First Draft, comments welcome

The Direct Estimation of the Event Matrix in the Input-Output Framework in the Case of a Major Catastrophe

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Abstract

Input-Output analysis (I-O) is increasingly used in studies of sudden catastrophes such as earthquakes, hurricanes, or, in the case of the Netherlands, major flooding. It enables us to focus at the desired level of investigation, whether local, regional or national.

Several issues are at stake. The reason is that, by definition, catastrophe affects the existing networks and interconnections in a fundamental way. Certain elements of the existing structures may be lost, some possibly forever, while other may survive. One task the I-O modeller faces after the disaster is to construct a plausible basis for the recovery and reconstruction efforts. Here the literature has introduced the concept of the so-called Event Matrix. The problem is that the concept of I-O on the basis of a well-founded 'Event Matrix' theory is still in the developing stage.

Above all, in our view one should think about the basic principles that should guide us in providing more transparency and comparability. Here we find a number of proposals, but yet no generally accepted theory. It is the purpose of this paper to put forward specific *point of departure* in disaster analysis. In doing so, we make one crucial choice: we shall start from I-O fundamentals. We shall approach the problem with the construction of an I-O matrix on the basis of insight about the production capacities that remain active in the affected region. It is here that we introduce the concept of a "Basic Equation" as a necessary first step in the construction of an Event Matrix. This enables us to directly estimate the situation at various stages after the flooding. We shall illustrate the method with an example.

Key words: natural disasters; initial shock; Input-Output analysis; Event Matrix; Basic Equation.

1. Introduction

The literature on natural disasters, and in particular the studies covering economic cost assessment part, is extensive. Several scientific research centres have devoted their efforts to investigate this field. A substantial part of the literature concerning the economic dimension of natural disasters addresses earthquakes and their consequences. A certain dominance of authors from the US and Japan is observed (Cole *et al*, 1993; Cochrane, 1997a; Jones, 1997; Rose *et al*, 1998; Shinozuka, 1998; Okuyama *et al*, 2002; Cole, 2003) mostly concentrating on the issue of earthquakes, although also contributions from European research should be mentioned (among others, Parker *et al*, 1987; van der Veen *et al*, 2003), contributing flooding issues to the field.

For the estimation of economic loss many authors start from the basic input-output (I-O) model developed by Wassily Leontief in the 1930s and 40's. The model has undergone a number of revisions and extensions, but in essence it has remained the same[1]. It is true, of course, that the production processes and inter-industrial relations have undergone extensive reconsideration, as well as the selection of the major players, such as the government, consumers and the external parties. Nonetheless, the basic framework does not seem to have changed much. This, in a sense, is a pity. We now are forced to adopt perhaps a rigid straightjacket for modelling what is happening in the real world.

I-O is particularly useful in studying the flow of goods and services. It allows us to distinguish various types of direct and indirect effects, based on extensions of accepted multiplier analysis. However, there remain several aspects that are not fully covered yet. In this paper we shall point out that the basic theoretical framework in fact is quite *flexible* and after some modifications allows us to focus directly on the issues at stake. We suggest to look at the fundamentals trying to inject specific dynamics in what basically looks like a rather static framework. Our approach generates its own data needs, and it will rather be data availability, accuracy, and so on, that determine which result ultimately will be within reach.

[1] See Miller and Blair (1985) or Leontief (1986) for an introduction in methodology and methods.

Much work in disaster analysis is case-oriented. Primarily the nature of such work often forces the researcher to develop specific methods quite closely tied to the case. In this respect, we will propose a more general position. This generalised approach may hopefully provide a connecting bridge between empirical application of the model and its theoretical foundation.

Our paper is organised as follows. In Section 2 we shall start with some background, discussing some aspects in search for common grounds in input-output analysis type of research. Section 3 puts forth the I-O equation, which will be our point of departure. Hereafter, we introduce what may be called the 'Basic Equation'. In Section 4 we fill the 'Basic Equation' with a number of assumptions to illustrate how a real I-O system can be subtracted from it. We round it up with the proposal about the construction of an Event Matrix. We leave nevertheless more detailed implications on these findings for further examination. Section 5 provides some numerical examples to accompany the theoretically developed model of the previous sections. Section 6 closes with preliminary conclusions.

2. Disaster Analysis within I-O: Looking for a Starting Point

As far as we are concerned here with a large-scale devastation brought by disasters, we should realise that this brings about also serious alterations in the entire economy. Therefore, incremental analysis is of no use here. At the same time, it is absolutely important to trace this major metamorphose. Nevertheless, the reviewed literature suggests that the 'starting points' for the I-O type of analysis are scattered, though it is recognised that it is crucial for the whole process of modelling to understand what exactly is happening *immediately after* the disaster. The literature ascertains that there is a great need for a good understanding of the post-disaster situation. Okuyama (2003, p.12), e.g., points to the uncertainty that appears as a result of a disaster:

> "Uncertainty arises after a disaster because first, the extent and range of direct damages are unknown right after the event; second, the trends of economic activities, especially the fluctuation of demand, become unclear in the short run; and third, the influx of demand injections for recovery and reconstruction activities makes the long-run forecast of economic growth in the region difficult.

