Understanding the shortcomings of commodity-based technology in input-output models: an economic-circuit approach *

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ABSTRACT. The Make-Use Model serves as a basis for most national accounting systems as the SNA and is acknowledged as the most suitable model for interregional analysis. Two hypotheses are traditionally made featuring either industry-based (IBT) or commodity-based (CBT) technologies. While IBT can be easily interpreted in terms of a demand-driven economic circuit, it will be shown that: 1) CBT cannot be interpreted as a demand-driven economic circuit because this involves computing the inverse of a matrix (the matrix of industry output proportions) which is either impossible or generates negative terms; 2) the only way to obtain a plausible explanation of CBT is to convert it into a supply-driven model. This provides a new reason for rejecting CBT: either IBT is adopted but violates Kop Jansen and ten Raa's axioms, or CBT is chosen but must be converted into a poor or unrealistic supply-driven model.

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1 INTRODUCTION

Most national accounting systems around the world are based on the rectangular input-output model developed by Stone (1961) and adopted by the United Nations and the OECD in the *System of National Accounts* (United Nations, Department of Economic and Social Affairs, 1968, 1993, 1999; Blades, 1989; van Bochove and Bloem, 1987; Vanoli, 1994; Lawson, 1997). The SNA is a big improvement on the former square industry-by-industry model: a rectangular model distinguishes commodities and industries, and does not require that they be equal in number. The rectangular model can also be usefully applied to interregional or multiregional economics: Oosterhaven (1984) showed that the rectangular model, despite a few disadvantages, has some important advantages over the square models when interregional tables are constructed [1]. This excerpt from Oosterhaven (1984, pp. 580-1) is particularly eloquent: "Positive points in its favour are (1) the direct relation with the way in which firms, as the main sources of input-output data, record their external relations, (2) avoiding the reconciliation of sales and purchase data, (3) avoiding the transfer of secondary products, (4) its suitability to deal directly with changes in demand for goods and services and (5) the possibility for combining data on interregional transport of goods with sectoral cost or technology data into multiregional format. Negative points are (1) the separate requirement of sales and purchase data, (2) the lesser amount of consistency checks, which relates to the absence of a reconciliation need and (3) its need for additional model assumptions about sector and market shares.".

However, being rectangular, the model can be constructed in more than one way. For rectangular models as those handled by the SNA, two main hypotheses can be considered, the industry-based technology hypothesis and the commodity-based technology hypothesis [2]. What they mean exactly will be recalled later, for now the reader needs simply to understand that they are normally alternative. Some countries opt for the SNA and the commodity-based hypothesis for their national accounting system, but the United States use the industry-based technology [3]. These two hypotheses must normally satisfy four axioms (ten Raa, 1988; Kop Jansen and ten Raa, 1990): material balance, financial balance, price invariance, invariance of scale; these axioms will be explicitly listed later. The commodity-based technology satisfies the four axioms but the industry-based technology violates the last three: for ten Raa (1988), the violation of these axioms is an obvious reason to abandon the industry-based hypothesis. Hence the choice of the commodity-based technology by the SNA seems unquestionable. However this hypothesis raises some difficulties: mainly some negative flows can appear when the model is solved because it requires computing the inverse of a matrix (the matrix of industry output proportions). This forces us to have a square model with the same number of commodities and industries (a pity for a rectangular model...) and generates negative flows, always problematic.

As the United Nations and OECD begin to draw up a new set of tables for all countries after a long interruption, it is time to look again at the validity of the commodity-based model. In this

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1 See DBS (1969), Oosterhaven (1980, 1981), Statistics Finland (1980) or Polenske (1980) about the construction of interregional input-output tables in Canada, Netherlands, Finland and USA in that historical order.

2 The many ways of building rectangular interregional tables from national tables set out by Oosterhaven (1984) must not be confused with these two polar hypotheses.

3 The difference lies essentially in the choice of the technology assumption but also in the treatment of imports (Jackson, 1998). However, it is not the aim of this paper to discuss further how the US approach differs from the SNA approach. See also (Kuboniwa, Matsue and Arita, 1986).

paper, an original approach is chosen: the economic circuit. While the industry-based model can easily be interpreted in terms of a demand-driven economic circuit, it will be shown (on the national or one-region case for simplicity) that 1) the commodity-based model cannot be interpreted as a demand-driven economic circuit because this involves computing the inverse of the matrix of industry output proportions; and 2) the only way to obtain a plausible explanation of the commodity-based model is to convert it into a supply-driven model. This provides a new reason for rejecting the commodity-based model: either the industry-based model is adopted but it violates Kop Jansen and ten Raa's axioms (1990) or the commodity-based model is chosen but one must accept to convert it into a poor or unrealistic supply-driven model, which precludes traditional input-output analyses.

2 THE ECONOMIC CIRCUIT IN THE TRADITIONAL LEONTIEF AND GHOSH MODELS

Although old-established, the traditional square model of input-output economics needs to be recalled here to show that both its versions -- the Leontief one ("demand driven") and the Ghosh one ("supply driven") -- may -- and indeed must be economically interpreted in terms of economic circuit. Viewing the model as an economic circuit is neither complicated nor strange: the idea is simply to ensure that the model can be developed in such a way that it is possible to pass from the direct effects (read into the matrix of coefficients) to the total effects (read into the inverse matrix). Even if there is a connection between this approach and graph theory, it is not really a circuit in the sense of graph theory from one vertex i to itself via many others, j, k, as might be the circuit $i \to j \to k \to i$ [4] but a global circuit from the aggregate of sectors to the aggregate of sectors (that comprise all the arcs from any sector to any sector). The economic circuit interpretation will be developed in what follows and the reader will understand why, when it is impossible to close the economic circuit correctly, the model becomes a sterile, economically meaningless exercise.

