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An Algorithm for Balancing Commodity-Flow Systems

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Abstract

In many cases there is a need to balance commodity-flow systems within a short period of time and with limited resources. A case in point is a comprehensive revision of the national accounts where the whole time-series of national accounts data is revised. In order to maintain supply and use tables and investment matrices consistent with the revised national accounts it is necessary to apply some sort of automatic balancing. The RAS algorithm normally cannot accomplish the task, since the row and column sums in this case are not a priori balanced. The border totals are endogenous variables in the balancing process called for.

The paper proposes an algorithm for automatic balancing which, contrary to RAS, can balance a commodity-flow system whose row and column totals have not already been balanced in advance. The algorithm allows the compiler to utilize information about the relative reliability of individual column totals in the balancing process but does not require the specification of a complete variance matrix like the Stone algorithm. The algorithm has been applied in connection with the recent revision of the Danish capital stock estimates.

1. Introduction

In many cases there is a need to balance commodity-flow systems within a short period of time and with limited resources. A case in point is a comprehensive revision of the national accounts where the whole time-series of national accounts data is revised. In order to maintain supply and use tables and investment matrices consistent with the revised national accounts it is necessary to apply some sort of automatic balancing. The RAS algorithm normally cannot accomplish the task, since the row and column sums in this case are not a priori balanced. The border totals are endogenous variables in the balancing process called for.

Some people would even argue that automatic balancing will yield a result which is superior to any manual balancing without an explicit optimisation of a distance function. The most prominent spokesman for this viewpoint was Stone, see e.g. Stone et al. (1942), whose joint article with Champernowne and Meade from 1942 was pathbreaking in introducing the idea of optimal balancing of the national accounts. The idea is that automatic balancing that minimizes a loss function will always be better than a manual approach which is unsystematic and whose criteria are more or less implicit. Another argument is that a manual approach has to limit itself to adjusting a limited number of flows and therefore is likely to "overshoot" on the product flows being touched in the course of balancing, whereas an automatic approach can handle all the products simultaneously.

This view, however, is not favoured by the majority of people involved in the actual compilation of national accounts, supply and use tables plus input-output tables. Based on the experience that a lot of errors in primary statistics are spotted in the course of a predominantly manual balancing process, compilers are usually convinced that a (mainly) manual balancing process gives results of higher quality than those resulting from a purely automatic balancing of the accounts. From that point of view the resources involved in manual balancing are justified as being used for a very efficient consistency check on the accounts. However, when supply and use tables have to be balanced again following a comprehensive revision of the national accounts, the data set has already been through the close scrutiny of a (mainly) manual balancing process once, and the risks of an automatic balancing procedure are consequently smaller.

Whether one holds one or the other view concerning the current production of national accounts, some sort of automatic balancing is indispensable when a lot of periods have to be rebalanced following a comprehensive revision. The same is true if supply and use tables are compiled in connection with provisional versions of the national accounts whose production time is much shorter than is the case for the final version.

The practical applications of the Stone et al. algorithm have been surprisingly few and typically of an ad hoc nature which did not produce any official estimates, see e.g. van der Ploeg (1982), Barker, van der Ploeg and Weale (1984), Stone (1984) and Smith, Smith, Weale and Satchell (1998). Furthermore, the applications appear to have been restricted to very small national accounting systems, systems that are far smaller than any real-world commodity-flow system. Until 20 years ago the limiting factor may simply have been computational power, since the Stone algorithm is computationally heavy. More recently, it seems unlikely that lack of computational power alone can explain the scarcity of practical uses of the algorithm. It is more likely to be because the algorithm is not so well suited to handling supply and use tables since it does not distinguish the different price levels of the System of National Accounts and because it is anyway unrealistic to require relative uncertainties (variances) to be specified for anything below the column totals in the supply table at basic prices and the column totals of the use matrix at purchasers' prices.

The only application of the Stone algorithm in current production of official statistics to our knowledge is the use in connection with the yearly national accounts in Italy for years other than benchmark years, see Pedullà (1995). Note, however, that this application concerns an extremely aggregate version of the national accounts namely a 42 by 42 symmetrical input-output table. This is very far from being a commodity-flow system that really exploits all the knowledge about the technical nature of products and the distribution circuits in the economy. Such a system must distinguish at least 222 products (CPA 3-digit level) and preferably many more. All in all it would seem that the Stone algorithm is best suited to relatively small accounting systems such as institutional sector accounts being drawn up after the supply and use table have already been balanced. Stone (1984) features such an application to the UK institutional sector accounts.

For commodity-flow systems another algorithm seems to be called for, namely one that can efficiently balance the supply and use tables of the SNA with their six or seven different levels of prices, while allowing knowledge about relative uncertainties of the variables to be taken into account. The present paper presents an algorithm which is specifically designed with commodity-flow systems or supply and use tables in mind. A simplified version was applied to the Danish national accounts when official figures for the stock of fixed capital had to be calculated backward all the way to 1966 following a major revision of the national accounts. In that connection, investment matrices containing about 700 product balances for capital goods covering the years 1966-1992 had to be balanced again. Given the resources available for the project, it was unrealistic to perform this task without an automatic balancing approach.