> …on the other hand, the degree of uncertainty over time requires a careful treatment."

That's why we would like to focus precisely on that issue in this paper. In this context, we suggest to "put on new glasses" and discover new features of the I-O model.

In this paper we shall address necessary background for the notion of an 'Event Matrix' to structure the modelling of a shock. In the literature dealing with the economic consequences of a natural catastrophe the notion of an 'Event Matrix' is used to introduce system into our thinking about the impacts of an exogenous event on the I-O system. That is, this special type of matrix helps us study the effects of a catastrophe until the time horizon set for analysis. However, the foundation of the concept of an Event Matrix needs additional support. Probably Cole (1993, p.4-7) was the first to introduce the notion of an Event Matrix:

> "In the most general case, the event matrix will be a set of tables corresponding to entries in the original IO table which specifies i) the extent of damage to internal and external components, ii) the goal for recovery and iii) the time scale for recovery. The details [of how an event matrix is specified] will depend on the situation under investigation."

Our concern then is contributing to the precision of the notion of an Event Matrix. In order to specify our point of view in particular, we would like to 'split' the elements of the definition above given by Cole (1993) into two stages: stage one would encompass element i), while stage two would consist of the elements ii) and iii). This will allow us to operate within a two-step procedure framework. In this contribution we shall focus on the first stage, as well as provide some insights about modeling possibilities for the second stage.

As a result of a disaster the existing ties within an economy are disrupted. This implies that we have to think in novel ways about the current notions of equilibrium and disequilibrium. Thus, in our view, stage one should form *the basis for systematic accounting* for the actual physical damage brought by a disaster. This means that we first make a summary statement on what is left after an outbreak of a catastrophe. This results in a kind of bookkeeping accounts, which, however, do not reflect the feasible economic interactions. So, we need a second step to extract information regarding the economic structure from these ‘bookkeeping’ operations. Only hereafter stage two in constructing a fully-fledged Event Matrix can be addressed. How an economy reacts after a disaster basically depends on three sets of factors: the level and severity of the damage incurred, the economy’s resilience[2], and the external factors. Cochrane (1997b, pp.243-244)[3] points out that combinations of these broadly defined factors stipulate the gains and losses ratio of a shock brought by a disaster and thus identify the recovery path of each particular economy towards a (new) equilibrium. In this paper we will not go as far as what might be called a full operationalisation of the required concepts. Rather, this paper should be viewed as providing a set of building blocks for later work.

The literature seems to agree that a precise *theoretical* starting point for disaster research is often missing. Even where it is assumed ‘obvious’, the basic issues of disaster implications always seem to require some additional attention. For example, there is no accepted formula for representation of disrupted ties within an I-O table as a result of disaster. We will focus therefore on the way the I-O modelling framework can help us in analysing the immediate effects of a catastrophe.

3. The Derivation of an After-Shock Equation

We mentioned that a well-founded theoretical point of departure is of prime importance in disaster research. But if we are trying to employ the I-O framework for analytical purposes, we immediately run into a major problem. I-O essentially is based on the concept of sectoral balances embedded in a circular flow. At present, we do not have a well-accepted theory of what happens if specific parts of this circular flow suddenly malfunction. However, the existing theoretical framework offers possibilities that have not been explored up to now. Below we shall propose a way to make a head start in modelling the phase immediately after the disaster in an I-O context. We shall focus on the question how those parts of the economy that are still intact will adapt to the new circumstances.

[2] See e.g. Green (2003) for the explicit discussion of resilience, vulnerability and related notions.
[3] Also Rose and Lim (2002) discuss a similar issue (p.12): “More sectorally diverse economies are better able to withstand the shocks of business interruption losses”.

As we know, in an I-O table only the part of the inter-industry flows that deals with production is represented in terms of fixed coefficients[4]. In a normal situation a firm sells part of its product to other manufacturers and part of it to the final users. The proportions between these parts are the result of many factors, some technical, some institutional, and some traditional. It is not easy to say what will happen if a particular firm is confronted with the fact that *some* of its customers (or suppliers) are not there anymore. Thinking about such cases will have to be based on the relevant behavioural, historical and technical circumstances, in the light of the possibilities that remain after the disaster.