Denote x_j as the output of sector j, f_i as the final demand of commodity i, w_j as the value added of sector j; z_{ij} indicates how much of commodity i is bought by sector j, that is, the flow from i to j. All quantities are computed in units of money. Matrix $\mathbf{Z}$ is homogenous by rows and columns. The central equation of traditional input-output economics (Leontief, 1936) is:

(1) $\mathbf{x} = \mathbf{A}\,\mathbf{x} + \mathbf{f}$

where $a_{ij} = \dfrac{z_{ij}}{x_j}$ is the technical coefficient, $\mathbf{A} = \mathbf{Z}\,\langle \mathbf{x} \rangle^{-1}$ denoting the matrix of technical coefficients. The model can be resolved simply as:

(2) $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{f}$

In the Leontief model, the quantity $\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + ... + \mathbf{A}^k + ... \underset{k \to \infty}{\to} (\mathbf{I} - \mathbf{A})^{-1}$ can be traditionally interpreted as the sum of, respectively, the effect of final demand of any commodity j (say, cars), the direct effect of the intermediate demand between any pair of sectors j and i (say, cars that need steel), the indirect effect of the intermediate demand between any pair of sectors j and i via any sector l (say, cars that need steel and steel that needs energy), etc. So, how is equation (1) obtained? The first possibility was chosen by Leontief (1985): from the accounting identity $\mathbf{Z}\,\mathbf{s} + \mathbf{f} = \mathbf{x}$, where $\mathbf{s}$ is the sum vector, substituting $\mathbf{A}$ into it, Leontief obtains directly (1) [5]. The second possibility consists into interpreting equation (1) in terms of

[4] See Ponsard (1972) and Lantner (1974) for a treatment of input-output analysis by the graph theory.

[5] This is known to be similar to the static General Equilibrium even if it is the "Leontief

economic circuit. Consider a surge in final demand; the initial increase in final demand for commodity *j* generates an equal increase in the output of sector *j*: $\Delta f_j^{(0)} \to \Delta x_j^{(0)} = \Delta f_j^{(0)}$ [6]. This, in turn, generates an increase in demand for input *i*: $a_{ij}\, \Delta x_j^{(0)}$. So, the total increase of the output of sector *i* is: $\Delta x_i^{(1)} = \sum_{j=1}^{n} a_{ij}\, \Delta x_j^{(0)}$. This continues at steps 2, ..., etc., and at step *k*: $\Delta x_i^{(k)} = \sum_{j=1}^{n} a_{ij}\, \Delta x_j^{(k-1)}$ that is, $\Delta \mathbf{x}^{(k)} = \mathbf{A}\, \Delta \mathbf{x}^{(k-1)}$ and the economic circuit is closed. Equation (1) can be retrieved by integration. The solution of the model is found by computing $\Delta \mathbf{x}^{(k)} = \mathbf{A}^k\, \Delta \mathbf{x}^{(0)} = \mathbf{A}^k\, \Delta \mathbf{f}$, thus the total increase in output is given by $\Delta \mathbf{x} = \sum_{k=1}^{n} \Delta \mathbf{x}^{(k)} = \left(\sum_{k=1}^{n} \mathbf{A}^k\right) \Delta \mathbf{f} \underset{k\to\infty}{\to} (\mathbf{I}-\mathbf{A})^{-1}\, \Delta \mathbf{f}$; equation (2) is retrieved by integration. So, this interpretation describes a circular process: production by one sector generates demand for some intermediate commodities, itself described by the technical coefficients, which in turn generates production by the relevant sectors (remembering that the one-to-one, or bijective, sector-product correspondence is assumed: a product is produced by one and only one sector and each sector produces one and only one product) and the model can be described as *demand-driven*. This is well known, but not exactly contained in the first possibility above. The economic circuit interpretation is closer to Sraffa's "production of commodities by means of commodities" (Sraffa, 1960). The interpretation in terms of economic circuit is elementary, even if it is generally overlooked. Whatever, if the interpretation as a closed economic circuit fails, the model loses all economic meaning: it is still possible to find (1) by the first possibility (direct substitution of the economic coefficients into the accounting identity) but the economic interpretation of the model no longer holds: this is not so with Leontief model, but it could be for others, as will be shown.

The alternative version of the model is *supply-driven* (Ghosh, 1958) [7]. Allocation coefficients $b_{ij} = \dfrac{z_{ij}}{x_i}$ are assumed to be stable. The central equation of the model is:

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closed model" -- where no exogenous elements are considered as added value or final demand -- that correspond to a static GE: a homogenous system is solved, $(\mathbf{I} - \mathbf{A})\, \mathbf{x} = \mathbf{0}$, that has a solution $x_i = f(x_1)$ only if $|\mathbf{I} - \mathbf{A}| = 0$ (where the "numéraire" commodity x_1 is chosen arbitrarily); equation (1) is the "Leontief open model". In this interpretation the Walrassian "tâtonnement" is a-temporal because no exchanges are made before equilibrium. When equation (1) is found following the first interpretation, all actions (buying, selling) of all sectors take place at the same time, the equilibrium described by the inverse matrix $(\mathbf{I} - \mathbf{A})^{-1}$ is reached instantaneously.

[6] It is known that to be plausible, this second interpretation must assume all sectors to have in stock -- ready to be immediately distributed -- all the quantities of commodity that the final or intermediary demands may need. For example, here sector *j* must have in stock at least an amount of commodity *j* equal to $\Delta xj^{(0)}$, ready to be distributed to final demand. Most authors choose such an interpretation of the Leontief model in terms of successive rounds; for example see Gale (1989, p. 300), Miller and Blair (1985, p. 18-9), United Nations (1999, p. 8-9), Weale (1984, p. 44).

This approach is also similar to the interpretation that can be made for Markov chains: assuming that each step takes a certain time, it is even possible to calculate how much time it takes to reach a given level of effect (indicated by $\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + ... + \mathbf{A}^k$), while reaching equilibrium (indicated by $(\mathbf{I} - \mathbf{A})^{-1}$) takes an infinite time; Lantner (1974) has introduced the idea of calendar of effects; see also (de Mesnard, 1992) for an asynchronous analysis. Note that this approach must not be confused with the dynamic Leontief model.

