An Effective Allocation of Investment Resources Using a Linear Programming Model Integrated with a SAM

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Abstract

Despite the crucial significance of appropriate investment in economic development of regions, a great many of them suffer from serious investment shortages. To this end, this paper proposes a procedure for an effective investment allocation: a Linear Programming Model linked to Social Accounting Model. Since it seems that the priorities of production activities in a country or a region is an essential step, its equation is taken into account as the objective function. Several constraints such as job creation for different levels of human force, income distribution inequality, supply and demand constraints for products are considered in the model. The economic sectors of the region for a certain proportion of products are ranked with respect to the above objective function and constraints to specify the level at which each sector is to be expanded in any level of investment. The model is examined with the SAM of the Golestan province in Iran for the year 1993/1994. The results are flexible for any level of investment (public or private) for a given year.

Keywords: Social Accounting Matrix, Linear Programming, Investment Allocation, Iran, Golestan Province

Introduction

Almost all of the development theories such as Smith (ed. 1904), Keynes (1936), Myrdal (1957), and Solow (1956) consider capital as the main factor for economic growth and development. In addition, in view of experiment, there are a number of recent pieces of evidence such as Yue (1999) and Gripaios *et al.* (1997) that confirm the role of investment in economic growth and development. But the shortage of investment has been recognised as one of the problems of developing countries. Hence, an adequate procedure for investment allocation seems to be able to affect economic growth.

Plenty of research have been carried out to compare production sectors. For instance, in a case study in Sudan, Hassan (1994) compared production sectors on growth and income distribution through SAM multipliers analysis. Havinga *et al.* (1987) also used agricultural sector of Pakistan SAM multipliers analysis to compare the production sectors through the result of an exogenous increase in demand on the total production, income distribution and households consumption. In addition, in the Kickapoo valley`s case study, the actual and proportional income distribution impacts on household income groups as a result of change in institutional income was analysed by Leatherman and Marcouiller in 1996. Furthermore Thorbecke and Jung`s (1996) case study on Indonesia, SAM multiplier was used to analyse and measure the impact of different production activities (sectors) on poverty alleviation. Therefore, as shown above, the SAM multiplier analysis seems to be quite common in case studies.

Thus this paper intends to propose a technique for optimum allocation at any level of investment on alternatives production sectors. For this purpose, different groups of production sectors with certain levels of production are ranked with respect to GRP and constraints of the planning model. By estimating the necessary investment (through a relation between outputs of sectors and the required investment) for different level of the optimum GRP, the required investment for all collection of sectors are estimated. To demonstrate the result of considering investment effectiveness in its allocation, the results of allocation are compared in two opposite conditions, i. e., considering or ignoring the investment effectiveness. Finally, it is demonstrated that it is possible to find an optimum way for any level of investment allocation with respect to the aims of planning.

The linear programming model including of the objective function and all constraints is introduced in the next section. Then the collections of economic sectors are ranked by the proposed model through a discussion to achieve the highest GRP with respect to or irrespective of the required investment. Finally, the results of discussion are classified as conclusion.

Linear Programming Model

A Linear Programming Model linked to a Social Accounting Model is used in this paper. The GRP of the region is taken into account as the objective function. Several constraints in terms of job creation for different groups of human force, income distribution inequality, supply and demand constraints for products are considered.

The social accounting matrix of Golestan Province in Iran for the year 1993/1994 is used to estimate the related coefficients as shown in Table 7. The matrix consists of seven accounts designated as production factors, production activities, households, other institutions, Investment/ saving, government and rest of the world including rest of Iran. The matrix consists of 54 rows and columns, 9 production factors groups, 27 production activities sectors, 10 households groups, 5 other institutions, and one row and column for other accounts.

The GRP of the region can be divided into two devices. The first part includes the value added concerning private or public production factors that is generated in the region and are examined by relation (1). The M^{v1} is a row vector in which M^{v1}_j concerns the vertical sum of the block M^{v1}, associated with the production factors' income in production activities, in matrix M_n

the Leonties matrix's inverse of the SAM. Thus, $\mathrm{M}^{\mathrm{v}1}{}_{\mathrm{j}} = \sum_{\mathrm{i}=1}^{9} m_{i,j}$ reveals the impact of a one unit exogenous final demand generated in the j^{th} sector of the region on the production holders' income and Y^* is a subvector of Y associated with the exogenous final demand for products of production sectors including Y_{10}, Y_{11}, Y_{12}, …, Y_{36}. Hence, GRP_1 explores changes in the total income generated as a result of responding to the exogenous final demand for products of the region:

$$\begin{aligned} GRP_1 = M^{v1} \times Y^* &= \sum_{i=1}^{9} m_{i,10} \times Y_{10} + \sum_{i=1}^{9} m_{i,11} \times Y_{11} + \sum_{i=1}^{9} m_{i,12} \times Y_{12} \Lambda + \sum_{i=1}^{9} m_{i,36} \times Y_{36} = \\ M_1^{v1} \times Y_{10} &+ M_2^{v1} \times Y_{11} + M_3^{v1} \times Y_{12} + \Lambda + M_{27}^{v1} 1 \times Y_{36} \end{aligned} \tag{1}$$