For this reason, it is advisable to adopt a modelling framework that seems flexible enough to face such choices. Clearly, many issues are involved. Let us now first take a look at the standard open I-O model, and let us see if this is a suitable candidate for a good starting point. We have:

$$x^0 = A^0 x^0 + f^0, \qquad [1]$$

where x^0, f^0 and A^0 respectively stand for total output, final demand and the matrix of input coefficients in the initial situation before the flood. Let's also introduce the labour market. It plays an essential role in shock analysis, because disturbances in labour markets may be a prime cause of long-term delays in economic growth. Therefore, we shall try to find a way to incorporate its volatilities and effects into our model. Standard, we have:

$$L^0 = l^0 x^0 \qquad [2]$$

Here L^0 represents labour supply and l^0 the vector of direct labour input coefficients in the situation before the flood[5].

If a disaster strikes, this will, above all, affect the production levels x^0. However, via disturbances in x^0 also the sectoral demands for labour will be affected. A shift in these labour demands immediately will translate into a shift in final demand f^0. Therefore, we

[4] That is, in the A^0 coefficients table only the vertical entries are fixed in proportions, the horizontal are not (see further Miller and Blair, 1985)
[5] Below we shall define a sector as consisting of a set of identical firms. Thus, if δ percent of the firms is lost, also δ percent of the sector's capacity is lost. This definition in later work, of course, should be replaced by a more appropriate one that recognises the aggregation issues involved in defining a 'sector'.

have to select a framework that allows us to address these points appropriately. The same is true when we would have started with a change in labour availability. Evidently, substitution and adaptation of production processes must follow. Therefore, we have to work out a strategy to incorporate such effects in a flexible, simple and direct way in the I-O framework. Though I-O often is considered as a rather rigid model in terms of its assumptions, we shall venture some steps along this road, while adhering strictly to the rules.

Let us assume that we have information at the level of the individual sector or firms, and let us say that because of the flood the firms' (or sector's) productive capacity has been reduced to $100\gamma_i$ percent of the pre-flood capacity ($0 \leq \gamma_i \leq 1$). This means that *if all necessary inputs are available*, the sector will operate only at $100\gamma_i$ percent of its earlier capacity. So, if a sector operates at, say, 50 percent, this will mean that its input requirements, including those for labour, are reduced to this level. At the same time this also means that outputs will drop accordingly.

We now shall try to reconstruct the state of affairs in the economy, directly after a flood, on the basis of those parts that are *not* directly damaged and, in principal, remain active. In line with this we propose the following. We shall assume that a reduction in the sector's labour input requirement directly translates into a corresponding reduction in final demand as given in [1][6]. In this way we keep in line with the idea of *balances*: from the point of view of our analysis 'feeding those workers' is not a problem[7]. With this in mind, we shall rewrite the model ([1], [2]) as:

$$x^0 = A^0 x^0 + \left[\left(\frac{f^0}{L^0} \right) l^0 \right] x^0 \qquad [3]$$

Now let us go to the inter-industry transactions matrix M^0. We define it as:

[6] However, we shall assume that proportions within the final demand basket remain the same.
[7] Evidently, in reality the economy still has to feed those workers that are now unemployed. That, however, is a matter of post-flood assistance, and shall be dealt with in subsequent work

$$M^0 \equiv \left[A^0 + \left(\frac{f^0}{L^0} \right) l^0 \right] \hat{X}^0 \qquad [4]$$

where $\hat{X}^0$ is a diagonal matrix with sectoral outputs x_i^0 at its main diagonal. The term in brackets on the right-hand side thus consists of two matrices, both of which have fixed coefficients. Post-multiplication by $\hat{X}^0$ gives us in *nominal values* both the inter-industry use of goods, and the purchases of final demand by the workers, separately recorded for each sector. The submatrix $[(f^0 / L^0)\, l^0\,]\hat{X}^0$ stands for workers' real wage, modeled as a package of goods consumed, of dimension $[n \times n]$. Provided all workers have the same preferences, vertical proportions are the same for all sectors, whereas between the sectors proportions differ according to the number of employees. Below we shall develop our argument starting from a supposed detailed knowledge of the geography of disrupted production as provided, e.g., by today's GIS databases. We shall start from a representation in absolute numbers. This form has certain advantages above other ones. For example, it tells us directly which links exist between intermediate deliveries and final demand.

Now let us suppose that a big flood occurs. The extent of the shock can be simulated based on the GIS databases that contain information on the physical location of (the firms making up) the sector (see for example Bočkarjova *et al*, 2004b). In the original situation, the columns of matrix M^0 above represent all interactions in the economy. As a next step, we incorporate our knowledge about the extent of the damage. If sector i has lost $100\gamma_i$ percent of its capacity, we shall interpret this in the sense that, immediately after the shock, this particular sector is able to produce $100(1-\gamma_i)$ percent of its potential output if the inputs required to maintain this production level are available, and can be used in the traditional way[8].