[7] I do not discuss its plausibility here even if the Ghosh model is often seen as less plausible (for a discussion of this point, see: Bon, 1986, 2000; Chen and Rose, 1986, 1991; Deman, 1988, 1991; de Mesnard, 1997, 2002; Dietzenbacher, 1989; Gruver, 1989; Helmstadter and Richtering, 1982; Miller, 1989; Oosterhaven, 1988, 1989, 1996; Rose and Allison, 1989,

(3) $\mathbf{x}^{/}\,\mathbf{B} + \mathbf{w}^{/} = \mathbf{x}^{/}$

This solves as:

(4) $\mathbf{x}^{/} = \mathbf{w}^{/}\,(\mathbf{I} - \mathbf{B})^{-1}$

This could be also interpreted as an economic circuit. The initial increase $\Delta w_i^{(0)}$ in the value added of an industry i generates an equal increase in the output of this industry, $\Delta x_i^{(0)} = \Delta w_i^{(0)}$; this then generates an increase in the supply of sector j: $b_{ij}\,\Delta x_i^{(0)}$. So the total increase in the output of sector j is $\Delta x_j^{(1)} = \sum_{i=1}^{n} b_{ij}\,\Delta x_i^{(0)}$, that is, at step k: $\Delta x_j^{(k)} = \sum_{i=1}^{n} b_{ij}\,\Delta x_i^{(k-1)}$, or in matrix terms, $\Delta\mathbf{x}^{(k)/} = \Delta\mathbf{x}^{(k-1)/}\,\mathbf{B}$, and (3) is retrieved by integration. The model solves as: $\Delta\mathbf{x}^{(k)/} = \Delta\mathbf{x}^{(0)/}\,\mathbf{B}^k = \Delta\mathbf{w}^{/}\,\mathbf{B}^k$ and the increase in total output becomes $\Delta\mathbf{x}^{/} = \sum_k \Delta\mathbf{x}^{(k)/} = \Delta\mathbf{w}^{/}\left(\sum_k \mathbf{B}^k\right) = \Delta\mathbf{w}^{/}\,(\mathbf{I} - \mathbf{B})^{-1}$: equation (4) can be retrieved by integration.

The model is every bit as coherent as the demand-driven one, but one must remember also that the Leontief and the Ghosh models are incompatible. As $\mathbf{B} = \hat{\mathbf{x}}^{-1}\,\mathbf{A}\,\hat{\mathbf{x}}$, allocation coefficients cannot be stable if technical coefficients are also stable: a star denoting aggregates after a change ($\mathbf{x}$ changing into $\mathbf{x}^*$), if $\mathbf{A}$ is stable, $\mathbf{A}^* = \mathbf{A}$, then $\mathbf{B}^* = \hat{\mathbf{x}}^{*\,-1}\,\mathbf{A}\,\hat{\mathbf{x}}^* \neq \mathbf{B}$.

3 THE MAKE-USE MODEL

In the rectangular models such as the SNA, two rectangular homogenous matrices are considered [8]. Denoting x_i as the output of industry i, w_j as the value added of industry j, q_i as the total production of commodity i, and e_i as the amount of commodity i sold to final demand. The Use matrix, denoted $\mathbf{U}$, with industries as columns and commodities as rows and with final demand as a supplementary column and value added as a supplementary row, indicates how much of each commodity each industry buys in order to produce: u_{ij} is the quantity of input i used by industry j. For example, for two industries and three products:

$$\begin{array}{cc}
\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \end{bmatrix} & \begin{array}{cc} e_1 & q_1 \\ e_2 & q_2 \\ e_3 & q_3 \end{array} \\
\begin{array}{cc} w_1 & w_2 \\ x_1 & x_2 \end{array} &
\end{array}$$

The Make (or Supply) matrix, denoted $\mathbf{V}$, with industries as rows and commodities as columns, indicates how much of each commodity each industry is producing: v_{ij} is the quantity of commodity j produced by industry i. For example:

$$\begin{array}{cc}
\begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \end{bmatrix} & \begin{array}{c} x_1 \\ x_2 \end{array} \\
\begin{array}{ccc} q_1 & q_2 & q_3 \end{array} &
\end{array}$$

Four accounting identities are given:

(5) $\mathbf{x} = \mathbf{V}\,\mathbf{s}$

(6) $\mathbf{x} = \mathbf{U}^{/}\,\mathbf{s} + \mathbf{w}$

(7) $\mathbf{q} = \mathbf{U}\,\mathbf{s} + \mathbf{e}$

Sonis and Hewings, 1992) unless we consider it as a price model (Dietzenbacher, 1997); but the model must be recalled for the clarity of exposition.

[8] The reader must not confuse between the Leontief or Ghosh models and the square models: even if the Leontief or Ghosh models are square, Make-Use Models can also be square when the number of commodities is equal to the number of sectors.

(8) $\quad \mathbf{q} = \mathbf{V}' \, \mathbf{s}$

Technical coefficients are defined as:

(9) $\quad \mathbf{A}^u = \mathbf{U} \, \hat{\mathbf{x}}^{-1}$

Two alternative hypotheses are posited about how the matrix **V** must be read, the industry-based technology and the commodity-based technology each generating two alternative models. It is possible to set out the complete solution to these models: each hypothesis generates two balance-accounting identities (commodities-by-commodities and industries-by-industries) and four total-requirement matrices (commodities-by-commodities, commodities-by-industries, industries-by-industries and industries-by-commodities); this will not be done here: see Miller and Blair (1985, pp. 159ff) or Aidenoff (1970) [9]. Denoting by $\mathbf{A}(\mathbf{U},\ \mathbf{V})$ the matrix of direct commodity requirements formed when one of the two polar hypotheses are chosen, Kop Jansen and ten Raa's four axioms (1990) are the following:

