INPUT-OUTPUT PROPERTIES OF AN ALTERNATIVE MATHEMATICAL STRUCTURE FOR THE ANALYSIS OF MARKETS AND INSTITUTIONS

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INPUT-OUTPUT PROPERTIES OF AN ALTERNATIVE MATHEMATICAL STRUCTURE FOR THE ANALYSIS OF MARKETS AND INSTITUTIONS[1]

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Abstract : This paper is based on an alternative to standard 'rational choice'' assumptions in economics, which assert that the human mind is isomorphic with a utility function and the firm with a technological cost function, expressed over a range of real numbers. The alternative is based on discrete mathematics and computational psychology. The mathematical architecture is an extension and elaboration of ACT-R (Anderson 1993, Anderson & Lebiere 1998), a leading computer-based model in experimental cognitive psychology. Mathematically, this is a two level architecture with a 'symbolic' level of structures and expressions in discrete logic, and a 'subsymbolic' or statistical level which selects which structures or productions at the 'symbolic' level are activated. The technique involves the construction of sets and relations representing human behavior and the physical environment, together with appropriate index sets, to express a discrete mathematics of both 'choice' and 'production'. On the 'choice' side, this enables discrete modeling of varieties of emotional and sensational appraisals complemented by cognition, corresponding to a variety of results in both experimental psychology and experimental economics. On the 'production' side, economic organization is expressed in discrete form as (1) classes of instrumental, or technical, economies, (2) logic-based institutional arrangements (Hohfeld 1964, Lindahl 1977, Knight & North 1997, Hage 1997) as constraints on behavior, and (3) selection and control of production behaviors compliant with ACT-R in teams and hierarchies. In conjunction with media of exchange, whose function is to ensure discrete partitions and subpartitions, these can define a complete economic system. Cognitive and economic production rules take the form of expressions in a formal mathematical language (Davis, Sigal & Weyuker 1994), with instrumental economies and institutional arrangements playing the role of 'syntax', and hierarchically arranged individual goal portfolios the role of 'semantics'. Productions of an economy expressed in such a 'language' underlie row vectors in a Leontief-type input-output table (Leontief 1986).

INTRODUCTION

Input-output analysis is commonly, although not exclusively, identified with general equilibrium analysis[2], in which prices and quantities of goods (or services) are the variables in focus. This paper, in contrast, reflects developments in an alternative

[1] This paper is subject to further revision.

[2] Leontief (1966) described the input-output methodology as 'an adaptation of the neo-classical theory of general equilibrium to the empirical study of the quantitative interdependence between interrelated economic activities'.

economics based on computational psychology, in which information processing and cognitive behavior selection, rather than prices and quantities, play a primary role. In such a model, both production and consumption are based on information processing and cognitive selection of behavior. For example, 'competition' is a cognitive phenomenon, in which sellers attempt to place information and impressions which will elicit purchasing behavior in the minds of potential consumers, based on a mental model of how consumer cognition works, including what consumers have learned elsewhere, and how a particular product may fit into various personal goal hierarchies.

Psychology-based marketing is based on empirical findings from experimental psychology, e.g. that (a) the human attention span is typically limited to 7±2 chunks of information at a time[3], and (b) how a choice is framed[4], and the context in which it occurs (e.g. in conjunction with what other alternatives[5]) more accurately predict choice behavior than logical deduction from a previously constructed set of global preferences as neoclassical theory presumes[6].

For these purposes, we need models which can account for human behaviors and information flows relating to both the production of goods and services and their transfer to users, including the organization, production and distribution of symbolic content. The one mirrors the other - real and symbolic encode and decode to the other. Every product, service, skill and consumption has counterparts in human learning and information processing, often involving large numbers of people, or it could not occur. The input-output framework has properties which can help organize this information in an analytically productive fashion.

The Role of Mathematics

It is important that models be expressible mathematically. The enormous influence of neoclassical economics is in no small part attributable to the apparent mathematical sophistication of its models - even when empirical support for the underlying assumptions is weak or non-existent[7]. Models expressed only in words can be interesting and important, but they permit a much less precise conversation.

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[3] Ashcraft, Mark H. (1994), *Human Memory and Cognition*, New York, NY : HarperCollins.

[4] Kahneman, Daniel & Amos Tversy (1984). Choices, Values, and Frames. *39 American Psychologist* 341-350.

[5] Simonson, Itamar & Amos Tversky (1992). Choice in Context : Tradeoff Contrast and Extremeness Aversion. *29 Journal of Marketing Research* 281-95.

[6] Neoclassical assumptions implicitly represent a choice of a particular mathematical approach, i.e. they are necessary to generate continuous variables on the basis of which calculus-based analysis can be framed. Traditional approaches to mathematics education may have encouraged the perception that mathematics is coextensive with the progression arithmetic-algebra-analytical geometry-calculus, but this is not the case.

[7] Rabin, Matthew (1998), "Psychology and Economics", *36 Journal of Economic Literature*, 11-46.

This paper outlines, using an informal mathematical presentation, some techniques using set theory and discrete logic[8], involving logical production rules on sets which generate structured subsets. The same base sets, and the same logical techniques, can be used for all types of production rules, and at both aggregated and disaggregated levels.

Part I discusses logical production rules as a common mathematical structure for both productive economic activity and computational psychology. It then sets forth the mathematical nature of 'structure'. A notional input-output structure for the recording industry is an example used as an illustration in various sections of the paper. Part II introduces 'instrumental economies' - arranging the physical world through technological production rules - and 'institutional arrangements' - arranging other people's behavior through legal or social production rules (laws, social norms, conventions) as the basic constituents of economic activity. It then sets out a method to formulate them in common mathematical terms as products of the same underlying sets. Part III identifies mental models of consumption and production as basic structures for the chain of economic processes summarized by an input-output row vector. Part IV presents economic process as mental model prediction, confirmation, adaptation or disappearance. Part V outlines some applications of this approach and future work of particular interest to the author : competition as cognitive processes; the role of institutional arrangements; the nature and organization of transactions between time periods; and the distribution of income.

I. THE PRODUCTION OF STRUCTURE

Production Rules

The term 'production rule' can have a number of connotations. It could be taken to mean the kind of instructions a recipe or a technology consists of - say making bread, at home, or commercially. Cognitive psychologists use the term production rules to refer to 'productions' of associations between concepts, such as 'pluck' and 'note' on a guitar, or 'rock music' and 'electric guitar'. In other fields, such as cybernetics[9], terms such as 'transition' are used to describe similar phenomena, such as going from state 1 to state 2 to state 3, etc., such as an electric keyboard which plays a tune from memory.

In all cases what is meant is a statement of logical implication, such as $A \rightarrow B$ or $C \leftarrow D$. '$\rightarrow$' is the same as *IF ... THEN .... .* On either side of the implication there can be multiple terms, such as *[E & F]* or *[G OR not-H]*. Sequences of such rules occur when the consequent of one rule is subsequently the antecedent in another (in computer programming, often linked to a *GOTO* statement). For example :

[8] Bourbaki, Nicolas (1968). *Elements of Mathematics : Set Theory*. Paris : Hermann.

[9] Ashby, William Ross (1961). *An introduction to cybernetics*, London : Chapman & Hall.

$$A \rightarrow B$$
$$B \rightarrow C \; OR \; D$$
$$C \; OR \; D \rightarrow H$$
$$D \rightarrow J$$

so that, following the logic chain through,

$$A \rightarrow H \;\; OR^{10} \;\; J$$

Where the terms in such formula are sets, production rules in the nature of operations to generate other sets are defined. There are three main possibilities.

One is an identity transition in which the set remains unchanged.

A second is the generation of subsets of a particular set. Subsets may be specified by a condition or constraint which elements must meet, such as the set of individuals who are members of female rock bands.

A third is the generation of a Cartesian product set, in which elements are selected from each of two or more sets (which may be identical sets) to form the set of all possible combinations of pairs (in general, *n*-tuples, where *n* is the number of base sets : e.g. $(A \; X \; A \; X \; B \; X \; C) \rightarrow \{a, a, b, c)$ for $a \in A, b \in B, c \in C)$. Subsets of such products define relations or mappings[11].

Consider two or more sets with multiple elements, *A*, *B* and *C*, such that $a_i \in A$, $b_j \in B$ and $c_k \in C$. Various products could be formed from these sets, such as $A \; X \; A$, $A \; X \; C$ or $A \; X \; B \; X \; C$ where a product chooses one element from each set, and where there the product set is the full set of such pairs or triplets, such as (a_1, a_1), (a_{27}, c_{12}), or (a_{11}, b_{22}, c_{33}). Importantly, subsets of these products can compose *structures*.

