A STRATEGY FOR PRODUCING HYBRID REGIONAL INPUT-OUTPUT TABLES*

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ABSTRACT. In this paper, I argue that the most effective means for targeting portions of a regional input-output model for superior data is to compare the sensitivity of the region's Leontief inverse to proportional changes in sector technology. Once a sector is identified as being critical, I suggest that modelers seek superior data for a limited number of cells in the direct-coefficients matrix associated with the sector. In particular, modelers should seek data on intermediate inputs and outputs, labor income, and intrasectoral shipments. They should limit additional gathering of regional technology data to that for cells identified as critical by a weighted version of West's (1981) measure. Following the insertion of superior data I propose that modelers formally reconcile the various data sources that are used. This sequence of tasks should be performed recursively until superior-data search resources are depleted or until all major sectors are covered. The paper ends with a test using the 1972 Washington State input-output table that repeats this sequence of tasks recursively through 13 of the 52 sectors. The results show much promise.

1. INTRODUCTION

Hybrid input-output models combine nonsurvey techniques for estimating regional direct requirements tables with superior data, which are obtained from experts, surveys, and other reliable sources (primary or secondary). Such data can be added at any stage of model construction. To date, hybrid model production has been defined only in very general terms. Elsewhere (Lahr, 1993), I have pointed out directions that could prove fruitful in developing a more detailed approach.

In a search for an approach that recognizes critical portions of input-output models ripe for superior data in a manner more compatible with survey work, I first review measures that identify <u>sectors</u> that are most critical to the accuracy of the Leontief inverse. I then compare these measures analytically for their ability to predict sectors critical to the 1972 Washington State Input-Output Table.

In Section 3, I combine the findings of Sections 2 with that of other literature to develop a quasi-complete superior-data search strategy for producing hybrid regional input-output tables. In Section 4, I test the strategy by employing the 1972 Washington State input-output table. Tests are of models both closed and open with respect to households. Following this, I conclude with implications of the findings.

2. IDENTIFYING CRITICAL SECTORS FOR SUPERIOR-DATA COLLECTION

The best-known measure (Jensen and West, 1980; Hewings and Romanos, 1981; West, 1981; Hewings and Jensen, 1986) forming the basis of building hybrid regional input-output models identifies cells that are most critical to the accuracy of output multipliers. For a recent contribution and nearly complete review of this line of literature, please see Casler and Hadlock (1997). There is some doubt, however, about the practicality of its cell-by-cell approach when used as a superior-data search procedure. The rationale behind these objections is that it may not be worth the effort to obtain superior data for a single cell in a given sector, data for a collection of cells in another sector may be much more valuable and certainly take nearly the same amount of effort to acquire. Furthermore, Isard and Langford (1971) inform us that data on regional imports for individual cells are much more difficult to obtain than they are by sector (either rowwise or columnwise). In addition, a set of literature (Miernyk, 1970; Bourque, 1971; Conway, 1975; Afrasiabi and Casler 1991) has established that proper accounting of trade patterns is critical to the accuracy of regional input-output models. It, therefore, follows that identifying sectors rather than individual cells as targets of survey work is likely to be more fruitful.

By "important sector," model constructors, generally mean sectors that are most critical to model accuracy. The measures that traditionally have been used to identify such sectors are known as "key sector" measures. Key sectors are sectors that have the most total linkages in an economy. The most accepted key-sector measures are those that combine Leontief and Ghoshian linkages and those focusing upon hypothetical extraction measures. In comparing these measures, Lahr (1992) identifies the theoretical inadequacies and the empirical impracticalities of those that combine Leontief and Ghoshian linkages (Jones, 1976; Beyers, 1976; Hübler, 1979; Lovisek, 1982). Hence, it appears that, of the two, hypothetical extraction (Paelinck et al., 1965; Miller, 1966; Meller and Marfàn, 1981; Cella, 1984; Szyrmer, 1992) should be favored. Hypothetical extraction measures are calculated by finding the effects of eliminating a sector from the economy. For more details, consult Lahr and Miller (1997). In addition, de Mesnard (1997) and Dietzenbacher (1997) provide evidence that should rekindle interest in a Leontief/Ghoshian hypothetical extraction approach.

In our specific case, however, by "important sector" we really mean a sector for which superior data will significantly improve nonsurvey model accuracy. Sectors that have technology that is at high variance with that represented in the nonsurvey model and that also have large total linkages will be identified correctly by hypothetical extraction approaches. But what of sectors with minimal technology variance and large total linkages or those with high technology variance and medium-to-small total linkages? Unfortunately, hypothetical extraction approaches are not designed to ascertain such differential importance of changing the technology of sectors. Instead, they measure error only due to the total elimination of a sector, not increases or decreases in its interconnectivity with the rest of the economy through differences in technology or trade patterns.[1] Therefore, we are seeking a measure that identifies the effect on the economy's total linkages of potential changes that manifest themselves through trade-pattern or technology differences.

[1] Furthermore, linkages are highly nonlinear (West 1981, 1982). Hence, by eliminating an entire sector we would undoubtably be miscalculating the measure that we need.

Additional assumptions of most regionalization techniques deal only with technology and trade patterns. The most popular nonsurvey regionalization approaches strictly adjust for imports by producing sector, working on rows of the national direct-requirements matrices, by assuming that technology in the nation is spatially invariant. This set of assumptions has proven somewhat accurate. Subsequently, when nonsurvey regional direct-requirements matrices are multiplied by the diagonal matrix of outputs, reasonable estimates of intermediate outputs by sector result. Through similar matrix operations, however, very reasonable estimates for intermediate inputs by sector are <u>not</u> necessarily to be expected. This is because by regionalizing strictly across rows, imported proportions of intermediate inputs are not well estimated. That is, the <u>use</u> of imports by industry is likely to be badly estimated by most nonsurvey models. Consequently, the problem of identifying "important sectors" for the most common situation is one of identifying sectors that are likely to induce the largest <u>changes</u> in total linkages when survey data replaces nonsurvey data for the sector's direct requirements. Essentially then, we are interested in determining the effects on the Leontief inverse of changes in a single sector's direct requirements. This problem is answered through a line of literature that started as early as Sherman and Morrison (1949, 1950) and Woodbury (1950).

Tolerable Limits to Intermediate Inputs

A large body of European literature as early as Yershof (1965) and as recently as Xu and Madden (1991) has developed in input-output analysis on a topic known as the "tolerable limits of change." The tolerable-limits approach is based on the calculation of the amount that direct-requirements must be perturbed in order to effect a one- percent change in output of a specified sector. Jilek (1971) made the first advance for hybrid model development by showing analytically and empirically that the magnitude of a direct-requirements coefficient is critical to determining its ability to effect a sizable change in sectoral output. Sell (1980) extended this work to show analytically that there is a strong relationship between the size of intermediate inputs and the potential for a sector to generate error in its output multiplier. Realizing that "it is often easier to collect a complete input (and/or output) pattern for selected industries" than it is to collect information on individual inputs for various industries, Schintke and Staglin (1988) extended Yershof's measure of tolerable limits to investigate how much entire a sector's technology needs to be perturbed to effect a pre-stipulated percentage change in the sector's output. They specified it as

(1) $$d_i(\rho) = \frac{1}{\left\|\mathbf{A}_j\right\| \cdot \left[\left\|\mathbf{B}_j\right\| \cdot \rho + 100 \cdot \mathrm{Max}_k\left(\frac{\left\|\mathbf{B}_k\right\| \cdot x_j}{x_k}\right)\right]}$$

where the maximum absolute amount of the relative errors in sector k's gross output, x, is prespecified as ρ percent, $\|\cdot\|$ denotes the maximum absolute column-sum norm, and $\mathbf{A}_j$ and $\mathbf{B}_j$ denote vectors of sector *j*'s columns in the direct- and total-requirements matrices, respectively. This measure is unsatisfactory for the purposes here because in a sense it identifies the sector that experiences the greatest change in output when its technology is changed by ρ. Instead, we seek the effect of a perturbation across the entire economy, not just a single sector. Although a sector with low tolerable limits likely greatly affects the entire economy when its technology is changed, low tolerable limits is neither a necessary nor sufficient condition to affecting the entire economy. This is because the effects of changing a sector's technology may not be concentrated in the sector itself and, in fact, may be fairly uniform across the sectors of the economy. That is, when the effects are less concentrated, the sector that we wish to identify may not be that identified by the tolerable-limits approach.

The Effects of Intermediate-Input Variation on the Leontief Inverse

A different direction of attack to this problem is provided by the combination of Sell (1980) and West (1982). As mentioned before, Sell (1980) tells us that a strong direct relationship exists between the magnitude of the proportion of intermediate inputs and the potential of the sector to generate error in its output multiplier. This suggests that a measure similar to that of West (1981) but extended to the vector of intermediate inputs would be reasonable. Sell, using a national model, was working on the assumption that (near) perfect information on imports and exports by producing sector is known. This assumption regarding trade patterns is overly strong for prototypical hybrid regional models, generally nonsurvey models are formed by the rows-only regionalization of national technology.