- material balance: $\mathbf{A}(\mathbf{U},\ \mathbf{V})\ \mathbf{V}'\ \mathbf{s} = \mathbf{U}\ \mathbf{s}$;
- financial balance: $\mathbf{s}'\ \mathbf{A}(\mathbf{U},\ \mathbf{V})\ \mathbf{V}' = \mathbf{s}'\ \mathbf{U}$;
- price invariance: $\mathbf{A}\ (\langle \mathbf{p} \rangle\ \mathbf{U},\ \mathbf{V}\ \langle \mathbf{p} \rangle) = \langle \mathbf{p} \rangle\ \mathbf{A}(\mathbf{U},\ \mathbf{V})\ \langle \mathbf{p} \rangle^{-1}$ for all price vectors $\mathbf{p} > \mathbf{0}$;
- and scale invariance: $\mathbf{A}(\mathbf{U}\ \langle \mathbf{k} \rangle, \langle \mathbf{k} \rangle\ \mathbf{V}) = \mathbf{A}(\mathbf{U},\ \mathbf{V})$ for all vectors of scale factor $\mathbf{k} > \mathbf{0}$.

3.1 Industry-based technology

Following Aidenoff (1970) or Miller and Blair (1985, p. 165-166) [10], the total output q_j of a commodity j is supplied by industries i in fixed proportions, i.e., the *commodity-output proportion* is fixed (termed as *technology based on industries*):

(10) $\quad \mathbf{D} = \mathbf{V} \, \hat{\mathbf{q}}^{-1}$

In other words the input structure of an industry does not depend on the products that it produces. This hypothesis corresponds simply to a fixed market share of all industries, which may be realistic in the short run, and for Miller and Blair (1985, p. 166), it is also suitable for by-products (products whose production is linked to the main product, such as cars and automobile parts) [11]. The commodity-by-commodity identity is found by substituting (9) in (7), that is, $\mathbf{q} = \mathbf{A}^u\ \mathbf{x} + \mathbf{e}$, then by substituting (10) in (5), $\mathbf{x} = \mathbf{D}\ \mathbf{q}$, giving:

(11) $\quad \mathbf{q} = \mathbf{A}^u\ \mathbf{D}\ \mathbf{q} + \mathbf{e} \Leftrightarrow \mathbf{q} = \mathbf{A}^I(\mathbf{U},\ \mathbf{V})\ \mathbf{q} + \mathbf{e}$

by denoting $\mathbf{A}^I(\mathbf{U},\ \mathbf{V}) = \mathbf{A}^u\ \mathbf{D}$ the matrix of direct commodity requirements when the industry-based technology is chosen. Note that $\mathbf{A}^I(\mathbf{U},\ \mathbf{V}) = [\mathbf{U}\ \hat{\mathbf{x}}^{-1}]\ [\mathbf{V}\ \hat{\mathbf{q}}^{-1}] = \mathbf{U}\ \langle \mathbf{V}\ \mathbf{s} \rangle^{-1}\ \mathbf{V}\ \langle \mathbf{V}'\ \mathbf{s} \rangle^{-1}$. By defining final demand in terms of industries' output, $\mathbf{f} = \mathbf{D}\ \mathbf{e}$, and by premultiplying (11) by **D**, the industry-by-industry identity is $\mathbf{x} = \mathbf{D}\ \mathbf{A}^u\ \mathbf{x} + \mathbf{f}$, which could be denoted $\mathbf{x} = \tilde{\mathbf{A}}\ \mathbf{x} + \mathbf{f}$, with $\tilde{\mathbf{A}} = \mathbf{D}\ \mathbf{A}^u$. And there is no requirement for **U** and **V** to be square for industry-based technology to compute direct matrices (even if **D** must be square when computing the commodity-by-industry inverse

[9] There is also a mixed hypothesis (ten Raa, Chakraborty and Small, 1984). Kop Jansen and ten Raa (1990) list other types of hypotheses, while the axiomatic of the rectangular model is developed; see also (ten Raa, 1995, pp. 87-100). On the connection between interregional models and rectangular model, see (Oosterhaven, 1984).

[10] Here I follow the most common presentation of the model. Sometimes, the hypotheses are presented in a reverse order, invoking the input structure instead of the output structure.

[11] Even if the true by-product model is different: all secondary products are by-products and are considered as negative inputs in the mixed-technology model (ten Raa, Chakraborty and Small, 1984, p. 88)

matrix; see Miller and Blair 1985, p. 171). However, the model violates the axioms of financial balance, price invariance and scale invariance, respecting only the material balance axiom.

3.2 Commodity-based technology

Again following Aidenoff (1970) or Miller and Blair (1985, p. 165), the total output x_i of any industry i is composed of commodities j in fixed proportions, i.e., the *industry-output proportion* is fixed (termed *technology based on commodities*), and the input structure of a commodity does not depend on the industry that actually produces the commodity:

(12) $\mathbf{C} = \hat{\mathbf{x}}^{-1}\,\mathbf{V}$

For Miller and Blair (1985, p. 166) this hypothesis is applicable to subsidiary products (secondary products -- that are primary for other sectors -- produced by the same technology as the primary product of the industry, such as automobiles and buses). The 1993 System of National Accounts prescribes the use of commodity-based technology. The commodity-by-commodity identity is found from (12): $\hat{\mathbf{x}} = \mathbf{V}\,\mathbf{C}^{-1}$ if $\mathbf{C}$ is square. By premultiplying by $\mathbf{s}^{\prime}$ and combining with (8), one obtains:

