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INPUT-OUTPUT IN MIXED MEASUREMNTS

By Ezra Davar

ABSTRACT

One of the main reasons that input-output is not usually used for the solution of meaningful economical problems, despite the fact that a majority of countries in the world compile empirical input-output, is the existence of a discordance between theoretical and empirical accounts of input-output and real economical activities.

It is well known that there exist two versions of input-output: theoretical (Walrasian) and empirical (Marxian-Leontievian). Theoretical input-output is characterized by the separable presentations of prices and quantities for both goods and primary factors. In this case, there are two types of prices for goods - supply (cost of production) and demand - such that in order to establish an equilibrium state their equality is required. Also, prices are real, meaning that the measurement of prices for goods is money ($) per unit real (physical) measure of goods (meter (M), ton (T), and so forth), namely $/M or $/T and for factors, for example labor $/H (H-hour of work).

Empirical input-output is characterized by the “quantities” in monetary terms. In this case, prices and quantities are not separated and they are amalgamated in one magnitude. This means that empirical input-output has a uniform measurement, namely money ($) measurement, not only for goods and factors, but also for the categories (private and public consumption, investment, export) of final use. A price in this case is called a latent price, determined as a quantity of money per unit $, whose measurement would be $/$.

At the same time, in practice there are goods and factors for which the real (physical) measurement can be defined without difficulty, for example: water (m^3), electricity (cwt.), some branches of industry and labor. On the other hand, there are goods and factors for which it is not possible to define real measurements, for example: some machinery industries, finance, business, and trade services, or fixed capital. Hence, they are only measured in money terms. Therefore, there is a necessity to consider (compile) input-output in mixed measurements, where one part of the goods and factors can be measured in real (physical) measurements and another in monetary terms.

Following the energy crisis that occurred at the beginning of the seventies attempts have been made to use mixed (hybrid) input-output for practical purposes. However, these attempts did not take important theoretical problems into account.

Therefore, in this paper the theoretical problems of input-output in mixed measurements will be discussed and both quantitative and price models will be presented. Such a new version of input-output will allow us broaden the use of input-output for practical goals.

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1. Introduction

One of the main reasons that input-output is not usually used for the solution of meaningful economical problems, despite the fact that a majority of countries in the world compile empirical input-output, is the existence of a discordance between theoretical and empirical accounts of input-output and real economical activities.

There are two types of input-output (I-O): Theoretical (Walrasian) and Empirical (Marxian-Leontievian) (Davar, 1994; Leontief, 1941 & 1986, Walras, 1954). The theoretical I-O is characterized by its separate presentation of prices and quantities for both goods and primary factors. In addition, there are two types of prices for goods: supply price (cost of production), which is obtained on the basis of prices of factor services and demand price, which is obtained by means of demand curves (functions) of goods. Hence, in order to establish an equilibrium state, their equality is required. Also, prices are real, meaning that the measurement of price for goods is money ($) per unit of physical measure of commodity (meter (M), ton (T), and so on) or for factor (H-hour of work), namely $/M, $/T, and $/H respectively.

Empirical I-O is characterized by the "quantities" representation in monetary terms. Here prices and quantities are not separated, but instead are presented in one magnitude – in monetary terms. This means that empirical I-O has a uniform measurement ($) not only for commodities and factors, but also for the categories of final uses. An external price for all them, in this case, is a latent price. This means that the measurement of latent price for the monetary quantity of commodities and primary factors, is money ($) per unit measure of monetary terms ($). Hence its measurement would be $/$.

In practice, on the other hand, there are goods and factors whose real measurement is not difficult to define, For example: water (m^3), electricity (kw), some industrial commodities and labor. However, it is not always possible to define the real measure of a certain commodity or factor, as in: the machinery industry, finance, business, trade services and fixed capital. Therefore, there is a need to compile a mixed I-O, where one part of commodities and factors is measured in real terms and the other in monetary terms (Leontief, 1974).

The first attempts at using mixed (hybrid) I-O for practical purposes occurred following the energy crisis in the beginning of seventies (Miller and Blair, 1985). Unfortunately, they were some theoretical problems, which were not covered.

In this paper, therefore, the problems of mixed I-O will be discussed, and both quantitative and price models will be presented. For the simplification, here we assume that goods' absolute (real) prices are uniform, but they are dependent on the total quantities. This means that real prices of goods are identical for all branches and categories of final uses, but the magnitude of prices change in consequence of total quantities changing by a certain given rule, for example according to demand curves (functions) of goods. The same is also true for the real prices of primary factors measured in real terms. In addition, the demand price of goods equals their supply price (cost of production). Also, the latent price for both goods and primary factors in an equilibrium state will equal one.