Structure

Structure is a widely used term which can be given a precise meaning in discrete mathematics, for present purposes, as applied to an economy. I will first state it abstractly and then give some intuitively accessible examples.

A *structure* is a subset of a product of sets, where each subset in the structure (product subset) intersects with at least one other such subset.

[10] An unrestricted logical OR includes & - i.e. one or the other or both.

[11] Gratzer, George (1979), *Universal Algebra*, New York: Springer-Verlag.

For example, a Leontief input-output table has a certain structure defined by intersecting horizontal and vertical partitioning[12] of inputs and outputs. The processing sector of an input output table consists of two sets : a set of industries $i \in I$, and a set of coefficients $c \in C$. Form two identical product sets $I \times C$, $I \times C$, each element of each product set consisting of an (i, c). A subset of one of these sets is a structure called a row vector (i_r, c_r), because each pair contains a shared element (there is an intersection), the industry identifier. A subset of the other product similarly represents a column vector (i_m, c_m) structure. The structure of the table itself is the subset of pairs $[(i_r, c_r), (i_m, c_m)]$ such that for each cell $c_r = c_m$. In other words, the $[(i_r, c_r)$ & $(i_m, c_m)]$ *intersect* because they share a common element. In matrix algebra these intersections are drawn attention to with subscripts identifying rows and columns (e.g. c_{11}, c_{12}, etc.).

It may be asked, why describe structures in terms of a subset of a larger product set when it is only the final result, i.e. the 'structure' which may be of interest? The general answer is, that the larger product set is the 'choice' set from which particular selections are made. The point of intersection of a supply curve and a demand curve are not the only points of analytical interest. The points in supply and demand curves are each subsets of a planar space, and how one arrives at a point of intersection is important and interesting, too. Possibilities which are precluded may be uninteresting, moderately interesting, or very interesting. Quantity of a good or service demanded if the price were zero may typically be precluded by reality (market supply is usually zero), but is nonetheless interesting because it can occur in some contexts (e.g. downloading popular music from Napster, listening to commercial radio play). It is the basis for the 'free rider' phenomenon.

Any abstraction focuses our attention on a particular set of possibilities, which may mask others. The Pareto criterion for example, is based on a presumptive supremacy of market transactions as determinative of an efficient pattern of outcomes. On this basis, a decision of a pharmaceutical company to emphasise treatments for AIDS, rather than vaccinations, is efficient because the main market for vaccines would be in poor countries, especially in Africa, which cannot afford high drug prices; and because an effective vaccine would limit future demand for existing treatment drugs. Similarly, US-style private health care would be considered more efficient because it is market based, whereas European or Canadian 'universal' public health care is by the same test inefficient. However, the US system uses a greater proportion of GDP while providing health care to a smaller proportion of the population; and while the US ranks well in health measures such as life expectancy and infant mortality, it ranks behind several countries which spend less per capita on health care to achieve better results[13]. In addition, the Pareto criterion is indifferent to distributions of wealth and income, regardless of how skewed, other than as expressed through market transactions. Such

[12] A partition on a set is a set of subsets in which each element of the set belongs to only one subset. Each partition of a set defines a particular equivalence relation for that set of subsets.

[13] United Nations (1999). *Human Development Report*.

examples reveal the Pareto criterion as a philosophical rather than objective choice of performance standard for an economy, a sector, or a transactional system.

A further example of structure as a subset of a larger set of possibilities : consider the materials required to build a house, as delivered by various suppliers to a building site. Now imagine all the possible ways they could be combined, including, perhaps, non-use of some items. A very small subset of these possibilities contains structures in which people could live, where the plumbing works, etc.. This is a subset with, for example, intersections at roof, wall and floor joints, and between pipes, taps and drains. The subset we call 'houses' is chosen with effort and skill, i.e. logical production rules of the various construction trades such as bricklayer, carpenter, electrician, plumber, etc..

An economy is also a set of outcomes chosen with skill and effort[14]. Input-output is a symbolic representation of certain aspects of the structure of a functioning economy. This paper sets out one way of specifying additional structure in an economy, as expressions in an *economic grammar*.

This paper will also outline how row vectors can express 'grammars' of production and distribution, in which instrumental economies such as specialization, and institutional arrangements such as forms of property, play the roles of 'syntax', and individual goal sets expressed as production rules play the role of 'semantics'; and how these can together be formulated as logical expressions in a formal mathematical language[15]. The 'grammar' expresses both the structure of real production and acquisition of goods and services, and the symbolic human cognitive processing which mirrors or encodes it.

An Example : The Recording Industry

A 'product' and an 'industry' can be defined in informational rather than, or than just, physical terms. Consider, for example, the 'inputs' and 'outputs' of the recording industry. For an 'output' which is a set of new owners of original CDs, many direct and indirect 'inputs' are required. A record label (establishment) buys blank CDs, packaging materials, and recording and duplicating equipment, and skilled production staff. It contracts with songwriters and performers. It contracts with delivery services to ship its products to broadcast outlets and to retail stores. It relies on the production and widespread purchase of standard format CD players. People hear the songs on radio or TV and may go to retail outlets to buy the CDs with the songs they like on them.

[14] In contrast to a house, most economies are not from a single plan, but from decentralized sets of plans. In both cases the resulting structure is from the application of sets of logical production rules. Decentralized processes may involve 'self-organization', i.e. rules to which the parts themselves require each other to conform (Haken, 1996). Contrast an industry standard such as Windows or VHS arrived at 'bottom up' as opposed to an EU-mandated standard for truck (lorry) axles on highways, which is 'top down'.

[15] A formal language consists of a symbol set ('alphabet') from which strings are composed, and production rules for subsets or sequences of strings which form valid expressions in the language.

Modified Recording Industry Vector
(focus industry/establishment in italics)

A	B	C	*D1*	D2	E	F	G	H	I	J	**{O}**
CDs	Pack-aging	Perfor-mers	***Prodn & Dupln (1)***	Prodn & Dupln (2) comp-etitor	Trans-port-ation	Broad-cast-ing	CD player mfrs.	Retail	Govt.	Indivi-duals	**Set of Users**

A	&	B	&	C	&	D1	&	not-D2	&	E	&	F	&	G	&	H	&	I	&	J	→	**{O}**

The vector at the bottom presents a simplified production rule statement for this industry.

If potential buyers go online to the Napster, Gnutella or FreeNet websites and download only the songs they like, for free, this is a different sequence of connections with a different structure. The business model implicit in the above sequence of production rules is to that extent falsified.

Exchange depends on more than physical production and distribution : it depends on effective property rights - the conditional right and effective ability to exclude - on a basis which has been widely learned in the relevant population. Exclusion is relatively straightforward in the case of objects. In the case of intellectual property such as copyright, it is the pattern or information in any manifestation from which exclusion is sought, rather than a particular physical object.

Thus the dilemma for a record label is not merely to make a product and set a price. A successful business model requires an orchestrated arrangement of behaviors on the part of many players :

- compliance with CD and CD player standards across many manufacturers, broadcast media and individuals
- effective copyright and contract laws to exclude competitors from recording or duplicating the same material
- broadcast outlets for pre-purchase exposure to potential buyers

- methods to protect downstream integrity of copyright, such as restricted distribution channels, encryption or widespread respect for copyright[16]

- availability of compatible CD players

- individual pre-purchase behavior as a necessary element, such as going to a CD store, or placing a phone or internet order

A	B	C	*D1*	D2	E	F	G	H	I	J	**{O}**
CDs	Pack-aging	Perfor -mers	***Prodn & Dupln (1)***	Prodn & Dupln (2) comp-etitor	Trans-port-ation	Broad -cast-ing	CD player mfrs.	Retail	Govt.	Indivi -duals	**Users**
blank discs	boxes, labels	reh - earsed music			truck trans-port	air play	CD play-ers	dis-play & sales		go shop-ping	
Prop-erty + contr-act + stand-ards	Prop-erty + contr-act + stand-ards	Prop-erty + contr-act + stand-ards	***Prop-erty + intell-ectual prop-erty + contr-act + stand-ards + autho r-ity (orgn)***	Intell-ectual prop-erty	Prop-erty + contr-act	Prop-erty + intell-ectual prop-erty + contr-act + stand-ards	Intell-ectual prop-erty + stand-ards	Prop-erty + intell-ectual prop-erty + contr-act	Rules + enforc ement	Prop-erty + intell-ectual prop-erty + contr-act + stand-ards	(Hier-archi-cal goal port-folios)

A & B & C & D1 & not-D2 & E & F & G & H & I & J ↔ **{O}**

The economically significant behaviors include both action and restraint. Two main classes of behavioral phenomena are considered first :

1. Instrumental economies - arranging the physical world through technological production rules.

[16] The author Stephen King has posted the first chapter of a new novel on the internet for free downloading. He will write additional chapters if enough people pay $1.00 for the chapter. At the time of writing, such voluntary payments represented about 2/3 of the number of downloads.