We shall start from matrix M^0 as defined above (see formula [4]). We recall that the elements of M are the sum of the flows of intermediate deliveries and the imputed parts of final demand. We have, for each individual element of M^0 *just before* the shock:

[8] That is, as prescribed by the i^{th} column of the coefficients matrix A^0 in [1].

$$M_{ij}^{0} = Z_{ij}^{0} + F_{ij}^{0}, \qquad [5]$$

where $Z_{ij}^{0} = A_{ij}^{0} x_{j}^{0}$ - nominal inter-industry transaction matrix, and where F_{ij}^{0} is the i^{th} element of $\left(\frac{f^{0}}{L^{0}}\right) l_{j}^{0} x_{j}^{0}$.

So, using [5], we have:

$$M^{0} = \begin{bmatrix} M_{11}^{0} & \cdots & M_{1n}^{0} \\ \vdots & & \vdots \\ M_{n1}^{0} & \cdots & M_{nn}^{0} \end{bmatrix} = \begin{bmatrix} Z_{11}^{0} + F_{11}^{0} & \cdots & Z_{1n}^{0} + F_{1n}^{0} \\ \vdots & & \vdots \\ Z_{n1}^{0} + F_{n1}^{0} & \cdots & Z_{nn}^{0} + F_{nn}^{0} \end{bmatrix} \qquad [6]$$

We now use our assumption that the elements of M^{0} stand for units that can be identified from the knowledge of the GIS data system, and that we possess exact information of the amount of productive capacity that is lost. The immediate consequence of this loss is that the sector now runs into the problem of having to decide which parts of its output should have intermediate demand as its destination and which part should go to final consumption. For expository reasons we shall for the moment assume that both categories of demand are affected in the same way percentage-wise.[9]

So, with the fraction of productive capacity that is *lost* in sector *i* denoted by γ_i, we now can write:

$$M^{0} = \begin{bmatrix} \gamma_1 (Z_{11}^{0} + F_{11}^{0}) + (1-\gamma_1)(Z_{11}^{0} + F_{11}^{0}) & \cdots & \gamma_n (Z_{1n}^{0} + F_{1n}^{0}) + (1-\gamma_n)(Z_{1n}^{0} + F_{1n}^{0}) \\ \vdots & & \vdots \\ \gamma_1 (Z_{n1}^{0} + F_{n1}^{0}) + (1-\gamma_1)(Z_{n1}^{0} + F_{n1}^{0}) & \cdots & \gamma_n (Z_{nn}^{0} + F_{nn}^{0}) + (1-\gamma_n)(Z_{nn}^{0} + F_{nn}^{0}) \end{bmatrix} \qquad [7]$$

We can re-write this equation, splitting it into two parts. Thus we can single out the elements that are not there anymore from the I-O entries that represent firms that remain active. We have:

[9] For the general case, this can be modified straightforwardly.

$$M^0 = \begin{bmatrix} \gamma_1(Z_{11}^0 + F_{11}^0) & \cdots & \gamma_n(Z_{1n}^0 + F_{1n}^0) \\ \vdots & & \vdots \\ \gamma_1(Z_{n1}^0 + F_{n1}^0) & \cdots & \gamma_n(Z_{nn}^0 + F_{nn}^0) \end{bmatrix} + \begin{bmatrix} (1-\gamma_1)(Z_{11}^0 + F_{11}^0) & \cdots & (1-\gamma_n)(Z_{1n}^0 + F_{1n}^0) \\ \vdots & & \vdots \\ (1-\gamma_1)(Z_{n1}^0 + F_{n1}^0) & \cdots & (1-\gamma_n)(Z_{nn}^0 + F_{nn}^0) \end{bmatrix} \quad [8]$$

We now turn to the second term on the right, which gives us the information we possess on the parts that have *not* been lost. If we denote the total of the i^{th} row of the second matrix on the right-hand side by the symbol x_i, we have, in input-output 'fashion', the following.

$$\begin{bmatrix} (1-\gamma_1)Z_{11}^0 + & \cdots & + (1-\gamma_n)Z_{1n}^0 \\ \vdots & & \vdots \\ \vdots & & \vdots \\ (1-\gamma_1)Z_{n1}^0 + & \cdots & + (1-\gamma_n)Z_{nn}^0 \end{bmatrix} + \begin{bmatrix} \sum_i (1-\gamma_i)F_{1i}^0 \\ \vdots \\ \sum_i (1-\gamma_i)F_{ni}^0 \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} \quad [9]$$

The above equation system evidently 'looks like' an I-O system. But is it? For example, the balances now neatly add-up horizontally, but does looking at the individual column of the matrix to the left make sense in terms of a production process?

