Optimization model in Input output analysis and computable general equilibrium by using multiple criteria non-linear programming

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Abstract.

This paper develops an optimization theory on aggregation of input output (IO) analysis and computable general equilibrium (CGE) theory, in contrast to the situation extant, where it is only partially described .An overall general equilibrium on IOCGE is therefore developed and presented, based on a model using multiple criteria non-linear programming (MCNP). Here, demand and supply models are considered as two parts of a single, composite flow IO model. This understanding implies that each IO model has dual counterpart in the demand and supply model. Through consistent use of this counterpart principle significant CGE model are developed for both model types; and a balance of the price between demand and supply can be stated in a simpler way. Specifically, multiple criteria non-linear programming for perfect aggregation of the balance between the output, as well as the price of demand and supply .In this context, equilibrium is obtained when there is a best compromise solution among the MCNP. This process enables us to establish feedback not only between supply and demand but also between theory and application. It is a great advantage and an important tool for planners and decision-makers.

Keywords: Computable general equilibrium; Input output analysis; Multiple criteria non-linear programming; Weighting method; Best compromise solution

JEL classifications: C50; C62; C67; C68; D57; D58

1.Introduction

In the years since Leontief first proposed his classic Input output (IO) models, IO is shown as a general equilibrium system (Leontief, 1953). A general equilibrium theory has to consider the quantities and the prices of both demand and supply at the same time (Arrow & Hahn, 1971); the IO

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theory would have to be the same. But so far the use of IO within the framework of the general equilibrium model has limited to one side for both aspects. From the very beginning, IO techniques do not wholly describe a general equilibrium system (Christ, 1955).

Optimization model has been used in CGE analysis since 1971. Specifically, the purpose of using the optimization methods for CGE is to find the general equilibrium point of the overall economic system along with attaining the objection from supply and demand. There are three theorems can often used in the optimization model of CGE Models: Fermat’s Theorem (Fermat 1629), Lagrange Multipliers Rule (Lagrange 1788), Kuhn-Tucker Theorem (Kuhn & Tucker 1951). The process of the Lagrange multiplier method, which is the most familiar way to solve the optimization in CGE is divided into three part: optimization in the model of demand, optimization in the model of supply, equilibrium in demand and supply. The first two parts has no interior relationship with each other and always regard some function, such as price, interest rate, wage of unit labor as fixed rate. Optimization model in IOCGE by using multiple criteria nonlinear programming is derived from well known linear models. When we deal with the conditional extremum problem with the inequality in the constraint, the multiple criteria nonlinear programming can be applied with Kuhn-Tucker Theorem. In this paper we aggregate the dual counterpart and the equilibrium within a overall mathematic programming. Since the objective function of the proposed method is nonlinear, we shall call it multiple criteria nonlinear programming.

This paper will proceed as follows: Section 2 will constitute the basic concepts of general equilibrium structure in IO analysis. Section 3 will elaborate the definition and algorithm of the proposed MCNP model for the IOCGE .The conclusions will be outlined in Section 4.

2.The general equilibrium model structure of IOCGE

The general equilibrium model commonly includes: model of demand, model of supply, model of price and model of equilibrium. While the framework of IO model has been limited to supply side for both aspects. In order to enable the IOCGE model to search the equilibrium point it is necessary to consist the dual counterpart of IOCGE structure (Qiyun Liu, 1993).

Figure 1 depicts the general equilibrium structure of IOCGE. Each formulation in this figure is given in money terms and is determined as follows:

The model of Supply can be outlined as:

$$X^S = (I - A)^{-1} Y^D \qquad (1)$$

Where, X^S is the vector of gross input (total production), in order to use primary inputs (supply). Y^D is the vector of final demand, A is the direct input coefficients.

The model of demand is more widespread amongst the models of input-output, in theory as well as in practice. By means of this model it is possible to determine gross input (total production) of supply when the final uses (demand) are given. In this case, supply adapts itself to demand. In other words, demand becomes the major factor and supply has a minor function. This is characteristic from the demand side of quantities in general equilibrium. Thus the supply prices are identified with them.

The model of demand can be outlined as:

$$X^D = N^S (I - R)^{-1} \qquad (2)$$

Where, X^D is the vector of gross output (total production), in order to meet final uses (demand). N^S is the vector of primary input, R is the direct distribution coefficients .

