Backward and Forward dispersion approach in a bi-regional Social Accounting framework

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Abstract

The aim of the paper is to provide a method of analysis that can give a further insight into the interactions among industries and institutional sectors, in two different region. An application that relies on a regional data base, inspired by the bi-regional Social Accounting Matrix, illustrates how macro multipliers ruling the multi-sector multi-industry interactions can be defined and evaluated. This feature greatly helps in showing the impact of the structure of macroeconomic variables since all the possible behaviours of the economy are determined by those multipliers: either those patterns that have emerged, because have been activated by the actual shock, and those that have kept latent. The identification of macro multipliers allows for the consistent definition of forward and backward dispersion, a tool especially efficient in the study of propagation since it is not confined to predetermined structures of macroeconomic variables and still allows for the determination of "summary" measures of dispersion through industries and sectors.

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1 Introduction

Some decades ago national accounts suffered of a fundamental dichotomy between income-final demand accounts and output-interindustry flows accounts. As Richard Stone pointed out (Stone, 1979) we faced two different and separated accounting systems which acted independently with almost no information exchange.

In the last decades the national accounting schemes have completely realized the integration of the two aspects and one side provides the information support to the other, causing the progressive integration in the actual accounting practice. Though integrated, the accounting scheme remains flexible and open. Its matrix representation constitutes a consistent nucleus that can be extended according the aims of the research. The Social Accounting Matrix is the result of this expansion that can be moved forward to include a greater set of economic and social phenomena at a substantial degree of detail.

From the SAM approach emerges a model of circular income flow which is more articulated than the usual one: each macroeconomic flow variable, conveniently disaggregated, generates a second flow variable through the use of a structural matrix and progressively so until the loop is closed. Final demands determine total outputs and value added by industry; the latter generates domestic incomes by factor which compose disposable incomes by institutional sectors; these give rise to final demands closing the loop. In multi-regional framework the income circular flow is separated among areas analyzed. The linkages among the region are the same but the structural matrix is partitioned by origin and destination of flows.

For facing these progresses in the design of a data base which provides meaningful sectorization of the major macroeconomic variables, flexible tools of analysis are needed, to get a deeper insight in the propagation phenomena characterizing regional, sectoral and industrial interactions. In these phenomena the scale, but especially, the structure of macroeconomic variables play a major role. The traditional tools for studying propagation are those provided by impact multipliers and linkage multi regional analysis. These tools, however, design procedures that do not give a complete account of the effects of the changing structures of macro-variables.

The propagation analysis we propose is based on a decomposition that allows for the identification and quantitative determination of aggregated macro multipliers regional, which lead the economic interactions, and the structures of macroeconomic variables, that either hide or activate these forces. The analysis will be applied to an extended income-output bi-regional loop that can be quantitatively tested forwarding a shock on a given macro-variable and observing the effects on another macro-variable within the loop. It will identify the most efficient structure, without confining on the equi-distributed unitary shock. "Summary" measures will be found, consistent with the multi-regional, multi-sectoral and multi-industry framework, that will allow to measure the degree of interaction among sector and industry components.

In section 2 the discussion on impact multipliers and linkage analysis is briefly referred to, in order to restate the "statistical" purpose of summary measures of linkage. In section 3 we describe the data base for our application inspired to the bi-regional SAM, where there are two region, industries and institutional sectors. Section 4 shows the extended circular flow loop on which the analysis will be performed. In section 5 proposes a "statistical" approach where the structural matrix is conveniently refined to obtain a synthetic representation of the interactions. In the same section measures of intraregional and interregional backward and forward dispersions are stated with reference to the dominating macro multipliers. A summary representation is then provided, which relies on the concepts of backward and forward dispersions, with the aim of determining the strength of bi-regional, multi-sector and multi-industry interactions.

Appendix A shows formalization of an extended version of the income circular flow (multi-sectoral model based on bi-regional Social Accounting Matrix) where the interactions between industries and institutions could be specified and evaluated, by two regions. Appendix B shows tables and graphs for interregional analysis.

2 Some considerations on multiplier and linkage analysis in bi-regional framework

The original Input-Output (I-O) problem consists in the search for an equilibrium output vector for the two region and n I-O sectors of the economy. Since in the following section income will be disaggregated by institutional sectors, in order to avoid misinterpretation, we will use the term industries for producing sectors, and the sector for institutional sectors. Such vector conveniently faces the predetermined final demand vector $\mathbf{f}^i$ by industries, and the induced industrial demand. In bi-regional model we have also the distinction between origin and destination for region of the flows (M and I are two regions).

The equilibrium output vector is given by

$$\mathbf{x} = \left[\begin{array}{c} \mathbf{x}^M \\ \hline \mathbf{x}^I \end{array} \right] = \left[\begin{array}{c|c} \mathbf{R}^{MM} & \mathbf{R}^{MI} \\ \hline \mathbf{R}^{IM} & \mathbf{R}^{II} \end{array} \right] \left[\begin{array}{c} \mathbf{f}^M \\ \hline \mathbf{f}^I \end{array} \right] \tag{1}$$

where $\mathbf{R} = [\mathbf{I} - \mathbf{A}]^{-1}$ and $\mathbf{A}$ is the technical coefficients matrix, and generally exists, as in general the technology can be expected to be productive, i.e. the technology is such that a part of total output is still available for final uses, after the intermediate requirements have been satisfied. In this case, $\mathbf{A}$ satisfies the Hawkins-Simon conditions. The $\mathbf{R}^{ij}$ matrix is usually referred to as the Leontief multipliers matrix and its elements, r_{ij}^{ij}, show the direct and indirect intraregional -for $i = j$- or interregional -for $i \neq j$-, requirements of industry output i per unit of final demand of product at industry j, in the two regions. Extensive use is made of matrix $\mathbf{R}$ within the traditional multipliers analysis and a substantial part of linkage and key sectors analysis is based on it. $\mathbf{R}$ matrix provides, in fact, a set of disaggregated multipliers that are recognized to be most precise and sensitive for studies of detailed economic impacts. These multipliers recognize the evidence that total impact on output will vary depending on which industries are affected by changes in final demand, into two region. The i^{th} total output multiplier measures the sum of direct and indirect input requirements needed to satisfy a unit final demand for goods produced by industry i.

Research on linkage analysis dates back to the definitions elaborated by Rasmussen of "summary measures for the inverse matrix" (Rasmussen, 1956). Developments in research have provided various definitions of linkage (Hirschman, 1958) which have led to the indicators called nowadays "forward linkages" and "backward linkages". These indicators are applied to the technical coefficients matrix, to the Leontief inverse or to the matrix of constant market shares (Ghosh, 1958) according the purposes of the research.[1]

Refer to the origins through the Rasmussen definition, for example in region M, he noted that the sum, $r_{.j}^{MM}$, of i^{th} column elements

$$r_{.j}^{MM} = \sum_{i=1}^{m} r_{ij}^{MM} \tag{2}$$

corresponds to the total increase in domestic output from all industries in region M needed to match an increase in the final demand for the product of industry j by one unit in region M.
Similarly the sum, $r_{i.}^{MM}$, of row elements i.e.

$$r_{i.}^{MM} = \sum_{j=1}^{m} r_{ij}^{MM} \tag{3}$$

gives the increase in intraregional output of industry i needed, in order to cope with a unit increase in the final demand for the product of each industry in region M. We can take the interregional multipliers when the effect on output in one region (M) that is caused by increase in final demand in another region (I) (Miller and Blair, 1985).

For generic element of the matrix R^{ij} in one region, we can take the average, of $r_{.j}^{ij}$, and they will represent an estimate of the (direct and indirect) increase in output to be supplied by an industry chosen at random if final demand for the products of industry j expands by one unit, in the same region (intraregional effects) or in another region (interregional effects):[2]

[1]However, from a modelling viewpoint, the fixed technical coefficient assumption is conflicting with the constant market shares hypothesis, since a model based on fixed technical coefficients will imply non constant market shares, and a model with constant market shares will imply varying technical coefficients. This is the reason why we will confine ourselves to the *leontievian* approach based on the concept of fixed technical coefficients

[2]Rasmussen (1956)ibidem p.130.

$$\left(\frac{1}{m}\right) \cdot r^{ij}_{.j} \qquad (j = 1, 2, .., m) \tag{4}$$

Similarly

$$\left(\frac{1}{m}\right) \cdot r^{ij}_{i.} \qquad (i = 1, 2, .., m) \tag{5}$$

can be regarded as the average increase in output to be supplied by industry i if the final demand for the products of an industry chosen at random is increased by one unit, in the same region when $i = j$ (intraregional effects) or in another region when $i \neq j$ (interregional effects).

For performing consistent interindustry comparisons, we need to normalize these averages by the overall average defined as

$$\frac{1}{m^2}\sum_{j=1}^{m}\sum_{i=1}^{m} r^{ij}_{ij} = \frac{1}{m^2}\sum_{j=1}^{m} r^{ij}_{.j} = \frac{1}{m^2}\sum_{i=1}^{m} r^{ij}_{i.} \tag{6}$$

and thus consider the indices

$$\pi^{ij}_{j} = \frac{\frac{1}{m} \cdot r^{ij}_{.j}}{\frac{1}{m^2} \cdot \sum_{j=1}^{m} r^{ij}_{.j}} \tag{7}$$

and

$$\tau^{ij}_{i} = \frac{\frac{1}{m} \cdot r^{ij}_{i.}}{\frac{1}{m^2} \cdot \sum_{i=1}^{m} r^{ij}_{i.}} \tag{8}$$

The aim of the direct and indirect, intraregional or interregional, backward linkage index π^{ij}_{j}, the *power of dispersion* in the Rasmussen definition [3], is to measure the potential stimulus to other industries from a demand shock in any industry j, in one of two region. If $\pi^{ij}_{j} > 1$ an industry will need a comparatively large production increase to meet a unit increase in final demand for the products of industry j, in one of two region. When $\pi^{ij}_{j} < 1$ industry j relies heavily on the system of industries and vice versa. π^{ij}_{j} can be considered an index of the, intraregional or interregional, power of dispersion for industry j. This index describes the relative extent to which an increase in final demand for the products of industry j is dispersed

[3] Rasmussen (1956) ibidem p.135.

throughout the system of industries, in bi-regional context. The index also expresses the extent of the expansion caused in the system of industries by expansion in industry j.