This paper consists of four sections. Following the introduction, the second section discusses the quantitative demand model of input-output in mixed measurement. In the third section, the supply price model will be presented. Finally, conclusions will be presented in the fourth section.

2. The Quantitative Demand Model of Mixed Input-Output

Assume that the first *n*' branches' commodities are measured in real terms and (*n*-*n*') branches' in monetary terms. Also, the first *m*' primary factors' are measured in real terms and (*m*-*m*') in monetary terms. In this case, the two parts combine to give a quantitative demand model. The first part relates to the commodities in real terms and it has the following form

$$\underline{x}_i^d = \Sigma \underline{x}_{ij} + \Sigma \underline{x}_{ij'} + \Sigma \underline{y}_{ir} , \qquad (i=1,2,...,n') \qquad (1)$$

and the second part relates to the commodities in monetary terms and

$$x_{i'}^d = \Sigma x_{i'j} + \Sigma x_{i'j'} + \Sigma y_{i'r} , \qquad (i'=n'+1, n'+2,...,n) \qquad (2)$$

Or in matrix and vector notation

$$\underline{x}^d = \underline{A}_{ij}\underline{x}^d + A_{ij'}x^d + \underline{y}^d , \qquad (3)$$

$$x^d = A_{i'j}\underline{x}^d + A_{i'j'}x^d + y^d , \qquad (4)$$

where

$\underline{A}_{ij}$ - [$\underline{a}_{ij}$] - is the square matrix (*n' * n'*) of the direct input coefficients in real terms (M/M);

$A_{ij'}$ - [$a_{ij'}$] - is the matrix (*n'* * (*n-n'*)) of the direct input coefficients in terms (M/$);

$A_{i'j}$ - [$a_{i'j}$] - is the matrix ((*n-n'*) * *n*) of the direct input coefficients in terms ($/M);

$A_{i'j'}$ - [$a_{i'j'}$] - is the square matrix (*n' * n'*) of the direct input coefficients in monetary terms ($/$);

$\underline{x}^d$ - is the vector (*n'* * 1) of total output in real terms (M);

x^d - is the vector ((*n-n'*)* 1) of total output in monetary terms ($);

$\underline{y}^d$ - is the vector (*n'* * 1) of total final uses in real terms (M);

y^d - is the vector ((*n-n'*)* 1) of total final uses in monetary terms ($).

There are four unknowns $\underline{x}^d, x^d, \underline{y}^d, y^d$ and two equation systems (3) and (4). Therefore, if the system is solvable, we could obtain only two unknowns when another two unknowns are given. If we take into account that Input-Output models allow us to obtain total output *x* when total final uses y are given and vice versa, there are four possibilities:

1) When $\underline{x}^d$ and x^d are given. By substituting them in (3) and (4) we obtain $\underline{y}^d$ and y^d *;*

2) When $\underline{x}^d$ and y^d are given. By substituting them in (4) we obtain x^d

$$x^d = (I-A_{i'j'})^{-1} (A_{i'j}\underline{x}^d + y^d), \qquad (5)$$

if $(I-A_{i'j'})^{-1}$ exists. And by substituting the value of x^d in (3) we obtain $\underline{y}^d$.

3) When x^d and $\underline{y}^d$ are given. By substituting them in (3) we obtain $\underline{x}^d$

$$\underline{x}^d = (I-\underline{A}_{ij})^{-1} (A_{ij'}x^d + \underline{y}^d), \tag{6}$$

if $(I-\underline{A}_{ij})^{-1}$ exists. And by substituting obtaining value of $\underline{x}^d$ in (4) we obtain y^d.

4) When $\underline{y}^d$ and y^d are given. In this case these two systems (3) and (4) are considered as one whole system and by substituting given $\underline{y}^d$ and y^d we obtain $\underline{x}^d$ and x^d

$$X^d = (I-A)^{-1}Y^d, \tag{7}$$

if $(I-A)^{-1}$ exists.

where

X^d - is the column vector $[\underline{x}^d, x^d]$;

Y^d - is the column vector $[\underline{y}^d, y^d]$;

A - is the combination of matrices $\underline{A}_{ij}$, $A_{ij'}$, $A_{i'j}$, and $A_{i'j'}$, namely

$$A = \begin{pmatrix} \underline{A}_{ij} & A_{ij'} \\ A_{i'j} & A_{i'j'} \end{pmatrix}$$