Define 1.*Direct input coefficient*

$$A_{ij} = \frac{X_{ij}}{X_j} \qquad (3)$$

Define 2.*Direct distribution coefficient*

$$R_{ij} = \frac{X_{ij}}{X_i} \qquad (4)$$

Where, X_{ij} is the square matrix ($n \times n$) of the inter-branch flows of intermediate uses. Each element of this matrix is the output of the branch i earmarked for branch j, and is the input of the branch j to branch i, X_j is the row vector ($1 \times n$) of gross output (total production), X_j is the column vector ($n \times 1$) of gross output (total production).

The model is no less wide spread and is used more in theory than in practice. By means of this model it is possible to determine the gross output of demand when the primary input are given. In this case, demand adapts itself to supply. In other words, supply becomes the major factor and demand has a minor function. This is characteristic from the supply side of quantities in general equilibrium. Thus the demand prices are identified with them.

The model of price is determined as follows:

The model of supply price:

$$P^s = (A_g + A_v + A_M)(I - A)^{-1} \tag{5}$$

$$A_g + A_v + A_M = A_N$$

Where, P^s is the price of supply, A_g is the direct input coefficients of fixed assets depreciation, A_v is the direct input coefficients of labor income & welfare, A_M is the direct input coefficients of social profits & Taxes. A_N is the added value per output.

The model of demand price:

$$P^D = (I - R)^{-1}(R_z + R_w + R_k + R_f) \tag{6}$$

$$R_z + R_w + R_k + R_f = R_Y$$

Where, P^D is the price of demand, R_z is the direct distribution coefficients of fixed capital formation, R_w is the direct distribution coefficients of consumption, R_k is the direct distribution coefficients of accumulation, R_f is the direct distribution coefficients of exports, R_Y is the final distribution coefficient.

The model of general equilibrium:

$$X^s = X^D \tag{7}$$

$$P^s = P^D \tag{8}$$

If we want to get the general equilibrium in the model, we must satisfy the following condition:

$$
\begin{aligned}
&(P^S)^T = P^D \\
&\because (P^S)^T = [(I-A)^{-1}]^T \bullet A_N^T \\
&P^D = (I-R)^{-1} \bullet R_Y \qquad (9) \\
&\therefore A_N = R_Y^T \bar{H} \\
&\bar{H} = (I-R^T)(I-A)
\end{aligned}
$$

Where, P^s is the price of supply, P^D is the price of demand, A is the direct coefficient, R is the distribution coefficient, A_N is the added value per output, R_Y is the final distribution coefficient.

3.Input-output computable general equilibrium by using multiple criteria non-linear programming

3.1 Basic structure of IOCGE

There are many optimization models we can constitute to attain the equilibrium solution of the economic system, such as operations research & mathematical programming, Mathematical analysis, control theory (Qiyun liu, 1993). While Lagrange multiplier method and the linear & nonlinear equation are familiar in calculation of CGE, the multiple criteria programming model in the IOCGE is a largely unexplored subject.

The MCNP model of IOCGE is stated as :

$$
\begin{aligned}
&\max : L_S = f_S(X^S, Y^D) \\
&\max : L_D = f_D(X^D, N^S) \\
&\begin{cases}
X^S - (I-A)^{-1} Y^D \le 0 \\
X^D - N^S (I-R)^{-1} \ge 0 \\
BX^s \le \bar{B} \\
F(X^D, N^S) \ge 0 \\
X^s = X^D \\
X^D \ge 0, N^S \ge 0 \\
X^s \ge 0, Y^D \ge 0
\end{cases}
\end{aligned}
\qquad (10)
$$

Where, X^S is the vector of gross input (total production) , in order to use primary inputs(supply). Y^D is the vector of final demand, A is the direct input coefficients. X^D is the vector of gross output (total production) , in order to meet final uses(demand), N^S is the vector of primary input,

R is the direct distribution coefficients, B is the resource consuming coefficients, $\bar{B}$ is the gross resource available. $f_S(\bullet)$ is the revenue function from supply side, $f_D(\bullet)$ is the total utility function from the demand side, $BX^s \le \bar{B}$ is the constraint of resource.