West's (1981) cell-sensitivity formula includes a proportional perturbation factor. That is, unlike the hypothetical extraction approach, it allows the analyst to adjust the proportion by which a cell is altered to effect a change in all output multipliers. By continuing the assumption of spatially invariant national technology, these proportions can be viewed as the error in coefficient estimates that are due to the mis-estimation of imports use. Therefore, if West's (1981) formula could be generalized in some way to apply to the column of each sector (intermediate inputs), the solution to our problem would seem to have been found. Fortunately, such a generalized formula was provided as early as Evans (1954) and Dwyer and Waugh (1953) and as recently as Maaß (1980) and West (1982). The derivation of Evans's (1954) version of the formula is provided in the Appendix B. The final formulation of this variant is the most efficient to calculate and can be expressed as

(2) $$\mathbf{E}=[(\mathbf{I}-\mathbf{BP})^{-1}-\mathbf{I}]\mathbf{B}$$

where **B** is the Leontief inverse, **I** is an identity matrix, and **P** is the matrix of perturbations of the direct-requirements matrix.

To find out the effect that a percentage change in a sector's information has on the rest of the economy, all cells of **P** should be to zero except for the column and row for which we want to test sensitivity to imports and calculate **E**. By letting $\mathbf{P}=[p_{ij}a_i,]$ (where the a_{ij}'s are the elements of the direct-requirements matrix), the values of the nonzero p_{ij}'s can be set to proportions that represent likely deviations of each of the sector's direct-requirement coefficient from its true value.

One disadvantage of **E** is its matrix form. Comparing the **E** matrices for each sector is likely to be extremely difficult. If each sector's error matrix could be expressed as a scalar, comparison of the importance of sectors would be much simpler. A means of creating such a scalar would be to premultiply $\mathbf{E}_i$, the error matrix for sector i, by a transposed vector of ones, **1**, and to postmultiply by a vector of regional outputs (value added), **x**, or some other set of economic weights like final demand, earnings, or employment

(3) $$E_i=\mathbf{1}'\,\mathbf{E}_i\mathbf{x}$$

3. A SIMPLE EMPIRICAL TEST OF THE MEASURE'S VALIDITY

Although the measure in the last section has appeal, there is no evidence that it can be valuable in targeting sectors for superior-data collection when estimated from a nonsurvey model. Furthermore, although it seems conceptually sound to assume that column-only perturbations will provide reasonable sectoral rankings, I have not offered evidence to show that the addition of row perturbations of the same sector will not provide significantly more accurate rankings. Hence, I decided to test how well sectors can be targeted with the measure by comparing the results of the measure using data strictly from a nonsurvey model to those of the total linkage difference[2] between the nonsurvey and survey-based models. Naturally, sectors with the largest total linkage differences between survey-based and nonsurvey models are those that should be targeted for superior-data collection efforts. Consequently, if we find that by using E in Equation (3) we can obtain a ranking of sectors that is very similar to that arrived through an intersectoral comparison of the total linkage difference between the survey-based and nonsurvey models, we should deem E useful.

I used the 1972 Washington State Input-Output Table closed to households plus its equivalent nonsurvey data from the Regional Science Research Institute (but using survey-based household and labor information). The RSRI model was aggregated from the 494-sector level to the 52 sectors of the Washington State survey-based model using regional weights and by applying RPCs prior to aggregation. Since according to Table 1, E reproduces well the sectoral ranking of the total linkage difference,[3] I deem the measure to be useful. I come to the conclusion that it "reproduces well" the sectoral rankings of total linkages by examining the rank correlations reported using both Spearman's P and Kendall's τ. Pearson's correlation coefficient is estimated for comparison purposes. Pearson's correlation coefficient measures the correlation of the actual values of the linkage measures, not just the ranks.

The fit of the rankings for the two measures is even better for the top-ranked sectors. Since we should be most interested in surveying such sectors first, these findings reveal even more promise for E.

In the above test, I strictly perturbed the columns of each sector by 30 percent. This amount was somewhat arbitrarily chosen, being the maximum that any sector could be perturbed

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[2] Lahr (1992) has determined that for situations such as this the total linkage measure developed by Meller and Marfàn (1981) is most appropriate. The Goshian equivalent or some combuination thereof could also be used.
[3]The absolute value of the difference between the total earnings linkages produced by the survey-based and

without having its column in the direct-requirements matrix sum to greater than unity. But what if the 30 percent value was too large? And what if the assumption that rowwise imports estimates are reasonable is incorrect? In answer to the first question, I estimated *E* five times reducing the column perturbations by 4 percent each time. The rankings were identical to those obtained through the original 30 percent perturbation. By perturbing row coefficients as well as those in columns and by varying each set of coefficients by 5 percent, I did obtain different sectoral rankings, however. The highest Spearman's ρ and Pearson's correlation coefficient (.943 and .816, respectively) were achieved when the row elements of the direct-requirements table were perturbed by 10 percent and the column elements were simultaneously perturbed by 30 percent. Hence, as assumed, the RPCs used were relatively accurate (off by an average of only 10 percent). In fact, when the ranked list of sectors from this table was compared to one that did not vary the row elements at all (only perturbing the column coefficients by 30 percent), no significant difference between them was found (the ρ and Pearson' correlation coefficients were .987 and .914, respectively). For simplicity, I therefore stuck to using column-only perturbation of 30 percent.

Unsure of the set of weights to use for the tests, I used three: survey-based final demand, nonsurvey output, and employment. Table 1 shows the results of these tests and compares them to the total linkage difference between the survey-based and nonsurvey model weighted by final demand. Surprisingly, employment weights reveal the best results with a Spearman rank correlation coefficient of .75 and Kendall's τ is .57. These reveal significant association between the two sets of data. Moreover, nine of the highest-ranked sectors using *E* weighted by employment are among the top ten that should be actually targeted for superior data based on absolute total linkage difference. Results using the other two weights fare only marginally less well. Hence, from this example it would appear that nonsurvey tables do a reasonable job in identifying sectors that are most important to target for superior-data collection.

4. A STRATEGY FOR BUILDING A HYBRID REGIONAL INPUT-OUTPUT TABLE

Many of the steps in the procedure that I suggest here are very different from West (1990), who has laid out the best outline to date. First, by following the advice of Stevens and Lahr (1992) aggregation error inherent in West (1990) is avoided. Second, I forward the notion

nonsurvey tables.

that hybrid model constructors should use the most accurate means of regionalizing that they have available and that they use the most accurate technology data available for their region. If a dated detailed survey-based model is available for the region, it should be used as the base model from which superior-data collection strategies should stem. Third, as discussed in the previous section, I assert that sensitivity analysis of sectors to variation in imports be performed to identify sectors (i.e., row-column combinations), not cells, that should obtain superior data. Finally, the use of different data sources means varying data reliabilities, the combination of survey data with nonsurvey data is no exception. Consequently, I also assert that formal reconciliation of the various data sources is imperative. The following paragraphs describe a detailed set of procedures that can be used to develop a hybrid regional input-output model.

Step 1, Preparation of Initial Nonsurvey Regional Direct Requirements.

This step comprises the first two in West (1990): developing a technology table and adjusting it to account for regional trade patterns. In developing the technology table, West (1990) lists the following important steps: selecting a base technology table, updating the technology (if necessary), and adjusting the updated table so that international trade is accounted for in a manner that is consistent with the regionalization technique.

The main difference between this step and the two in West (1990) is the regionalization approach. Instead of using the modified location quotient approach described in West (1980), which is perhaps appropriate for the Australian setting in which West and his associates work, I suggest employing a regionalization scheme that allows for cross-hauling, which is the norm in interregional trade rather than the exception.

Step 2, Identifying Sectors for Superior-data Collection.

By omitting aggregation, we can immediately produce a prototype model. The first step in this process is to identify the sectors that should gain superior data. If the technology table is one "borrowed" from the national input-output table and if the region being modeled comprises only a small proportion of the total economy of the nation, then several sets of sectors should be given top consideration for superior-data collection. They are the household-labor sector, resource production sectors (e.g., agriculture, forestry, fishing, and mining) and any aggregate sectors such as those denoted as "miscellaneous" or "not elsewhere classified" or others which, due to regional data problems, are severely aggregated. If funds for data collection are severely limited, however, only one or a few of these sectors will get needed superior data.

If national technology is used, the sectors mentioned in the above paragraph should first be targeted for superior-data collection. But if data-collection funds are severely limited, which of these sectors should be targeted for survey work? Also, if funds are more than sufficient to enable collection of data in all of these sectors or if regional technology is otherwise obtained for these sectors, which sectors should receive superior data using the remaining resources?

Since it has long been established that a prime factor in the instability of regional direct requirements coefficients is regional trade, I have opted to focus on this as a means for identifying sectors for superior data. By using the combined-coefficient sensitivity concept of West (1982) as described in the last section, sectors that make the regional table most sensitive to changes in trade can be easily identified. Since rankings of the sectors using this approach are altered with any coefficient correction (West, 1982), the process of identifying sectors should be performed recursively after the model has been enhanced by each sector's survey data.

Step 3, Identifying Individual Cells for Data Collection.