2. Institutional arrangements - arranging other people's behavior through legal or social production rules (laws or social norms or conventions).

II. TECHNOLOGICAL AND INSTITUTIONAL ARRANGEMENTS

Instrumental Economies

An 'economy' (cf. 'diseconomy') is an arrangement which produces comparatively more of something : 'economy' is inherently a relative term, implying at least two arrangements for comparison purposes. For example, settled agriculture is an instrumental economy. It reduces the variety of species of plants in a plot. At harvest time, this eliminates most of the walking between plants in the wild, and all of the walking required to find them. A barn-raising is an instrumental economy. Because many hands enable it to be built sooner, the owner gets more or earlier use of it for dry storage.

The 'more' of something which an instrumental economy produces involves *repetition*.[17] What is being 'economized' on is, ultimately, our own behavior, the only thing we have to change or maintain the world to our liking, and the only thing an individual can directly control. If each product - say a CD with music recorded on it - had to be made from scratch for each listening without tools or previously learned skills, there would be no economies. An economy comes from repeating something previously learned, or repeated use of effort stored in the structure of a object or appliance.

A primary rationale for coordination between individuals is achieving outcomes which, directly or indirectly, one cannot achieve alone. Coordinated action therefore depends on the presence of instrumental economies. The presence of one or more instrumental economies is a necessary precondition to exchange : the presence of instrumental economies means there could be advantages to exchange, their absence means there could not[18]. But, as will be seen, it also requires institutional arrangements.

'Capital spreading' involves using the same physical capital (tools, equipment) or human capital (skills) to make multiple items, or an item for use in multiple time periods, etc. Buildings are repeatedly used over time. Management is spread over many establishment activities. Closely linked to 'specialization', spreading involves multiplying the number of things which are done using a particular tool, machine or skill set.

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[17] For a new invention the comparative term at the beginning is zero, for it did not previously exist. However, it is not a true (successful) instrumental economy unless it is then repeated (repeatedly applied or produced).

[18] Even a one-of-a-kind item, such as an original Van Gogh, is of interest because its presence persists, i.e. is repeated, across time periods.

'Specializing' refers to reducing the variety of things which are done by individuals or things. This is usually the basis for 'spreading', either directly as a service or as embodied in a physical object, or both. In the case of humans, it works because it economizes on the limited attentional, learning and performance capacities of humans[19], alone or in teams. In the case of things, it works because the similarly specialized physical properties which the tool can alter occur repeatedly in the environment, such as trees in relation to an ax, or water in relation to a bucket. An important property of many technological economies is arranged repetition, i.e. standardization - so that male and female stereo plugs fit together, for example. In the case of a machine, specialization economizes on overall set-up behaviors. Set-up economies are extended to multiple functions in the case of 'programmable' machines.

'Coding' (and decoding) refers to repeating pattern at different energy levels. A computer simulation of aircraft take-off and landing operates at a much lower (more economical) energy level for pilot training than operating an aircraft; it may also be safer. A pilot's training decodes into more effective operation of an actual aircraft when that occurs.

'Combining' refers to the ability to achieve or preserve instrumental economies by combining various outputs of specialization, coding and capital spreading. In this way, individual instrumental economies are not isolated to single application classes. An assembled band of different instruments and musicians is an example. An input-output table is an aggregated way of summarizing the combinatorial properties of an economy or sector which is based on specialization.

Institutional Arrangements

We live in a world which is structured in many physical ways. However, no physical law ensures, for example, that people will respect copyrights or refrain from chatting during a concert performance. Institutional arrangements are based on human rules, which may be formal laws (promulgated by governments) or informal norms based on social sanction[20] (e.g. other members of the audience). These rules supplement the constraints of physical laws. The rules on which institutions are based are also instrumental economies - to restrict the range of human behavior, i.e. by standardization ('every one is subject to the same rules').

An important class of rules arranges organized restraint. 'Property' involves everyone except the owner suppressing all or virtually all their behaviors with respect to the subject matter of the property, which can be a thing or a pattern (most entertainment conglomerates aggressively restrict the use by anyone else of their copyright items or

[19] This applies to 'natural' limitations (not everyone has the unusual height generally required for professional basketball) as well as to learned ones (in a lifetime one can only master at most a few professions).

[20] Rule sets often operate together as a grammar, e.g. slightly different rules apply to the first and subsequent members of a queue.

trademarks). This produces the result that, in principle, the owner has exclusive cognitive control over its use. Human or civil rights involving avoidance of harm to others are similar.

Other rules provide a framework for joint action where one or both participants supply the specific behavioral content. A contract involves specific reciprocal promises : if you pay $X, I will do Y studio sessions, and vice versa. Individuals adopt their own specific rules within a legally sanctioned framework.

Authority[21] involves one person selecting other people's behavior by symbolic communication, such as those of a conductor of an orchestra.

In each of these cases, part of the cognitive logic is contingent third party sanctions for non-compliance. 'Third parties' may be government agencies or member's of a group to which one, even if temporarily, belongs (peer group, fellow members of a queue).

Money

A special class of institution is 'money', a generalized form of property. Conventionally, money is thought of as a medium of exchange and a store of value (medium of exchange across time). These are its instrumental functions in an exchange context. The concept of money as a unit of account can be the source of much misunderstanding. It is sometimes presumed that money is a reliable economic metric, such that a dollar measures or indexes economic value in a consistent way, much the same way a meter or a degree Celsius measure length or relative heat content, respectively. But this is not the case. The same person may pay, or the same seller may charge, a different price for the same thing in different places or at different times - in the case of airlines, computer programs may change prices for seats and routes from minute to minute.

The notion of money as a 'unit of account' has led some to propose that it can be used to place a 'value' on anything, including life and death, or hypothetical products or situations (such as peace and quiet on city streets) for which there are no 'markets' in which to buy and sell the item. However, where the same item cannot be bought and sold at all, let alone at an identical price, money measures are, at best, highly context specific. For example, where risks of injury in, say automobile use, and the costs of compliance with some regulation to reduce accidents are both actuarially known, one can calculate the cost point at which the regulation effectively stops protecting life and limb - but no life can be replaced at that or any other price. A similar point applies to 'life' insurance.

For reasons such as these, money is not a uniform or consistent metric of 'value', for a single person or enterprise, or between two or more persons and enterprises.

[21] Note that the root of 'authority' is 'author'. No particular 'power' relationship is necessarily implied.

However, the primary functional role of money is partition - division into subsets of finite sets. Goods, services and assets are not unlimited. If money is to be exchangeable for them, it, too, must be limited. In simple terms, to perform its exchange function without price inflation or deflation, the volume of money must correspond to the exchange volume and velocities it is required to support. This is, of course, consistent with input-output method, since both rows and columns are partitions into cells[22]. In addition, the partitioning requirement of money is reflected in non-negativity stability conditions in an input-output table.

Money income is similarly partitioned in a given time period, among enterprises (against which shareholders or creditors may have claims) and individuals. In practice, money partitions are enforced by solvency tests.

Because of the partition-enforcement function of money, it is almost always incorrect to treat money as a pure cardinal quantity, i.e. a rational number whose denominator is unity. In fact, all money transactions are conducted, even if implicate, on the basis of fractions which are less than 1, i.e. of available financial resources. People cannot spend more than they their financial resources (income, monetary assets, credit) permit. Firms must garner revenue and financial capital (including borrowings) in excess of outlays; insolvency or bankruptcy require reorganization or disappearance. This means that each purchase reduces the set of things which can be bought in the time period defined by those financial resources (e.g. that paycheck). Identifying money quantities as fractions - as input-output tables expressed in money terms do - draws attention to partition as one of the most important functions which money plays.