There are three reasons for this model:(1) CGE and IO are prepared and used in certain function, not the fuzzy or random function .All those can be combined with mathematical programming. This allow us to link CGE and IO with MCNP.(2) Even if CGE&IO models exist in physical terms, it is not possible to use the models to equate unitility with money terms. When we use the weighting method to solve MCNP, the problem can be solved easily.(3) MCNP can be easily rewritten to dynamic programming, which can be used as dynamic CGE models.(4)we can get the best compromise solution from the MCNP model .The solution is the global solution to the demand and supply side at the same time, which can meet the basic assumption of CGE in economic system.

3.2Existance of the best compromise solution of IOCGE

When we use the computer-based algorithm to solve IOCGE, we firstly use the weighting method to change the multiple criteria programming to the single criteria programming. The original MCNP model ban be rewritten as :

$$\max : \omega_1 f_S(X^S, Y^D) + \omega_2 f_D(X^D, N^S)$$

$$\begin{cases} X^S - (I-A)^{-1} Y^D \le 0 \\ X^D - N^S (I-R)^{-1} \ge 0 \\ BX^s \le \bar{B} \\ F(X^D, N^S) \ge 0 \\ X^s = X^D \\ X^D \ge 0, N^S \ge 0 \\ X^s \ge 0, Y^D \ge 0 \end{cases} \qquad (11)$$

Then MCNP can be solved as single objective nonlinear programming. Because the selected weighting is positive(we often select $\omega_1 = 0.5, \omega_2 = 0.5$) ,the optimal solution in model (11) is equal to the best compromise solution in model (10)(see proof 1).The objective function in model(11) is differentiable and continuous with the feasible region is bounded. So the existence of the optimal solution in model(11) can be proven, as well as the best promise solution in model(10)(see proof 2).

Proof 1. *T*his is an application of Kuhn-Tucker Condition result to the noninferior solution in multiple criteria programming.

Proof 2. *T*his is an application of Kuhn-Tucker Condition result to the optimal solution in Linear programming .

3.3Computer-Based Algorithm of IOCGE

Figure 2 depicts the framework of the algorithm. A heuristic IOCGE algorithm can be outlined as:

Step 1: Get the basic parameters ,such as the direct input coefficient ,the direct distributing coefficient ,the function of the revenue and the utility etc. The process presents requirements for Input-output tables. The production part in the input-output tables of today is presented in detail. In contrast to this ,the demand and factor parts are highly aggregated .We cannot expect to gain very good results from this process if change are not made to give correct proportions between these parts(Ezra davar,1989).

Step 2: Constitute the MCNP model.

Step 3: Use the software "Lingo" to solve the IOCGE .During calculation of IOCGE, we find the scale of calculation is so enormous that we have to try the different toolbox to find the best, such as Matlab, MINOS, LOQO and so on. Finally ,we use the "Lingo" software (see http://www.lindo.com) to do and get the fitful result. Projection Method , one kind of Simplex Method ,is the kernel algorithm in "Lingo" which can deal with the variable beyond 1×10^8 units and is the best algorithm for our IOCGE. Now Lingo (5.0 version) is unlimited for installation on constraint ,variable, nonlinear variable and integer variable along with the limit of generator memory is 1031.5 M.

Step 4: Sensitivity analysis for the IOCGE. Another advantage of using the "Lingo" is that it has a special package to calculate the sensitivity analysis result.

4.Conclusions

The discovery of new dual counterpart characteristics of IO models enables us to construct the IOCGE model by using MCNP. These models ,along with existing ones, generate general structure of models which enable us to establish linkage and feedback between demand and supply, with respect to both quantities and prices. The IOCGE model suggested in this paper, based on the above-mentioned models enable us to find the general equilibrium at the same time in one model as to establish feed back between supply and demand. We can use the software for the large-scale computation in order to attain the best compromise solution. This is an important tool for planners and decision-makers.