To keep data-collection costs for each sector to a minimum and yet maintain accuracy, I have identified (Lahr, 1993) several pieces of information should be obtained for all targeted sectors. These are intermediate inputs and outputs as a proportion of total regional sector output, intrasectoral flows as a proportion of total regional sector output, total regional sector output, and regional labor income. Intermediate input proportions and total regional sector output are necessary to produce regional input proportions, which are essential to proper regionalization (Lahr, 1992). Szyrmer and Lahr (1992) show that intermediate output and total regional output are necessary to estimate properly the RPCs. In addition, Phibbs and Holsman (1981) find that intrasectoral flows generally are one of the larger sets of flows into any given sector. Since Jensen and West (1980) also find that large cells tend to the most critical to model accuracy, I decided to make the cells on the diagonal of the technology matrix targets for superior data since they tend to be the largest cells in indirect requirements matrices of high order. Finally, it is now well-established (Stevens and Trainer, 1976; Giarratani and Garhart, 1987) that accuracy in the measurement of the household-labor sector is also critical. Hence, if such data are not available at a sufficiently disaggregate areal level from state or local government sources, they should be obtained via survey work.

For targeting cells of a sector for superior data, I used a measure introduced by West (1981). Of course, the measure was applied to every cell in a selected sector's row and column since a firm should be able to furnish such information. I weighted West's (1981) single-coefficient change estimates by information using value-added as suggested by Jackson (1991).

I suggest only estimating West's (1981) measure for a selected sector's column and row as opposed to all cells of the direct-requirements matrix, as was previously suggested (West, 1990), for algorithm efficiency. The reasoning here is that calculations must be made for each cell. In a typical regional input-output mode, this amounts to at least several hundred calculations for a single sector's row and column.

Step 4, Insertion of Superior Data. '

The data for cells identified in Step 3 are inserted into the region's technology table (not the regionalized version). Values for cells that are not survey-based are proportionally adjusted so that each sector's intermediate known input and output levels are met.

Step 5, Biproportional Regionalization.

In order to regionalize using RAS, the intermediate inputs total must sum to intermediate outputs total. Convergence of the nonlinear RAS technique is achieved when estimated margin totals are within a "reasonable" measure of tolerance. I suggest that the bases for these tolerances be the relative reliabilities of the corresponding margin data for the row/column. Survey data are the most reliable. Superior data used to estimate intermediate outputs are generally next most reliable. Superior data on intermediate inputs somewhat less reliable. Nonsurvey estimates are considered to be highly unreliable. Using this ranking of reliability, sales and purchases data are reconciled simultaneously with the different data sources.

The process continues by iterating through Step 2 to 5 until funds for superior-data collection are depleted or the difference between the survey and nonsurvey results appears to consistently be very small. The matrix that is obtained from the last use of Step 5 is the ultimate matrix to be used for impact or other input-output-based analysis.

For my examples, I set the tolerance limits that terminate the RAS procedure as follows: (1) if I used data from the survey-based model, I assigned tolerances of 1 unit (in my case million dollars of shipments) to intermediate inputs and outputs; (2) if the sector had no survey-based data and it was a manufacturing sector, I assigned intermediate outputs a tolerance of 30 percent of their initial estimate (the RPC estimates that I used are more accurate for manufacturing sectors); (3) if the sector had no survey-based data and it was a manufacturing sector, intermediate inputs were given a tolerance of 100 percent of the initial estimate; and (4) if the sector had no survey-based data and it was not a manufacturing sector, I set the tolerance for intermediate inputs and outputs to 100 percent of the initial estimate.

In the case of my examples, I “surveyed” 13 sectors (the number was arbitrarily set at 25 percent of the number of sectors in the model). I tested the approach against the model both open and closed with respect to households. A discussion of the matrix comparison measures—Mean Absolute Deviation (MAD), Weighted Absolute Deviation (WAD), Standardized Total Percentage Error (STPE), r, Root Mean Squared Error (RMSE), and Thiel’s U—is included in Appendix C.

5. INTERPRETATION OF RESULTS

I hypothesize (1) that error should monotonically decrease as more sectors get superior data and (2) that the percent change in error should approach zero as more sectors get superior data. That is, the two hypotheses are (1) each additional sector “surveyed” reduces error and (2) there are decreasing marginal returns to accuracy from superior data.

Figures 1 to 3 show test results when the model is closed with respect to households and Figures 4 to 6 show the open-model results. It appears that the first hypothesis holds: error is reduced by additional partial survey work. This hypothesis is violated most in technology matrices (Figures 1 and 4) and second most in Leontief inverses (Figures 2 and 5). In these cases, by examining only the WAD measure, which weighs error in larger coefficients disproportionately [as suggested by Jensen and West (1980)], not only are fewer violations observed but those that still exist are less severe. The WAD measure is not as appropriate as some of the other measures for examining error in multipliers (Figures 3 and 6), because no rationale exists for specifying that results for larger sectors need to have lower proportions of error than those for smaller sectors. Hence, the WAD results for multipliers should be ignored;

incidentally Figure 3 shows that WAD error change takes a radically different direction than that of other measures.

The second hypothesis (diminishing marginal returns to superior data) appears to hold well for the model closed to households (Figures 1 to 3). That these figures reveal a vacillating diminishing trend to the change in error is no surprise; there may be little difference between the “survey” data that is inserted and the nonsurvey data it is replacing. The results shown in Figures 4 and 5 are somewhat disappointing, however. These figures reveal an early five-sector period (the third through seventh sectors chosen) with little or no decrease in hybrid model error. It is disappointing to learn that the funds needed to obtain such data could yield so little partitive accuracy.

The results for open-model multipliers (Figure 6) are more favorable with error change consistently less than zero. Regardless of the model type, however, “surveys” for only four sectors were required to reduce error to 50 percent (an STPE error of 24 to about 11 in the closed model) of its original value. For the closed model these sectors were: FIRE, Trade, Services, and Construction; all of which are very aggregate sectors. Aluminum replaced Construction as a major error-reducing sector in the open model.

How do these results compare to the case where, in each successive round of survey work, the sector with the greatest absolute deviation in total linkages between the closed survey-based and nonsurvey models is known? Figures 7 through 9 reveal that, in general, similar results are obtained. The absolute amount of error remaining for each successive sector surveyed is constantly less for all measures than it was in the for the measures supporting Figures 1 through 3, however. Surveys of only two sectors—FIRE and Construction—were required to reduce error for multipliers to about 36 percent of its original value (to an STPE error of about 5.5). The two late downward spikes in Figures 7 and 8 are from technology changes in Dairy Products and Logging. The trade adjustments in Aluminum account for the largest downward spike for output multipliers in Figure Table 2 is a comparison of the open- and closed-model sectors that my approach suggests should be “surveyed” (not including the household labor sector in the 52-sector model, which is the top-ranked sector). These should be compared with the sectors that would be modeled if the recursive superior-data collection technique had been employed with perfect information on the differences between the survey-based and nonsurvey models (see Table 2, bottom). In both cases, the nonsurvey model identifies 7 of the top 13 sectors to be targeted. On the other hand, several critical sectors not identified by my approach are resource-based sectors: Fisheries, Mining, Vegetables and Fruits, and Forestry. I earlier made the point that such sectors should automatically be considered for receiving superior data since they have technology that is likely to very different from the national average technology represented in the nonsurvey model. I chose not to do so because of the severe aggregation in the survey-based Washington State model, which would have left few sectors to test the approach presented here.

6. CONCLUSIONS

It appears that the algorithm developed in this paper is successful in drastically improving both the partitive and holistic accuracy of nonsurvey tables. In particular, there was evidence vindicating a priority to the collection of superior data for the household/labor sector, resource production sectors, and any aggregate goods-producing sectors denoted as “miscellaneous” or “not elsewhere classified.” Perhaps because of loose tolerances for sectors that did not get superior data in the RAS procedure, it turns out the hypotheses that (1) each additional sector “surveyed” reduces error and (2) there are decreasing marginal returns to superior data are not found to be strictly true. The results did not necessarily prove the two hypotheses to be wrong, on the other hand. For example, error did not tend to increase as more sectors were “surveyed,” and there were general improvements in the various components of the tables as the process proceeded. Hence, there is hope that similar approaches, (e.g., alternatives with more stringent RAS tolerances) might favor the hypotheses. In any case, more testing must be performed to validate variations of the procedure outlined in this paper.

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TABLE 1: Rankings of Sectors for Survey Work: Actual Versus Estimates

