz}$, and
$\overline{\lambda}_{k_t} = \lambda_{sz}$.

Then, if $b \in B$ defaults an amount ε on its asset deliveries (or violates its capital requirements) its net gain will be at most

$$(\varepsilon\nabla\Pi^b_s(\pi^b_s)/\theta_j) - \varepsilon\lambda^b_{sz}$$

For $\lambda^b_{sz} = \overline{\lambda}_{sz}$ the expression becomes negative. Similarly for capital requirements and $h \in H$. □

10.4 Proof of Proposition 3

Let at the original MECBD bank b buys asset j and sells j'. From the existence argument, $\theta_j > c$, $\forall s \in S^*$, $l \in L$. Define $J^b = \{j \in J : b^b_j > 0\}$ and $L^b = \{j \in J : q^b_j > 0\}$. Since $\theta_j > 0$, $\forall j \in J$, and $\sum^b$ is bounded below by ε, $\forall b \in B\ \exists\ b, b'\ \forall j \in J \ni J^b \cap L^{b'} \neq \phi$ (or equivalently $J^{b'} \cap L^b \neq \phi$). Otherwise, all banks would be transacting in the same asset markets rendering them lopsided. Thus, $\exists b, b'$, $\forall j \in J$, involved in reverse transaction. Else, one bank would either violate its capital requirement excessively or would not satisfy its budget constraint.

From the F.O.C. of optimisation, $\forall s \in S^*$, assuming $\lambda^b_{sz} = +\infty$,

$$(\nabla\Pi^b_s(\pi^b_s)/\theta_j) = (\nabla\Pi^b_s(\pi^b_s)/\theta_{j'})(1+\rho)$$

If LHS $>$ RHS then b could have borrowed $\varepsilon\theta_j$ more on the interbank market, bought ε units more of asset j, sold $(\varepsilon\theta_j/\theta_{j'})(1+\rho)$ more units of asset j' to defray its loan and improve its profits. Alternatively, if LHS $<$ RHS, then b should have spent $\varepsilon\theta_j$ less on asset j, thus borrowing $\varepsilon\theta_j$ less from the interbank market, sold $(\varepsilon\theta_j/\theta_{j'})(1+\rho)$ less of asset j' improving its profits. Note that this last option is feasible by (A2).

Now consider a change in k. Suppose asset investments remain the same. Then either $\overline{m}^b$ or d^b should change to satisfy the new capital requirements. If $\overline{m}^b$ changes then the argument follows the proof of proposition 5. If d^b changes then market clearing requires ρ to change as well. For profits to remain unchanged, by indecomposability some b is buying as well as selling , some assets, say, j and $j^{'}$. Thus $(\theta_j/\theta_{j'})$ should fall. But then j should have a seller who buys another $j^{''}$. So, $(\theta_{j'}/\theta_{j''})$ must also fall. Thus, we arrive out some $\widehat{j}$ which has already been mentioned, and then $(\theta_j/\theta_{j'})(\theta_{j'}/\theta_{j''})...(\theta_{j^*}/\theta_j) = 1$ should be falling, a contradiction. In the case of bankruptcy the previous argument is strengthened since $\nabla\Pi_s^b(\pi_s^b)/[D_{sz}^h]^+ < 0.\Box$

10.5 Proof of Proposition 4

(i) Assume not. since $h \neq \widehat{h}, \sigma^h(\cdot) \neq \sigma^{\widehat{h}}(\cdot)$. W.l.o.g. consider the case that only $v_{sb_1}^h \mu^{h^{b_1}} \neq v_{sb_2}^h \mu^{h^{b_2}}$. Such $s \in S$ exists since either $e_{sl}^h \neq e_{sl}^{\widehat{h}}$ or $u^h(\cdot) \neq u^{\widehat{h}}(\cdot)$ for some $\overline{s} \in S^*$. Then, $\Pi_{\overline{s}}^{b_1}(\cdot) \neq \Pi_{\overline{s}}^{b_2}(\cdot)$, for $\overline{s} \in S^*$, say $\Pi_{\overline{s}}^{b_1}(\cdot) > \Pi_{\overline{s}}^{b_2}(\cdot)$. Since everything other action is the same, Mr. $\widehat{h}^{b_2}$ could have reduced his investment in b_2 by $\triangle$ and invest instead in b_1 in $t = 0$. His net gain would be $\triangle(\Pi_{\overline{s}}^{b_1} - \Pi_{\overline{s}}^{b_2}) > 0$. From (5^h), he could have used his extra cash to improve his repayment rates, or if $\triangle(4^h) > 0$ to increase his consumption and thus improve his pay off, a contradiction.

(ii) Assume not, i.e. bank equity prices are the same. Since $u_{\overline{s}}^{b_1}(\cdot) \neq u_{\overline{s}}^{b_2}(\cdot)$ for some $\overline{s} \in S^*, \sigma^{b_1}(\cdot) \neq \sigma^{b_2}(\cdot)$. W.l.o.g. consider the case that $v_{\overline{s}}^{b_1} > v_{\overline{s}}^{b_2}$. This implies from (3^b) that $\Pi_{\overline{s}}^{b_1} < \Pi_{\overline{s}}^{b_2}$. Let $\widehat{h}$ reverse his action of (i) and so improve his payoff, a contradiction.