Moreover, a transactional price is never one fraction but (at least) two asymmetric fractions - for each of the buyer and the seller, affecting their subsequent financial capacities.

Expressions in Discrete Mathematics

Both instrumental economies and institutional arrangements may be expressed in discrete mathematics using the same symbols. They are both composed as products of the same base sets[23] :

[22] Because each cell represents some volume (i.e. set) of transactions.

[23] More detailed specifications in the Appendix.

Base Sets
(All Discrete)

Set		Subset
B	Behavior set	b
I	Individuals	i
S	States (sets of physical properties)	s
C	location Coordinates	c
T	Time	t

In each case, the sets can be expressed in a variety of ways, providing elements are identified consistently. Behaviors, for example, could be a simple catalog of actions, say playing a sequence of guitar chords, or the recording technicians manipulation of recording equipment to produce a master; or they could be a more technical catalogue such as that compiled by linguistics to produce particular sounds in the mouth, or oral cavity laryngal control to produce notes of a certain pitch and intensity. States could be objects (keyboards, string or percussion instruments); or detailed sets of properties (e.g. electronic sound analysis for mixing).

In addition, the following conventions are used :

–	NOT (absence of specified behaviors)
{ }	set
[ ] , ( )	Cartesian product
P(B)	power set, i.e. set of all subsets (e.g. of set B) important for excluding all or most combinations, e.g. of behavior, or individuals

Specific actions taken by individuals are products of subsets of each of the base sets :

What done?	by whom?	to what?	where?	when?
Set of Behaviors	Set of Individuals	Set of physical States	Set of locations (Coordinates)	Set of Times
{b}	{i}	{s}	{c}	{t}

Product : [{b}, {i}, {s}, {c}, {t}]

Sometimes, as in institutional arrangements involving property, it is the *absence* of behaviors (interference by others) which is of interest :

What NOT done?	by whom?	to what?	where?	when?
–Set of Behaviors	Set of Individuals	Set of physical States	Set of locations (Coordinates)	Set of Times

[{–b}, {i}, {s}, {c}, {t}]

Anything that happens (or doesn't) as the result of human action or inaction is describable by a [{±b}, {i}, {s}, {c}, {t}] quintuplet, or, as a shorthand, 'bisct'.

Combinations of these sets and products may be used to express the organization of instrumental economies (technological arrangements) and institutional arrangements (of human behaviors) in the same symbol sets. For production rule purposes, they are 'chunks' or 'terms', so that one 'bisct' producing another can be expressed by logical statements.

Since we are interested in predictability, we are interested in how we might determine one of the b, i, s, c or t terms from knowledge of the others, perhaps including knowledge of other 'bisct' product subsets.

Instrumental Economies in Discrete Mathematics

A skill set is a particular behavior set, normally associated with a particular class of objects, associated with a particular individual; those of an electrician, for example. These are skills which will have been learned, such as through instruction or apprenticeship, or both. The learning produces the skills, and the skills produce a stream of outcomes in electrification of areas or facilities. Each is some set of (b, i, s, c, t) products, shorthanded to 'bisct' in the following charts. { } indicates a set of the enclosed 'bisct's, () a single such product. To avoid clutter, most distinguishing subscripts or superscripts are omitted.

Instruction		Learning		Skill set		Applications
$\{bi_Psct\}_{teach}$	$\rightarrow$	$\{bi_Rsct\}_{learn}$	$\rightarrow$	$\{bi_Rsct\}_{apply}$	$\rightarrow$	$n(bisct)^1 + m(bisct)^2 + ...$

Here individual P has instructed individual R whose learning set included errors and other behaviors which will (hopefully) be eliminated from the skill set which produces precise outcomes for a variety of customers.

A machine, such as a machine to blister package CD jewel boxes, and a quantity of standardized raw material, such as clear sheet plastic, are instrumental economies produced by 'bisct' sets to produce a stream of packaging :

Manufacture of the IE		Apply IEs	Outputs
$\{bisct\}_{machine}$, $\{bisct\}_{plastic,}$ $\{bisct\}_{recorded\ CDs}$	$\rightarrow$	$[(bisct)^{machine}, n(bisct)^{plastic}$ $m(bisct)^{recorded\ CDs}]\rightarrow$	$q(bisct)^{packaged\ CDs}$

This is also an example of a simple combination of 'inputs' to produce a type of 'output'.

Institutional Arrangements in Discrete Mathematics

Three types of institutional arrangement are : property, contract and authority.

The basis of property is the suppression of behaviors by everyone except the 'owner' with respect to the subject matter of the property. This 'produces' ownership, cognitive control, in principle, of behaviors with respect to that property. If the property is a moveable object or created pattern, the 'ownership' applies to that object or pattern, anywhere. If it is immoveable property, land or buildings, it applies only within the applicable physical boundaries. So for a moveable object :

$$(-P(B), P(I-i_{owner}), s_{property}, C, \{t\}) \quad \rightarrow \quad (\{b\}, i_{owner}, s_{property}, C, \{t\})$$

But how is the behavior suppression produced? In simplistic form, legal rules are commonly of this logical type :

IF [compliance] THEN [no sanction]
&
IF [non-compliance] THEN [sanction]

and these contingencies are supposed to produce compliance, i.e.

IF
[IF [compliance] THEN [no sanction]
&
IF [non-compliance] THEN [sanction]]
THEN
[compliance]

The terms of this logical formula can be expressed in 'bisct' form where the sanction is (or is not) administered by a designated third party, such as a court.

A contract is a rule set which parties legislate for themselves by a rule of unanimity to meet conditions which will attract third party enforcement. Contracts often involve the bilateral exchange of property. They are of the general form :

$$(A \rightarrow B) \ \& \ (C \rightarrow D)$$

where, for example, $(A \rightarrow B)$ signifies that Virgin Megastores transfers ownership of a Red Hot Chili Peppers CD to Guido and $(C \rightarrow D)$ signifies that Guido pays Virgin Megastores $20. Both terms are necessary for a contract. In each case, $(A \rightarrow B)$ and $(C \rightarrow D)$ involve only a change in the i element of the 'bisct', e.g.

$$[(bi_{Virginr}\, s_{RHCP\ CD}\, ct) \rightarrow (bi_{Guido}\, s_{RHCP\ CD}\, ct)] \ \& \ [(bi_{Guido}\, s_{\$20}\, ct) \rightarrow (bi_{Virgin}\, s_{\$20}\, ct)]$$

with $(-P(B), P(I-i_{owner}), s_{property}, C, \{t\})$ implied with respect to both the CD and the money before and after the transfer.

In practice, most contracts are 'agreed' in symbolic (encoded) form at one time period, and performed (decoded into actions) in one or more later ones. Once a contract has been agreed, the standard contingencies are supposed to produce compliance, i.e.

IF [contract in valid form]
&
IF
[IF [compliance] THEN [no sanction]
&
IF [non-compliance] THEN [sanction]]
THEN
[compliance]

Institutions of *authority*, defined as the selection of one person's behavior by means of symbolic communication from another, is the basis of organization, particularly organization with hierarchy. The 'scope of authority' is a set of behaviors which the authority may select by symbolic communication, such as a conductor leading an orchestra. Once a relation of authority is established, such as by contract, the standard contingencies are supposed to produce compliance, i.e.

IF
[IF [compliance] THEN [no sanction]
&
IF [non-compliance] THEN [sanction]]
THEN
[compliance]

Combining Instrumental Economies and Institutional Arrangements

Instrumental economies are, in practice, paired with, or supplemented by, institutional arrangements. Farmers do not in general plant so that others may harvest. Hence rules of property. In market economies, especially Anglo-American ones, there is an extended Lockean notion that the owner of effort, whether individual or enterprise, is entitled to 'own' (exclude others from) the benefits. Organization implies either (a) hierarchy, i.e. authority, or (b) self-coordination of individuals through application of jointly understood rule sets, as in team sports and queues, or both. Rules of exclusion - property, or human rights, including exclusion from harm by others - would, unsupplemented by contract and consent prevent transfer or cooperation.

Institutional arrangements are 'conservative'. Property protects cognitive control. Contract permits change in one 'bisct' term, ownership identity, while leaving the others unaffected[24]. Authority preserves organization. For example, when the machinery and raw materials belonging to an enterprise are converted to products, the institutional arrangement of property attaches 'automatically' to the products as they are produced. When they are sold, the property 'protection' is transferred to the new owner : only the set of excluded individuals (correspondingly, 'owners') changes, by adding the previous owner and subtracting the new one. Where intellectual property is embodied in a product, the ownership of that pattern stays with the product, even when ownership of the item changes. However, the permitted (not excluded) behaviors (b term in the 'bisct') will be modified, e.g. to permit playing a CD for private listening.