The intraregional or interregional forward linkage τ_i^{ij}, the *sensitivity of dispersion* in sens of the Rasmussen definition, measures the degree at which on industry output is used by other industries as an input in one region or in another region. In the case of $\tau_i^{ij} > 1$, for given increases in demand, industry i will have to increase its output more than other industries. Index τ_i is an index of sensitivity of dispersion for the industry i. This index expresses the extent to which the system of industries relies on industry i and the degree to which industry i is affected by an expansion in the system of industries.

It has to be stressed, however, that the Rasmussen definitions were of statistical nature, since both measures were mean values of either outputs or final demands of industries chosen at random in one region. For each of these measures, in fact, he elaborated a coefficient of variation in fact a standard deviation. In particular, for the power of dispersion we get

$$\sigma_{.j}^{ij} = \frac{\sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(r_{ij}^{ij} - \sum_{i=1}^{m} r_{ij}^{ij})^2}}{\frac{1}{m}\sum_{i=1}^{m} r_{ij}^{ij}} \qquad (j = 1, \ldots, m) \qquad (9)$$

and for the sensitivity of dispersion:

$$\sigma_{i.}^{ij} = \frac{\sqrt{\frac{1}{m-1}\sum_{j=1}^{m}(r_{ij}^{ij} - \sum_{j=1}^{m} r_{ij}^{ij})^2}}{\frac{1}{m}\sum_{j=1}^{m} r_{ij}^{ij}} \qquad (i = 1, \ldots, m) \qquad (10)$$

Nevertheless the original statistical approach of the Rasmussen analysis progressively disappeared and the interpretation of his measures have definitely become deterministic.

It has to be stressed,however, that all these measures, built starting from sub matrix $\mathbf{R}^{ij}$, are not independent on the structure of either total output vector, on which we observe the effects, nor on the structure of final demand

vector on which we impose the unit demand shock.
The column sum of $\mathbf{R}^{ij}$ sub matrix in equation [1] implies the consideration of a set of final demand vectors of the type, in region i:

$$\mathbf{f}_1^i = \begin{bmatrix} 1 \\ 0 \\ 0 \\ . \\ 0 \end{bmatrix}, \mathbf{f}_2^i = \begin{bmatrix} 0 \\ 1 \\ 0 \\ . \\ 0 \end{bmatrix}, \cdots, \mathbf{f}_m^i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ 1 \end{bmatrix} \tag{11}$$

while the sum of row elements in equation [1] implies the consideration of a final demand structure of the type in region i:

$$\mathbf{f}^i = \begin{bmatrix} 1 \\ 1 \\ 1 \\ . \\ 1 \end{bmatrix} \tag{12}$$

We can expect that these measures hold for demand vectors of varying scale but with the same structures of equations [11] or [12]. However neither the demand vector nor its changes will ever assume a structure of this type. This is why some authors come to the drastic conclusion that "linkage should be never used" (Skolka, 1986).

On the other hand it is a common opinion that the structure of final demand produces the most different effects on the level of total output (Ciaschini, 1988c). Given a set of non zero final demand vectors, whose elements sum up to a predetermined level, but with varying structures, we will have to expect that the corresponding level of total output will also vary considerably.

For these reasons we cannot confine our knowledge of the system to the picture emerging from measures which can only show what would happen if final demand assumed a predetermined and unlikely structure.

3 The data base: Towards a Social Accounting Matrix for Marche

The basic organization of the data base that has been built, is inspired by social the accounting matrix scheme and follows the matrix presentation of regional economic accounts. The income circular flow is quantified and connects data on the production process (final demand, total output and value added generation) gathered by branches which play the role of industries, with data on the distribution process (factor allocation of value added, primary and secondary distribution of incomes) collected by institutional sectors.

The matrix can be broken up into quadrants which can be further divided into blocks. A brief sketch of blocks in each of the six sub matrices, as shown in Table [1], can be easily described as follows:

- quadrant I - production, primary allocation, secondary distribution and capital formation blocks in region M;
- quadrant II - production, secondary distribution of incomes entering in region M;
- quadrant III - production, secondary distribution of incomes entering in region rI;
- quadrant IV - production, primary allocation, secondary distribution and capital formation blocks in region rI;
- quadrant V - production, primary allocation, secondary distribution and capital formation blocks referred to Public Administrations;
- quadrant VI - operations with the rest of the world block.

Accounts are given in rows and columns corresponding to eight denominations namely Output, Wage and Salaries, Other Incomes, Households, Corporations, Capital formation, Public Administrations and Rest of the World.

Each Quadrant in Table [1], then, gives account of the intraregional and interregional flows of the two region and their allocation in different blocks

Table 1: Bi-regional *SAM* table for the whole economy

		Area *M*						Area *rI*								
		Output	Wages & Salaries	Other incomes	Households	Corporations	Capital Formation	Output	Wages & Salaries	Other incomes	Households	Corporations	Capital Formation	Public Administration	Rest of the World	Totals
Area *M*	Output	$B^{M,M}$			$C^{M,M}$		$I^{M,M}$	$B^{M,rI}$			$C^{M,rI}$		$I^{M,rI}$	$C^{M,PA}$	E^{M}	Q_1
	Wages & Salaries	$Va_1^{M,M}$													$T^{M,RdM}$	Q_2
	Other incomes	$Va_2^{M,M}$														Q_3
	Househols		$Ld^{M,M}$	AR	$T^{M,M}$	$T^{M,M}$					$T^{M,rI}$	$T^{M,rI}$		$T^{M,PA}$	$T^{M,RdM}$	Q_4
	Corporations			AR	$T^{M,M}$	$T^{M,M}$					$T^{M,rI}$	$T^{M,rI}$		$T^{M,PA}$	$T^{M,RdM}$	Q_5
	Capital formation				$S^{M,M}$	$S^{M,M}$							$A^{M,rI}$	$S^{M,PA}$	$A^{M,RdM}$	Q_6
Area *rI*	Output	$B^{rI,M}$			$C^{rI,M}$		$I^{rI,M}$	$B^{rI,rI}$			$C^{rI,rI}$		$I^{rI,rI}$	$C^{rI,PA}$	E^{rI}	Q_7
	Wages & Salaries							$Va_1^{rI,rI}$							$T^{rI,RdM}$	Q_8
	Other incomes							$Va_2^{rI,rI}$								Q_9
	Househols				$T^{rI,M}$	$T^{rI,M}$			$Ld^{rI,rI}$	$AR^{rI,rI}$	$T^{rI,rI}$	$T^{rI,rI}$		$T^{rI,PA}$	$T^{rI,RdM}$	Q_{10}
	Corporations				$T^{rI,M}$	$T^{rI,M}$				$AR^{rI,rI}$	$T^{rI,rI}$	$T^{rI,rI}$		$T^{rI,PA}$	$T^{rI,RdM}$	Q_{11}
	Capital formation						$A^{rI,M}$				$S^{rI,rI}$	$S^{rI,rI}$		$S^{rI,PA}$	$A^{rI,RdM}$	Q_{12}
	Public Adminstration	$T^{PA,M}$		$AR^{PA,M}$	$T^{PA,M}$	$T^{PA,M}$		$T^{PA,rI}$		$AR^{PA,rI}$	$T^{PA,rI}$	$T^{PA,rI}$		$T^{PA,PA}$	$T^{PA,RdM}$	Q_{13}
	Rest of the World	Imp	$T^{RdM,M}$	$T^{RdM,M}$	$T^{RdM,M}$	$T^{RdM,M}$		Imp		$T^{Rdm,rI}$	$T^{Rdm,rI}$	$T^{Rdm,rI}$		T^{RdMPA}		Q_{14}
	Totals	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	

in order to describe the whole circular flow. Table [1] gathers data from 11 input output sectors [Agriculture, Oil, Energy, Metal & Chemical Products, Machinery and Cars, Food, Tobacco & Alcoholic Beverages, Manufacturing, Trade Transportation, Marketable Service, Non Marketable Services], 7 institutional sectors[4] [I_Income class, II_ Income class, III_Income, class IV_Income class, V_Income class, Corporations, Public Administration], 3 value added components [Wage and Salaries, Other incomes, Indirect Tax], 2 macro sectors [Rest of Italy, Rest of the world] for two regions. Last Quadrants (V and VI) describe flows between regions and the public administration and the rest of the world[5]

The results attained in bi-regional SAM encourage the attempt of building an extended version of the bi-regional income circular flow where the interactions between industries and institutions in two region could be specified and evaluated.[6]

[4]The Households Income Class are disaggregated for disposable income.

[5]For numerical determination see (Socci, 2004)

[6]See appendix A for the model.