The second part of GRP concerns the net indirect taxes received by the government in the region. This part denoting GRP_2, is recorded as the government income from production activities that is embedded in the exogenous part of the table. Since the net indirect tax depends on the level of the products of production sectors it can be formulated with respect to the level of these products:

$$GRP_2 = t \times X^* = t \times M^p \times Y^* \tag{2}$$

where X^* is a column subvector of X concerning the total products of the sectors including X_{10}, X_{11}, X_{12}, …, X_{36}. In addition, t and M^p are the row vectors in which t_j refers to the net indirect taxes received from a unit of goods or services produced in the j^{th} production sector and a submatrix of M corresponding intermediate transaction that measures the effect of one unit exogenous increase in final demand for products of the region on total products.

If:

$$C = t \times M^p \tag{3}$$

GRP_2 can be rewritten as:

$$GRP_2 = C \times Y^* \tag{4}$$

Where C is a row vector, $C_1, C_2, C_3 \ldots C_n$, denoted as the total net indirect tax receivable in the region from a unit increase in $Y_{10}, Y_{11}, Y_{12}, \ldots, Y_{36}$, respectively.

Finally, GRP of the region can be derived by summation of increasing GRP_1 and GRP_2 that are examined through equation (5).

$$GRP = GRP_1 + GRP_2 = (M_1^{v1} + C_1) \times Y_{10} + (M_2^{v1} + C_2) \times Y_{11} + (M_3^{v1} + C_3) \times Y_{12} + \Lambda + (M_{27}^{v1} + C_{27}) \times Y_{36} = g_1 Y_{10} + g_2 Y_{11} + g_3 Y_{12} + \Lambda + g_n Y_{27} \quad (5)$$

G is a raw vector in which g_j is place of $M^{v1}{}_j + C_j$.

The relations concerning to job creation for human force are considered as constraints of this model. These relations pertain to under HS diploma, HS diploma, undergraduate and postgraduate employment.

$$N_{1,1}Y_1 + N_{1,2}Y_2 + \Lambda + N_{1,n}Y_n \leq A_1 \quad (6)$$

$$N_{2,1}Y_1 + N_{2,2}Y_2 + \Lambda + N_{2,n}Y_n \leq A_2 \quad (7)$$

$$N_{3,1}Y_1 + N_{3,2}Y_2 + \Lambda + N_{3,n}Y_n \leq A_3 \quad (8)$$

$$N_{4,1}Y_1 + N_{4,2}Y_2 + \Lambda + N_{4,n}Y_n \leq A_4 \quad (9)$$

where N_{ij}`s show the total i^{th} group's human force that would be employed for a unit increase in the exogenous final demand for j^{th} sector's products. A_1, A_2, A_3 and A_4 refer to the maximum size of different educational groups of human force's supply in this wage level. The less and equal signs of relations enable us to prevent extra employment with respect to labour's supply of the region that may lead to some problems for the region due to immigration if assumed necessary.

To prevent from an undesired level of unemployment for different groups of human force, the relations (10) to (13) are used in the model. E_1, E_2, E_3 and E_4 refer to the level of minimum desire level of job creation for different groups of human force. The left hand side of relations (6) to (9) and (10) and (13) measure the size of demand for different groups of human force which are exactly the same.

$$N_{1,1}Y_1 + N_{1,2}Y_2 + \Lambda + N_{1,n}Y_n \geq E_1 \tag{10}$$

$$N_{2,1}Y_1 + N_{2,2}Y_2 + \Lambda + N_{2,n}Y_n \geq E_2 \tag{11}$$

$$N_{3,1}Y_1 + N_{3,2}Y_2 + \Lambda + N_{3,n}Y_n \geq E_3 \tag{12}$$

$$N_{4,1}Y_1 + N_{4,2}Y_2 + \Lambda + N_{4,n}Y_n \geq E_4 \tag{13}$$

Therefore, the two groups of relations denoted by (6) to (7) and (10) to (13) prevent extra demand for human force and undesired level of unemployment for different groups of human force, respectively. Thus, these groups of constraints can be used as an instrument for decision making on job creation in the region.