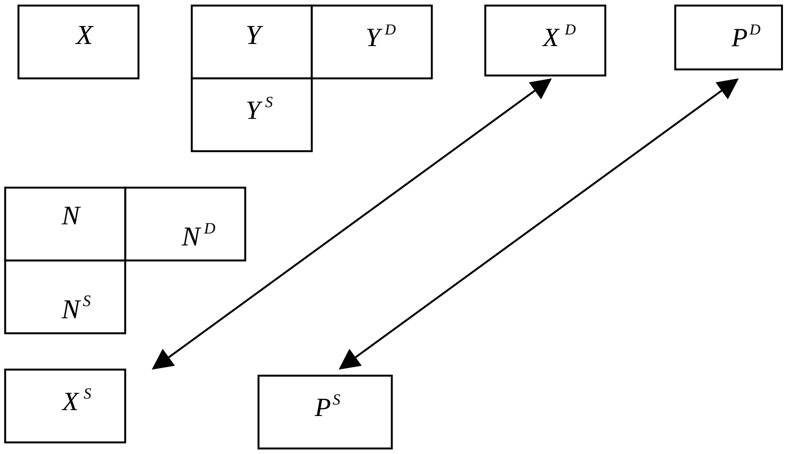

Figure 1. General structure of IOCGE

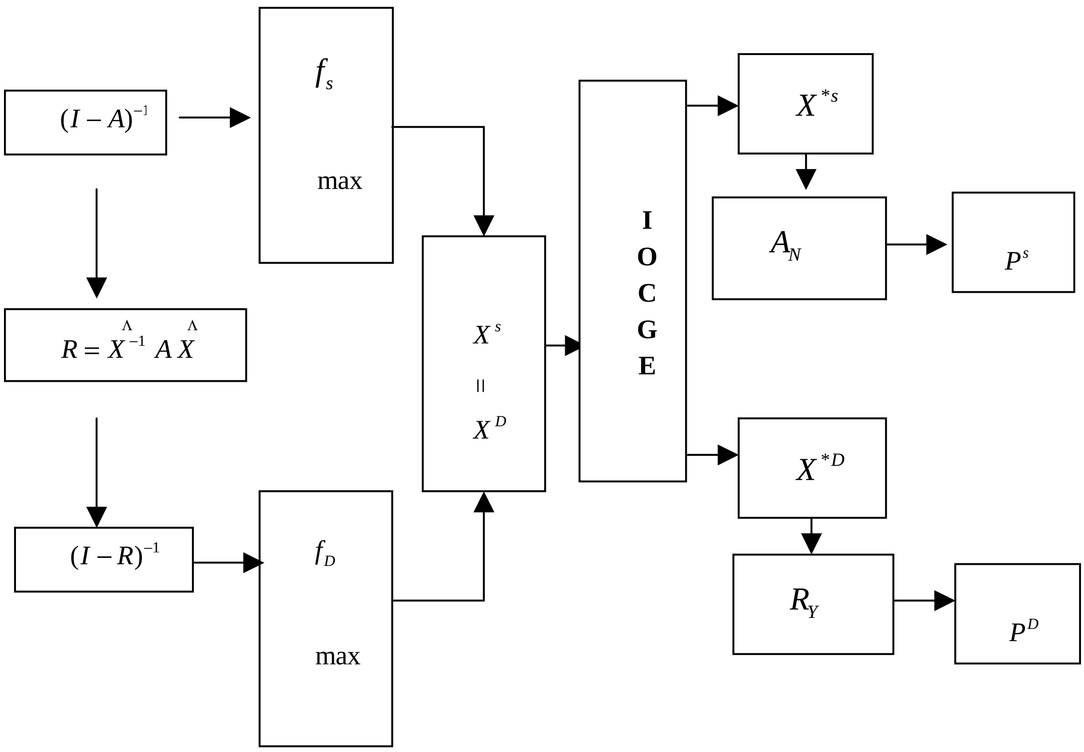

Figure 2 Calculation process of IOCGE

Acknowledgements

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Appendix

List of variables

Name	Description
X^S	Gross input of supply
Y^D	Final demand
A	Direct input coefficients
X^D	Gross output of demand
N^S	Primary input of supply
R	Direct distribution coefficients
B	Resource consuming coefficients
$\bar{B}$	Gross resource available
P^s	Price of supply
A_g	Direct input coefficients of fixed assets depreciation
A_v	Direct input coefficients of labor income & welfare
A_M	Direct input coefficients of social profits & Taxes
A_N	Added value per output
P^D	Price of demand
R_z	Direct distribution coefficients of fixed capital formation
R_w	Direct distribution coefficients of consumption
R_k	Direct distribution coefficients of accumulation
R_f	Direct distribution coefficients of exports
R_Y	Final distribution coefficient

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