The definition of $\underline{x}^d$ and x^d allows us to obtain demanded quantities of primary factors. For the factors measuring in real terms:

$$\underline{v}^d = \underline{C}_{kj}\underline{x}^d + C_{kj'}x^d, \tag{8}$$

and for the factors measuring in monetary terms:

$$v^d = C_{k'j}\underline{x}^d + C_{k'j'}x^d, \tag{9}$$

where

$\underline{v}^d$ - is the vector (m'*1) of total demand of factors in real terms (M);

v^d - is the vector ((m-m')*1) of total demand of factors in monetary terms ($);

$\underline{C}_{kj}$ - is the matrix (m'*n') of base-year direct input coefficients of primary factors in real terms (M/M);'

$C_{k'j}$ - is the matrix ((m-m')*n') of base-year direct input coefficients of primary factors in terms ($/M);

$C_{kj'}$ - is the matrix (m'*(n-n')) of base-year direct input coefficients of primary factors in terms (M/$);

$C_{k'j'}$ - is the matrix ((m-m')*(n-n')) of base-year direct input coefficients of primary factors in monetary terms ($/$).

If the obtained quantities of primary factors $\underline{v}^d$ and v^d are to remain within the framework of their supply curves, then there is a quantitative equilibrium and we can go on to the definition of prices.

To summarize this section, it is necessary to stress that a discussion four possibilities of the quantities demand model is useful from the point of view of practice. This is because, sometimes a total output of a certain branch is bounded, and for this branch it is preferable ***to give total quantity instead of final uses as it is usually done*** (Stone, 1961).

3. The Supply Price Model of Mixed Input-Output

Before describing the prices' equation system it is necessary to remember that in this paper we assume that absolute (real) prices are dependent on the total quantities (i.e., absolute prices are identical for all branches and categories of final uses, but they change according to changing of total quantities). The same is also true for the prices of primary factors measured in real terms. This means that absolute price for goods and primary factors, as mentioned above, are uniform.

The previous section described a quantitative demand system model by which, in all four cases, one can obtain required quantities of primary factors in order to satisfy a certain level of quantities of final uses of goods. Yet, simultaneously, two types of prices might be obtained: unique commodities prices (q^d and π^d) from the demand side by means of the aggregate demand curves of commodities, and unique prices for primary factors (s^s and λ^s) by means of the aggregate supply curves of primary factors. Therefore, it is necessary to verify whether there is accordance between them, as well as between quantities. However, in this paper we assume that demand prices of goods equal their supply prices (cost of production). Also, latent price of both goods π and primary factors λ in equilibrium situation are equal to one.

On this basis the supply price model for commodities with absolute prices may be written as

$$p_j^s = \Sigma\, p_i^s\, \underline{a}_{ij} + \Sigma\, \pi_{I'}^s\, a_{i'j} + \Sigma\, s_k^s\, \underline{c}_{kj} + \Sigma\, \lambda_{k'}^s\, c_{k'j}\,, \qquad (j=1,2,...,n') \qquad (10)$$

and for commodities with latent prices it may be written as

$$\pi_{j'}^s = \Sigma\, p_i^s\, a_{ij'} + \Sigma\, \pi_{I'}^s\, a_{i'j'} + \Sigma\, s_k^s\, c_{kj'} + \Sigma\, \lambda_{k'}^s\, c_{k'j'}\,, \quad (j'=n'+1,...,n) \qquad (11)$$

Or in matrix and vector notation

$$(p^s)' = (\underline{A}_{ij})'\, (p^s)' + (A_{i'j})'\, (\pi^s)' + (\underline{C}_{kj})'\, (s^s)' + (C_{k'j})'\, (\lambda^s)'\,, \qquad (12)$$

$$(\pi^s)' = (A_{ij'})'\, (p^s)' + (A_{i'j'})'\, (\pi^s)' + (C_{kj'})'\, (s^s)' + (C_{k'j'})'(\lambda^s)'\,, \qquad (13)$$

where

p^s - is the row vector (1*n') of absolute supply prices of commodities measured in real terms, i.e., the measurement of p^s is (\$/M);

π^s - is the row vector (1*(n-n')) of latent prices of commodities measured in monetary terms, i.e., the measurement of π^s is (\$/\$);

s^s - is the row vector (1*m') of absolute prices of primary factors measured in real terms, i.e., the measurement of s^s is (\$/M);

λ^s - is the row vector (1*(m-m')) of latent prices of primary factors measured in monetary terms, i.e., the measurement of λ^s is (\$/\$).