	Total Linkage Difference		West's (1982) Sensitivity by Weight Type					
			Final Demand		Output		Employment	
Industry #	Measure	Rank	Measure	Rank	Measure	Rank	Measure	Rank
1.	323	9	290	10	1,107	8	188	11
2.	124	22	239	14	291	19	263	7
3.	163	17	264	11	1,246	7	219	10
4.	15	44	33	43	53	40	26	36
5.	72	34	35	41	143	30	24	38
6.	155	18	178	20	562	12	117	16
7.	66	36	132	25	383	15	88	19
8.	128	21	224	17	300	18	78	22
9.	14	45	74	33	161	28	24	39
10.	13	46	101	29	129	32	39	31
11.	57	37	150	22	341	16	84	20
12.	12	47	7	51	8	52	3	52
13.	48	39	50	37	42	43	21	43
14.	129	20	57	36	183	25	39	32
15.	220	13	19	49	82	35	18	45
16.	143	19	263	12	506	13	121	15
17.	113	25	505	9	586	11	163	13
18.	5	49	225	16	172	27	71	24
19.	50	38	141	24	138	31	43	29
20.	15	43	30	45	56	37	22	41
21.	112	26	141	23	192	24	38	33
22.	78	33	181	19	209	22	66	25
23.	114	24	173	21	158	29	52	28
24.	243	12	73	34	217	21	99	17
25.	78	32	124	27	181	26	53	26
26.	29	41	8	50	26	46	7	50
27.	177	14	130	26	203	23	52	27
28.	33	40	4	52	10	51	5	51
29.	90	28	108	28	324	17	76	23
30.	117	23	24	47	37	44	21	42
31.	11	48	32	44	113	34	16	48
32.	567	7	562	8	454	14	97	18
33.	168	16	38	39	33	45	26	35
34.	66	35	33	42	54	39	17	46
35.	2	51	35	40	12	50	14	49
36.	90	29	23	48	18	48	24	40
37.	1	52	62	35	25	47	27	34
38.	2	50	42	38	17	49	16	47
39.	549	8	734	7	128	33	240	9
40.	82	31	100	30	44	42	25	37
41.	112	27	225	15	51	41	144	14
42.	22	42	82	32	55	38	39	30
43.	1,416	6	1,311	6	2,536	6	1,085	6
44.	273	11	221	18	678	10	168	12
45.	85	30	26	46	79	36	19	44
46.	173	15	92	31	278	20	80	21
47.	294	10	248	13	747	9	240	8
48.	3,389	3	4,399	2	6,017	5	1,939	5
49.	2,963	4	3,406	3	8,006	3	3,202	2
50.	3,905	2	2,768	4	9,903	2	2,526	4
51.	2,027	5	2,614	5	7,989	4	3,145	3
52.	4,634	1	9,399	1	25,245	1	6,249	1
ρ			.6538	(7.9)	.7237	(9.7)	.7487	(10.6)
τ			.4949	(27.0)	.5477	(29.9)	.5688	(39.0)
r			.9293		.9244		.9052	

Notes: Numbers in parentheses after Spearman's ρ is its t statistic and that after Kendall's τ is its z score. Both test the hypothesis that there is no relationship between the subject measure and the linkage difference measure. The linkage difference measure is the absolute difference between the total linkages for the survey-based and nonsurvey models, with total linkages as in Meller and Marfàn (1981).

Industries are defined in Appendix B.

TABLE 2: Sectors Targeted for Survey Work

The Recursive Search Technique

Open Model	Closed Model
1. Construction	FIRE
2. FIRE	Trade
3. Trade	Services
4. Services	Construction
5. Aluminum	Transportation Services
6. Sawmills	Aerospace
7. Transportation Services	Meat Products
8. Meat Products	Communications
9. Livestock and Products	Livestock and Products
10. Aerospace	Sawmills
11. Logging	Logging
12. Canning and Preserving	Field and Seed Crops
13. Paperboard and Other Paper Products	Aluminum

Maximum Absolute Deviation in Total Linkages

Closed Model

1. FIRE
2. Construction
3. Fisheries
4. Trade
5. Livestock and Products
6. Aluminum
7. Mining
8. Vegetables and Fruits
9. Services
10. Petroleum
11. Dairy Products
12. Forestry
13. Logging

APPENDIX A

STRUCTURAL MATRIX ERRORS IN INTERINDUSTRY RELATIONS ESTIMATES: A DERIVATION OF EVANS'S (1954) FORMULA

Using **A** and **I** to represent the *n*-order direct-requirements and identity matrices, respectively, consider a new matrix that is formed by a perturbation of **A** that is expressed as
$\mathbf{A^*} = \mathbf{A}\text{-}\mathbf{P}$, where $\mathbf{P}=[p_{ij}a_{ij}]$. Then,

$$(\mathbf{I}-\mathbf{A^*}) = (\mathbf{I}-\mathbf{A})-\mathbf{P}$$

which when letting $\mathbf{B}=(\mathbf{I}-\mathbf{A})^{-1}$ and $\mathbf{B^*}=(\mathbf{I}-\mathbf{A^*})^{-1}$ leads to either of the following:

$$\mathbf{B^*} = -\mathbf{B}(\mathbf{I}-\mathbf{PB})^{-1}$$

$$= (\mathbf{I}-\mathbf{BP})^{-1}\mathbf{B}$$

Subsequently, the matrix of errors produced in a Leontief inverse, $\mathbf{E}=\mathbf{B^*}-\mathbf{B}$, by a perturbation in a direct-requirements matrix is

$$\mathbf{E} = -\mathbf{B}[\mathbf{I}-(\mathbf{I}-\mathbf{PB})^{-1}]$$

$$= [(\mathbf{I}-\mathbf{BP})^{-1}-\mathbf{I}]\mathbf{B}$$

This is equivalent to that derived by West (1982), who showed that

$$\mathbf{E} = (\mathbf{I}-\mathbf{BP})^{-1}\mathbf{BPB}$$

and that by Dwyer and Waugh (1953), who showed that

$$\mathbf{E} = -\mathbf{BPB}(\mathbf{I}+\mathbf{PB})^{-1}$$

That there are two fewer matrix multiplications to be performed makes the Evans's formula more practical.

APPENDIX B

INDUSTRIES IN THE 1972 WASHINGTON INPUT-OUTPUT STUDY

# Industry Name	U.S. Standard Industrial Classification Code(s)
1. Field and Seed Crops	011, 013 (except 0133), pt. 019
2. Vegetables and Fruits	0133, 016, 017, pt. 019
3. Livestock and Products	02 (except 027), 075
4. Other Agriculture	018, 027, 07 (except 074, 075)
5. Fisheries	09 (except 097)
6. Meat Products	201
7. Dairy Products	202
8. Canning and Preserving	203, 2091, 2092
9. Grain Mill Products	204
10. Beverages	208
11. Other Foods	205-207, 2095-2099
12. Textiles	22
13. Apparel	23
14. Mining	10-14
15. Forestry	08, includes national and state forests
16. Logging	241
17. Sawmills	242
18. Plywood	2435, 2436
19. Other Wood Products	2431, 2434, 2439, 244-249
20. Furniture and Fixtures	25
21. Pulp Mills	261
22. Paper Mills	262
23. Paperboard & 0th. Paper Prod.	263-266
24. Printing and Publishing	27
25. Industrial Chemicals	281, 286-289
26. Other Chemicals	282-285
27. Petroleum	29
28. Glass Products	321-323
29. Cement, Stone, and Clay	324-329
30. Iron and Steel	331, 332, 3398, 3399
31. Other Nonferrous Metals	All other 33
32. Aluminum	3334, 3353-3355, 3361
33. Structural Metal Products	344
34. Other Fabricated Metals	34 (except 344)
35. Nonelectrical Motive Equip.	351-353
36. Machine Tools and Shops	354, 359
37. Nonelectrical Ind. Equip.	355-358
38. Electrical Machinery	36
39. Aerospace	372, 276

40. Motor Vehicles	371, 374, 375, 379
41. Ship and Boat Building	373, incl. Puget Sound Naval shipyard
42. Other Manufacturing	30, 31, 38, 39
43. Transportation Services	40-47, inc. Post. Serv. & public transportation
44. Electric Companies	491, pt. 493, incl. government enterprises
45. Gas Companies	492, pt. 493, incl. government enterprises
46. Other Utilities	pt. 493, 494-497, includes government enterprises.
47. Communications	48
48. Construction	15-17
49. Trade	50-59, includes state liquor stores
50. FIRE	60-67
51. Services	074, 097, 70-89 (excl. publicly owned schools & hospitals)
52. Households/Labor	

APPENDIX C

SOME MEASURES FOR COMPARING INPUT-OUTPUT MATRICES

C1. INTRODUCTION

Despite extensive evaluation of tables regional input-output derived via the various nonsurvey methods, there is still some question regarding the relative accuracy of the various techniques when compared to survey-based tables (Miernyk, 1976, p.54). In part, this is due to the age-old problem in regional analysis—the lack of proper and sufficient survey data to which estimates can be compared (Polenske, 1989; Miernyk, 1976, p. 54). In addition, even provided proper and sufficient "superior" data, comparisons are generally made using measures whose properties have not been investigated. That is, there is no proven strict relationship between the value of the measures and the amount of error inherent in an estimate, especially in the case of the application of input-output tables. Thus, the results of the comparisons have been, at best, inconclusive with regard to the goodness-of-fit of the estimates.

Measures of distance and association serve two purposes, (1) to measure the ability of models to produce accurate results and (2) to determine the statistical significance of the difference between the actual and estimated data (Knudsen and Fotheringham, 1986) [henceforth, K-F]. Unfortunately, other than K-F few researches have ever thoroughly studied measures of distance or association for matrix comparison problems. And, certainly, no one else has considered and compared a battery of such measures in a scientific way.

In total, Lahr (1992) found 14 different measures that have been used to determine the accuracy of I-O models. The reason so many have been used is that the tables, themselves, are unlike many other matrix-based models. The structure of I-O tables and the possible uses toward which they can be applied require a very stringent set of properties for a general-use comparison measure. Hence, researchers have chosen a measure they have used in other statistical applications or have used measures used by other I-O analysts, despite their possible impropriety.

In the literature, at most 6 of the 14 measures have been discussed and/or tested with regard to the validity of a few of their properties. Not even all desired traits have been discussed in these few measures. In fact, testing of the measures was only evident in one paper, Miller and Blair (1982). Rather those measures not perceived as being well-suited to a particular application are either altered slightly (creating a new measure) or are omitted from use. Hence, despite passage of over two decades, it is difficult to disagree with Miernyk (1975, p.34) “that there is some ambiguity in the measurement of projection error” and that “there is an opportunity for an innovative statistician to make an important contribution here.” I suspect, however, that it also may take work along the lines of that by Asami and Smith (1995), who used an axiomatic approach to identify appropriate measures.