Recording

A	B	C	*D1*	D2	E	F	G	H	I	J	**{O}**
CDs	Pack-aging	Perfor-mers	***Prodn & Dupln (1)***	Prodn & Dupln (2)	Trans-port-ation	Broad-cast-ing	CD player mfrs.	Retail	Govt.	Indivi-duals	**Users**
A &	B &	C &	D1 &		E &	F &	G &	H &	I &	J →	**{O}**
[{bisct} {-bisct} $)]^{A}	[{bisct} {-bisct} $)]^{B}	[{bisct} {-bisct} $)]^{C}	**[{bisct} {-bisct} $)]^{D1}**	[{bisct} {-bisct} $)]^{D2}	[{bisct} {-bisct} $)]^{E}	[{bisct} {-bisct} $)]^{F}	[{bisct} {-bisct} $)]^{G}	[{bisct} {-bisct} $)]^{H}	[{bisct} {-bisct} $)]^{I}	[{bisct} {-bisct} $)]^{J}	[bisct] {-bisct}$^{\{O\}}$

A &	B &	C &	D1 &		E &	F &	G &	H &	I &	J ↔	**{O}**

[24] A rental for a term of months or years changes two terms while preserving the others.

III. ARCHITECTURE OF COGNITION

Mental Models

Mental models - sets of production rules describing one's own and others' imagined, potential or intended behaviors - are fundamental[25]. All considered human action is based one or more mental models constructed (imagined) from memory, including, perhaps, prompts from external memory in the form of written, recorded or computer records[26].

At each stage of a processing sector in an input-output table there are partial mental models of upstream and downstream processes. Each producer must have some knowledge of its suppliers' capabilities and its customer's needs - often extending through to elements of consumer behavior - and it's own establishment's capacity to meet those needs within the scope of its particular product or service. Mental models encode / decode into external physical processes, or behaviors, individuals, objects or patterns, places and times - and vice versa[27].These mental models overlap (sets of production rules intersect) and, in a successful processing sector, coordinate to a coherent overall structure so that 'it all comes together' as a successful customer presentation of, say, a packaged CD.

Cognitive Psychology of Mental Models

Human beings acquire and process information in units styled 'chunks' : these are the 'atomic components of thought'[28]. A 'chunk' is any pattern processed as single unit. It could thus be a single digit, such as the number of days in a week, or the chunk constituting one's own memorized phone number. Beginning musicians think in chunks of individual notes and the actions required to produce them. Professional musicians process at the level of patterns of, say, chords, bars or stanzas. A CEO may think in chunks called sales targets, while a sales person thinks in terms of retail chains or stores.

Chunks as facts are learned as *declarative memory*. Human behavior depends on both declarative and *procedural* memory. Procedural memory is 'how to' memory which

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[25] See, e.g. Johnson-Laird, Philip N. (1983), *Mental Models*, Cambridge, MA : Harvard University Press; Moray, Neville (1999). Mental models of mental models of ..... *Attention and Performance XVII*. Cambridge, MA : MIT Press. 'Mental models' are also a cybernetic requirement, i.e. of any system with some separation of information and control of action : Ashby, W. Ross (1960), *An Introduction to Cybernetics*.

[26] Donald, Merlin (1991). *Origins of the modern mind : three stages in the evolution of culture and cognition.* Cambridge, MA : Harvard University Press.

[27] See, in general, Anderson, John R. (1995). Learning and Memory : An Integrated Approach. New York : Wiley; Ashcraft, Mark H. (1994), *Human Memory and Cognition*, New York, NY : HarperCollins.

[28] Anderson, John R. & Christian Lebiere (1998). *The Atomic Components of Thought*. Mahwah, NJ : Lawrence Erlbaum.

relates two or more chunks to each other. It takes the form of *production rules* of the form IF [chunk A] THEN [chunk D], or IF [chunk A OR chunk B OR chunk C] THEN [chunk D], e.g. IF [eminem OR Shania Twain] THEN [platinum CD sales].

Human memory and learning-based human performance have been predictively modelled and empirically confirmed in terms of production rules[29]. More specifically, human cognition corresponds to a two level mathematical architecture : one level, production rules in discrete logic, as just illustrated; and another level based on statistical quantities (such as repetition or recency) as the basis for selecting terms (chunks). These empirically based statistical quantities are the basis matching or conflict resolution in competition with other chunks; and then as the basis for selecting overall formulas containing those chunks as selected production rules.

For example, consider the choice of recording format - vinyl record, cassette tape, digital audio tape or CD - in the recording industry. Technically, all are valid methods for recording music. But what numbers of people in the intended market actively use playback equipment in this format, and which of those numbers meets or exceeds some threshold for economies of scale? Vinyl record playback is now rarely used, and is not available in a portable format. Digital audio tape players have not achieved significant penetration. Cassette players did but their use is in decline. CDs are now the leading format. So the CD format would definitely be chosen, and perhaps cassette tapes too, though that market is a declining one.

The mental models implicitly distributed across a processing sector vector are thus statistically chosen rules in logical form. The mental models distributed across an industry or product row vector express instrumental economies, institutional arrangements, and partitions. 'Coordination' occurs because these models - i.e. sets of production rules - are shared in overlapping (intersecting) sets among participants in the 'production process' described by a row vector. The intersecting mental models are one way of describing the structure of a row vector in a processing sectors.

The mental models implicit in actual business facilities and operations are inevitably somewhat imperfect; they are never completely predictive. Even if they are predictive in one time period, changes in technology or markets could undermine that in the next time period. All implicit mental models are necessarily experimental, an ongoing empirical test of the predictiveness, or continued predictiveness, of the business model and consumer behavior model implicit in real production (distribution, marketing) decisions. (There are nonetheless remarkable stabilities and predictabilities - a remarkable amount of structure - in modern economies.)

Business organizations can attempt to address unpredictabilities - insufficiently specified production rule sets, such as the effects of 'competition' - in one of two ways (other than

[29] Anderson, John R. & Christian Lebiere (1998). *The Atomic Components of Thought*. Mahwah, NJ : Lawrence Erlbaum.

abandoning the business). They can try to learn more - garner more information - to more predictively specify production rules, and adapt by changing internal structures (operations). For example, they may increase the 'granularity' of sets of production rules by disaggregating 'chunks', such as preparing a different mix of product for different retail locations. (In the record industry this might be based on patterns of returns of unsold CDs by store location). Alternatively, they can try to enhance predictability by attempting to add more structure to the external environment into which they sell, such as by moving from simple distribution through retailers to selling with negotiated display arrangements with the same retailers; or adding distributed cognitive structure through advertising (storage in the memories of potential customers for recall to attention) and contemporaneous in-store promotion (visual / auditory attention capture).

In modern economies, strategies of these types are as common as the 'price' competition on which neoclassical models are based. Among the more profitable businesses, they may be even more common.

But what about mental models of consumer behavior?

Human Goals, Emotions and Consumption

Psychological research is increasingly able to specify in an orderly way how humans make choices which cannot be based on information alone, but involve emotional appraisals in relation to basic human goals. Modern marketing takes advantage of this knowledge[30]. A particular form of structuring relates to product placement in the *goal hierarchy* of target consumers.

Evolutionary psychology has identified 5 'top level' evolved human goals :

1. Self-preservation and personal well-being.
2. Sexual access, preferably exclusive.
3. Control of material resources.
4. Status, particularly as a factor influencing or signaling control of material resources or sexual access, or both. This can take the form of (a) rank, (b) membership, or (c) combined rank and membership (e.g. social class).
5. 1 - 4 for one's genetic relatives, based on perceived degree of shared genes.