Prices of primary factors, absolute prices and latent prices, s^s and λ^s are given, which are determined according to quantities of primary factors obtained by the solution of systems (3) and (4). Therefore, there are two unknowns p^s and π^s, and two equation systems, (12) and (13). By solving them, generally, we could obtain two unknowns when another two unknowns are given. This is the well-known case. There are two versions: a) In the first version, by substituting s^s and λ^s in (12) we obtain p^s by means of π^s

$$(p^s)' = (I-(\underline{A}_{ij})')^{-1}\, [(A_{i'j})'\, (\pi^s)' + (\underline{C}_{kj})'\, (s^s)' + (C_{k'j})'\, (\lambda^s)']\,, \qquad (14)$$

if $(I-(\underline{A}_{ij})')^{-1}$ exists. And by substituting the value obtained for p^s in (13) we determine the value of π^s

$$(\pi^s)' = (A_{ij'})'\, (I-(\underline{A}_{ij})')^{-1}\, [(A_{i'j})'\, (\pi^s)' + (\underline{C}_{kj})'\, (s^s)' + (C_{k'j})'\, (\lambda^s)'] + (A_{i'j'})'\, (\pi^s)' +$$

$$+ (C_{kj'})' (s^s)' + (C_{k'j'})'(\lambda^s)' , \quad (15)$$

that is if certain conditions are satisfied. And, finally, in order to determine the value of p^s we must substitute the value obtained for π^s in (14).
b) In the second version, these two systems [(12) and (13)] are considered as one whole system

$$(P^s)' = (\mathrm{I} - A')^{-1} C' (S^s)' \quad (16)$$

if $(\mathrm{I} - A')^{-1}$ exists.
where

P^s - is the row vector $\{p^s, \pi^s\}$;
S^s - is the row vector $\{s^s, \lambda^s\}$;
C - is the combination of the matrices $\underline{C}_{kj}$, $C_{k'j}$, $C_{kj'}$, and $C_{k'j'}$, similar of A

$$C = \begin{pmatrix} \underline{C_{kj}} & C_{kj'} \\ C_{k'j} & C_{k'j'} \end{pmatrix}$$

If we take into account the assumption here that demand prices of commodities equal their supply prices (cost of production), the prices of commodities obtained are equilibrium prices. In brief we can therefore conclude that instead of that direct input coefficient of commodities and primary factors that are constant, supply prices of factors changed depending on the change of total quantities, and consequently also supply prices of commodity change.

4. Conclusions

In this paper Input-Output in mixed measurement is suggested in order to bring input-output analysis closer to real economic life. In this suggested approach, input-output combines two types of branches of economics, one which might be measured in real (physical) terms (water, electricity, labor) with one which might be measured in money terms (financial, services, fixed capital) only; similar to practical economics. In the second section, the quantitative model of input-output in mixed measurement is discussed. It was shown that there are four possibilities in solving such input-output systems: 1) when given total output of commodities are measured in both physical and money terms then their final demands are obtained by the solution; 2) when the total output of physically measured commodities and the final uses of commodities measured by money are given, and the final output of the first and the total output of second are obtained; 3) when the final uses of physical and the total output of money commodities are given, and the total output of the first and the final uses of second are obtained; and finally 4) when the final uses of both are given and the total output of them are obtained. In the third section the supply price model was discussed. There are two suggested versions of the definition of commodities' prices when prices of services are obtained according to the solution of previous quantitative model. The first version proposes the stage solution, and the second is considered as one whole model, similar to the usual price's model of input-output system. Finally, such a new version of input-output will allow us widen the use of input-output for practical purposes.

References

Davar, E. (1994), *The Renewal of Classical General Equilibrium Theory and Complete Input-Output System Models*, Avebury, Aldershot, Brookfield USA, Hong-Kong, Singapore, Sydney

Leontief, W. (1941). *The Structure of American Economy*, 1919-1929. Cambridge, Harvard University Press, (second Edition 1951, New York, Oxford University press)

Leontief, W. (1986) I*nput-Output Economics*, Second Edition, New York, Oxford University Press

Leontief, W. (1974) Structure of the World Economy, *The American Economic Review,* December

Miller, R.E. and P.P. Blair, (1985), *Input-Output Analysis: Foundations and Extensions,* New-York, Prentice-Hall, USA;

Stone, R. (1961), *Input-Output and NationalAccounts,* Paris, DECD;

Walras, L. (1954) *Theory of Pure Economics*, Translated by W. Jaffe, Allen &Unwin, London