We recall that [9] is the best survey of still existing productive capacity that we possess about the economy in its totality. It can be viewed as a first effort at "bookkeeping" of the disaster aftermath. Let us call this the *Basic Equation*. However, this equation is indeed only a "bookkeeping" identity. It cannot be interpreted as an operative I-O system immediately after the flood. In fact, only looking at [9] may severely overestimate real world productive possibilities. We should realise that we need a two-step procedure here. In fact, in [9] a smaller I-O system is embedded that realistically informs us about the new situation. This smaller system can provide a basis for an analysis from the perspective of an *Event Matrix*. Below we shall introduce the basic issues, i.e. "digging up" the embedded viable I-O system in terms of a numerical example. However, a full exploration will have to be the subject for later and more detailed work.

4. Introducing Coefficients

There are several ways in which we can proceed now. For reasons of space we shall only take an example and explore two of them.

4.1. Homogeneous Shock

Let us take a further look at the system [9]. First consider vector *X*. As pointed out, it 'has the look' of a total output vector[10]. Given that it is a natural question to ask, which coefficients matrix to associate with it. Here again we have several possibilities. Let us see what the old coefficients matrix A^0 in [1] tells us, and let us ask if final demand as given by [9] is compatible with an economy that produces a total output vector $X=[x_i]$ employing technology *A*. Under technology *A* we have:

$$\begin{bmatrix} (1-\gamma_1)Z_{11}^0 + & \cdots & +(1-\gamma_n)Z_{1n}^0 \\ \vdots & & \vdots \\ (1-\gamma_1)Z_{n1}^0 + & \cdots & +(1-\gamma_n)Z_{nn}^0 \end{bmatrix} = \begin{bmatrix} (1-\gamma_1)a_{11}^0 x_1^0 + & \cdots & +(1-\gamma_n)a_{1n}^0 x_n^0 \\ \vdots & & \vdots \\ (1-\gamma_1)a_{n1}^0 x_1^0 + & \cdots & +(1-\gamma_n)a_{nn}^0 x_n^0 \end{bmatrix} =$$

$$= \begin{bmatrix} a_{11}^0 & \cdots & a_{1n}^0 \\ \vdots & & \vdots \\ a_{n1}^0 & \cdots & a_{nn}^0 \end{bmatrix} \begin{bmatrix} (1-\gamma_1)x_1^0 \\ \vdots \\ (1-\gamma_n)x_n^0 \end{bmatrix} \qquad [10]$$

Because we assume that the system is subject to a homogeneous shock, all its sectors are to be hit to the same extent. This means that $\gamma_i = \gamma$ for all *i*. Then [9] becomes

$$\left[\begin{vmatrix} a_{11}^0 & \cdots & a_{1n}^0 \\ \vdots & & \vdots \\ \vdots & & \vdots \\ a_{n1}^0 & \cdots & a_{nn}^0 \end{vmatrix}\right] \begin{bmatrix} (1-\gamma)x_1^0 \\ \vdots \\ \vdots \\ (1-\gamma)x_n^0 \end{bmatrix} + \begin{bmatrix} \sum_i (1-\gamma)F_{1i}^0 \\ \vdots \\ \sum_i (1-\gamma)F_{ni}^0 \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} \qquad [11]$$

[10] Recall in I-O analysis total output is obtained by solving equations like [3.1] above. Here the 'total output vector' has simply been obtained by addition of the terms on the left-hand side of the basic equation [3.9].

Or

$$(1-\gamma)A^0x^0 + (1-\gamma)f^0 = (1-\gamma)\,x^0 \qquad [12]$$

However, this is nothing but equation [1], premultiplied by the factor (1-γ), with total output being X=(1–γ)x^0. So, this special input-output equation shows that output just declines at the same rate for each sector, and no indirect effects are observed. The same conclusion we find *inter alia* in Cochrane (1997a, p.2).

4.2. Non-homogeneous Shock

Things become more complicated if we take the case where all industries suffer different level of disruption. In our notation this means that $\gamma_i \neq \gamma$. Then we will get:

$$\left[\begin{vmatrix} a_{11}^0 & \cdots & a_{1n}^0 \\ \vdots & & \vdots \\ \vdots & & \vdots \\ a_{n1}^0 & \cdots & a_{nn}^0 \end{vmatrix}\right]\begin{bmatrix} (1-\gamma_1)x_1^0 \\ \vdots \\ \vdots \\ (1-\gamma_n)x_n^0 \end{bmatrix} + \begin{bmatrix} \sum_i (1-\gamma_i)F_{1i}^0 \\ \vdots \\ \sum_i (1-\gamma_i)F_{ni}^0 \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} \qquad [13]$$

Here a number of issues come up to the surface. Similarly to the question posed in the beginning of the paper about the new distribution of the displaced final output between the intermediary and final demand, here we are interested to know, is new output, X, compatible with the final demand in [13]. Next, we would like to see the change in total output level with respect to the pre-disaster level. Thus, what is the relation between X^0 and X?