The mean income level for new human force of the region relation is another constraint that will be considered in this model. This constraint is shown as relation (14). $b_{1,j}$ measures the role of a unit exogenous final demand for products in sector j on the mean income for new employed human force of the region. B_1 denotes the minimum desire level that is specified as constraint for the mean income of the new human force that are employed in the region. In addition, the greater or equal sign of the relation let the mean income of private production factors holders of the region be specified more than or at least equal to a minimum desire level in the resource allocation process.

$$\mu = b_{1,1}Y_{10} + b_{1,2}Y_{11} + \Lambda + b_{1,n}Y_{36} \geq B_1 \tag{14}$$

Non-equation (15) concerns the new employed income distribution inequality. The relative mean deviation index, *I*, is used that can be written as a linear form with some preparation. *N* refers to the number of new employment and M^{pc} components exhibit the total difference of sectors production factors' per capita income from the average level per capita income of the region due to a unit exogenous final demand for goods and services produced in these sectors. Thus, $b_{2,j}$ reveals the impact of a unit increase in exogenous final demand for goods or services that is produced in the j^{th} production sector on income distribution inequality of new employed of the region. In addition, B_2 shows a maximum acceptable income distribution inequality for the human force that will be employed in the region.

$$I = (1/N\times\mu)M^{pc} = (1/N\times\mu)(M_1^{pc}\times Y_{10} + M_2^{pc}\times Y_{11} + \Lambda + M_{27}^{pc}\times Y_{36}) =$$
$$b_{2,1}\times Y_{10} + b_{2,2}\times Y_{11} + b_{2,3}Y_{12} + \ldots + b_{2,n}\times Y_{36} \le B_2 \qquad (15)$$

Non- equation (16) is used as constraint of the model to consider the supply and demand for products of production sectors of the region. Hence, non-equation (16) is representative of 27 constraints for products of 27 production sectors in which d_i, the maximum possible products of sector i, is shown in Table 7. Thus, non-equation (16) is considered as another constraint of the model.

$$X^* = M^p \times Y^* \le d_i \qquad ,i = 1,\Lambda\ ,n \qquad (16)$$

Finally, since all of the decision variables are as exogenous final demand for products of sectors, however, Y_is would be greater than or at least equal to zero, as shown in (17).

$$Y_i^* \ge 0 \qquad ,i = 1,\Lambda\ ,n \qquad (17)$$

Table 1 Production sectors of the region

Title of sectors	j: Sectors No.	Title of sectors	j: Sectors No.	Title of sectors	j: Sectors No.
Farming	1	Textile industries	10	Water, Electricity and Ga	19
Traditional Livestock	2	Carpet	11	Construction	20
Modern Husbandry	3	Wood Products	12	Communication	21
Modern Hen-breeding	4	Publication & Paper	13	Transportation	22
Fish-breeding	5	Chemical Products	14	Bank and Insurance	23
Forestry	6	Non-Metals Products	15	Education	24
Fishery	7	Metal Products	16	Health	25
Mining	8	Machinery Products	17	Public Services	26
Food Processing Industries	9	Other Industry	18	Personal Services	27

Ranking of economic sectors

As mentioned above, investment is recognised as an important factor in most economic growth theories. It is also considered as a barrier in the way of progress in developing countries. In other words, most of these countries are facing shortage of investment for their development programs. Consequently, it is crucially important to find a more effective allocation for investment in developing countries.

There are numerous of plans that have not been implemented properly due to insufficient investment in developing countries. In addition, since different parts of any plan are interrelated the desired aims will be achieved when a plan is implemented thoroughly. But it seems it is possible to find an optimum allocation for any level of investment.

To this end, this paper intends to propose a technique to find the optimum allocation for any level of investment on alternatives production sectors of a region or a country. To do so, different groups of the economic sectors with certain levels of products have been ranked with respect to their priority in viewpoint of the GRP maximisation of the region (as the objective function of planning process) considering the above constraints. In the second step, the required investment, which is to be made to be on the production sectors leading to the highest level of GRP in comparison to other available groups, will be considered.

The linear programming model is used through different stages. The above model is solved in the first stage excluding non-equations (16). In other words, there is no supply and demand constraint in the first stage. Since the number of decision variables (products of sectors) is greater than the number of constraints of the model, it is expected that the model have zero solutions for at least a group of sectors whose number is at least equal to the surplus of the number of decision variables from those of the constraints. The optimum solutions of the model include both zero and non-zero solutions, requiring the optimum level of products of sectors that leads to the optimum value of GRP. In fact, since the sectors that are associated with non-zero solutions are selected with respect to the condition of the model irrespective of the necessary investment, these sectors are considered as the first highest rank sectors for GRP maximisation in comparison to other sectors. As demonstrated below, attention is also to be paid to the relation of the non-zero solution (level of products) of these sectors for planning purposes.