4 Intraregional and interregional relationships: summary approach

In this section we will explicitly consider the intraregional and interregional interaction between industries and institutional sectors operating on the structural matrices composing the loop in equations [20-29](see Appendix A). We will also utilize the singular value decomposition in the attempt of finding a "summary" measures of propagation (see section 2).

The interactions among industries and institutional sectors, in each region, can be appreciated if one considers the direct and indirect effects of disposable incomes on industry outputs. From the extended income output circular flow we determine the structural sub matrix $\overline{\mathbf{R}}^{ij}$ that links a unit change in disposable income by institutional sectors in one region to total output by industries in same region or another region.

a)*Intraregional analysis: region M*

If we consider intraregional effect for region M, we have

$$\overline{\mathbf{R}}^{MM} = \mathbf{R}^{MM} \cdot \mathbf{D}^{MM} \tag{13}$$

where $\mathbf{D}^{MM} = [\mathbf{F}^{MM} + \mathbf{K}^{MM}]$ gives the link between disposable income and final demands shown in equation [25] and $\mathbf{R}^{MM}$ is given in equation [29].The intraregional loop, disposable income of domestic institutional sectors and domestic output will be given

$$\mathbf{x}^{MM} = \overline{\mathbf{R}}^{MM} \cdot \mathbf{y}^{MM} \tag{14}$$

Its numerical determination is given in Table [2]. Two additional rows and columns show totals and quadratic moduli ($||\mathbf{x}||$) of the row (column).

We can perform the singular value decomposition (Lancaster, 1985) of data in the table and determine the intraregional macro multipliers (Ciaschini and Socci, 2003).

Considering that matrix product $(\overline{\mathbf{R}}^{MM})^T \cdot \overline{\mathbf{R}}^{MM}$ is the matrix of the deviations from zero of the effects of a unit domestic shock and that the square roots of its eigenvalues are the singular values of matrix $\overline{\mathbf{R}}^{MM}$, we can conclude that each singular value in Table [3] can be interpreted as the share of the deviations related to the associated eigenvectors. If we determine the cumulated percentage shares, we see that the first two singular values cover

Table 2: Direct and indirect effects of disposable incomes on industry outputs in region M

	I	II	III	IV	V	VI	VII	$x_{i.}$	$\|\|x\|\|$
x_1	0.16	0.14	0.13	0.13	0.12	0.11	0.11	0.91	0.35
x_2	0.15	0.14	0.13	0.12	0.12	0.10	0.12	0.88	0.34
x_3	0.09	0.08	0.08	0.08	0.07	0.06	0.06	0.53	0.20
x_4	0.33	0.36	0.37	0.38	0.40	0.43	0.31	2.58	0.98
x_5	0.33	0.42	0.49	0.54	0.62	0.76	0.40	3.57	1.40
x_6	0.30	0.27	0.25	0.24	0.22	0.18	0.21	1.66	0.64
x_7	0.07	0.06	0.05	0.05	0.05	0.04	0.04	0.35	0.14
x_8	0.95	1.05	1.11	1.20	1.31	1.48	0.86	7.96	3.06
x_9	2.13	1.91	1.74	1.67	1.56	1.30	1.43	11.74	4.49
x_{10}	0.30	0.26	0.23	0.21	0.19	0.14	0.17	1.49	0.58
x_{11}	0.13	0.13	0.12	0.12	0.12	0.12	1.05	1.80	1.09
$x_{.j}$	4.95	4.82	4.70	4.75	4.79	4.72	4.76		
$\|\|x\|\|$	2.44	2.30	2.19	2.20	2.20	2.18	2.06		

Table 3: Intraregional Macro multipliers and percent sum region M

	Macro Multipliers	Percent sum
s_1^{MM}	5.76	77%
s_2^{MM}	0.92	89%
s_3^{MM}	0.82	99%
s_4^{MM}	0.02	100%
s_5^{MM}	0.00	100%
s_6^{MM}	0.00	100%
s_7^{MM}	0.00	100%

the 89 per cent of total deviations. This means that we can confine our analysis of intersectoral and interindustry intraregional interactions to the first two macro multipliers to get results valid for the 89 per cent of the cases. Rather than considering matrix $\overline{\mathbf{R}}^{MM}$, which can be decomposed into the sum of seven "impact" components each one determined by a intraregional macro multiplier:

$$\overline{\mathbf{R}}_0^{MM} = s_1^{MM} \cdot \mathbf{u}_1^{MM} \cdot \mathbf{v}_1^{MM} + s_2^{MM} \cdot \mathbf{u}_2^{MM} \cdot \mathbf{v}_2^{MM} + \ldots + s_7^{MM} \cdot \mathbf{u}_7^{MM} \cdot \mathbf{v}_7^{MM} \quad (15)$$

we can refer to matrix

$$\overline{\mathbf{R}}_0^{MM} = s_1^{MM} \cdot \mathbf{u}_1^{MM} \cdot \mathbf{v}_1^{MM} + s_2^{MM} \cdot \mathbf{u}_2^{MM} \cdot \mathbf{v}_2^{MM} \quad (16)$$

where components greater than 2 have been neglected with the aim of obtaining "summary" measures. Now the economic interactions are completely determined by the first two aggregated intraregional macro multipliers s_1^{MM} and s_2^{MM}.

We note that in matrix $\overline{\mathbf{R}}_0^{MM}$, vectors

$$s_1^{MM} \cdot \mathbf{u}_1^{MM} = \begin{bmatrix} s_1^{MM} u_{1,1}^{MM} \\ s_1^{MM} u_{2,1}^{MM} \\ s_1^{MM} u_{3,1}^{MM} \\ . \\ . \\ . \\ s_1^{MM} u_{11,1}^{MM} \end{bmatrix}, \qquad s_2^{MM} \cdot \mathbf{u}_2^{MM} = \begin{bmatrix} s_2^{MM} u_{1,2}^{MM} \\ s_2^{MM} u_{2,2}^{MM} \\ s_2^{MM} u_{3,2}^{MM} \\ . \\ . \\ . \\ s_2^{MM} u_{11,2}^{MM} \end{bmatrix} \tag{17}$$

are the result of splitting the two intraregional macro multipliers into the eleven output sector. These two vectors represent both how each of the intraregional macro multipliers affects outputs and how each industry output is affected by the two intraregional macro multipliers, which quantify the magnitude of industry-sector interactions: As we stressed in section 2, the

Table 4: Intraregional Forward dispersion: i.e. impacts on industry outputs of intersectoral interactions, in terms of intraregional macro multipliers

	First impact component ($\mathbf{u}_1^{MM} \cdot s_1^{MM}$)	Second impact component ($\mathbf{u}_2^{MM} \cdot s_2^{MM}$)	Forward Dispersion (modulus)	Percent forward dispersion (%)
x_1	0.35	0.01	0.35	3%
x_2	0.33	0.02	0.33	3%
x_3	0.20	0.00	0.20	2%
x_4	0.97	-0.05	0.98	7%
x_5	1.34	-0.20	1.35	10%
x_6	0.63	0.03	0.63	5%
x_7	0.13	0.01	0.13	1%
x_8	3.00	-0.34	3.02	23%
x_9	4.47	0.18	4.47	34%
x_{10}	0.57	0.02	0.57	4%
x_{11}	0.63	0.81	1.02	8%
modulus	5.76	0.92	13.07	

aim of the intraregional *sensitivity of dispersion* in the Rasmussen definition,

τ_i^{MM}, -which generated the concept of the forward linkage- measures the extent to which industries draw upon industry i and the degree of relevance of each industry as a supplier.

As we see from Table [4], the expansion, in region M, of the i^{th} industry output is quantified by vector $[s_1^{MM}\ u_{1i}^{MM},\ s_2^{MM}\ u_{2i}^{MM}]$ and its module. It is to be noted that the industry expansion effect is measured with reference to the two intraregional macro multipliers independently from the fact that such multipliers have been activated by a change in domestic final demand or a change in domestic disposable incomes influencing domestic final demands. This feature allows for a generalization of the sensitivity of dispersion concept. This concept can be used both in the case that the model is limited to the Leontief inverse and to case were a larger output/income model is used that includes also the income distribution process. In order to avoid misinterpretation we will define the intraregional forward dispersion, fd_i^{MM}, as the change in the value of the sales by industry i (to face a demand vector generated by an increase in disposable income in all sectors). The percent intraregional forward dispersion can be easily obtained dividing intraregional forward dispersion by its total value.
Table [4] produces an ordering of industries according the forward of dispersion: Industry 9 Transport and Trade (34%), 8 Manufacturing (23%), 5 Machinery and Cars (10%), 11 Service non market (8%), 4 Metal & chem. Products (7%), 6 Food (5%), 10 Service market (4%), 2 Oil (3%), 1 Agriculture, (3%), 3 Energy (2%), 7 Tobacco and Alcoholic Beverages (1%).

On the other hand vectors

$$\begin{aligned} s_1^{MM} \cdot \mathbf{v}_1^{MM} &= [s_1^{MM} \cdot v_{1,1}^{MM}, \ldots, s_1^{MM} \cdot v_{1,7}^{MM}] \\ s_2^{MM} \cdot \mathbf{v}_2^{MM} &= [s_2^{MM} \cdot v_{2,1}^{MM}, \ldots, s_2^{MM} \cdot v_{2,7}^{MM}] \end{aligned} \tag{18}$$

split the same two intraregional macro multipliers into the seven institutional sectors and represent how the change in sectoral disposable domestic income influences the two intraregional macro multipliers.