(13) $\mathbf{x} = \mathbf{C}^{\prime\,-1}\,\mathbf{q}$

Once again, substituting equation (9) in (7) gives $\mathbf{q} = \mathbf{A}^{u}\,\mathbf{x} + \mathbf{e}$ and finally from (13) $\mathbf{q} = \mathbf{A}^{u}\,\mathbf{C}^{\prime\,-1}\,\mathbf{q} + \mathbf{e}$. This can be denoted $\mathbf{q} = \mathbf{A}^{C}(\mathbf{U},\,\mathbf{V})\,\mathbf{q} + \mathbf{e}$, where $\mathbf{A}^{C}(\mathbf{U},\,\mathbf{V}) = \mathbf{U}\,\hat{\mathbf{x}}^{-1}\,[\mathbf{V}^{\prime}\,\hat{\mathbf{x}}^{-1}]^{-1} = \mathbf{U}\,\hat{\mathbf{x}}^{-1}\left[\hat{\mathbf{x}}\,(\mathbf{V}^{\prime})^{-1}\right] = \mathbf{U}\,(\mathbf{V}^{\prime})^{-1}$ is the direct requirement matrix when the commodity-based technology is chosen. Note that the derivation of the commodity-based technology requires the number of commodities to be equal to the number of industries because the inverse of $\mathbf{C}$ has to be computed: Make and Use matrices must be square even to compute direct matrices, which is a highly restrictive condition. While it is possible to generate the balance-accounting identities of industry-based technology without computing the inverse of $\mathbf{D}$, the same is not true of commodity-based technology without computing the inverse of $\mathbf{C}$. Conversely, the model fulfills the four axioms.

4 ECONOMIC CIRCUITS AND MAKE-USE MODELS

As with the Leontief model, the Make-Use Model can be interpreted in terms of economic circuit. Here, it is not a complete economic circuit in the traditional sense, from industries to consumers and conversely, but a more limited one, between industries. Everything is dependent on the plausibility of the circular process as described by the alternative hypotheses: either the process is plausible and the solution of the model is economically meaningful or it is not.

4.1 The closed economic circuit under the industry-based technology hypothesis

The interpretation in terms of economic circuit works well for the industry-based technology. Consider a variation in final demand $\Delta e_j^{(0)}$ for commodity j. There is an equal need for commodity j: $\Delta q_j^{(0)} = \Delta e_j^{(0)}$ which generates an increase in the production of industry i: $d_{ij}\,\Delta q_j^{(0)}$; so, in total, industry i has to produce: $\Delta x_i^{(1)} = \sum_{j=1}^{m} d_{ij}\,\Delta q_j^{(0)}$. Then, the additional production of industry i generates the need for intermediate goods, which is for commodity l: $a_{li}^{u}\,\Delta x_i^{(1)}$. The total intermediate demand for commodity l is: $\Delta q_l^{(1)} = \sum_{i=1}^{n} a_{li}^{u}\,\Delta x_i^{(1)}$. The economic circuit is closed and begins again with this demand for commodity l. At step k, one has: $\Delta x_i^{(k)} = \sum_{j=1}^{m} d_{ij}\,\Delta q_j^{(k-1)}$, that is, $\Delta \mathbf{x}^{(k)} = \mathbf{D}\,\Delta \mathbf{q}^{(k-1)}$, and $\Delta q_l^{(k)} = \sum_{i=1}^{n} a_{li}^{u}\,\Delta x_i^{(k)}$, that is,

$\Delta\mathbf{q}^{(k)} = \mathbf{A}^u\,\Delta\mathbf{x}^{(k)}$. Finally, $\Delta\mathbf{q}^{(k)} = \mathbf{A}^u\,\mathbf{D}\,\Delta\mathbf{q}^{(k-1)}$ and the model of (11) is recovered by integration: both possibilities -- the mathematical one and the economic one -- explained for the Leontief model hold. In graphical terms, the economic circuit is as in figure 1. Obviously, the process could also begin with demand made on an industry instead of demand for a commodity.

4.2 The interrupted economic circuit under the commodity-based technology hypothesis

With commodity-based technology, industries demand commodities by means of technical coefficients, but these commodities are assumed to be produced by industries in accordance with the industry output proportions, c_{ij}. If we are to translate this in terms of economic circuit, the process could begin with a final demand for commodity *j*: $\Delta e_j^{(0)} \to \Delta q_j^{(0)} = \Delta e_j^{(0)}$. There are two cases.

1) If the number of industries is different from the number of commodities, which is the general case, then obviously the inverse of **C** cannot be computed [12] but the difficulties are not only a question of being able to compute the inverse of a matrix or otherwise. When the inverse of **C** cannot be computed, it is obvious to say that **C** does **not** indicate which industry will produce this commodity: **C** only indicates how commodities are produced by each industry. To understand what happens, the reader might consider the following rectangular example:

$$\mathbf{C} = \begin{array}{c} \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0.3 \end{bmatrix} \begin{array}{l} 1 \\ 1 \end{array} \quad \text{Industries} \\ \text{Commodities} \end{array}$$

From, $\Delta q_3 = 1000$ for example, one cannot determine how much x_1 or x_2 will be increased (even if **C** is square). No information is available in **C** to determine whether it is industry 1 or industry 2, or both, that will increase their output. One could decide that it is a particular industry i_0 that has to produce this commodity *j*, that is, $\Delta q_j^{(0)} \to \Delta x_{i_0}^{(1)} = \dfrac{\Delta q_j^{(0)}}{c_{i_0 j}}$, but this is arbitrary. See figure 2.

2) When the number of commodities (miraculously) equals the number of industries [13] it is mathematically true that $\mathbf{x} = \mathbf{C}'^{\,-1}\,\mathbf{q} \Leftrightarrow \mathbf{q} = \mathbf{C}'\,\mathbf{x}$; this does not mean that **x** is determined by **q**

[12] It is not a matter of computing pseudo-inverses or other artifices of computation.