C2. NOTATION

At this point, I set out the notation that will be used throughout this appendix. Following the notation of Miller and Blair (1985), a_{ij}, is the *i*th element down the *j*th column of matrix A, the technology matrix, **A**, which has an estimate **Å** with elements designated $\mathring{a}_{ij}$. Likewise, μ and μ are the vectors of actual and estimated multipliers, and μ_i and μ_i are the respective multipliers of a specific sector *i*. Throughout I have not separately designated the formulae for transactions or total requirements matrices which, besides mere variable-notation changes, have formulae identical to those of the technology matrix.

C3. THE ERROR MEASURES

Standardized Total Percent Error

Of the measures used in this paper, the first to be used in I-O applications was the weighted average percentage error measure (Leontief, 1966, p. 244). Somehow Leontief’s use of the measure was forgotten, because it was not used again until Sawyer and Miller (1983), where it was apparently reinvented as the mean absolute deviation as a percentage of the mean coefficient (MPMC), and Miller and Blair (1982, 1983) where it was given its most commonly used, current name—standard total percentage error (STPE). In an unpublished, paper Szyrmer (1984) used this measure under yet another name, mean normalized deviation. It has the formula

$$100\frac{\sum_j \sum_i \left| a_{ij} - \mathring{a}_{ij} \right|}{\sum_j \sum_i a_{ij}}$$

Its only major drawback is that it may not be exceptionally sensitive to high-valued cells.

Correlation Coefficient (r)

The correlation coefficient, *r,* isolated from the rest of the regression parameters and statistics has been used as a measure of association. To make it a distance measure, Szyrmer (1984) transformed it into what he called the disparity index, defining this "new" measure as 50(1*r).* Hence, at perfect correlation its value is 0, at no correlation its value is 50 and at perfect negative correlation its value is -50. To keep the direction of the values of *r* in line with those of the other measures, I have merely subtracted its value from 1.

Mean Absolute Difference

In 1974, Smith and Morrison, and Morrison and Smith introduced the use of the mean absolute difference (MAD) measure to I-O analysis. Its formula is

$$100\frac{\sum_j \sum_i \left| a_{ij} - \overset{\circ}{a}_{ij} \right|}{q^2}$$

As with many other measures, there is no penalty in this measure for having error in high-valued coefficients as opposed to low-valued ones. In addition, its magnitude changes with the order (size) of the I-O tables being evaluated. In answer to the first of these two problems, I developed the weighted absolute difference (WAD), which is discussed later.

Index of Inequality (Theil's U)

A measure originally developed in Theil et al. (1966) was first used in the input-output literature by Stevens and Trainer (1976). This measure, commonly called Theil's index of inequality (U), can be broadly interpreted as a standardized root mean squared error. (The root mean squared error measure is discussed later in this appendix.) If multiplied by 100 percent has an interpretation that is very similar to that of the STPE measure. It has the advantage of yielding an overall distance proportion as well as three other proportions: bias (U), variance (U), and covariance (U). These extra error proportions are valuable in showing the researcher the patterns of the differences between two matrices. The formula for index of inequality is

$$\left[\frac{\sum_j \sum_i \left(a_{ij} - \mathring{a}_{ij}\right)^2}{\sum_j \sum_i a_{ij}^{\ 2}}\right]^{.5}$$

This measure is probably used to its full potential in a paper by Stevens, Treyz, and Lahr (1989), where the authors examine the accuracy of various RPC-estimating techniques. Stevens, Treyz and Lahr, however, do not look at any of the I-O tables or their components that are being analyzed in this study. Its main drawback is that, as with most of the other measures, it does not penalize more heavily for error in higher-valued coefficients.

Root Mean Squared Error (RSME)

The Root Mean Squared Error or Euclidean Metric Distance is defined as

$$\frac{\left[\sum \sum \left(a_{ij} - \mathring{a}_{ij}\right)^2\right]^{.5}}{q^2}$$

Harrigan, McGilvray, and McNicoll (1980) introduced this well-known measure to the I-O literature. As with others such as MAD, this measure does not yield any idea of the relative difference between two matrices, but rather only the average total difference. Hence, one cannot really determine how bad or good an estimated matrix is when compared to the actual. To correct for the latter problem, Theil et al. (1966) developed the index of inequality, which was discussed earlier.

Weighted Absolute Difference (WAD)

To my knowledge Lahr (1992) is the first to use the WAD. It is designed to make up for the problems of most of the other measures. Its formula is

$$\frac{\sum_j \sum_i \left(a_{ij} + \mathring{a}_{ij}\right) \cdot \left|a_{ij} - \mathring{a}_{ij}\right|}{\sum_j \sum_i \left(a_{ij} + \mathring{a}_{ij}\right)}$$

That is, the (a_{ij}-$\mathring{a}_{ij}$)-term weights the absolute difference term so that the errors of large cells are emphasized. In this way the measure is extremely sensitive to error in large cells, something that, so Jensen and West (1980) have told us, is critical. The advantage of this measure is that if either of the matrices is nonzero for a cell, the measure's value is not undefined. There is one problem with this measure, it does not necessarily express proportional error as do STPE or Theil's U.

FIGURE 1: Error Change for Sequence Surveyed—Model Closed to Households
Direct Requirements

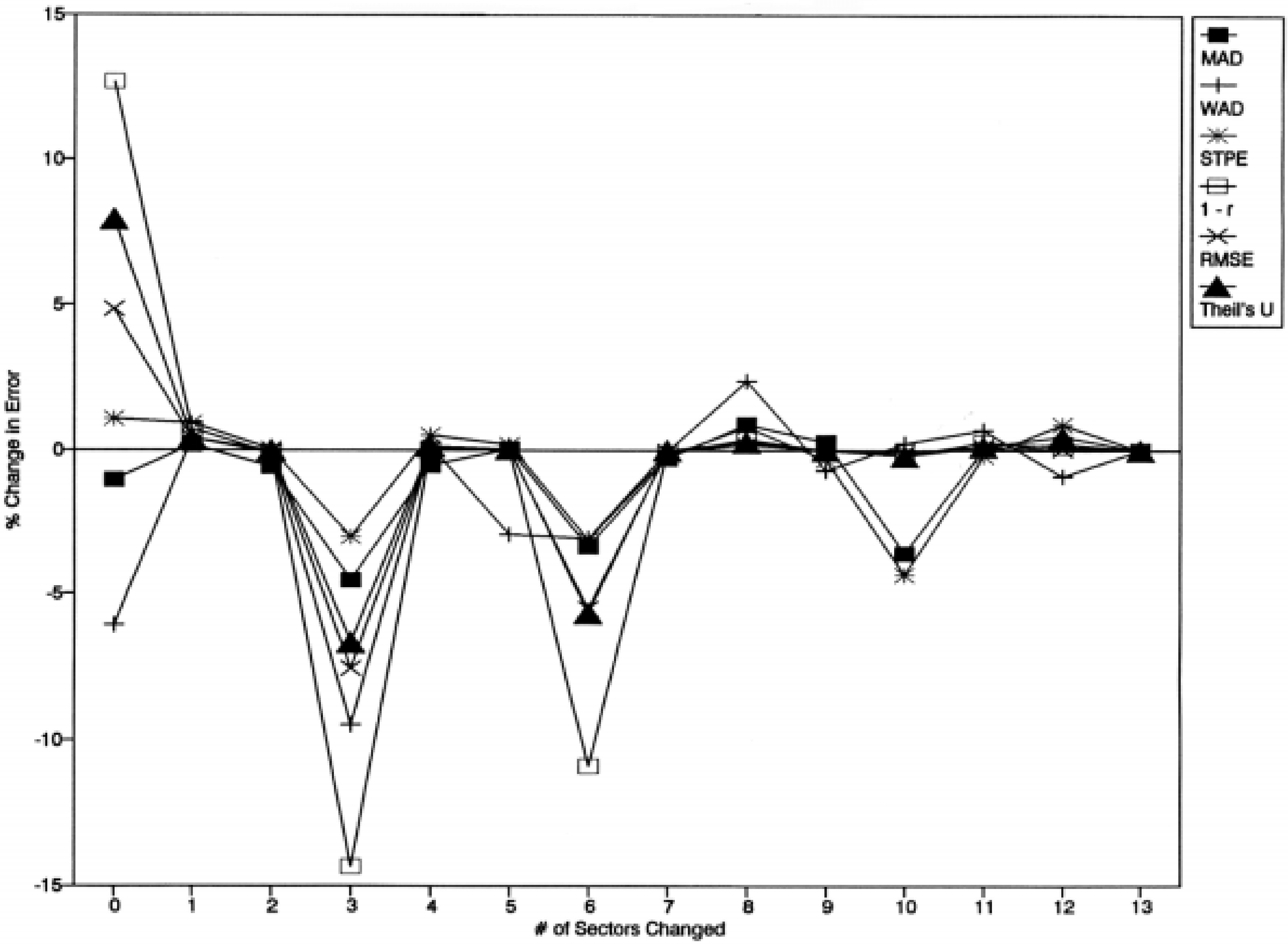

FIGURE 2: Error Change for Sequence of Sectors Surveyed—Model Closed to Households
The Leontief Inverse