Again from section 2, the aim of index π_j^{MM}, the power of dispersion in the Rasmussen definition -which generated the concept of intraregional backward linkage- was that of measuring the extent to which an increase in

domestic final demand for products of industry j is dispersed throughout the system of industries.
If we introduce in the interindustry model, institutional sectors and income distribution, final demand will no more be exogenous but explained by income distribution. Whatever multisectoral macro variable will it be, the index will quantify the degree of relevance of each component of such macro variable in stimulating the multipliers.

If the model under analysis had been the loop between domestic final demand and output vectors [18] would have well represented the intraregional backward linkage i.e. the expansion caused by an expansion in industry j. By analogy we can define intraregional backward dispersion, bd_j^{MM}, as the change in the value of the purchases by those industries that produce goods according the consumption patterns of domestic income sector j. Intraregional backward dispersion can be also determined in percent terms as in Table [5].

Table 5: Intraregional Backward dispersion:i.e. impacts of a unit disposable domestic income shock on economic interactions, in terms of intraregional macro multipliers

	First impact component $(\mathbf{v}_1^{MM} \cdot s_1^{MM})$	Second impact component $(\mathbf{v}_2^{MM} \cdot s_2^{MM})$	Backward Dispersion (modules)	Percent backward dispersion %
I	2.38	0.10	2.39	15%
II	2.28	0.00	2.28	15%
III	2.19	-0.08	2.19	14%
IV	2.19	-0.14	2.20	14%
V	2.19	-0.22	2.20	14%
VI	2.09	-0.37	2.12	14%
VII	1.88	0.79	2.03	13%
modules	5.76	0.92		

We note that the fourth column of Table [4] corresponds to the modules of the rows of table $\overline{\mathbf{R}}_0^{MM}$ and that the same column in Table [5] gives the modules of the columns of table $\overline{\mathbf{R}}_0^{MM}$, which at his turn approximates to be $\overline{\mathbf{R}}^{MM}$.

We can give a graphical representation of each element in the four vectors. We will define the axis of the first intraregional macro multiplier, on which we measure the elements of vectors $s_1^{MM}\mathbf{u}_1^{MM}$, $s_1^{MM}\mathbf{v}_1^{MM}$ and the axes of the second macro multiplier, where we measure the elements of vectors $s_2^{MM}\mathbf{u}_2^{MM}$, $s_2^{MM}\mathbf{v}_2^{MM}$. Then we will represent the couples ($s_1^{MM}\mathbf{v}_{1,i}^{MM}$, $s_2^{MM}\mathbf{v}_{1,i}^{MM}$) i=1,. . .,7, with seven arrows, showing how the change in disposable income impacts on intersectoral interactions (intraregional backward dispersion), in terms of the two intraregional macro multipliers; and couples ($s_1^{MM}\mathbf{u}_{1,i}^{MM}$, $s_2^{MM}\mathbf{u}_{1,i}^{MM}$) i=1,. . .,11, with eleven dots, showing how intersectoral interactions impact on industry outputs (intraregional forward dispersion).

Figure 1: Sector and industry interactions region M - intraregional Backward and Forward dispersions (absolute levels)

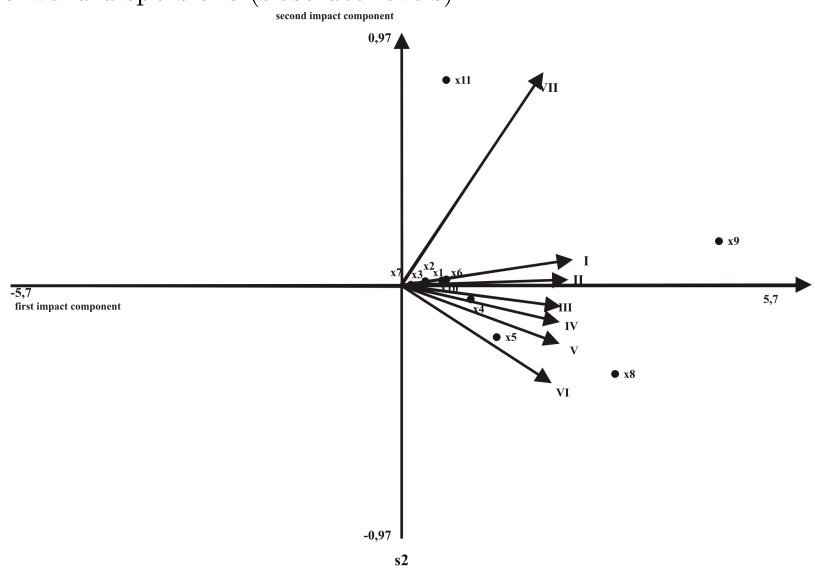


Figure [1] shows that, in addition to the information based on the modules of the vectors, some further information can be achieved referring to the directions of each vector. In order to perform consistent comparisons, independently from the unit measures effects of outputs and incomes, we need to standardize data in Table [2] taking the deviations from the mean values and dividing by the standard deviations. We note that the singular value decomposition of standardized data will result in the eigenvalue decomposition of matrices $(\overline{\mathbf{R}}^{MM})^T \cdot \overline{\mathbf{R}}^{MM}$ and $\overline{\mathbf{R}}^{MM} \cdot (\overline{\mathbf{R}}^{MM})^T$ which represent the correlation matrices of sectoral incomes and industry outputs respectively.

We will then get the diagram in Figure [2].

Figure 2: Sector and industry interactions -Intraregional Backward and Forward dispersions standardized

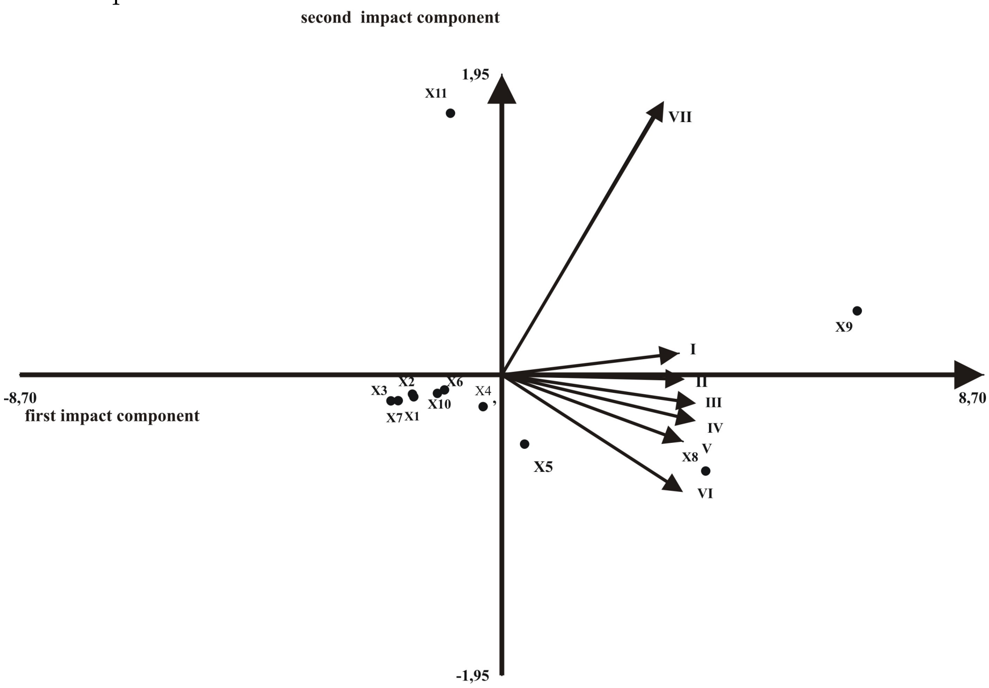


Figure [2] allows for the identification of clusters of industries that move together, i.e. respond linearly, to intersectoral interactions as quantified by the two macro multipliers. This is done considering that the angular distance of two dots will represent the correlation coefficient since:

$$Corr(\mathbf{x}_i^{MM}, \mathbf{x}_j^{MM}) = \cos\beta = \frac{\mathbf{x}_i^{MM} \cdot \mathbf{x}_j^{MM}}{|| \mathbf{x}_i^{MM} || \cdot || \mathbf{x}_j^{MM} ||}$$

in fact, if two industries "move together", we have to expect that they will be located on the same line, relative to the two intraregional macro macro multipliers.

From Figure [2], as for correlation coefficients we can identify a set of industries clusters:
1^{st} cluster: positive correlation (about 1) characterizes industry 1 Agriculture, 2 Oil, 3 Energy, 6 Food, 7 Tobacco and Alcoholic and 10 Services market;
2^{nd} cluster: positive correlation (about 0.9) between industries 9 Transport

and Trade and 8 Manufacturing; negative correlation with others industries
3^{nd} cluster: low negative correlation between 5 Machine and Car and industries 1, 2, 3, 4, 6, 7 and 10; positive correlation with industries 8 and 9;
4^{nd} cluster: low positive correlation between 11 Services non market and industries 1, 2, 3, 6, 7, and 10; low negative correlation with 4, 8, 9 and high negative correlation with 5.

For what concerns the backward dispersion, the modulus of each vector labelled I, II, III, IV, V, VI, VII, represent the stimulus forwarded to the interindustry interactions by a unit change in disposable income by institutional sector. From Figure [2] we note that in our example the effects of disposable incomes of institutional sectors from I to VI are highly correlated, more than 90 per cent in terms of correlation coefficient. Only sector VII, Administration, seems to exhibit a different pattern.