Each person has an acquired, learning-based portfolio of more specific goals in each of these categories. These more specific goals are ordered and re-ordered in conditional, context-based hierarchies, in which important individual differences occur. Some sacrifice self-preservation and well being for the obsessive pursuit of wealth or status, or for their country or ethnic group. Some submit to vows of perpetual poverty, chastity and

[30] See, e.g., recent issues of the journal *Psychology and Marketing*.

subservience in the pursuit of a superior afterlife, a putative form of self-preservation. Some make sacrifices for their children; others abuse or even kill them. For some, loss of status may make life not worth living. In all cases, people make specific choices which compose and recompose a portfolio of resulting states within these five evolved goal classes. The combination of results in individual portfolios may often be distinctive, but the individual components (e.g. particular states of health, trade or profession, marital history) are likely to occur across numbers of individuals. Individuals may in practice put any one of these five top level goals ahead of any other, such as placing status or wealth ahead of health or family - perhaps until the threat to one of the latter is clear. Most people confronted with an imminent threat to their physical well-being will give priority to personal survival. However, for most people in advanced economies, most of the time, physical well-being permits priority to be given to other goals.

Goals are capable of empirical investigation and may be specified in logical 'bisct' form. Goals are states associated with oneself (e.g. (i_{self}, s, c, t)) constructed in imagination from recent or long-term memory, and usually associated with some possible method (behavior set) for pursuing them.

For example, personal well-being, or its absence, can be defined in terms of physiological states {s}, {c}, {t} associated with that individual, or a transaction from one to the other and by means of what actions that transaction may be accomplished (e.g. satisfying hunger or thirst).

Sexual access implies reciprocal behavior in the form of genital contact [$(bisct)^1$ & $(bisct)^2$] between two individuals. 'Exclusive' sexual access implies that does not occur with any other individuals.

Status can mean membership in some set (e.g. eligibility as a pensioner, acceptance into a social circle). These imply behavior on the part of members of that or some other set - payment of a pension, inclusion in a group's social activities.

Status can also mean rank or authority. This is often associated with control of material resources, from rank in a queue to obtain some benefit, to winning a competition, such as an Olympic event to the effective power to direct others.

Direct control of material resources involves control through property ownership, contract, or contract-based authority.

In the modern world, we are encouraged to pursue almost all personal goals through consumer purchases. For example :

1. *Personal well-being* : processed foods, pain killers or, in some countries, gun ownership for self-defence.
2. *Sexual access* : cosmetics, bikinis, diamond rings.
3. *Control of material resources* : lottery tickets, stock market investments.

4. *Status* : peer group clothing, university degree, expensive car.
5. *for genetic relatives* : some or all of the above for one's children.

These product concepts imply more or less implicit production rules : e.g.

IF [product A] THEN [status with defined peer group]

The primary goal a product appeals to can have an important bearing on its 'demand' characteristics, notably the price (fractions of income or financial resources) it will fetch. For example, footware serves an important functional purpose for everyone of protecting the feet. However, for many teenagers what matters is the status statement - peer conformity (group membership) or superiority statement (rank, such as by association with an elite athlete who has 'endorsed' the product) thought to be embedded in particular brands or styles of footware. To produce a sale - and particularly to produce a higher price (partitional share) - a producer must design and manufacture (create and implement production rules for) effective symbolism as well as effective foot cover. It is in the nature of 'status' that it, and the means to achieve it, are in more limited supply than the means to protect one's feet[31].

Production Rules as Grammars : A Mathematical Economic Language

The outputs of instrumental economies (products and services) and of institutional arrangements (specific forms of property, contract, technical standards, etc.) can be expressed as logical production rules on 'bisct's and their underlying sets, as can entries in personal goal hierarchies. This means they can also be expressed in a formal mathematical language[32], by coding each 'bisct' (which is a relational mapping (b, i, s, c, t)) as a symbol string with the same characters i.e.' **bisct**'.

A 'language' in discrete mathematics comprises

(1) a set **M** of symbols, including starting, terminal (**M'**) and non-terminal (**M''**) symbols;[33]

(2) a set **M*** of all strings of those symbols of some finite length; and

(3) production rules between strings of the form **abc** $\rightarrow$ **cde** which generate subsets of strings which are 'valid' (syntactical in the sense of conforming to the

[31] At the 1996 Summer Olympics in Atlanta there was a controversial shoe company billboard which said, "You don't win Silver. You lose Gold."

[32] Davis, M. D., R. Sigal and E. J. Weyuker (1994). *Computability, Complexity and Languages*, Boston, Academic Press.

[33] The reason for 'starting' and 'terminal' symbols is that starting and ending points are needed for chains of logical implication, which is what a mathematical language comprises.

production rules for the language; and 'semantic', i.e. meaningful, i.e. effectively goal-related).

A natural language such as English begins with symbol strings from an alphabet of 26 letters plus a space to distinguish one string (word) from another. Of the set of all possible letter strings of some finite length, only a small subset are valid words, e.g. 'man' and 'name' but not 'nam'. Words may then be assembled into strings of some finite length. Only a subset of such word strings meet syntactical requirements for a valid sentence, such as subject-verb-object. But even the subset of word strings which are syntactically correct contains expressions which are nonsense, such as Noam Chomsky's famous 'Colorless green ideas sleep furiously.' Only a further subset are semantic, i.e. convey meaning between two speakers of the language.

An institutionalised instrumental economy - a machine, an enterprise, a product, a particular form of human capital - is production on a 'bisct' or string of 'bisct's. Each expression must start from a string which contains sct; intuitively, some initial state, at some place and point in time, to which some behavior is applied.

Take all the labels **±b**, **i**, **s**, **c**, **t**. Add to this the logical symbol **&** and the set **{r}** of rational numbers (to represent 'prices'). This is the 'vocabulary' of the language.

Specify that only the following may be included in expressions :

(a) letter strings of the form **±bisct**;

(b) a number which immediately precedes a **±bisct**;

(c) an **&** which follows a **±bisct** and which precedes either a number or a **±bisct**.

Production rules in the language are then of the form

$$\mathbf{r_m \pm bisct \rightarrow r_n \pm bisct}$$

or terms of strings of **r ±bisct**s connected by an **&**.

Call the set of all such strings **N***. Only a subset $\mathbf{M^* \subseteq N}$ of the results those strings represent what is physically possible : one can't do different things in different places at the same time (although one can, of course, *not* be in lots of different places at the same time, a natural feature which 'property' takes advantage of.). Only a subset $\mathbf{L^* \subset M^*}$ involves instrumental economies.[34] Only a subset $\mathbf{K^* \subseteq L^*}$ of those produce institutional consequences (e.g. property rights; sanctions for rule non-compliance). Call this subset

[34] An instrumental economy is identified by comparing two productions with a common **r ±bisct** output term, but where $\mathbf{r_m \pm bisct > r_n \pm bisct}$ or where $\mathbf{s_1 \supset s_2}$ or $\mathbf{sct_1 \supset sct_2}$ in $\mathbf{\pm bisct_1}$ and $\mathbf{\pm bisct_2}$, respectively.

'syntactic'. Only a further subset **J* ⊆ K*** will successfully 'connect' with customers' context specific goal hierarchies. Call this subset 'semantic'. The production rules (technologies and associated legal rules) which produce these subsets comprise an *economic grammar*.

In summary, the production rule set for a language of economic production would include three levels of syntax and one level of semantics, starting from a set **N*** of all possible string combinations :

(a) a subset **M* ⊂ N*** of the physically possible (e.g. same thing not in two places at the same time);

(b) a subset **L* ⊂ M*** which are correct in form because they express institutional economies;

(c) a further subset **K* ⊆ L*** which are correct in form because they comply with institutional arrangements;

(d) a further subset **J* ⊆ K*** which conform as product presentations to context specific goal hierarchies of customers, i.e. are semantic.

A language is functionally important as an interpersonal source of symbolic isomorphisms between real and cognitive phenomenon. That function can only be performed, of course, if expressions in the language are *recognized* by the relevant parties. Each formal language in discrete mathematics corresponds to an automata (an extension or generalization of mechanism) which recognizes, i.e. acts on inputs expressed in, that language. 'Semantic' means effective in controlling that mechanism. For example, an expression in a 'language of institutions' is recognized when people attribute institutional significance to each other's behavior : a police officer recognizes an offence; a subordinate recognizes a lawful command; an offer is accepted, making a binding contract; a check is honored by a bank; a ceremony is recognized as a marriage; a divorce is recognized, or not, in another jurisdiction. A full expression in an economic grammar expresses a relation among technological production, institutional arrangements, and individual goal hierarchies.