First, consider final demand. Let us call this new level of f 'maximal' final demand. But does this fit the sales pattern of final demand? We shall compare this final vector in [13] with the composition of pre-disaster f^0. Should we impose the initial (pre-disaster) final demand structure, this will result in a new x and a new employment figure. Let us have a look at various options of economic response patterns to a catastrophic event.

Restriction 1. Let's assume that after the outbreak of a disaster the actual economic system is rigid, and its structure stays the same, i.e. technical coefficients do not change. Therefore, as we know the maximum output level x and the a_{ij}'s, this time we will rearrange the basic input-output equation [1], making the final demand an unknown. We will assemble the resident capacity coefficients $(1-\gamma_i)$ with the respective elements of x^0. To ease the matrix notation, let's introduce $(I - \Gamma)$ for the diagonal matrix with $(1 - \gamma_i)$ at the i^{th} location on the main diagonal:

$$(I - \Gamma) = \begin{bmatrix} 1-\gamma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1-\gamma_n \end{bmatrix} \qquad [14]$$

Then we should get the following equality:

$$(I - \Gamma)\, x^0 = A^0 [(I - \Gamma)\, x^0] + f^1 \qquad [15]$$

Now let us express the final demand as a variable of interest through the known ones:

$$f^1 = (I - \Gamma)\, x^0 - A^0 [(I - \Gamma)\, x^0] \qquad [16]$$

Now again we face several decisions. First of all, we have to compare f^1 with final demand in [3. in the 'just after disaster' situation, and take a look at the difference. Secondly, suppose this f^1 is feasible. We have to ask if it is economically acceptable and if it is in line with the traditional preference pattern.

Restriction 2. Following the argumentation above, we provide the system with a restriction over the structure of final demand. We suggest, that its structure does not change just after a disaster. Thus, in mathematical terms this means that we have to model output on the basis of the following formula, replacing final demand in the standard formulation by $[\varepsilon f^0]$,

$$x^2 = A^0 x^2 + \varepsilon f^0, \qquad [17]$$

where ε is the is a scalar, so that $0 \leq \varepsilon \leq 1$.

Finally, suppose that in the post-disaster situation government policy is aiming at pursuing final demand maximisation. Let us see what kind of effects will be rippled throughout the economy then. In our notification this will end up in the maximisation problem such as

$$\begin{cases} \boldsymbol{max}\ \{\varepsilon f^0\}: \quad \rightarrow \quad x^2 = A^0 x^2 + \varepsilon f^0, \\ \text{s.t.} \qquad\qquad\qquad x^2_i \leq x^0_i\,(1-\gamma_i) \end{cases} \qquad [18]$$

In effect if we rearrange the objective function, mathematically it will suggest that the output levels should also decrease by the same proportion as the final demand, i.e.

$$x^2 = \varepsilon\, x^0, \qquad [19]$$

and we obtain

$$\varepsilon\, x^0 = (I - A^0)^{-1}\, \varepsilon f^0 \qquad [20]$$

it therefore can be concluded that

$$\varepsilon = max\ \{(1-\gamma_i)\}, \qquad [21]$$

which says that in the optimum in the after-shock situation, provided the restrictions on the production and final demand structures, both output and final demand should shrink by the proportion of the mostly disrupted sector. This sector will evidently act as ‘bottleneck’ for the whole economy. Consequently, final demand (especially for the less damaged sectors) will also be far under the highest possible level taken the restricted response of an economy to a shock.

4.3. Allowing More Flexibility

On the other hand, suppose that producers are able to adjust to the new circumstances via shifts in production technology. Let us see how in case of extreme flexibility the new technology matrix will look like. We start again from the *Basic Equation*, and we will try to determine a new coefficients matrix, starting from our knowledge of maximum possible output after a disaster x. We now identify 'flexibility' as a situation when an economy is capable of an extreme adaptability to employ its at the moment restricted output. Thus, we fix x. We then have:

$$\begin{bmatrix} (1-\gamma_1)Z_{11}^0 + & \cdots & +(1-\gamma_n)Z_{1n}^0 \\ \vdots & & \vdots \\ (1-\gamma_1)Z_{n1}^0 + & \cdots & +(1-\gamma_n)Z_{nn}^0 \end{bmatrix} = \begin{bmatrix} \tilde{a}_{11}\, x_1 & \cdots & \tilde{a}_{1n}\, x_n \\ \vdots & & \vdots \\ \tilde{a}_{n1}\, x_1 & \cdots & \tilde{a}_{nn}\, x_n \end{bmatrix} \qquad [22]$$

As far as we have fixed for the time being the output level at its new post-disaster maximum level, the only unknown now is the technology, i.e. $\tilde{a}_{ij}$ coefficients. This procedure results in a new coefficients matrix, to be called $\tilde{A}$. The link between $\tilde{A}$ and A^0 can be established via a new matrix E, such as:

$$\tilde{A} = E\, A^0 \qquad [23]$$

If we confirm along this path, we shall face the task to define a proper set of properties for the matrices $\tilde{A}$ and E to satisfy to be acceptable as I-O based technology matrices.