Figure [2], in addition, allows for a cross comparison sectors/industries which can identify the "strength" of the link between sectors and industries in terms of cross correlation coefficients.

Table 6: Cross Correlation coefficients between industries and sectors in region M

	I	II	III	IV	V	VI	VII
x_1	1.00	1.00	0.99	0.98	0.98	0.95	0.89
x_2	1.00	1.00	0.99	0.99	0.98	0.95	0.89
x_3	1.00	1.00	0.99	0.98	0.98	0.95	0.89
x_4	0.87	0.85	0.82	0.80	0.77	0.71	1.00
x_5	-0.64	-0.68	-0.71	-0.74	-0.77	-0.82	-0.19
x_6	1.00	0.99	0.99	0.98	0.97	0.95	0.90
x_7	1.00	1.00	0.99	0.99	0.98	0.95	0.89
x_8	-0.98	-0.99	-0.99	-1.00	-1.00	-1.00	-0.75
x_9	-1.00	-1.00	-0.99	-0.99	-0.98	-0.96	-0.88
x_{10}	1.00	0.99	0.99	0.98	0.97	0.95	0.90
x_{11}	0.44	0.49	0.52	0.55	0.59	0.67	-0.05

Table [6] shows high positive correlations between sectors I, II, III, IV, VI and VI and industry 1, 2, 3, 6, 7 and 10; sector VII and industries 4 and 10. Positive correlations between industries 4 and 10 and sectors I, II, III, IV, VI and VI; sector VII and industries 1, 2, 3, 6 and 7. It shows low positive correlations between industry 11 and sectors I, II, III, IV, VI and

VI. While it shows high negative correlations between industries 8 and 9 and sectors I, II, III, IV, VI and VI. Table shows negative correlation between sector VII and industries 8 and 9; low negative correlations between sector VII and industries 5 and 11.

b)*Intraregional analysis: region I*

In this case our analysis concerns the region I. When the shock is relative to institutional sector income of region I we can find synthetic indexes that shows the linkages between industries and institutional sectors. If we consider intraregional effects for region I, we have

$$\overline{\mathbf{R}}^{II} = \mathbf{R}^{II} \cdot \mathbf{D}^{II} \tag{19}$$

Its numerical determination is given in Table [7]. We can perform the singular values decomposition of data from table [7] and determine the intraregional macro multipliers (singular values), table [8]. The first two macro multipliers account for about 98 per cent of the intraregional phenomena. Considering only s_1^{II} and s_2^{II} macro multipliers approach above mentioned, we can find synthetic index for intraregional phenomena. Forward dispersion index results are calculated as above, table [9], using vectors $\mathbf{u}_1^{II}$ and $\mathbf{u}_2^{II}$ and they are splitted into first and second components relative to macro multipliers ($s_1^{II} \cdot \mathbf{u}_1^{II}$ and $s_2^{II} \cdot \mathbf{u}_2^{II}$). In table [9] we observe that intraregional

Table 7: Direct and indirect effects of disposable incomes on industry outputs in region I

	I	II	III	IV	V	VI	VII	$x_{i.}$	$\|\|x\|\|$
x_1	1.10	1.04	1.01	0.93	0.81	0.67	0.98	6.5	2.5
x_2	1.41	1.33	1.30	1.20	1.05	0.87	1.28	8.4	3.2
x_3	0.51	0.49	0.48	0.44	0.39	0.33	0.47	3.1	1.2
x_4	3.23	3.10	3.07	2.88	2.59	2.23	3.02	20.1	7.7
x_5	2.26	2.19	2.18	2.07	1.90	1.69	2.15	14.4	5.5
x_6	1.49	1.40	1.36	1.25	1.09	0.89	1.32	8.8	3.4
x_7	0.30	0.28	0.27	0.25	0.22	0.18	0.26	1.8	0.7
x_8	4.67	4.49	4.44	4.17	3.76	3.24	4.33	29.1	11.1
x_9	11.15	10.57	10.33	9.55	8.41	7.02	10.01	67.0	25.6
x_{10}	1.44	1.35	1.32	1.20	1.04	0.85	1.27	8.5	3.2
x_{11}	2.87	2.73	2.67	2.47	2.18	1.83	3.68	18.4	7.1
$x_{.j}$	30.4	29.0	28.5	26.4	23.4	19.8	28.8		
$\|\|x\|\|$	13.3	12.7	12.4	11.5	10.2	8.6	12.3		

Table 8: Intraregional Macro Multipliers and percent sum region I

	Macro Multipliers	Percent sum
s_1^{II}	30.90	96%
s_2^{II}	0.97	98%
s_3^{II}	0.48	99%
s_4^{II}	0.15	100%
s_5^{II}	0.00	100%
s_6^{II}	0.00	100%
s_7^{II}	0.00	100%

Table 9: Intraregional Forward dispersion: i.e. impacts on industry outputs of intersectoral interactions, in terms of intraregional macro multipliers

	First impact component $(\mathbf{u}_1^{II} \cdot s_1^{II})$	Second impact component $(\mathbf{u}_2^{II} \cdot s_2^{II})$	Forward Dispersion (modulus)	Percent forward dispersion (%)
x_1	2.50	0.02	2.50	4%
x_2	3.22	0.01	3.22	5%
x_3	1.19	0.00	1.19	2%
x_4	7.65	0.04	7.65	11%
x_5	5.47	0.04	5.47	8%
x_6	3.36	0.02	3.36	5%
x_7	0.67	0.01	0.67	1%
x_8	11.06	0.09	11.06	16%
x_9	25.57	0.19	25.57	36%
x_{10}	3.24	0.02	3.24	5%
x_{11}	7.05	-0.95	7.11	10%
modulus	30.90	0.97	71.06	

forward dispersion in per cent terms among industries. On the other side, we consider Backward dispersion index, table [10], from vectors rows $\mathbf{v}_1^{II}$ to $\mathbf{v}_2^{II}$. As above we can give a synthetic representation through the graph, figure [3], where absolute level data are used.

Also in this case, in order to perform consistent comparisons, independently from the unit measures effects of outputs and incomes, we need to standardize data in Table [7] taking the deviations from the mean values and dividing by the standard deviations. We will then get the diagram in Figure [4]. With reference to correlation coefficients three industry clusters can be identified:

Table 10: Intraregional Backward dispersion: i.e. impacts of a unit disposable domestic income shock on economic interactions, in terms of intraregional macro multipliers

	First impact component $(\mathbf{v}_1^{II} \cdot s_1^{II})$	Second impact component $(\mathbf{v}_2^{II} \cdot s_2^{II})$	Backward Dispersion (modulus)	Percent backward dispersion %
I	13.33	0.17	13.33	16%
II	12.67	0.17	12.67	16%
III	12.43	0.17	12.43	15%
IV	11.52	0.16	11.52	14%
V	10.20	0.15	10.20	13%
VI	8.59	0.13	8.59	11%
VII	12.31	-0.89	12.34	15%
modulus	30.90	0.97		

1^{st} cluster: Positive correlation (about 1) characterizes industry 1 Agriculture, 2 Oil, 3 Energy, 6 Food, 7 Tobacco and Alcoholic and 10 Services;
2^{nd} cluster: Negative correlation (about -1) observed between industry 4 Metal and Chemical and 1 Agriculture, 2 Oil, 10 Services;
3^{nd} cluster: low (positive and negative) correlation from 11 Services non market and the other industries.

From Figure [4] we note highly correlated among all institutional sectors, but only for sector VII seems to exhibit a different pattern.

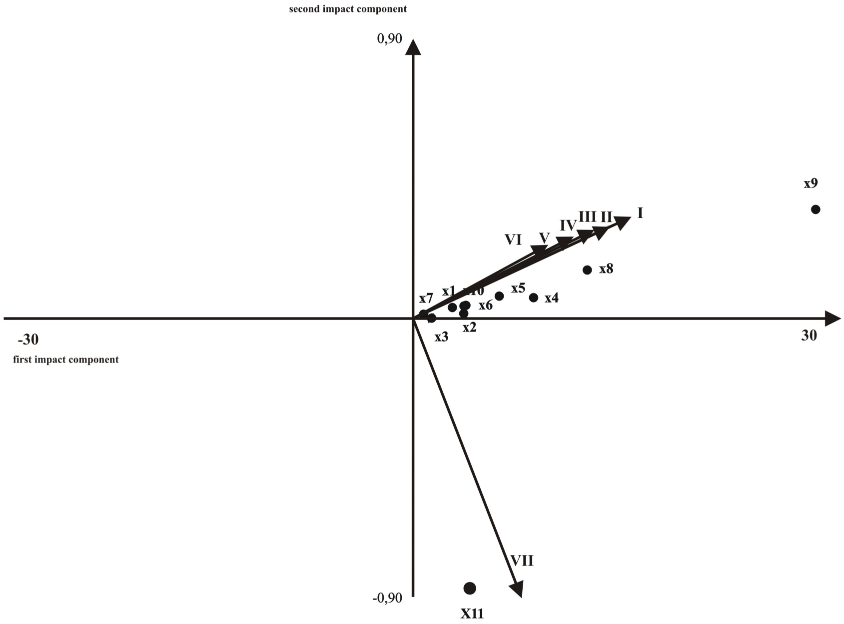
We finally see that three clusters are formed between the institutional sectors and the industries. First cluster shows a high positive cross correlation between all the institutional sectors and industries 4, 8 and 9. Second cluster presents a high negative cross correlation between all the institutional sectors and the industries 1, 2, 3, 5, 6, 7 and 10. Last cluster sees a low cross correlation among the industry 11 and the institutional sectors.

b)*Interregional analysis: M vs I and I vs M region*

We can perform the singular values decomposition on matrix $\overline{\mathbf{R}}^{M,I}$ and $\overline{\mathbf{R}}^{I,M}$ and determine the intraregional backward and forward dispersion. Table [11] shows intraregional macro multipliers M vs I and I vs M region.