In the second stage, a new solution can be obtained for the highest rank sectors by adding the supply or demand constraints on products from non-equation (16) associated with these sectors to the linear programming model of the previous stage. Since any constraint such as a barrier prevent the maximisation of the objective function, hence the optimum GRP obtained in the second stage is less than or equal to that of the first stage. Consequently, the results for the sectors are considered as the second highest rank.

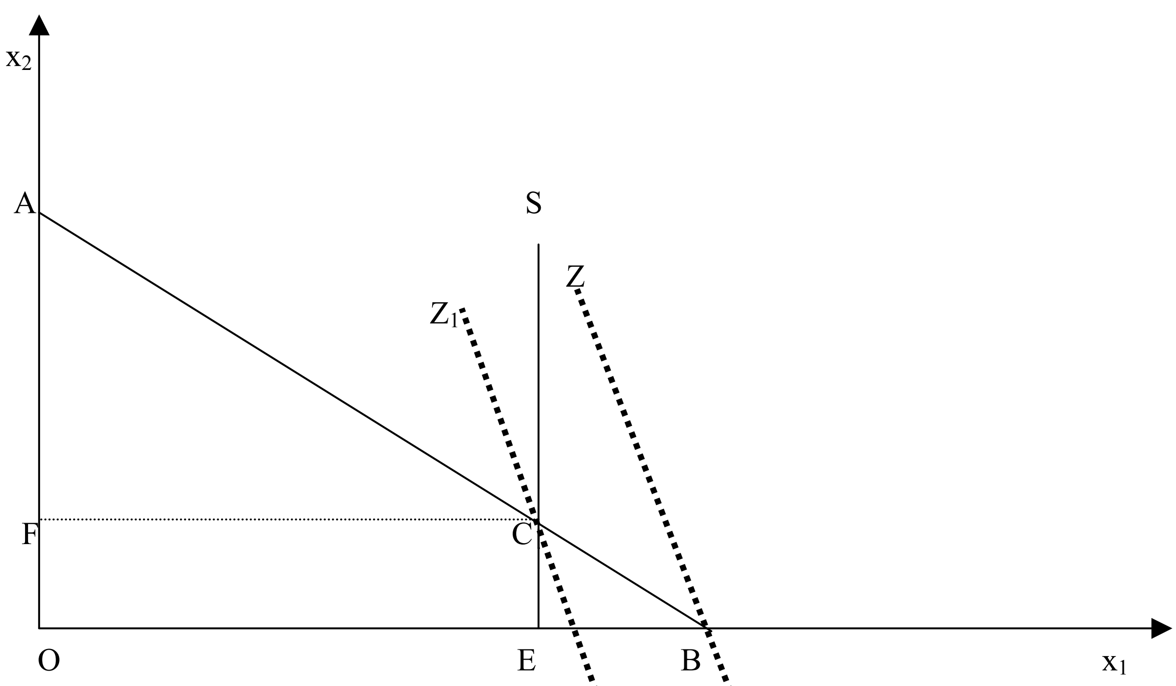

Figure 1 The effects of supply and demand considering on maximum solution

The above stages can be illustrated through a very basic model in Figure 1. In this figure, x_1 and x_2 are two decision variables that display the level of the outputs of sectors I and II, respectively. The line AB runs as the constraint of the model. Thus, triangle OAB specifies the feasible area of the model. Z is assumed to have the slope of the objective function. In this simple linear programming model, B, the intersection of the feasible area with the highest available level of the objective function slope line, shows the optimal solution of the model. Hence, the optimum solution for the model is determined as x_1 = OB and x_2 = 0. In other words, sector I is recognised as the first rank sector.

In the second stage, it is assumed that due to supply or demand constraint of the sector I, it is not possible to produce more than OE units in this sector. Hence, the ES is another constraint of the model and the feasible area for the solution of the model changes into trapezium OACE. Therefore, C, the intersection of the feasible area with the highest level of the objective function slope shows the position of the optimum solution of the model. As a result, the x_1 = OE and x_2 = OF are considered as the optimum solutions of the model. In fact, the latter group of solutions are obtained when, due to supply or demand constraint, it is not possible to produce on the B position (x_1 = OB and x_2 = 0) that is associated to the Z the first highest rank. Hence, the C position can be considered as the second highest rank.