[13] It would be deception to claim that, as the inverse of **C** must be computed, the number of industries must be equal to the number of commodities. On the other hand, why to develop a rectangular model if it is to transform it into a square model by aggregation, even if it is with two matrices, just to be able to use it? Actually, there is no reason to have the same number of industries and commodities. The only justification is that the name of an industry comes from the name of its main product. But it is not a good one because many industries may have the same main product, e.g. cars, while they may not have the same secondary products; an aggregation of all of them will change the picture. For example, Fiat produces mainly cars but secondarily aircraft, while BMW also produces mainly cars but bikes as its secondary production; aggregating both leads to an industry that produces mainly cars, plus aircraft and bikes as its secondary product. It is better to aggregate as little as possible; at least the justification of a square model must not be mathematical (computing the inverse of a matrix) but economic. Note that ten Raa (1995, pp. 97-8) has discussed how coefficients can be found in the rectangular case.

from **C**, i.e., $\Delta\mathbf{q} \to \Delta\mathbf{x} = \mathbf{C}'^{\,-1}\,\Delta\mathbf{q}$ is false, only the contrary remains true: $\Delta\mathbf{x} \to \Delta\mathbf{q} = \mathbf{C}'\,\Delta\mathbf{x}$. Care is required with the meaning of the equals sign: in this last expression, "=" does not mean that the right-hand side "equals" the left-hand side but that the left-hand side implies the right-hand side [14]. To explain this, consider the following square example:

$$\mathbf{C} = \begin{array}{c} \begin{array}{cc} \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.1 & .06 & 0.3 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} & \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \end{array} \\ \text{Commodities} \end{array} \quad \text{Industries}$$

Computing the inverse of **C** is mathematically correct:

$$\mathbf{C}^{-1} = \begin{array}{c} \begin{array}{cc} \begin{bmatrix} 2.25 & -1.0625 & -0.1875 \\ -0.25 & 2.0625 & -0.8125 \\ -0.25 & -0.4375 & 1.6875 \end{bmatrix} & \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \end{array} \\ \text{Industries} \end{array} \quad \text{Commodities}$$

However, this indicates that, say, $\Delta q_3 = 1000$, generates $\Delta x_1 = -250$, $\Delta x_2 = -437.5$ and $\Delta x_3 = 1687.5$. Even if $\mathbf{C}^{-1}$ seems to be able to indicate which industry has to produce each commodity, the appearance of negative terms "out of thin air" proves that an illicit operation has been performed. These negative terms have long been misunderstood: many authors have tried to eliminate them or to test if they are due to errors of measurement (see ten Raa and van der Ploeg (1989), or Steenge (1990) with the introduction of a transition matrix between $\mathbf{A}^u$ and **C**, or Almon (2000)). Ten Raa (1988) rightly argues that negative terms are not due to errors in the data but to the model and he concludes that the commodity-based model must be abandoned.

The inverse matrix $\mathbf{C}^{-1}$ necessarily has many negative terms. As **C** is not negative by hypothesis, and as $\mathbf{C}\,\mathbf{C}^{-1} = \mathbf{I}$, then for the off-diagonal terms of **I**, the following formula holds: $\sum_k c_{ik}\,\sigma_{kj} = 0$ for all *i* and all *j*, where σ_{kj} is the $\{k, j\}$ term of $\mathbf{C}^{-1}$. So at least: there exists a *k* such that $\sigma_{kj} < 0$ for all *j*, i.e., there is one negative term per column of $\mathbf{C}^{-1}$, that is, per industry. But as one could have written $\mathbf{C}^{-1}\,\mathbf{C} = \mathbf{I}$ equivalently, there is also at least one negative per row of $\mathbf{C}^{-1}$, that is, per commodity: finally, the inverse of **C** has at least one negative term per row and columns, that is, per industry and commodity. In terms of valuated-graphs theory, a negative coefficient corresponds to an arrow pointing in the reverse direction. This is the case with many (perhaps all) of the off-diagonal terms of $\mathbf{C}^{-1}$; in the above example, all off-diagonal terms are negative. So, returning to graph theory, in $\mathbf{C}^{-1}$, the negative coefficients $\{i, j\}$ (commodity *i* and industry *j*) point in the reverse direction, not $i \to j$ (commodity $\to$ industry) but $j \to i$ (industry $\to$ commodity). They do not describe the industry structure of commodities (which industries produce each commodity) but the commodity structure of industries (which commodities each industry is producing) as in **C**. To summarize, as σ_{ij} is normally the percentage of commodity *i* produced by each industry *j*, a negative σ_{ij} would indicate either what negative proportion of commodity *i* an industry *j* produces (nonsense), or what positive proportion of the output of industry *i* is devoted to commodity *j* (which breaks the circuit and is not useful for the commodity-based model).

Finally, computing the inverse of **C** is a valid matrix operation whenever $n = m$, but it 0is economically meaningless [15]. The paradox with the commodity-based technology model is that its matrix computation is correct but its economic-circuit interpretation is not: the first

[14] Computer programming languages often make this distinction between "=" ("equal") and ":=" ("put this value into that variable").

[15] It is not the case of $(\mathbf{I} - \mathbf{A})^{-1}$ in the Leontief model.

possibility explained for the Leontief model holds but not the second. However, it is possible to restore the circuit by converting it into a supply-driven model.

4.3 Commodity-based technology and push-process

As it is necessary for the economic circuit to enter matrix **C** by industries, it is possible to reverse the economic circuit, converting the model into a supply-driven one. Replace the technical coefficient matrix $\mathbf{A}^u$ by a matrix of allocation coefficients, $b_{ij}^u = \frac{u_{ij}}{q_i}$, that is:

(14) $\mathbf{B}^u = \hat{\mathbf{q}}^{-1}\,\mathbf{U}$

It follows from (8) and (12) that:

(15) $\mathbf{q} = \mathbf{C}'\,\mathbf{x}$

From (14), we obtain

(16) $\hat{\mathbf{q}}\,\mathbf{B}^u = \mathbf{U}$

and substituting this in (6) gives $\mathbf{x} = \mathbf{B}^{u\prime}\,\hat{\mathbf{q}}\,\mathbf{s} + \mathbf{w} = \mathbf{B}^{u\prime}\,\mathbf{q} + \mathbf{w}$; so, the equation of the model is obtained by substituting (15) in this last equation:

(17) $\mathbf{x} = \mathbf{B}^{u\prime}\,\mathbf{C}'\,\mathbf{x} + \mathbf{w}$

This could be denoted $\mathbf{x} = \mathbf{B}'\,\mathbf{x} + \mathbf{w}$, with $\mathbf{B} = \mathbf{C}\,\mathbf{B}^u = \hat{\mathbf{x}}^{-1}\,\mathbf{V}\,\hat{\mathbf{q}}^{-1}\,\mathbf{U} = \langle \mathbf{V}\,\mathbf{s}\rangle^{-1}\,\mathbf{V}\,\langle \mathbf{V}'\,\mathbf{s}\rangle^{-1}\,\mathbf{U}$ and transformed into a commodity-by-commodity equation by premultiplying (17) by $\mathbf{C}'$ and using (15), that is: $\mathbf{q} = \mathbf{C}'\,\mathbf{B}^{u\prime}\,\mathbf{q} + \varpi$, where $\varpi = \mathbf{C}'\,\mathbf{w}$ is the value added by commodity, or $\mathbf{q} = \tilde{\mathbf{B}}'\,\mathbf{q} + \varpi$, with $\tilde{\mathbf{B}} = \mathbf{B}^u\,\mathbf{C}$.