The two components, associated to the macro multipliers, bring the direct and indirect effects on the product of the two regions, when there are not variations on local institutional sectors income. Table [12] we observe

Figure 3: Institutional Sector and Industry interactions region I- intraregional Backward and Forward dispersions (absolute levels)



Forward dispersion. In particular, key industries into region M are 9 Transport and Trade, 8 Manufacturing and 4 Metal and Chemical, when the shock is given on income of institutional sectors into region I. While key industries in region I are 9 Transport and Trade, 8 Manufacturing, 5 Machine and Car, 4 Metal and Chemical and 11 Services non market. Table [13] shows interregional backward dispersion. institutional sectors I, II, and VII are important on output of the region M, while sector VI is not important. All institutional sectors are important for output of the rest of Italy, except the public administration. We can give a graphical representation of this phenomena. The horizontal axis relative to the first intraregional macro multiplier, while the vertical axes relative to the second intraregional macro multiplier. Figure [5] shows the link among industries of region M and institutional sectors of region (M vs I) and Figure [6] shows relationship industries region I vs institutional sectors of region (I vs M).

In order to perform consistent comparisons, independently from the unit measures effects of outputs and incomes, we need to standardize data taking the deviations from the mean values and dividing by the standard

Figure 4: Institutional Sector and Industry interactions -Intraregional Backward and Forward dispersions standardized

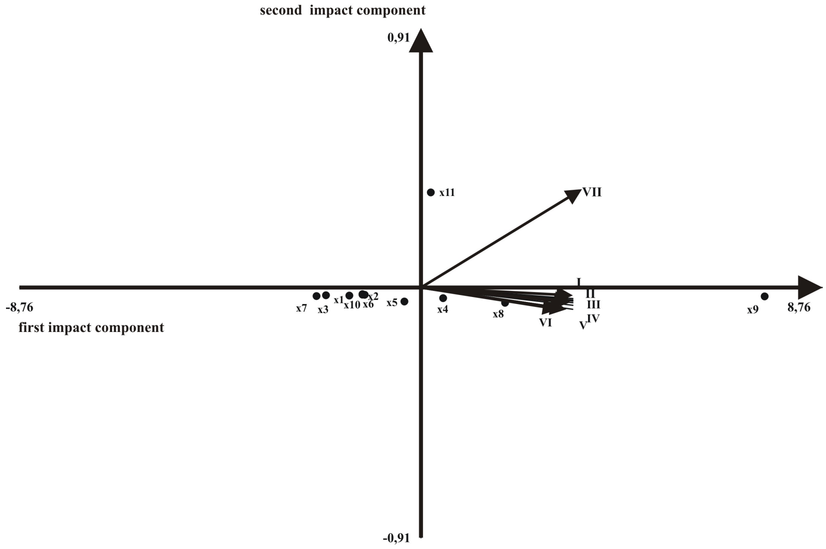


Table 11: Macro multipliers and percent sum interregional

	region M vs I		region I vs M	
	Macro Multipliers	Percent sum	Macro Multipliers	Percent sum
s_1	1.22461	99.597%	32.0874	99.277%
s_2	0.00296	99.838%	0.1920	99.871%
s_3	0.00174	99.979%	0.0245	99.947%
s_4	0.00025	100%	0.0171	100%
s_5	0	100%	0	100%
s_6	0	100%	0	100%
s_7	0	100%	0	100%

deviations. We note that the singular value decomposition of standardized data will result in the eigenvalue decomposition of interregional matrices, which represent the correlation matrices of sectoral incomes and industry outputs respectively. We will then get the diagram in Figures [7] and [8]. For interregional cross correlation among institutional sectors of region I and industries of region M we observe, graph [7]: all institutional sectors are negative cross correlation with industries 1, 2, 3, 4, 5, 6, 7, 10 and 11; all institutional sectors are positive cross correlation with industries 8 and 9. On the other side, output region I vs sectoral incomes region M, graph

[8], we have: negative cross correlation between all institutional sectors and industries 1, 2, 3, 5, 6, 7, 10 and 11; all institutional sectors are positive cross correlation with industries 4, 8 and 9.

5 Conclusions

The origin of linkage analysis, in the study of propagation phenomena through industries, was that of finding "summary" measures of dispersion and of applying them on interindustry data in a statistical way. However, in later developments, the original statistical approach has been progressively abandoned and the interpretation of these measures have definitely become deterministic. Further developments have proposed problem specifications, such that based on the assumption of constant market shares, conflicting with the hypothesis of fixed technical coefficients. On the other hand developments in national accounts have provided a consistent data base for the enlargement of the traditional Leontief framework to problems of income distribution on the lines explored by Miyazawa.

Our attempt has been that of taking inspiration from some of these developments to design measures of dispersion, either "summary" and "statistical", that can be applied both to a traditional Leontief framework and to an enlarged model, where income distribution can be also taken into consideration. The results have been discussed on the basis of a specific bi-regional model whose data base we have tried to render consistent, having in mind a bi-regional social accounting scheme.

The emerging enlarged income flow has been analyzed identifying the intraregional and interregional macro multipliers that "rule" the flow. Once identified these multipliers, that represent the potential scale of all the possible types of dispersions through industries and sectors, we evaluated both, intraregional and interregional, backward and forward dispersions with reference to them. This procedure generates a set of indices -in absolute and percent values- for the (intraregional and interregional) industry-forward-dispersion and (intraregional and interregional) sector-backward-dispersion which quantify, respectively, the change in the value of the sales by industry i, in region M or I, to face a demand vector generated by an increase in disposable income in all sectors in region M or I, and the change in the value of the purchases by those industries in region M or I that produce goods according the consumption patterns of income sector j in region M or I.

An extension of the method has also been provided in terms of a "summary" graphical representation. The standardization of data, in fact, produces a representation, explainable in terms of correlation analysis, which allows for an immediate interpretation of the strength of the mutual links among and between the disaggregated components of total output and disposable income. A synthetic picture of the working of intraregional and interregional sector-industry-interactions is then attained in graphical and quantitative terms.

References

Banca d'Italia, . (1995). *Indagine sul bilancio delle famiglie.* Banca Dati.

Bulmer-Thomas, V. (1982). *Input-Output Analysis in Developing Countries.* John Wiley and Sons Ltd, USA.

Casini Benvenuti, S. (2000). *Il nuovo sistema dei conti nazionali e regionali SEC 1995.* Franco Angeli,IRPET, Milano.

Ciaschini, M. (1988a). *Input Output Analysis Current developments.* Chapman and Hall London.

— (1988b). *Input output analysis final demand and approximate controllability problems of compilation of input-output table.* BADEN 14-15 March.

— (1988c). *Scale and structure in economic modelling.* Economic Modelling.

— (1993). *Modelling the structure of the economy.* Chapman and Hall.

Ciaschini, M. and Socci, C. (2003). *Macro multiplier approach to output generation and income distribution in a social accounting matrix.* Quaderni di Ricerca, Dipartimento di Istituzioni Economiche e Finanziarie, Università di Macerata, (19).

Dietzenbacher, E. (1989). *On the relationship between the supply-driven and the demand-driven input-output model.* Environment and Planning, A(21).

EUROSTAT (1996). *Système Europèen des comptes SEC95.* CEE, Luxembourg.

Ghosh, A. (1958). *Input-output approach to an allocative system.* Economica, 25(1).

Gorla, G. and Senn, L. (1991). *La matrice di contabilità sociale ed un sua applicazione per l'analisi economica.* In *I modelli input-output nella programmazione regionale: teoria e applicazioni* (edited by Bianco, L. and La Bella, A.). Franco Angeli, Milano.

HANSON-LAWNSON (1974). *Solving least squares problems.* Prentice Hall.

HIRSCHMAN, ALBERT, O. (1958). *The Strategy of Economic Development.* Yale University Press, New Haven.

ISTAT (1996). *Conti Nazionali Economici e Finanziari dei Settori Istituzionali - Anno 1996.* ISTAT, Roma.

KEUNING, S. J. (1992a). *National accounts, sams and sesame: a system of economic and social accounting matricies and extensions with application to the case indonesia.* Netherlands, Central Bureau of Statistics.

— (1992b). *Proposal for a social accounting matrix which fits into the next system of national accounts.* Economic System Research.

— (1994). *The sam and beyond: open sesame.* Economic System Research, 6(1).

KEUNING, S. J. AND RUIJTER, W. (1988). *Guidelines to the construnction of a social accounting matrix.* The Review of Income and Wealth.

KING, B. B. (1994). *What is a sam?* In *Social Accounting Matrix a Basis for Planning* (edited by PYATT, G. AND ROUND, J.). Wolrd Bank, Washington.

LAGER, C. (1988). *The use of a social accounting matrix for the comparative static equilibrium modelling in input-output analysis.* In *Input Output Analysis Current developments* (edited by CIASCHINI, M.). Chapman and Hall London, London.

LANCASTER, P. & TIESMENETSKY, M. (1985). *The Theory of Matricies.* Academic Press, New York, second ed.

LAVANDA, I., R. G. AND SORO, B. (1997). *La revisione delle tavole intersettoriali 1970-90: Metodi e procedure.* Rivista di Statisitca Ufficiale-Quaderni di ricerca, Franco Angeli.