In the above model, the objective function and all constraints except supply and demand of sectors were solved. In the first stage, sectors 16, 21, 24, 25 and 26 that are associated to Metal Products, Communication, Education, Health and Public Services were selected as the highest rank sectors in which the ratio of products of these sectors (as mentioned) should also be considered. For instance, based on these calculations, a collection of 17820, 93579, 15160, 2085 and 14215 million rials worth production in Metal Products, Communication, Education, Health and Public Services sectors, in that order, will maximise the GRP of the region considering the above constraints. In the second stage, by considering the supply or demand constraints of the region on products of these sectors as they are shown in Table 7, these five sectors' supply or demand constraints were added to the first stage model. By considering constraints for products associated to non-zero solution sectors in the first stage, a collection of sectors 6, 16, 21, 24, 25 and 26 were selected as the second highest rank, etc. All eligible sectors were specified as the non-zero solution set in a total of 13 stages. Sector 27, relating to Personal Services, was chosen in the 12th stage and hence, its supply and demand constraint being considered in the 13th stage. Therefore, the 13th stage solutions are with respect to the Personal Services sector's supply and demand constraint.

Table 2 Ranking of sectors based on objective function and constraints.

Rank	Sectors No.	entered	Droped
1	16,21,24,25,26	16,21,24,25,26	-
2	6,16,21,24,25,26	6	-
3	5,6,16,21,23,24,26	5,23	25
4	6,8,16,21,23,24,26	8	5
5	6,8,16,18,21,23,24,26	18	-
6	5,6,7,8,16,18,21,23,24,26	5,7	-
7	4,5,6,7,8,16,18,21,23,24,25,26	4,25	-
8	4,5,6,7,8,12,16,18,21,22,23,24,25,26	12,22	-
9	4,5,6,7,8,12,15,16,18,21,22,23,24,25,26	15	-
10	4,6,8,12,13,16,21,23,24,25,26	13	5,7,15,18,22
11	3,4,6,7,8,12,13,14,16,18,21,22,23,24,26	3,7,14,18,22	25
12	3,4,6,7,8,12,13,14,16,18,21,22,23,24,26,27	27	-
13	3,4,6,7,8,12,13,14,16,18,21,22,23,24,26,27	-	-

To conclude, according to Figure 1, each stage leads to a higher (or at least an equal) optimum value for GRP in comparison to the next one (position of B in compare to C) if there is no constraint for its sectors' supply or demand and it can be fully implemented. Thus the related GRPs are calculated for Table 3.

Table 3 The Maximum GRP resulted for different rank (1000 Rials)

Rank	GRP	Rank	GRP	Rank	GRP
1	246679642	6	230816799	11	220790636
2	235225805	7	225097617	12	220574601
3	232564759	8	224269886	13	220574601
4	231558224	9	224110824		
5	231261749	10	221355073		

However, the above results were obtained irrespective of any of the constraints on the new non-zero solutions at any stage. Since these constraints are determined independent from the role of these sectors in the model, the available result may change the sectors' priority in Table 3. Thus the maximum available GRP at different stages are calculated for the Table 4.

Table 4 The maximum available GRP, required investment and GRP/Investment relating different ranks

Rank	GRP*	Required Investment**	GRP / Investment***	Rank	GRP	Required Investment	GRP / Investment
1	33841812	534811	63.3	8	32362762	334368	96.8
2	20399124	325342	62.7	9	32257880	360084	89.6
3	30599891	288975	105.9	10	32208635	287270	112.1
4	32732684	342011	95.7	11	31043050	311121	99. 8
5	2637896	29381	89.8	12	30954247	312072	99.2
6	14245287	174598	81.6	13	30954247	312072	99.2
7	32768926	319557	102.5				

* (1000 Rials) ** (1,000,000 Rials) ***(thousands Rials GRP for per 1,000,000 Rials Investment)

As shown above, the maximum available GRP associated to sectors which were selected in the second stage is less than that resulted in the first stage. Therefore, since the resulted GRP concerned with the first stage is greater than that of the second one, irrespective of the required investment, the sectors that were selected in the first stage are preferred to those of the second stage, and the sectors selected at the first stage will be chosen to receive up to 33,841,812 thousands Rials level of GRP. Thus, the collection of sectors that were selected in the second stage would be ignored and a similar comparison is made done between the resulted GRP of the first and the third stages. If the third stage leads to a greater value for the GRP (e.g. X thousands Rials) in comparison to its first value, and a value from 33,841,812 thousands Rials to X thousands Rials for the GRP is planned, this collection of sectors will be preferred as the second rank.

But, in rare cases, all sectors lead to a GRP less than that of the first one; through a similar analysis, that is there is no the second best collection that can be considered as the second rank in this case study. Thus, Table 5 that is obtained from Table 4 that can be used when there is no shortage in investment has only one rank.

Table 5 Ranking the collection of economic sectors based on the maximum available GRP (regardless of investment effectiveness) (1000Rials)

Current Rank	Previous rank in Table 4	Sectors' No.	GRP
1	1	16,21,24,25,26	33,841,812

As to the classification of economic sectors based on maximum effectiveness of investment, it is necessary to consider the required investment for different collection of sectors, as displayed in Table 4. For this purpose, it is necessary to calculate the required investment associated to the collection of sectors, which are categorised as different ranks. Hence, in this calculation the required investments were applied to create a unit of output in different sectors of the region. In addition, dividing the GRP associated to any rank to their related required investment, the values of GRP resulted from one million Rials investment for different collections of sectors were obtained, see Table 4.