A supply of a commodity j generates an output from any industry as indicated by $\mathbf{B}^u$, in the Ghoshian way, and then the industries sell commodities in the proportions indicated by the coefficients c_{ij}. In terms of economic circuit, the initial increase $\Delta v_i^{(0)}$ in the value added of an industry i generates an equal increase in the output of this industry: $\Delta x_i^{(0)} = \Delta v_i^{(0)}$. By matrix **C**, this generates an increase in the supply of all commodities: $c_{ij}\,\Delta x_i^{(0)}$, that is, all told, the increase in the supply of commodity j is: $\Delta q_j^{(1)} = \sum_{i=1}^{n} c_{ij}\,\Delta x_i^{(0)}$. This supplementary supply of a commodity j induces an increase in the output of all industries l following $\mathbf{B}^u$: $\Delta q_j^{(1)} \to b_{jl}^u\,\Delta q_j^{(1)}$, so, in total, industry l increases its output of $\Delta x_l^{(1)} = \sum_{j=1}^{m} b_{jl}^u\,\Delta q_j^{(1)}$ and the economic circuit is closed (see figure 3).

At step k, one has: $\Delta q_j^{(k)} = \sum_{i=1}^{n} c_{ij}\,\Delta x_i^{(k-1)}$ and $\Delta x_l^{(k)} = \sum_{j=1}^{m} b_{jl}^u\,\Delta q_j^{(k)}$, that is, $\Delta\mathbf{q}^{(k)} = \mathbf{C}'\,\Delta\mathbf{x}^{(k-1)}$ and $\Delta\mathbf{x}^{(k)} = \mathbf{B}^{u\prime}\,\Delta\mathbf{q}^{(k)}$, so $\Delta\mathbf{x}^{(k)} = \mathbf{B}^{u\prime}\,\mathbf{C}'\,\Delta\mathbf{x}^{(k-1)}$. This complies with the corresponding model (17): the supply-driven commodity-based-technology model is consistent [16].

5 CONCLUSION

Most national accounting systems are based on Stone's Make-Use Model. The two traditional alternative hypotheses have been explored. The first one, industry-based technology, can be explained in terms of economic circuit even in the rectangular case and is a fairly conventional demand-driven model. The alternative hypothesis, commodity-based technology, is problematic because the inverse of **C**, the matrix of industry output proportions, must be computed which is impossible in the rectangular case while in the square case, it generates inexplicable negative terms. This problem, which authors have tried to correct empirically,

[16] All this is irrespective of the discussion about the artificial character of a supply-driven model: it is just to demonstrate that commodity-based technology is inconsistent, wavering between a supply-driven and a demand-driven-model.

suggests reversed circuits inside **C** that lack credibility. Consequently, the commodity-based model cannot be interpreted in terms of economic circuit: the problem of the negative terms generated in the direct requirement matrix is not simply an annoyance, but leads to rejection of the model. However, if this demand-driven commodity-based model is converted into a supply-driven one, it recovers its coherence even in the rectangular case, as an inverse matrix needs no longer be computed: it can be interpreted in terms of economic circuit. To summarize, the industry-based technology model is a demand-driven one while the commodity-based technology model needs to be reconstructed as a supply-driven model, which is a completely different thing.

Nevertheless, the supply-driven model has long been criticized as either poor or unrealistic (for a discussion of pros and cons, see: Bon, 1986; Dietzenbacher, 1997; Gruver, 1989; Miller, 1989; Oosterhaven, 1988, 1989, 1996; Rose and Allison, 1989 [17]). In de Mesnard (2002), all interpretations of the supply-driven model are systematically examined. In one case, one homogenous output and multiple inputs are considered as usual, with ordinary "output prices": the model's mathematical results are very poor (only values are found, a value being the product of a price by a quantity, but prices or quantities are not found) but the model is credible. In the other case, one homogenous input and multiple outputs are considered, with "input prices" (that is, the rather strange prices of the buyers); the model is the mathematical dual of the Leontief model but the meaning of the variables it not the same; the model lacks credibility because it is impossible to give a valid economic interpretation of input prices or economic agents (whatever they may be they are certainly not productive sectors). So, the supply-driven model is either poor or unrealistic. Moreover, whatever one's opinion of the supply-driven model, the new commodity-based hypothesis cannot be used for ordinary input-output economics because it is a supply-driven model, a model completely different model from the Leontief model: at best, it can be used only for what Dietzenbacher (1997) calls "cost-push exercises".

Remark that one must not deduce of these results that there is only one way to derive a clean commodity-based model (i.e., without negative terms) but it is not the aim of this paper to list and develop these other ways.

On the basis of this paper, the promoters of the System of National Accounts - the United Nations and the OECD - should take the opportunity of the introduction of new tables to reflect on the foundations of the SNA, even if this calls into question long years of established practice. The industry-based model, which was adopted by the United States, should be carefully reconsidered; it too has some drawbacks from an axiomatic standpoint: perhaps the price to pay will be to relinquish some of the four axioms listed by Kop Jansen and ten Raa (1990) or Rueda-Cantuche (2002). Scholars and specialists in National Accounting are between the devil and the deep blue sea: either they adopt the industry-based model which violates three of the four axioms, either they adopt the commodity-based model that respect the four axioms but they must accept to convert it into an unrealistic or poor supply-driven one.