LEE, K. C. (1986). *Input output multipliers with backward forward and total linkages'.* Paper presented at the 8th International Conference on Input-Output Technique -Sapporo.

Miller, R. E. and Blair, P. D. (1985). *Input-Output Analysis: Foundations and Extensions.* Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Miyazawa, K. (1970). *Input-Output Analysis and Structure of Income Distribution*, vol. 116. New York.

Oosterhaven, J. (1996). *Leontief versus ghoshian price and quantity models.* Southern Economic Jounrnal, (62).

Polenske, K. R. and Jordan, P. (1988). *A multiplier impact study of fishing.* In *Input Output Analysis Current developments* (edited by Ciaschini, M.). Chapman and Hall London.

Pyatt, G. (1991). *Fundamentals of social accounting.* Economic Systems Research, 3.

Rasmussen, P. (1956). *Studies in intersectoral relations.* North Holland, Amsterdam.

Skolka, J. (1986). *Input output multipliers and linkages.* Paper presented at the 8th International Conference on Input-Output Techniques - Sapporo.

Socci, C. (2004). *Una sam biregionale per le marche.* In *Matrici regionali di contabilità sociale e analisi di politiche economiche: il caso della Liguria, Toscana e Marche* (edited by Fossati, A. and Targetti, R.). Franco Angeli.

Stone, R. (1979). *Where are we now? a short account of the development of input ouput studies and their present trands.* Paper presented at the seventh International Conference on Input Output Techniques held in Innsbruck, Austria, April 9-13.

United Nation, . (1994). *System of National Accounts.* N.U., New York.

United Nations, . (1968). *System of National Accounts.* UNSO, New York.

A Appendix A: Bi-regional/Multi-sectoral model

Our distributive structural matrices will be given by
Bi-regional Gross value added generation(by industry)

$$\mathbf{v}(x) = \left[\begin{array}{c} \mathbf{v}^M \\ \hline \mathbf{v}^I \end{array}\right] = \left[\begin{array}{c|c} \mathbf{L}^M & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{L}^I \end{array}\right] \cdot \left[\begin{array}{c} \mathbf{x}^M \\ \hline \mathbf{x}^I \end{array}\right] \tag{20}$$

where $\mathbf{L}^M$ and $\mathbf{L}^I$ [11,11] give the value added by industry starting from the output vector and technical coefficients matrix for the two regions.

Bi-regional Gross value added allocation(by VA components)

$$^c\mathbf{v}(x) = \left[\begin{array}{c} ^c\mathbf{v}^M \\ \hline ^c\mathbf{v}^I \end{array}\right] = \left[\begin{array}{c|c} \mathbf{V}^{M,M} & \mathbf{V}^{M,I} \\ \hline \mathbf{V}^{I,M} & \mathbf{V}^{I,I} \end{array}\right] \cdot \left[\begin{array}{c} \mathbf{v}^M \\ \hline \mathbf{v}^I \end{array}\right] \tag{21}$$

where $\mathbf{V}^{ij}$ [3,11] represents the intraregional and interregional distribution of value added to the factors (components) .

Bi-regional Primary distribution of income(by Institutional sub-sectors)

$$^{si}\mathbf{v}(x) = \left[\begin{array}{c} ^{si}\mathbf{v}^M \\ \hline ^{si}\mathbf{v}^I \end{array}\right] = \left[\begin{array}{c|c} \mathbf{P}^{M,M} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{P}^{I,I} \end{array}\right] \cdot \left[\begin{array}{c} ^c\mathbf{v}^M \\ \hline ^c\mathbf{v}^I \end{array}\right] \tag{22}$$

where $\mathbf{P}$[7,3] represents the intraregional distribution factors' value added income to the sectors.[7]

Bi-regional Secondary distribution of income(by Institutional sub-sectors)

$$\mathbf{y}(x) = \left[\begin{array}{c} \mathbf{y}^M \\ \hline \mathbf{y}^I \end{array}\right] = \left\{ \left[\begin{array}{c|c} \mathbf{I}_1 & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I}_2 \end{array}\right] + \left[\begin{array}{c|c} \mathbf{T}^{M,M} & \mathbf{T}^{M,I} \\ \hline \mathbf{T}^{I,M} & \mathbf{T}^{I,I} \end{array}\right] \right\} \cdot \left[\begin{array}{c} ^{si}\mathbf{v}^M \\ \hline ^{si}\mathbf{v}^I \end{array}\right] \tag{23}$$

where $\mathbf{T}^{ij}$ [7,7] represents intraregional and interregional net income transfers among sub-sectors.

Bi-regional Final demand formation(by industry)

$$\mathbf{f}(x) = \left[\begin{array}{c} \mathbf{f}^M \\ \hline \mathbf{f}^I \end{array}\right] = \mathbf{F} \cdot \left[\begin{array}{c} \mathbf{y}^M \\ \hline \mathbf{y}^I \end{array}\right] + \mathbf{K} \cdot \left[\begin{array}{c} \mathbf{y}^M \\ \hline \mathbf{y}^I \end{array}\right] + \left[\begin{array}{c} ^0\mathbf{f}^M \\ \hline ^0\mathbf{f}^I \end{array}\right] \tag{24}$$

where $\mathbf{F}$ provide the consumption demand structure by industry and is given by the product of two matrices,

$$\mathbf{F} = \left[\left(\begin{array}{c|c} ^1\mathbf{F}^{M,M} & ^1\mathbf{F}^{M,I} \\ \hline ^1\mathbf{F}^{I,M} & ^1\mathbf{F}^{I,I} \end{array}\right) \cdot \left(\begin{array}{c|c} \mathbf{C}^M & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{C}^I \end{array}\right) \right]$$

[7]Interregional distribution factors' value added income is zero.

where ${}^1\mathbf{F}^{ij}$ [11,7] transforms the consumption expenditure by institutional sector into consumption by industry and $\mathbf{C}^{ij}$ [7,7] represents the consumption propensities by institutional sector.

K represents the investment demand and is given by

$$\mathbf{K} = \left[\begin{array}{c|c} {}^1\mathbf{K}^{M,M} & {}^1\mathbf{K}^{M,I} \\ \hline {}^1\mathbf{K}^{I,M} & {}^1\mathbf{K}^{I,I} \end{array}\right] \cdot \left[\begin{array}{c|c} \mathbf{s}^M & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{s}^I \end{array}\right] \cdot \left[\left(\begin{array}{c|c} \mathbf{I}_1 & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I}_2 \end{array}\right) - \left(\begin{array}{c|c} \mathbf{C}^M & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{C}^I \end{array}\right) \right]$$

where $\mathbf{K1}$[11,7] represents the investment demands to I-O industry and diagonal matrix **s** represents the share of private savings which is transformed into investment i.e. "active savings" in two regions. Finally, ${}^0\mathbf{f}^j$ is a vector of 11 elements which represents exogenous demand.
If we put $\mathbf{D}^{ij} = \mathbf{F}^{ij} + \mathbf{K}^{ij}$ equation[24] becomes

$$\mathbf{f}(x) = \left[\begin{array}{c} \mathbf{f}^M \\ \hline \mathbf{f}^I \end{array}\right] = \left[\begin{array}{c|c} \mathbf{D}^{M,M} & \mathbf{D}^{M,I} \\ \hline \mathbf{D}^{I,M} & \mathbf{D}^{I,I} \end{array}\right] \cdot \left[\begin{array}{c} \mathbf{y}^M \\ \hline \mathbf{y}^I \end{array}\right] + \left[\begin{array}{c} {}^0\mathbf{f}^M \\ \hline {}^0\mathbf{f}^I \end{array}\right] \tag{25}$$

substituting through the equations [20]-[24] in 25 we get

$$\mathbf{f}(x) = \mathbf{D} \cdot [\mathbf{I} + \mathbf{T}] \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L} \cdot \mathbf{x} + {}^0\mathbf{f} \tag{26}$$

We now turn to the output generation process which is ruled by the Leontief model.

Output generation

$$\left[\begin{array}{c} \mathbf{x}^M \\ \hline \mathbf{x}^I \end{array}\right] + \left[\begin{array}{c} \mathbf{m}^M \\ \hline \mathbf{m}^I \end{array}\right] = \left[\begin{array}{c|c} \mathbf{A}^{M,M} & \mathbf{A}^{M,I} \\ \hline \mathbf{A}_{I,M} & \mathbf{A}^{I,I} \end{array}\right] \cdot \left[\begin{array}{c} \mathbf{x}^M \\ \hline \mathbf{x}^I \end{array}\right] + \left[\begin{array}{c} \mathbf{f}^M \\ \hline \mathbf{f}^I \end{array}\right](x) \tag{27}$$

where $\mathbf{m}^j$ represents imports, $\mathbf{A}^{ij}$ the technical coefficients matrix, $\mathbf{f}^i(x)$ represents the demand vector. Imports have been considered as exogenous variable and in the model, which are used exports net variable defining $\mathbf{d}={}^0\mathbf{f}^i - \mathbf{m}^i$. Substituting equation 26 in 27 and solving for $\mathbf{x}^i$ we finally get

$$\left[\begin{array}{c} \mathbf{x}^M \\ \hline \mathbf{x}^I \end{array}\right] = \left\{ \left[\begin{array}{c|c} \mathbf{I}_1 - \mathbf{A}^{M,M} & -\mathbf{A}^{M,I} \\ \hline -\mathbf{B}^{I,M} & \mathbf{I}_2 - \mathbf{A}^{I,I} \end{array}\right] - \left[\begin{array}{c|c} \mathbf{D}^{M,M} & \mathbf{D}^{M,I} \\ \hline \mathbf{D}^{I,M} & \mathbf{D}^{I,I} \end{array}\right] \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L} \right\}^{-1} \cdot \mathbf{d} \tag{28}$$