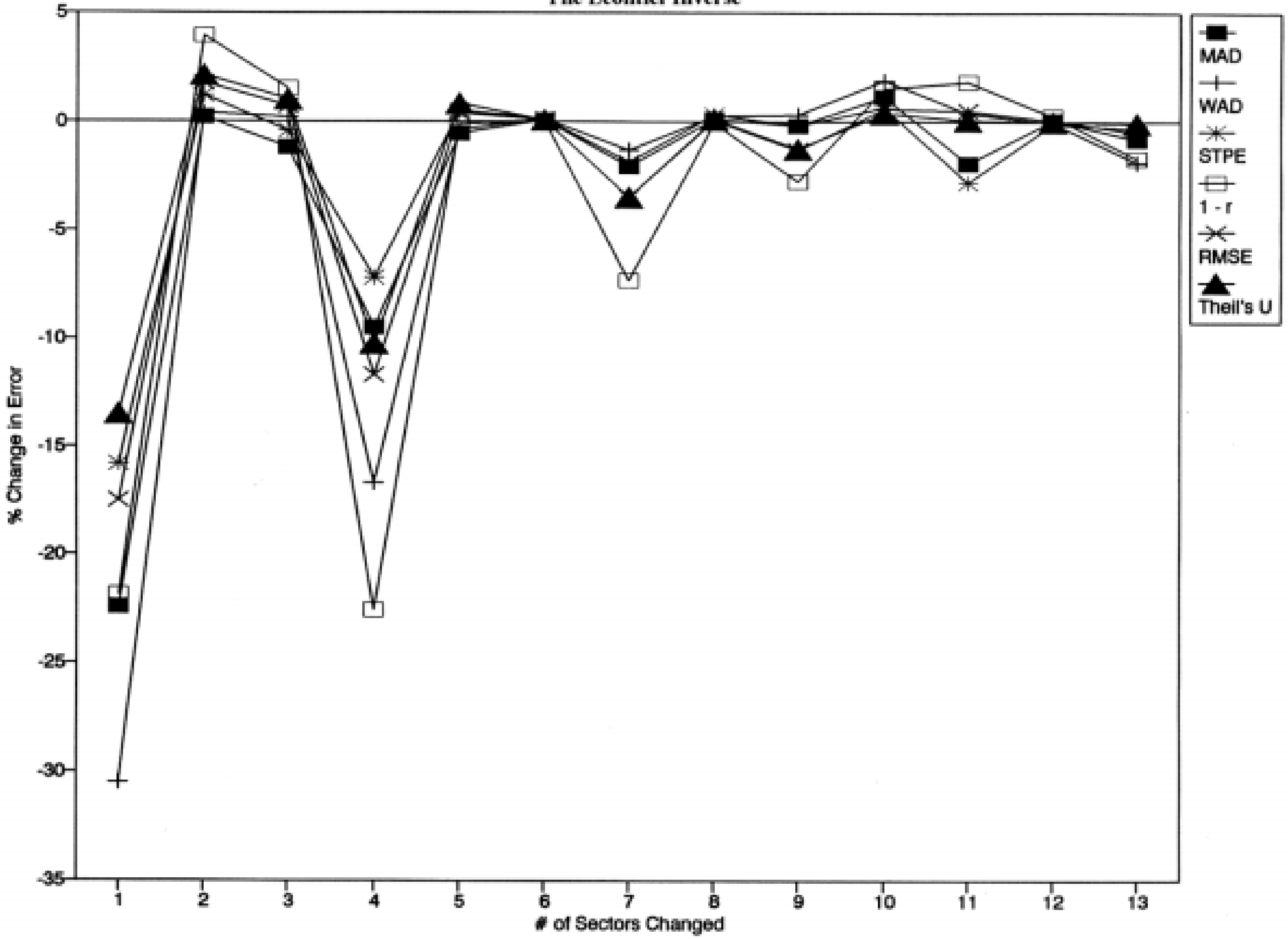

FIGURE 3: Error Change for Sequence of Sectors Surveyed—Model Closed to Households
Output Multipliers

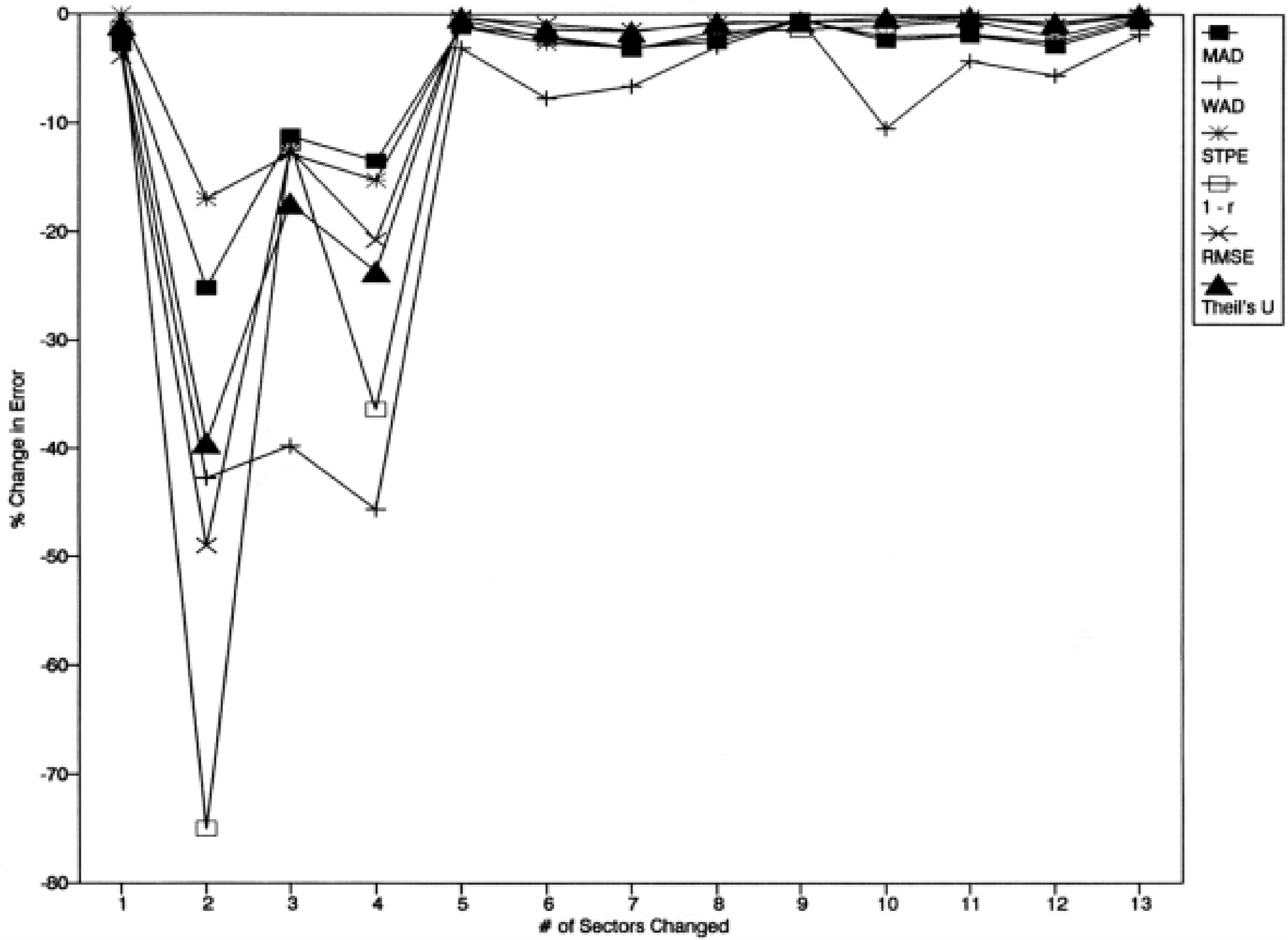

FIGURE 4: Error Change for Sequence of Sectors Surveyed—Model Open to Households Output Multipliers