IV. PREDICTION, CONFIRMATION, ADAPTATION OR DISAPPEARANCE

A developed economy, in contrast to ancient hunter-gatherer societies, is an environment whose engineering is shared with nature, not determined by it. It is our teaching various parts of the environment how we wish them to behave. The teaching-learning analogy is deliberate, for our actions store structured pattern in our environment - which includes other people - for subsequent repeated use; as our long-term memory stores structured pattern as encoded input from the environment. The two processes have the same logical form : just as our memories store systems of production rules associated with statistical

patterns, that is also what our engineered environment stores. For example, our memories store a popular song we like so that we can later recall it and reproduce it in song. A CD (in conjunction with a compatible CD player) does the same. Our road networks store routes compatible with automobile traffic between travelled destinations, including people's homes. So do our memories of how to get from one place to another; or our system for acquiring such information by studying maps, or seeking directions from others. The rules of the road, and driving skills[35], are stored

There is this difference : what we learn originates in real physical pattern in our environment, but what we 'teach' the environment is formulated first in our imagination (albeit by recombinant construction from acquired memory).

Applied to economic production, the investment process involves construction of mental models of how a proposed product will elicit purchasing behavior (based on implicit or explicit mental models of purchaser cognition). This process is, in a market economy, subject to a test : will 'revenues' cover 'costs'?. (In practice, this test is on an enterprise basis, rather than a product basis; it is in the discretion of enterprise management what revenue/cost tests, including profitability, it applies to a particular product or production process, or to groups of them.)

A production process, which can be described as a set of logical production rules subject to a revenue constraint described by its costs[36], is thus an ongoing experimental test of its predictability of other people's, and, ultimately, consumer behaviors.

There are three possible outcomes. First, the mental model may be confirmed with sales precisely matched to the model. Second, the mental model may be adapted on the basis of feedback or the lack of it. Adaptation could range from expanding production to meet unexpected market interest to withdrawing and reintroducing the product in a modified form (such as the launch and relaunch of the Mercedes 'A' class vehicle after it proved unstable in swerving to avoid large mammals). A third possibility is disappearance of the product line and, perhaps, enterprise.

Disappearance of a product or enterprise does not, however, eliminate the physical equipment, buildings, etc., the instrumental economies on which the production was based. What has occurred is merely institutional change - most obvious in the case of bankruptcy, where the ownership term in the physical assets of the business changes on liquidation, and the organization disappears. A similar phenomenon occurs when rental or

[35] Driving skills are typically 'overlearned' so as to become semi-automatic, economizing on limited attentional resources. Other examples are walking, speaking one's native language or riding a bicycle.

[36] 'Costs' may include profit requirements imposed by enterprise management or, indirectly, by stock or other financial markets. They may thus be in some sense 'arbitrary'. Sometimes profits are not even necessary, as indicated by the performance of stock prices for some new businesses, particularly in the technology 'sector' in the US.

owned premises are vacated. A new owner may or may not wish to use the premises in a similar fashion; if not they will be re-engineered, i.e. renovated.

The congruence or coincidence of mental models described in a production process specifies a set of conditions which must be satisfied subject to a cumulative financial constraint. It is suggested this is a realistic as well as predictive model of how a market / business economy operates. It is noted that 'price' is not, or not necessarily, a determining variable; it is, on a cumulative basis only a constraint. Similarly, the concept of market 'equilibrium' is unnecessary. Even if one were to label a congruence pattern which satisfied a constraint as an 'equilibrium', it is likely to be at most a fleeting one for products or enterprises. The economic grammar underlying an input-output table suggests why : even if a product like, say, Coca-Cola remains 'the same' over many decades, the packaging (e.g. bottles vs. cans, for cost, weight, recycling or vending machine purposes), distribution channels and other aspects of the 'grammar' change[37].

An economy is thus mathematically described by on cognitive processing 'chunks' composing production rules, with both the components of production rules and the rules themselves being selected by statistical measures. It is beyond the scope of this paper to describe them in detail : their further development is a project for future work. However, an empirically based intuitive example will illustrate.

In human learning as demonstrated by a great deal of data in the ACT-R framework[38], three critical statistical factors determining chunks and production rules which are stored in memory and recalled for performance are *frequency* (i.e. how often something is repeated in exposure or rehearsed), *recency* (i.e. how recently has there been exposure to this chunk or production rule) and *noise* (factors which interfere or distract). Similar factors often apply to consumer purchasing - habit and last purchase of a similar item if satisfactory often lead to repeat purchases of the same item, without active consideration of alternatives[39]. The counterpart for production rules for recency is current use : a the production facilities for a product for which there is no current market interest will fall into disuse. But recency is not enough : there must be a sufficient frequency of purchases to generate the revenues to meet the cumulative financial constraint.

An input-output framework is particularly suitable for considering the nature and connectivities of the 'grammars' of an industry row vector set of production rules. For many products or services must 'connect' not only locally, but downstream as well, as

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[37] It is noteworthy that Coca-Cola tried in the 1980s to change the product to New Coke. This failed. The original product was re-introduced as Coca-Cola Classic as an additional choice. New Coke was gradually withdrawn, and only the original product remains.

[38] E.g. Anderson, John R. & Christian Lebiere (1998). *The Atomic Components of Thought*. Mahwah, NJ : Lawrence Erlbaum. See also the ACT-R website at http://act.psy.cmu.edu/.

[39] See, e.g., Rabin, Matthew (1998), "Psychology and Economics", *36 Journal of Economic Literature*, 11-46.

components, as ‘product quality’ reflected in the care taken in the manufacturing process, as distribution, advertising, packaging or retailing which ultimately reaches the consumer.

V. SOME APPLICATIONS AND FUTURE WORK

This paper proposes a discrete mathematical structure of information processing and of goods and services production in, and underlying, an input-output framework. The two level mathematical architecture on which it is based has been proven out empirically in cognitive psychology in a wide range of domains, including complex matters such as learning mathematics, metaphor processing in natural languages and even scientific discovery[40]. It has not yet been extensively investigated as an economic model : it is presented here as a theoretical framework for future elaboration and investigation. The following are some aspects of particular interest to the author.

Competition

'Competition' is a often a somewhat amorphous concept in economics, accounting for a variety of phenomena, and depending, for example, on whether the perspective is that of a firm, or an observer of the market as a whole. Sometimes 'competition' refers to a putative process which will, for example, cause prices to be driven down (supplier competition) or up (competition among buyers in an auction format). Sometimes it is supposed to account for the introduction of new products or technologies. Sometimes it refers to new suppliers who will be attracted into production by sufficiently high expected profits or prices. The precise process by which these things are supposed to happen may be taken for granted rather than spelled out.

From the standpoint of an individual firm, 'competition' may refer to the presence or actions of others supplying the same or similar products. Those actions may be in the nature of lower prices, but others' style, performance, features, advertising, packaging, product warranties or restricted distribution channels may also be seen as limiting the sales of one's own products. Often businesses do not seek to compete on the basis of lower prices, but seek to match higher prices which some of their competitors (e.g. popular name brands like Nike) are able to charge.

That a concept like competition to which important functions are ascribed is used in many, often insufficiently specified ways, is unsatisfactory.

The framework presented in this paper enables a precise characterization of 'competition' as a distributed cognitive phenomenon, while in principle specifying the various constituents underlying it.

First, sellers must gain consumers' attention. No sales are possible if the product is not in attention at the time a purchase decision might be made. In practice, this is likely to be a process embracing multiple factors, including prior learning (e.g. through product advertising), selection of distribution channels, and product display. The 'grammar' of this is an expression of production rules across an input-output vector for that production

[40]. See footnote 38.

process or establishment. 'Competition' is an alternative grammar of presentation *which is selected or not by the same statistical variables*. This would be a precise counterpart of a decision in an ACT-R framework : selections among chunks and production rules which are similar but not identical.

The Institutional Component of Economic Grammars

The properties of economic activities as a formal language point to possibilities for investigating the roles of institutional arrangements in economic performance, such as :

(1) the notion of individual or organizational institutional competence, analogous to competence in a natural language, and how it is acquired and applied;

(2) an economy as a collective expression in an institutional proto-language, and comparisons between economies as comparisons between institutional 'cultures';

(3) institutional failures as 'syntactical' (control) failures.

The Organization of Transactions between Time Periods

Instrumental economies operate in real time, a succession of continuously ephemeral presents. In each time period, there are physical transactions among the past, the present and the future (the $(t_{...})$ and terms reflect points in time indicated by subscripts) :

Behavior Source	Behavior Application	Illustration
Past (t_{a-b})	Present (t_a)	Wear and tear on equipment
Past (t_{a-b})	Future (t_{a+c})	Physical capital or natural resources retained for future use
Present (t_a)	Present (t_a)	Current consumption
Present (t_a)	Future (t_{a+c})	Physical investment
Future (t_{a+c})	Present (t_a)	Real borrowing; promise to 'return the favor'

As noted below, the institutional arrangements associated with claims between time periods may play an important role in the distribution of economic benefits and costs.