The ratio of GRP to investment was applied to have a maximum effectiveness for different levels of investment. For this purpose, first, different rows of Table 4 were sorted decreasingly with respect to the ratio of GRP to investment. The highest collection of sectors in viewpoint of GRP to investment ratio is specified as the first rank collection that has the maximum effectiveness for investment. In the second stage, with respect to the related GRP, the rows whose GRPs were less than or equal to that relating the maximum GRP/Investment ratio were ignored. This is due to the possibility of achieving this value of GRP by less investment or allocating the investment with a higher effectiveness. Thus, the second stage obtained with respect to the second best GRP/Investment ratio, its GRP being greater than that of the first rank among the remaining rows. By continuing this procedure, other collections of sectors were specified as displayed in Table 6.

Table 6 Ranking the collection of sectors with respect to higher effectiveness of investment

(1000 Rials)

Current Rank	Rank in Table 4	Sectors' No.	GRP*	Required Investment+	GRP / Investment^
1	10	4,6,8,12,13,16,21,23,24,25,26	32208635	287270	112.1
2	7	4,5,6,7,8,16,18,21,23,24,25,26	32768926	319557	102.5
3	1	16,21,24,25,26	33841812	534811	63.3

* (1000 Rials) + (1,000,000 Rials) ^(thousands Rials GRP for per 1,000,000 Rials Investment)

A comparison of Table 5 and Table 6 reveals that there are differences among the results of these tables. The collection of sectors denoted as rank 1 in Table 2 is specified as rank 1 in Table 5 to maximise GRP of the region with respect to the constraints of the model. But the collection of sectors denoted as rank 10 in Table 2 is specified as rank 1 in Table 6 to maximise the GRP of the region for investment funds up to 287,270 millions Rials with respect to the same constraints considered for Table 5 collections. Hence, it can be demonstrated that 534,811 million rials investment is required to reach 33,841,812 thousand rials value of GRP by using the first rank collection of sectors in Table 5, whereas 301,890 million rials investment will be enough to reach the same level of GRP by selecting the first rank collection of sectors in Table 6 that is about 44 percent less than that required by using the first collection.

In addition, the results of Table 6 can be worked out by calculating the required investment. For this purpose, the required investment associated to a certain level of GRP can be calculated for other collections of sectors. For example, the required investment for 32,208,635 thousands Rials GRP can be obtained by 287270, 314093 and 509002 millions Rials investment through collection of sectors denoted as the first, second and third ranks, respectively.

Therefore, the initial funds of investment are advised to be made on the sectors denoted as rank 1 in Table 6. Thus, considering the determined values as outputs of these sectors leads to the highest efficiency for investment. Obviously, when higher funds of investment is available, the collection of sectors relating the second or third rank will lead to higher levels of GRP, respectively, though being less effective for investment.

Moreover, it can also be proved that any proportion of a linear programming solution will be optimum when compared with similar feasible solutions. This can be proved through multiplying all the right hand sides constraints' by a positive value, which leads to the same proportion change on the optimum solutions of the model (see Appendix A). For example, in the case of Figure 2, when, due to insufficient investment, allocation is less than that determined at the first stage, the optimum solutions of the model will lie on OE. But the optimum solutions of different levels of the second stage will lie on GC so that, if continued, the line will pass O, i.e., the intersection of axes. Consequently, in the case of two or three variables, the sets of optimum solutions lie on a straight line that passes through the intersection of axes.

Therefore, any proportion of the first stage products leads to a maximum GRP in the region in comparison with any other collections of products. For example, when the investment funds are less than 287,270 million rials, the most effective investment will be obtained by multiplying the share of each sector's investment in 287,270 million rials associated with the maximum GRP by the ratio of the proposed investment funds to 287,270 million rials .