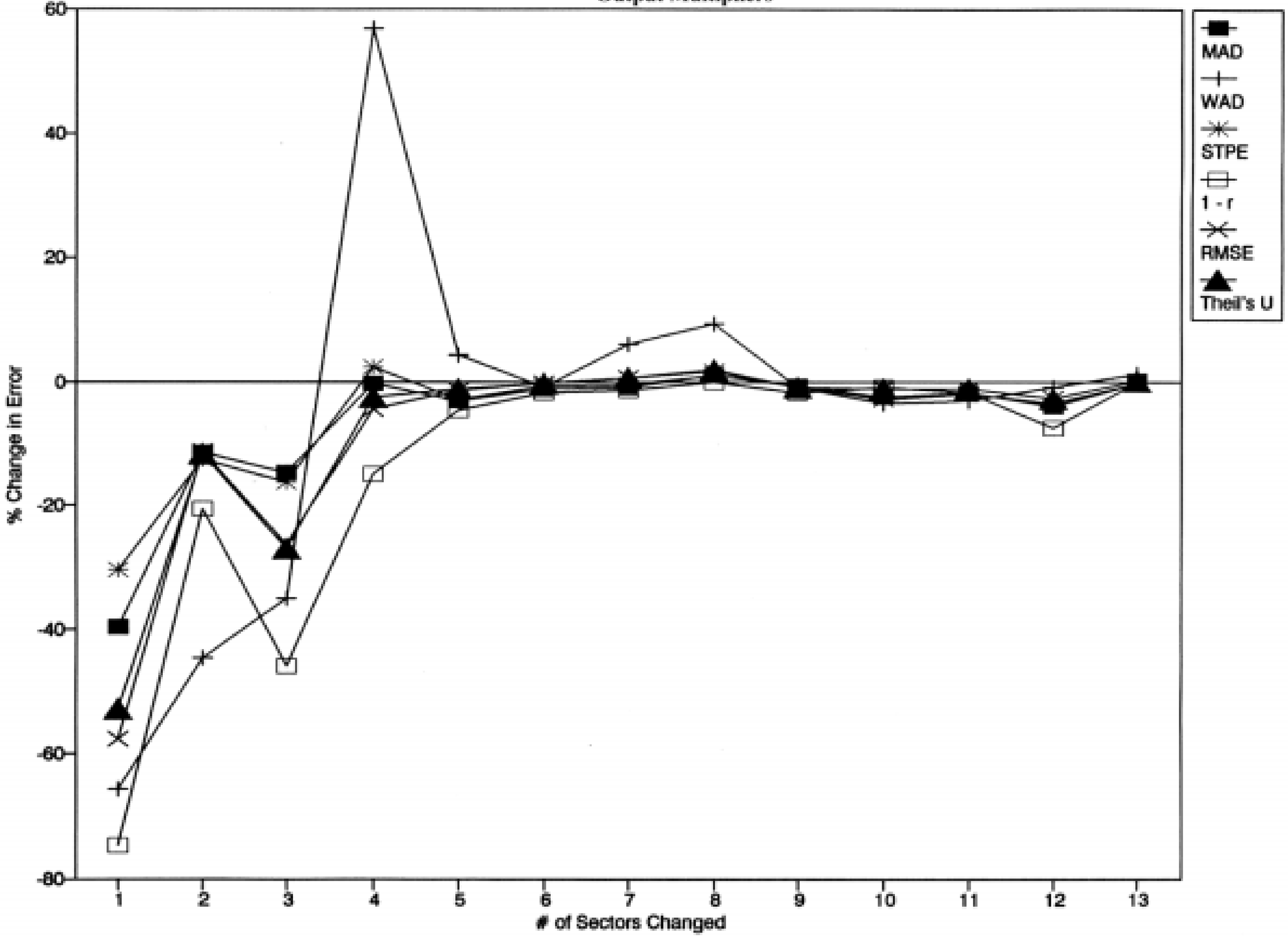

FIGURE 5: Error Change for Sequence of Sectors Surveyed—Model Open to Households Direct Requirements

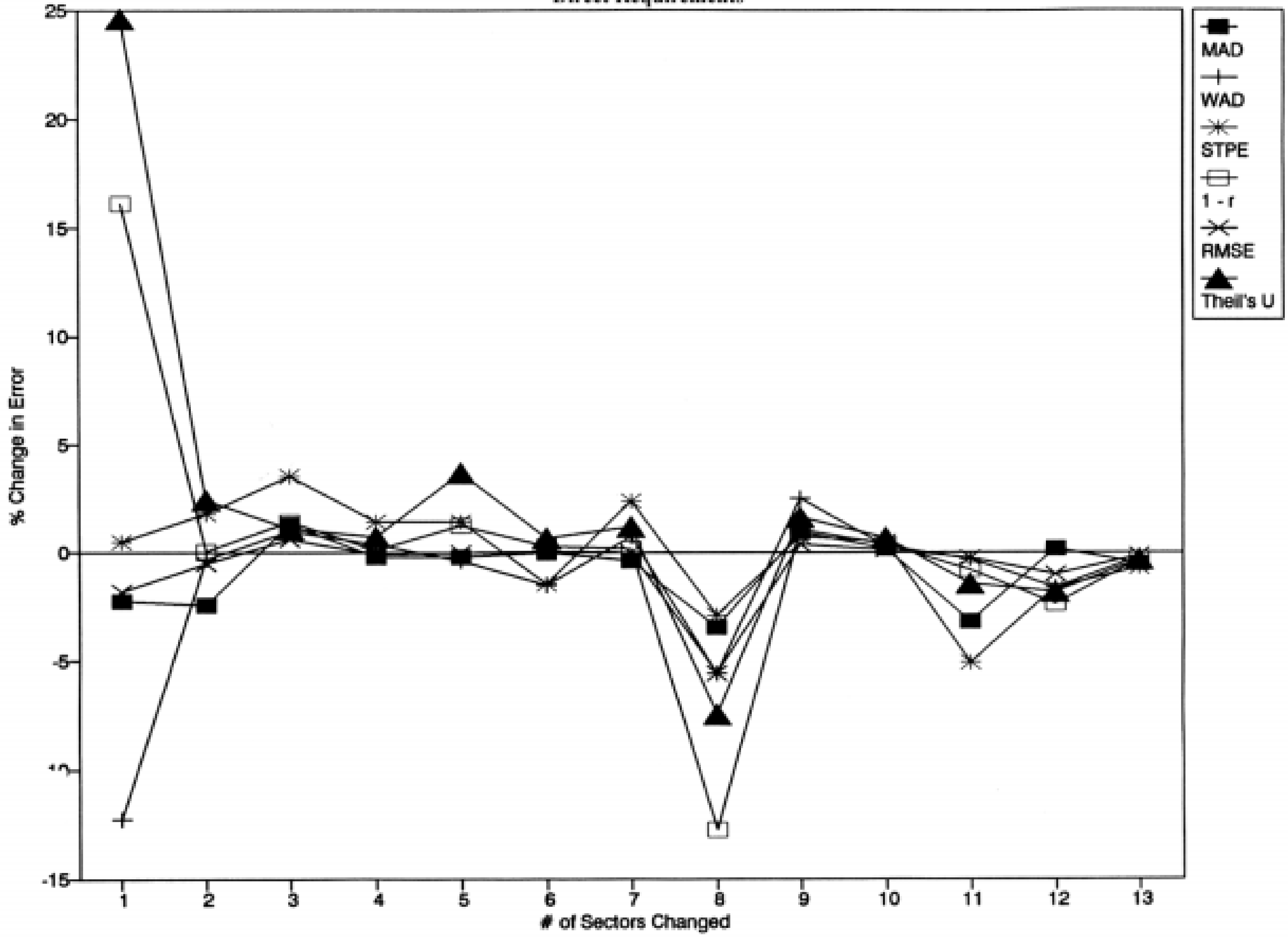

FIGURE 6: Error Change for Sequence of Sectors Surveyed—Model Open to Households
The Leontief Inverse

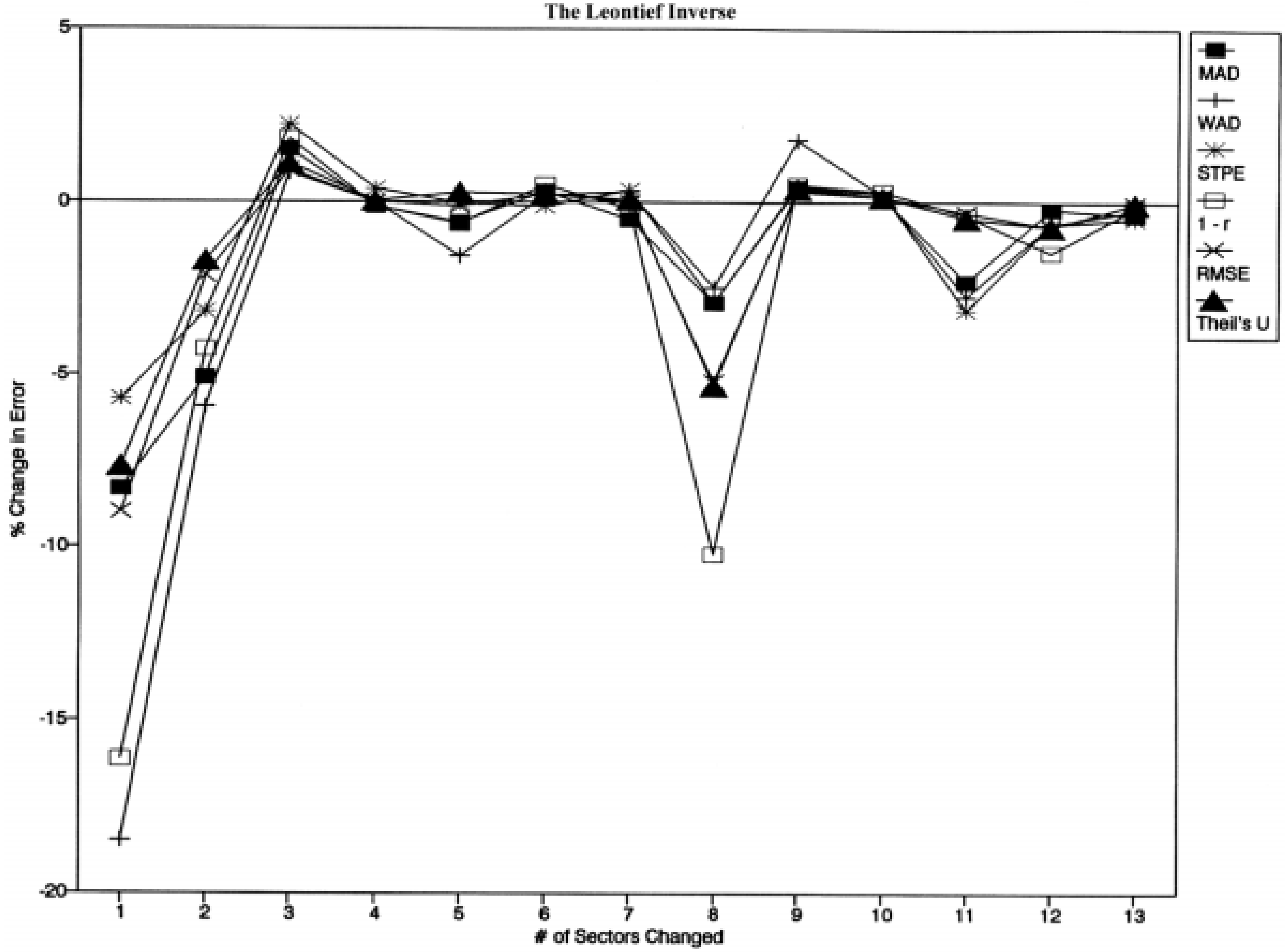

FIGURE 7: Error Change for Sequence of Sectors Surveyed—Total Linkage Difference Closed Direct Requirements