(iii) Consider the case in which $\overline{k}_t - k_t^{b_1} > 0$ but $\overline{k}_t - k_t^{b_2} = 0$. Then, let $\widehat{h}$ reduce his investment to b_1 by $\triangle$ and instead increase his investment to b_2 by $\triangle$, and thus increase his payoff, a contradiction. This act is affordable by $\widehat{h}$, since $\theta_s^{b_1} = \theta_s^{b_2}$. If $\Pi_{\overline{s}}^{b_1} \neq \Pi_{\overline{s}}^{b_2}$, for some $\overline{s} \in S$ then let $\widehat{h}$ scale down his investment in both b_1 and b_2 by $\frac{\triangle}{2}$ and borrow $\triangle$ less and so save $\triangle v_{\overline{s}b}^h \mu^{h^b}$, yet another contradiction.

(iv) See Geanakoplos (1990)

(v) It follows immediately from the asset span theorem and the linearity of prices. $\Box$

10.6 Proof of Liquidity Structure of Interest Rates Proposition

By the argument of the theorem all p's, θ's, r's, and ρ stay bounded. No h is left with unused cash at $t = 1$, i.e. $\triangle(5^h) = 0, \forall\, h \in H$. Otherwise he could have borrowed $\triangle/(1+r^b) > 0$ more and spend it on some $\overline{l}\overline{s}$ and improve his utility by $\nabla\Pi_{\overline{s}}^h(\chi_{\overline{s}}^h)\triangle/(1 + r^b)p_{\overline{s}\overline{l}} > 0$. Then, h could have used his left over cash $\triangle = \triangle(5^h)$ to defray his loans. Similarly, no $h \in H$ returns more than what he owes. Finally, if h defaulted then he could have used his unused cash to reduce his debt and improve his payoff by $\lambda_s^h\triangle$. Similarly, for $b \in B$ since $r^b > 0, \forall b \in B$ banks would not leave cash unused in order to improve their last period profits. (Recall, banks maximise $\Pi_s^b(\cdot), \forall\, s \in S$.

Thus, all cash is returned to the commercial banks and the equality follows $\forall s \in S$ (for $s = 0$ cash may be transferred for use in the beginning of the next period). $\Box$

10.7 Proof of Quantity Theory of Money Proposition

Banks that hold idle cash, say $\triangle$, either they could have deposited it in the interbank market, extend it in the credit market or invest it in the asset market and improve their payoff by at least $\nabla\Pi_s^b(\pi_s^b)\triangle\rho > 0$. Otherwise, they could have reduced their borrowing from the interbank market and thus save $\rho\triangle$ interest repayment. Likewise, households if they are borrowers will spend all of their cash; or else they should not have borrowed. If $r^b = 0$, $\forall b \in B$ still h should not have borrowed thus letting some b deposit with the interbank market and improve profits by $\rho\triangle$. This, would improve his payoff since $\sum_{h \in H} s_b^h = 1$, $\forall\, b \in B$. (Note that for this household h, if $s_b^h = 0$ then he could have bought $\varepsilon\ s_b^h$ shares and borrowed $(\triangle - \varepsilon\ \psi_b^h)$ less and improve his payoff by at least $\nabla\Pi_s^h(\chi_s^h)(\triangle - \varepsilon\ \psi_b^h)/p_{sl}$ for some $s \in S$, $l \in L$.

If default in the asset markets occurs then adjust the previous arguments in case of asset investments by $\lambda_s^b(1 - v_{sj}^b)\triangle$ to induce the payoff improvement.

By the same argument, at $s = 0$ all unused cash will be spent in the next period. □

10.8 Proof of Fisher Effect Proposition

It follows immediately from the optimisation conditions. □

10.9 Proof of Proposition 5

As in proposition 3, from the FOC of optimisation $\forall s \in S^*$ and for some $h \in H$, assuming no bankruptcy

$$\nabla\Pi_s^h(\chi_{sl}^h)/p_{sl} = (\nabla\Pi_s^h(\chi_{sl'}^h)/p_{sl'})(1 + r^b)$$

Now consider a change in m^{CB} (the same argument applies for changes in M^{CB} or μ^{CB}). Let the case not covered in proposition 3 occurs. By the liquidity structure of interest rate rates proposition r^b changes. To maintain the same consumptions, using indecomposability, $(p_{sl}/p_{sl'})(p_{sl'}/p_{sl''})...(p_{sl^*}/p_{sl}) = 1$, should be falling, a contradiction. *A fortiori*, if $[D_{sz}^h]^+ > 0$ agents by reducing their borrowing by ε and adjusting their consumption by ε/p_{sl} and $p_{sl}/p_{sl'}$ accordingly to satisfy the FOC. In such a case, for high enough λ_{sz}^h, his net gain in utility, $(\lambda_{sz}^h r^b - \nabla\Pi_s^h(\chi_{sl}^h)/p_{sl})\varepsilon > 0$.

Remark: Investors' indecomposability may be relaxed and then in case where the previous argument does not hold then liquidation of bank will necessarily leave some h with $\triangle(5^h) > 0$.□

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