Transactions in time give rise to assets and liabilities : arrangements which economise (or diseconomise) on behaviors across time periods. They can be physical, such as a building,

a machine or a forest; or environmental degradation. They can also be cognitive, such as intellectual property, human organisations, or the social capital of learned, and shared, rules for the conduct of economic life.

Assets and liabilities depend on reference levels, because 'economies' (or 'diseconomies') are relative.[41] Assets such as machine tools are substitutes for more time consuming behavior sets, such as trying to do the same task by hand. They therefore can be exchanged for other behaviors. Liabilities - such as a building which is under a work order - exhibit a behavior 'deficit'.

Physical assets can be owned as property; so, in principle, can organisations. Although shares in a corporation are in one sense a financial asset, they can also be interpreted as a pattern-of-behavior asset. One owns a share in the patterns of behaviors - economies and diseconomies - which the enterprise represents.

Income Distribution and Human Capital

Input-output frameworks not only present partitioning or production processes, but also partitions as the basis of income distribution, such logical structures representing variously institutionalised forms of human capital. Income pools accumulate in various places with various rules of access. Consider : employee stock options in Microsoft; salaries of U.S. professional basketball players; fees of orthopaedic surgeons; counter jobs at McDonald's; gambling in Los Vegas; academic tenure; unionised factory jobs; workers constructing a toll road; the owners of the road. The union representing most Hollywood actors is likely to go on strike over a dispute with media and entertainment conglomerates over 'residuals' - whether they should earn royalties for subsequent uses of productions in which they appear in television re-runs, video rentals, internet distribution, etc.

In neoclassical economics, income distribution, in theory, reflects productivity differences. However, it is difficult to explain why an hour of one person's effort is remunerated at many thousands of times that of others, even in the same economy, based on marginal productivity analysis. While productivities indeed play a role, there are also many institutional factors, such as those illustrated above, whose influence may even predominate. In addition, there are a variety of institution-based redistributive mechanisms both governmental, such as through taxation and transfers, and market, such as insurance and securities markets.

[41] An instrumental economy is a **bisct** which, for the same interval of time, produces a common set of states plus an additional state or states compared to some other **bisct**, such as more of the same output, or the same output with additional features.

CONCLUSION

Tableaux économiques have a long tradition in economics[42]. This paper outlines a discrete mathematical framework in the larger context of input-output as originated by Leontief and others. It sets forth a method in a prototypical two-level mathematical architecture to account for production, consumption and distribution in both a more realistic and a more complete fashion that neoclassical economic analysis. In particular, the method is consistent with empirically-based modern computational psychology, as neoclassical analysis is not.

The method also involves an analytical re-orientation of the function of 'money', or media of exchange more generally. That key function is setting the terms of and enforcing partition. This replaces the less specified concept of 'scarcity', which can be partially overcome in a modern economy through both technical instrumental economies, and institutional arrangements, both of which can be specified in the model.

Appendix

As set out in this paper, the principal 'unit' of analysis is an expression in 'bisct' form, composed (a) in the 'real' sector as a quintuplet of subsets of five base sets, and (b) in encoded form as sets of strings corresponding to those quintuplets.

The elements in each base set, or subsets of them, could be specified, and can be cognitively processed, at various levels of detail or aggregation. For example, each component in a 'bisct' could be prescribed in minute mechanical detail. Alternatively, a series of 'bisct's could be expressed in a sentence such as "Andre Agassi won the final of the French Open in 2000."

The following specifications illustrate the precise composition of sets at a fine level of granularity. Let the physical world as experienced by humans be partitioned into a series of sets, subsets and conventions as follows :

(1) *Human behaviors.* $\mathbf{B}$, the presence or absence of each human behavior $\mathbf{b} \in \mathbf{B}$ based on musculature, mechanically specified in arbitrarily small units, as in kinesiology;

(2) *Symbolic behaviors.* Let $\mathbf{B}$ be partitioned further into 'instrumental', $\mathbf{B'}$, and 'symbolic', $\mathbf{B''}$, behaviors on the following criterion : all elements of $\mathbf{B}$ based on mouth, tongue and jaw musculature which are also paired with vocalisation (i.e. phonemes) are 'symbolic', and all other behaviors are 'instrumental';

(3) *Sets of individuals.* $\mathbf{I}$, an index set of unique identifiers $\mathbf{i} \in \mathbf{I}$ for each human being;

[42] E.g. Blaugh, M. (1997). *Economic Theory in Retrospect, 5th ed*. Cambridge : Cambridge University Press; Schumpeter, Joseph A. (1954). *History of Economic Analysis*. New York : Oxford University Press.

(4) *Physical states (objects, patterns and mechanisms):* **S**, the presence or absence of states **s** ∈ **S**, defined as all sets of physical properties (**p, p** ∈ **P**) not elements of **B**, and which have either of two types of mathematical structure: topologies, ('objects', including each human body, or other patterns, such as a body of water which, roughly, preserve relative spatial relations) or mappings (set of logical connections or transitions) ('mechanisms');

(5) *Spatial coordinates.* **C** (a triplet **c, c, c** in three dimensional space);

(6) *Time.* **T**, time, in discrete units **t**;

(7) *Presence and absence* of a behavior or a state is indicated by '+' and '−' respectively;

(8) *Sequences of behavior/ series of states.* Let sequences be specified by finite strings, and let the corresponding set of strings be indicated by the superscript '*****', e.g. **B'***;

(9) *Power sets.* Let the power set (set of all subsets) of a set **A** be specified by **P(A)**;

(10) *Coding.* Let **B''*** ⊂ **B''*** comprise a set of strings of between which there is a one-to-one mapping with each element in any subset of **[P(B'*)** ∪ **P(S*)** ∪ **P(I*)** ∪ **P(C*)** ∪ **P(T)]** ∪ **[P(B'*)** X **P(I)** X **P(S*)** X **P(C*)** X **P(T*)]** ('X' indicates Cartesian product) which includes **B'** ∪ **S** ∪ **I** ∪ **C** ∪ **T**. (Intuitively, there is a 'word' for every elemental **b'**, **s**, **i**, **c** and **t**, and any other strings or subsets we care to specify.)

(11) *Rational numbers*. Sets of rational numbers **$/q**, where **$** is an integer from the series **1, 2, ... n** and **q** is a sum (∑) of some integer multiples of **$**.

(12) *Logical operators*. The following logical operations (after Bourbaki 1968) : ¬ (not); ∨ (or); & (and); → (implies); (place holder for a term, or 'substitution'); and ι (identification, i.e. *the* term which has the properties specified by a formula).

(13) *Set convention.* { } indicates a set of the objects contained within the brackets.

(14) *Generic subscript*. The subscript $_{\varphi}$ identifies some element of that set.

(15) *Generic superscript*. The superscript $^{\eta}$ identifies some particular combination or encoded string of subsets.

(15) Structures are the subsets which result from the imposition of *constraints* on a larger set of otherwise possible outcomes. Note that 'constrain' has a dual meaning in both English and mathematics: 'cause particular outcomes to happen' and 'prevent particular outcomes from happening'.

The intuitive concept of any combination of people engaging (or not) in any type of behaviour with respect to any state, anywhere, anytime can be expressed as the Cartesian product (all possible quintuples of elements) **P(B'*)** X **P(I)** X **P(S*)** X **P(C*)** X **P(T*)**. Thus any particular human behaviour in the human environment (which includes other people) is a subset of this product set : a '**bisct'** or *fully specified* subset, comprises a set of behaviours of specific individuals with respect to identified states at specified places and times. *Partially specified* subsets are also of interest, such as : (1) **bsct**, where the identity of the individual performing the behaviours (skills) is not important; (2) **{b}i{s}**, scope of actual or potential behaviours of an individual or group with respect to particular states in an environment (e.g. 'property'), place or time not yet specified; (3) **bs**, behaviours applied to a particular state; intuitively, 'technique' or 'technology', e.g. the transition $\mathbf{b}_{\varphi}(\mathbf{s}_0) \rightarrow \mathbf{s}_1$; (4) **sct**, a particular state at a particular place and time, not associated with any particular behaviour or individuals; intuitively, an object in the physical environment.

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