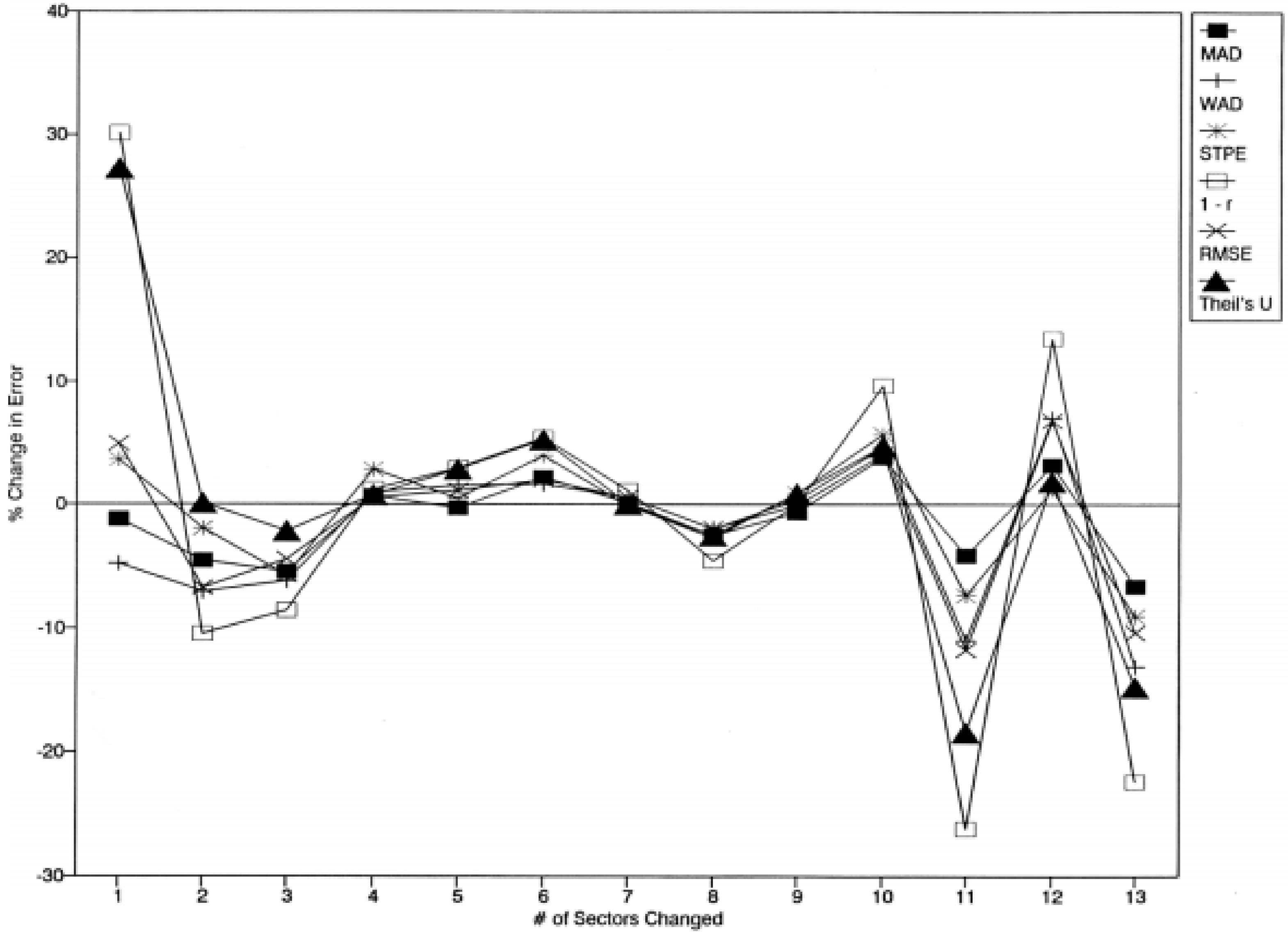

FIGURE 8: Error Change for Sequence of Sectors Surveyed—Total Linkage Difference Closed The Leontief Inverse

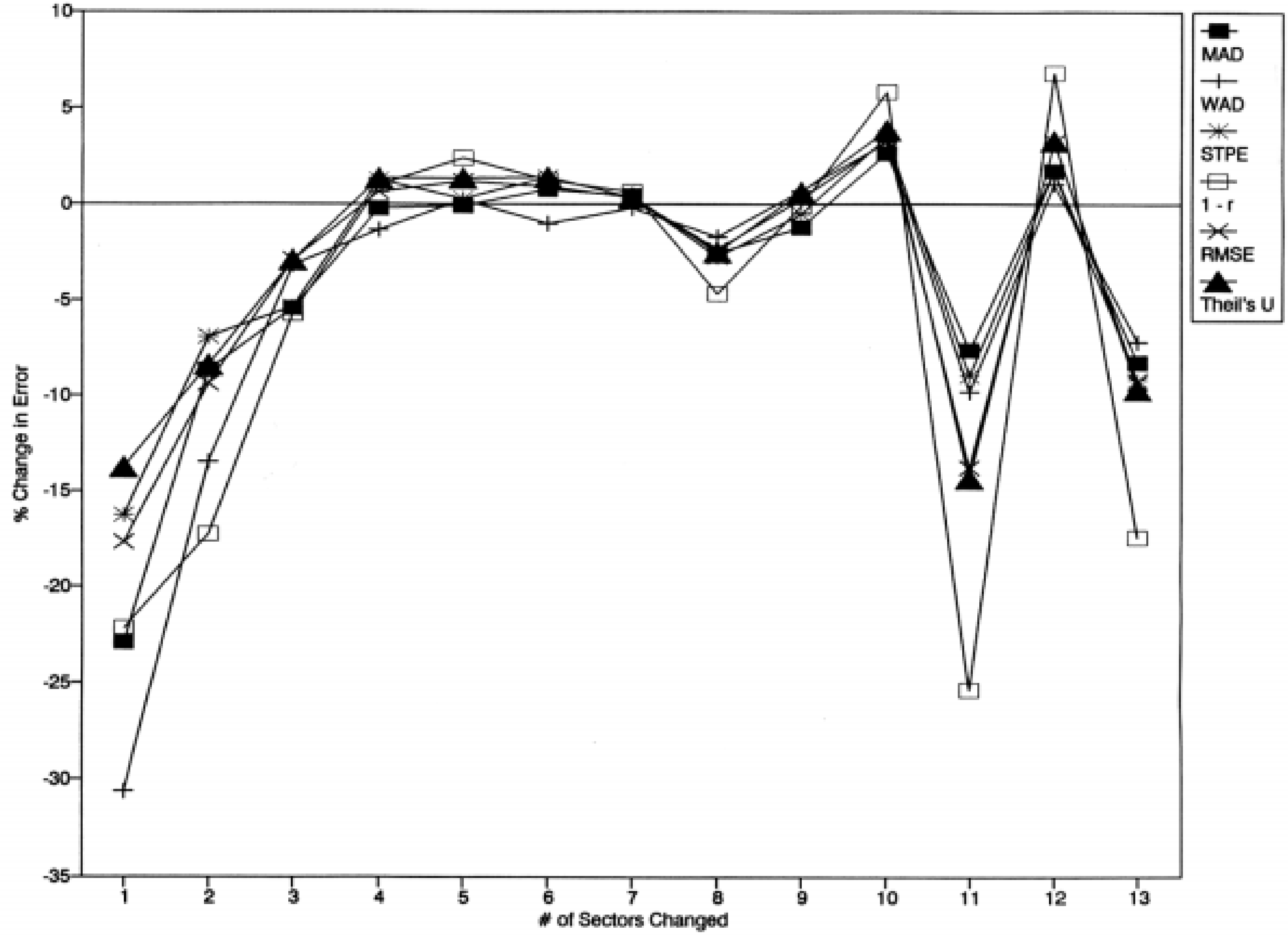

FIGURE 9: Error Change for Sequence of Sectors Surveyed—Total Linkage Difference Closed Output Multipliers