We can to write the last equation

$$\mathbf{R} = \left\{ \left[\begin{array}{c|c} \mathbf{I}_1 - \mathbf{A}^{M,M} & -\mathbf{A}^{M,I} \\ \hline -\mathbf{A}^{I,M} & \mathbf{I}_2 - \mathbf{A}^{I,I} \end{array}\right] - \left[\begin{array}{c|c} \mathbf{D}^{M,M} & \mathbf{D}^{M,I} \\ \hline \mathbf{D}^{I,M} & \mathbf{D}^{I,I} \end{array}\right] \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L} \right\}^{-1} \tag{29}$$

Matrix inverse **R** is composed from four sub-matrices and they show intra-regional and interregional, direct and indirect, effects.

$$\mathbf{R} = \left[\begin{array}{c|c} \mathbf{R}^{M,M} & \mathbf{R}^{M,I} \\ \hline \mathbf{R}^{I,M} & \mathbf{R}^{I,I} \end{array}\right] \tag{30}$$

B Appendix B: tables and graphs interregional analysis

Table 12: Interregional Forward dispersion

	M vs I				I vs M			
	first component $(u_1^{MI} \cdot s_1^{MI})$	second component $(u_2^{MI} \cdot s_2^{MI})$	fd_i (modulus)	fd_i (%)	first component $(u_1^{IM} \cdot s_1^{IM})$	second component $(u_2^{IM} \cdot s_2^{IM})$	fd_i (modulus)	fd_i (%)
x_1	0.17	0.00172	0.167	6%	2.99	0.081	3.00	4%
x_2	0.09	0.00026	0.085	3%	3.51	0.029	3.51	5%
x_3	0.10	0.00102	0.098	3%	1.37	0.018	1.37	2%
x_4	0.24	-0.00130	0.238	8%	8.79	-0.088	8.79	12%
x_5	0.22	-0.00015	0.225	8%	6.01	-0.067	6.01	8%
x_6	0.16	0.00090	0.161	6%	4.13	0.126	4.13	5%
x_7	0.04	0.00045	0.040	1%	0.75	0.0150	0.75	1%
x_8	0.64	-0.00124	0.635	22%	11.74	-0.00055	11.74	16%
x_9	0.93	0.00063	0.935	33%	26.21	0.0142	26.21	35%
x_{10}	0.09	0.00010	0.090	3%	3.29	-0.0198	3.29	4%
x_{11}	0.19	-0.00019	0.187	7%	6.31	-0.0033	6.31	8%
modulus	1.22	0.00296			32.09	0.192		

Table 13: Interregional Backward dispersion

	M vs I				I vs M			
	first component $(v_1^{MI} \cdot s_1^{MI})$	second component $(v_2^{MI} \cdot s_2^{MI})$	(modulus)	(%)	first component $(v_1^{IM} \cdot s_1^{IM})$	second component $(v_2^{IM} \cdot s_2^{IM})$	bd_i (modulus)	bd_i (%)
I	0.53	0.0016	0.53	17%	12.65	0.12210	12.652	15%
II	0.51	0.0008	0.51	16%	12.55	0.05694	12.546	15%
III	0.50	-0.0001	0.50	15%	12.47	0.00330	12.473	15%
IV	0.46	-0.0007	0.46	14%	12.38	-0.01582	12.384	15%
V	0.40	-0.0013	0.40	13%	12.27	-0.04712	12.265	14%
VI	0.33	-0.0018	0.33	10%	12.08	-0.12738	12.084	14%
VII	0.48	0.0006	0.48	15%	10.33	0.00124	10.332	12%
modulus	1.22	0.0030			32.09	0.19199		

Figure 5: Interregional Backward e Forward dispersion: region M vs region I (absolute level)

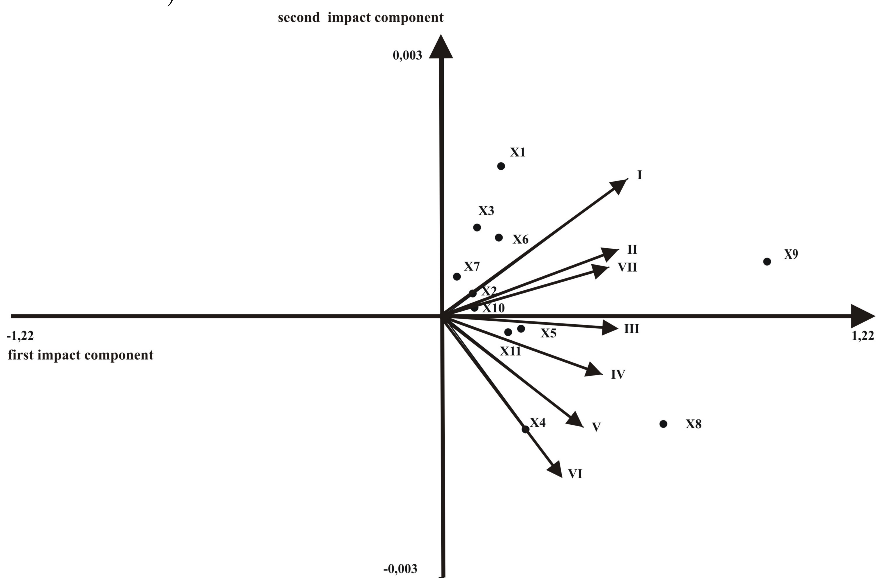


Figure 6: Interregional Backward e Forward dispersion: region I vs region M (absolute level)

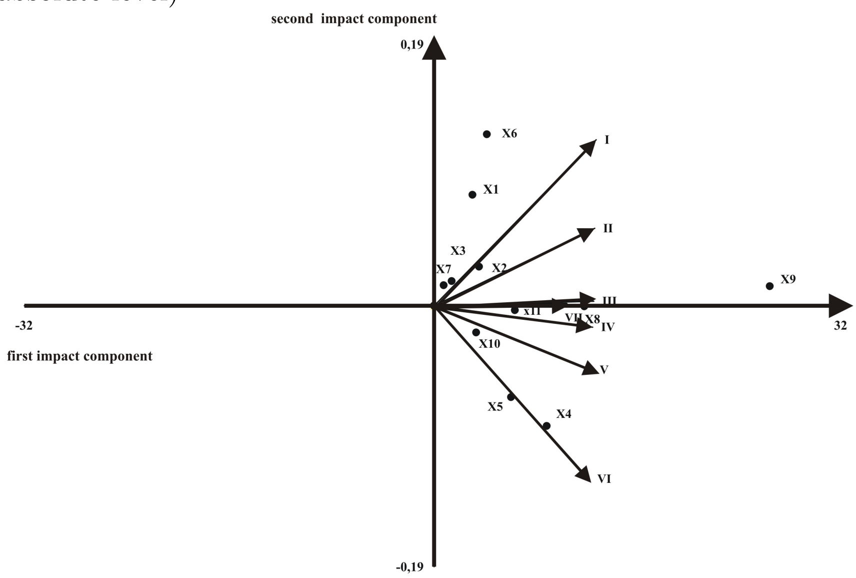


Figure 7: Sector in region I and industry in region M interactions - Interregional Backward and Forward dispersions standardized

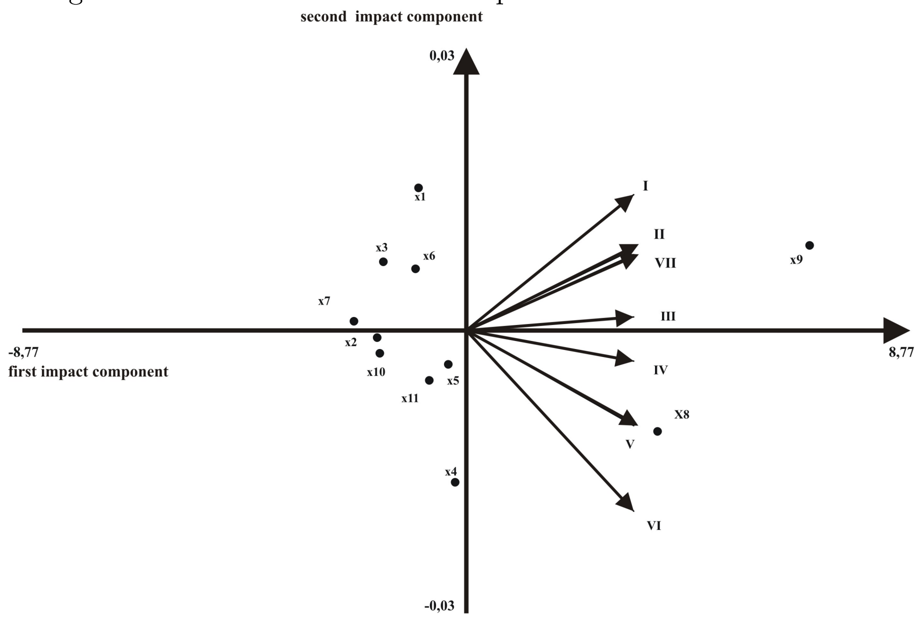


Figure 8: Sector in region M and industry in region I interactions - Interregional Backward and Forward dispersions standardized