It is important to observe here, that the approach we have presented is not an exhaustive coverage of the issue of disaster modelling within the I-O framework. It should rather be seen as the way of thinking about how things might work, applying it to the rich though not yet fully uncovered features of the I-O model.

This type of analysis can be extended and applied in bi- or multi-regional frameworks. By doing so one will be able to capture additional economic insights of the performed analysis. For expository reasons we shall below provide the 2×2 case.

5. A Numerical Example

The numerical example below may clarify part of the intuition behind the mechanisms introduced in the previous section. We shall explore the initial effects of an asymmetric shock on a simple 2×2 input-output model. Assume that the initial situation looks like:

$$\begin{array}{ccccc} \boldsymbol{Z^0} & + & \boldsymbol{f^0} & = & \boldsymbol{x^0} \\ \begin{pmatrix} 50 & 20 \\ 10 & 20 \end{pmatrix} & + & \begin{pmatrix} 80 \\ 40 \end{pmatrix} & = & \begin{pmatrix} 150 \\ 70 \end{pmatrix} \end{array} \qquad [24]$$

The corresponding coefficient matrix is:

$$A^0 = \begin{pmatrix} 0.333 & 0.286 \\ 0.067 & 0.286 \end{pmatrix} \qquad [25]$$

Suppose that knowledge of the labour inputs allow us rewrite the system as in [6]:

$$\begin{array}{cc} \textit{Sector } \boldsymbol{1} & \textit{Sector } \boldsymbol{2} \\ \multicolumn{2}{c}{\left(\begin{array}{cc|cc} 50 & 60 & 20 & 20 \\ 10 & 30 & 20 & 10 \end{array}\right)} \\ \text{production} & \text{final demand} \end{array} \qquad [26]$$

That is, to produce 150 units of the first good the industry needs three times as many workers as the second industry needs to produce its 70 units. We know from above the real wage in terms of final consumption demand that these workers claim. The individual wage bundles for the first sector then add up to 90: 60 units of the first good and 30 units of the second. Thus, full operation expenditures for the first industry constitute a total of (50 + 60) = 110 units of the first good and (10 + 30) = 40 units of the second. For the second industry a similar exercise can be done.

We now introduce an asymmetric shock caused by the flood: $100\gamma_1 = 50\%$ of sector 1 capacity is lost and $100\gamma_2 = 10\%$ of sector 2 capacity. Therefore, we can now rewrite the original (composite) transactions matrix as:

$$\left(\underbrace{\boxed{\begin{matrix}25 & & 30\\ & + & \\ 5 & & 15\end{matrix}}}_{\text{lost}} \; \underbrace{\boxed{\begin{matrix}25 & & 30\\ & + & \\ 5 & & 15\end{matrix}}}_{\text{active}} \; \underbrace{\boxed{\begin{matrix}|\,2 & & 2\\ & + & \\ |\,2 & & 1\end{matrix}}}_{\text{lost}} \; \underbrace{\boxed{\begin{matrix}18 & & 18\\ & + & \\ 18 & & 9\end{matrix}}}_{\text{active}} \right) \qquad [27]$$

This means we can separate the lost and the remaining active parts of the M^0 matrix (see equation [8]):

$$\left(\underbrace{\boxed{\overset{\textit{intermediate part lost}}{\begin{matrix}25 & 2\\ 5 & 2\end{matrix}} + \overset{\textit{demand part lost}}{\begin{matrix}30 & 2\\ 15 & 1\end{matrix}}}}_{\text{lost}} + \underbrace{\boxed{\overset{\textit{intermediate part left}}{\begin{matrix}25 & 18\\ 5 & 18\end{matrix}} + \overset{\textit{demand part left}}{\begin{matrix}30 & 18\\ 15 & 9\end{matrix}}}}_{\text{active}} \right) \qquad [28]$$