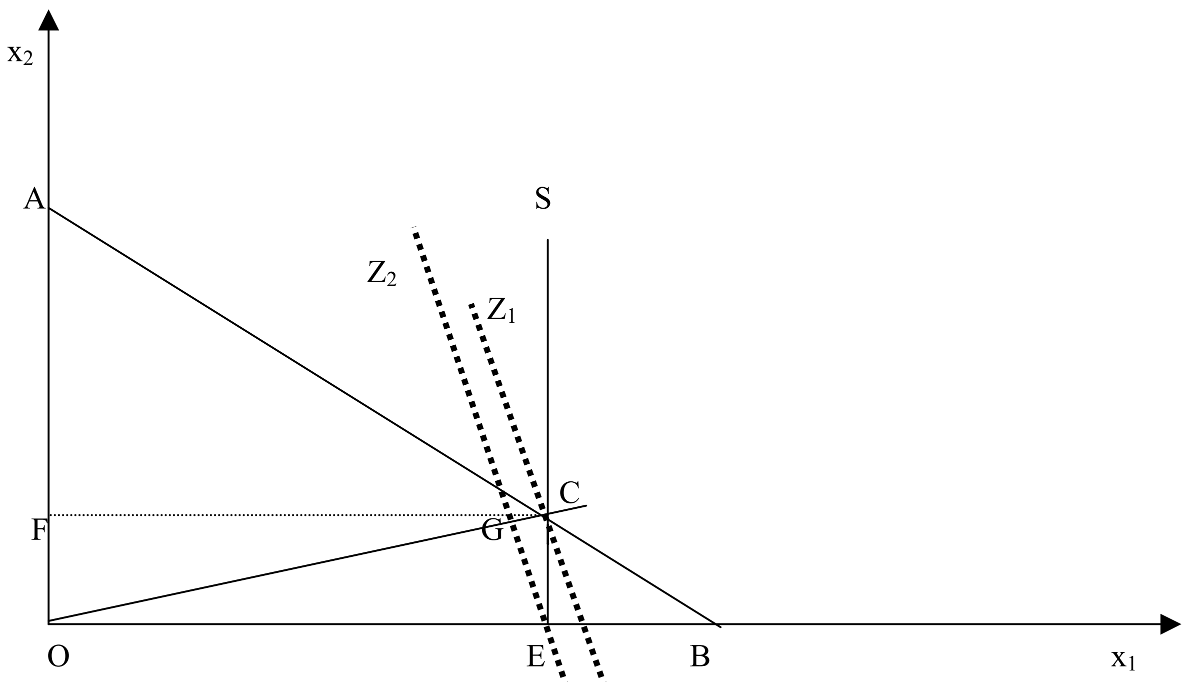

Figure 2: Optimum solutions of the model in the case of insufficient investment

In addition, in cases when a fund more than that specified as the required investment associated to maximum GRP of the first rank is available, a similar procedure should be used in allocation of this investment funds. Obviously, when a higher level of funds for investment is available, the planner should allocate them to obtain the highest value of GRP, the objective of the model. Hence, extending this procedure leads to allocating any level of investment funds up to 534,811 millions Rials, i. e., the maximum required investment (public or private) for the region with respect to other constraints specified in Table 6, based on an effectiveness for investment. As a result, the model is flexible to be applied for any level of investment.

Conclusion

To rank economic sectors, a Linear Programming Model linked to a Social Accounting Model has been applied. The GRP of the region is taken into account as the objective function. The job creation for different groups of human force, income distribution inequality as well as supply and demand constraints for products are considered as constraints of the model. Thus, the economic sectors were ranked based on maximum GRP value in two separate conditions, i. e., with respect to and irrespective of required investment. The results of the two cases were compared with each other. It was found that when the sectors are selected with respect to the required investment, this will lead to an effective investment to achieve the highest GRP.

Based on the result of this paper, the collection of sectors denoted as No. 16, 21, 24, 25, 26 with a certain level of outputs are recommended to achieve the highest effectiveness for investment. One of the advantages of this procedure is finding an effective allocation for investment that will be quite valuable for developing countries, which generally suffer from its shortage. In addition, this technique considers other relevant constraints that are important in the planning process. Finally, this technique can be applied to other production factors like water, land, and environment.

Appendix A

In a general linear programming maximisation form in which:

max $Z = C * X$ (18)

$s.t.$ $A * X \leq b$ (19)

$X \geq 0$ (20)

Z is the objective function, C is a 1×n decision variable coefficient vector, and X the n×1 decision variables vector. In addition, A, an m×n matrix, represents the resource required for X_i. Finally, b, the available resources, is an m×1 column vector. Because of Proportionality Assumption of the linear programming model in which there is no initial income or costs, it can be proven that if all the resources increased t times, then the optimum solution will equally increase.