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[17] For the interregional case, Oosterhaven (1984, p. 572) argues that "rows-only" input-output table and models are not plausible.

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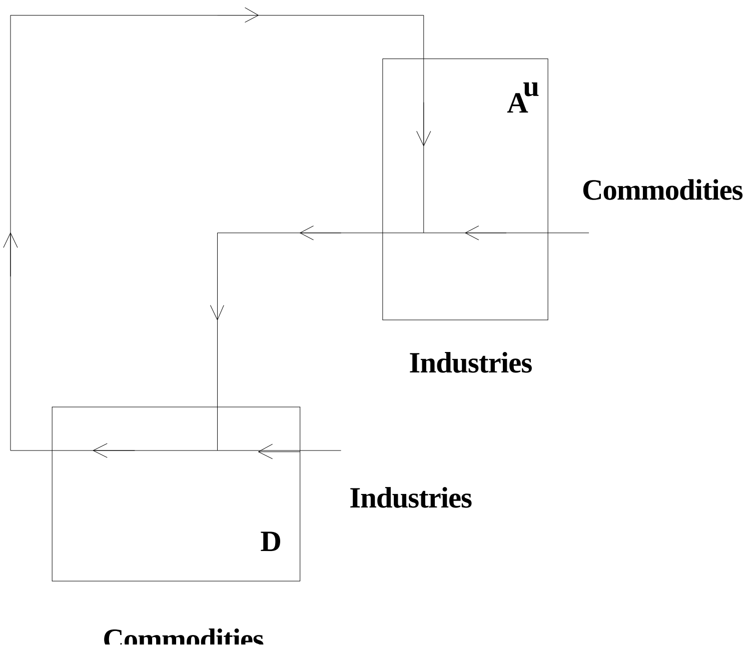

Figure 1. Symbolized economic circuit
of the demand-driven industry-based model
(two entries are possible: final demand for commodities, **e**,
or final demand made on industries, **f**)

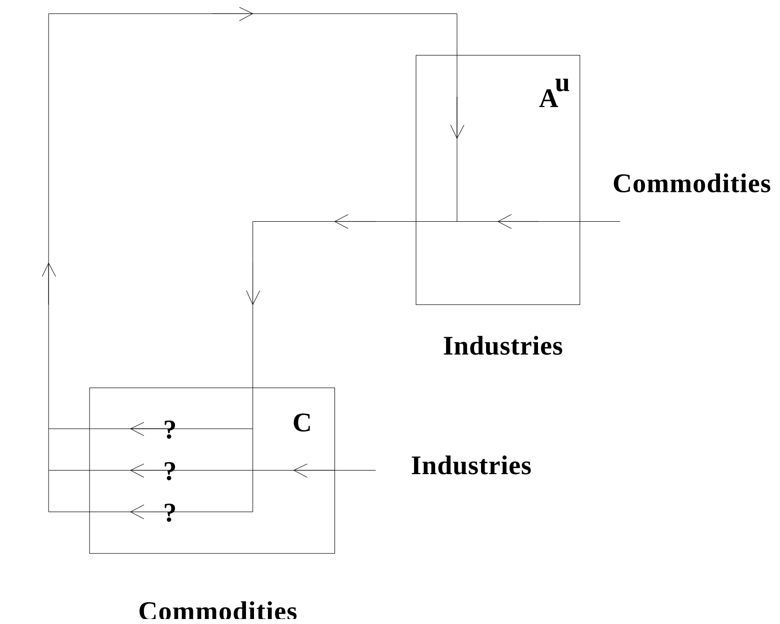

Figure 2. Symbolized undetermined circuits
of the rectangular demand-driven commodity-based model
(two entries are possible: final demand from commodities, **e**,
or final demand made on industries, **f**)

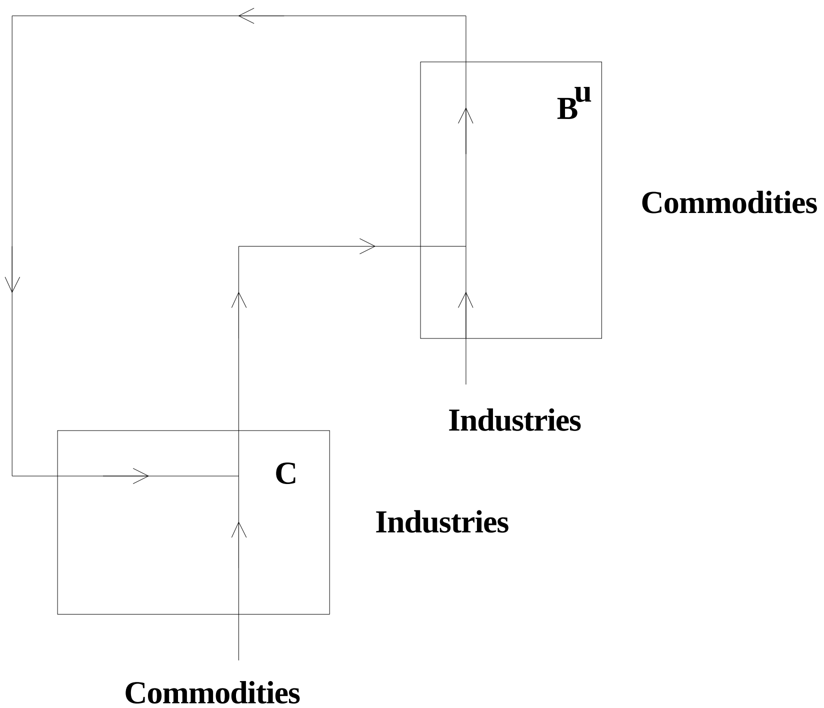

Figure 3. Symbolized economic circuit
of the rectangular supply-driven commodity-based model
(two entries are possible: value added from industries, **w**,
or value added on commodity, ϖ)