The lost production therefore can be represented as:

$$\overset{\textit{intermediate part lost}}{\begin{pmatrix}25 & 2\\ 5 & 2\end{pmatrix}} + \overset{\textit{demand part lost}}{\begin{pmatrix}32\\ 16\end{pmatrix}} = \overset{\textit{output part lost}}{\begin{pmatrix}59\\ 23\end{pmatrix}} \qquad [29]$$

and the remaining production as:

$$
\begin{array}{ccccc}
\textit{intermediate part} & & \textit{demand part} & & \textit{output} \\
\textit{active} & + & \textit{active} & = & \textit{active}
\end{array}
$$

$$\begin{pmatrix} 25 & 18 \\ 5 & 18 \end{pmatrix} + \begin{pmatrix} 48 \\ 24 \end{pmatrix} = \begin{pmatrix} 91 \\ 47 \end{pmatrix} . \qquad [30]$$

The latter identity is what we would like to call the *Basic Equation*. It provides us with insight what is left of the economy immediately after the shock. However, as pointed out earlier, [30] *cannot* be interpreted as an I-O model of the affected area. To see this, we only need to calculate the implicit input coefficients. This means, that the derived identity should be viewed as a starting point to isolate the (smaller scale) viable I-O system embedded in it. Here a number of options exist. In any case, we shall need additional argumentation to derive acceptable *real* balances from the Basic Equation. To accomplish that, we shall have to introduce the whole gamut of the behavioural, historical and other background information that should be – hopefully – available. That, however, must be left for later exercise.

It is of interest though to notice that in this case the relation between the demands of good 1 and 2 in [30] stays the same as in original matrix [24] - 2:1. If we suppose that the equality [30] is a viable input-output system, it's corresponding coefficient matrix would be:

$$A = \begin{pmatrix} 0.275 & 0.383 \\ 0.055 & 0.383 \end{pmatrix} \qquad [31]$$

Above we have discussed that most probably equation [30] can be merely considered a bookkeeping identity, and one has to impose additional assumptions to extract an *input-output system* from this Basic Equation. Therefore, we assume that production coefficients stay unchanged after the shock. This corresponds to the following 'embedded' I-O system (see Restriction 1, [15], section 4.2):

$$\begin{array}{ccccccccc} \boldsymbol{A^0} & \cdot & \boldsymbol{x^1} & + & \boldsymbol{f^1} & = & \boldsymbol{x^1} \end{array}$$

$$\begin{pmatrix} 0.333 & 0.286 \\ 0.067 & 0.286 \end{pmatrix} \cdot \begin{pmatrix} 75 \\ 63 \end{pmatrix} + \begin{pmatrix} 32 \\ 40 \end{pmatrix} = \begin{pmatrix} 75 \\ 63 \end{pmatrix} \qquad [32]$$

Compared to the pre-disaster final demand structure $\boldsymbol{f^1}$ shows that final demand has adjusted and it has higher preference towards the second good. On the one hand, it can hardly be imagined that consumers' preferences change so dramatically, shifting in favour of the good that can be produced relatively in bigger proportion than before the shock. On the other hand, this can be true if we assume that the economy is open and can find external markets for the good it now has in relative abundance.

Finally, let us assume now that in addition to that government aims at pursuing final demand maximization, provided the final demand structure remains the same as in the pre-disaster situation (i.e. consumer preferences are unchanged). In mathematical terms this means that we have to model on the basis of Restriction 2 (see formula [17]). Let's look at the figures:

$$\begin{array}{ccccccccc} \boldsymbol{A^0} & \cdot & \boldsymbol{x^2} & + & \boldsymbol{f^2} & = & \boldsymbol{x^2} \end{array}$$

$$\begin{pmatrix} 0.333 & 0.286 \\ 0.067 & 0.286 \end{pmatrix} \cdot \begin{pmatrix} 75 \\ 35 \end{pmatrix} + \begin{pmatrix} 40 \\ 20 \end{pmatrix} = \begin{pmatrix} 75 \\ 35 \end{pmatrix} \qquad [33]$$

As a result of such a policy we can observe that both variables drop by half – both final demand and the total output level shrink to the 50% capacity level for both industries. This suggests, that the economy is in a state when after an asymmetric shock some industries are capable of higher levels of production, but are restricted by other industries' outputs. In other words, this is the case of 'production bottleneck': the whole economy reduces its output to the level of the most hit sector. In our example, sector two maximally can produce 63 units of output, but is forced to supply on the market only 35. This means, that this sector suffers indirect economic loss of 44%.

6. Conclusions

In this paper we have proposed a theoretical foundation based on I-O methodology for problem approach of severe changes in economic structure. We put forward that it is extremely important to have a good basis for discussing the consequences of the catastrophe and the subsequent recovery period. We discuss what we view as a '*Basic Equation*'. This equation serves as a starting point for investigating various resilience issues and may lead to a better presentation of the concept of an 'Event Matrix'.

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