Proof: In the optimisation process in any iteration, $b^{\sim}$ shows the new value of decision variable, i. e., $X^{\sim}$, and can be written as:

$$b^{\sim} = B^{-1} * b \quad (21)$$

in which B is the basic matrix of the relevant iteration with m×n dimensions. Substituting equation (21) in equation (18) in any iteration, Z, can be written:

$$Z = C * B^{-1} * b \quad (22)$$

Consequently, if b is multiplied by t:

$$b_t = t * b \quad (23)$$

the new optimum solution will be as follows:

$$Z_t = C * B^{-1} * b_t = t * C * B^{-1} * b = t * Z \quad (24)$$

Table 7 Estimation of coefficients of the model for the Golestan Province of Iran for the year 1993/1994

Title of sectors	g_{j*}	N_{1j**}	N_{2j**}	N_{3j**}	N_{4j**}	b_{1j**}	B_{2j***}	d_{j*}	*Sec.no*
Farming	1.833	47.2	0.11	0.48	0.05	3.64	3.58	31855796	1
Traditional Livestock	1.912	46.69	0.14	0.48	0.06	2.56	4.17	1667870	2
Modern Husbandry	1.354	27.96	0.54	0.54	0.12	1.51	3.53	41229000	3
Modern Hen-breeding	0.868	11.75	0.13	0.31	0.07	1.51	2.12	10233650	4
Fish-breeding	0.853	16.75	1.26	1.49	0.03	2.11	1.63	32252361	5
Forestry	1.808	26.58	0.78	0.73	0.04	1.79	3.09	2772049	6
Fishery	1.498	20.43	0.21	1.41	0.10	2.62	2.73	1268760	7
Mining	1.787	30.77	0.66	0.58	0.05	2.93	3.07	13494300	8
Food Processing Industries	1.372	33.10	0.18	0.71	0.08	2.39	3.92	15925520	9
Textile industries	1.351	27.67	0.23	0.62	0.07	2.98	2.85	5585530	10
Carpets	1.567	82.82	0.13	0.39	0.05	2.66	4.81	1949786	11
Wood Products	1.576	29.77	0.49	1.09	0.11	2.87	2.93	6641249	12
Publication & Paper	1.516	26.82	2.83	0.70	0.06	2.87	3.73	3585400	13
Chemical Products	1.557	25.28	2.64	0.63	0.06	2.70	4.00	3153700	14
Non-metals Products	1.486	27.71	0.38	0.81	0.08	3.02	3.18	2925760	15
Metal Products	1.633	29.83	6.60	0.46	0.05	2.61	4.04	32678000	16
Machinery Products	1.440	28.67	0.31	0.70	0.07	1.78	3.00	48729000	17
Other Industry	1.288	17.97	0.36	0.38	0.09	1.96	2.34	272617	18
Water, Electricity and Gas	1.060	17.87	0.39	1.37	0.13	2.60	2.39	6184642	19
Construction	1.371	31.86	0.15	0.434	0.05	2.51	2.60	27866900	20
Communication	1.729	25.56	0.93	2.68	0.05	3.18	2.74	9363500	21
Transportation	1.683	31.85	0.16	0.54	0.05	2.62	3.59	17571970	22
Bank and Insurance	1.770	22.71	0.37	1.33	0.04	3.37	3.17	3159090	23
Education	1.770	18.89	0.16	15.05	0.51	3.25	3.16	56177580	24
Health	1.671	28.89	0.33	4.98	2.23	3.40	2.91	1192096	25
Public Services	1.790	38.35	0.18	4.90	0.14	3.44	2.65	86468900	26
Personal Services	1.782	28.68	0.13	0.48	0.09	0	4.43	56570550	27

*: for 1993/1994 1000 Rials
**: for1993/1994 100,000,000 Rials
***: for 1993/1994 $10^{(12)}$ Rials

Reference

Gripaios P, Gripaios R, Munday M (1997) The role of inward investment in urban economic development: The cases of Bristol, Cardiff and Plymouth. *Urban Studies* 34(4): 579-603

Hassan FMA (1994) Is adjustment with equitable growth possible? Evidence from a development country. *Canadian Journal of Development Studies* 15(2): 219-240

Havinga IC, Sarmad K, Hussain F, Badar G (1987) A Social Accounting Matrix for the Agricultural Sector of Pakistan. *The Pakistan Development Review* XXVI(4):

Keynes JM (1936) *The General Theory of Employment, Interest and Money*. MacMillan, London

Leatherman JG, Marcouiller DW (1996) Income distribution characteristic of rural economic sectors: Implications for local development policy. *Growth and Change* 27(4): 434-459

Myrdal G (1957) *Economic Theory and Under-Developed Regions*. Duckworth, London

Pyatt G, Round JI (1979) Accounting and fixed price multipliers in a social accounting matrix framework. *The Economic Journal* 89: 850-873

Smith A (ed. 1904) *An Inquiry into the Nature and Cause of the Wealth of Nations*, Vol. 1, Cannan E. (ed.) Methuen, London

Solow RM (1956) A Contribution to the theory of economic growth. *Quarterly Journal of Economics* LXX: 65- 94

Thorbecke E, Jung HS (1996) A multiplier decomposition method to analyze poverty alleviation. *Journal of Development Economics* 48(2): 279-300

Yue CS (1999) Trade, foreign direct investment and economic development of Southeast Asia. *Pacific Review* 12(2): 249-270