Real world supply and use tables are of highly varying size. In the product dimension they range from 123 products and industries in the UK, over 500 products and 90 industries in France, 800 products and 200 industries in the Netherlands to 980 products and 178 industries in Norway and even more product detail in Denmark, all compiled on a yearly basis. The benchmark input-output tables in the United States are based on supply and use tables containing slightly under 500 industries and products. The Danish national accounting system has opted for a high level of detail in the product dimension. Supply and use tables are compiled as an integral part of the final national accounts for a given year at the level of approximately 2750 products of which 2300 are goods and 450 services. There are 130 industries and 72 groups of final household consumption. For the provisional yearly and the quarterly accounts, however, a much more aggregate approach is used.

2. The supply and use tables and investment matrices of SNA 1993

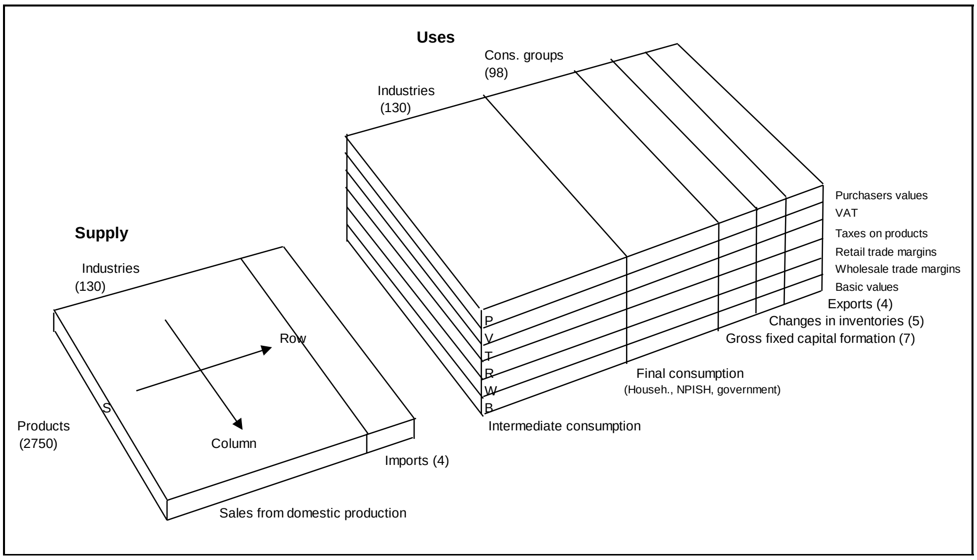
The supply and use tables of the 1993 System of National Accounts are illustrated in figure 1 with the supply and use tables underlying the Danish national accounts as example. It is seen that there are six different price levels involved

1) Basic prices
2) Wholesale margins
3) Retail margins
4) Taxes on products excluding VAT
5) Non-deductible VAT
6) Purchasers' prices

If transport margins are treated explicitly and not merged with wholesale margins, there will be one more use matrix and consequently seven layers of prices. The number of non-zero cells in the system is very large and changes to supply or use components at basic prices generally trigger several changes to the other value levels. Even with a moderate number of products, say 600, the system is seen to be very large.

The system includes many deterministic accounting constraints or identities. Examples are that the total of the cells of wholesale and retail margins must equal output at basic prices of the industries wholesale trade and retail trade. Similarly, the sum over all the cells in the VAT matrix must equal the total VAT revenue on an accruals basis.

Figure 1: Supply and Use tables



The investment matrices of the 1993 SNA are a cross classification of the investment columns in the use table in figure 1 by industry of investment. For each category of investment the investment matrices thus expand the investment column in the use table into a matrix.

3. Manual versus automatic balancing

In general, when the national accounts are compiled, what is available from primary statistics after they have been processed for national accounting purposes is:

Supply at basic prices broken down by product and domestic producer industries plus imports as well as the **totals** of the columns in the use matrix **at purchasers' prices.**

A purely manual balancing starts from commodity-flows that are initially unbalanced, and each and every product flow has to be balanced. This is clearly a very labour intensive method if the number of products is large.

A purely automatic balancing procedure allows the supply and use tables to be balanced using a well-defined algorithm. There is no manual interference at all, once obvious data errors in the initial estimates have been identified and amended. This kind of balancing may require computer power and appropriate software but is inexpensive in terms of manpower.

In the Danish national accounting system the balancing process is started by running the first two steps of the RAS algorithm before the manual balancing takes over. The great advantage of this approach is that one starts off with all the commodity-flows being balanced, even though this balancing has to be modified since it does not give sufficient weight to the initial estimates of the column totals but adjusts every demand type for a given product proportionately in the light of the overall supply/demand situation for that product.

Without running the second step of RAS (a "horizontal" distribution) before starting the manual work it would not be manageable to produce supply and use tables with 2750 products on an annual basis as an integral part of compiling the final national accounts estimates. Thus the balancing procedure actually used in Denmark for the final national accounts estimates is in fact only semi-manual. In the following sections it will nevertheless be referred to as "manual balancing". The reason is that all the strategic choices in that kind of balancing process are made by the team of national accountants working to balance the data files. There is no set of balancing instructions coming anywhere near something that could be called an algorithm.

4. Methods for automatic balancing

In the special case where the row and columns totals of an accounting matrix have been balanced, so that the sum of the row sums equals the sum of the column sums the RAS technique of biproportional adjustment can be used to estimate the "interior" of the accounting matrix, see Friedlander (1961), Department of Applied Economics (1963), Bacharach (1965) and Fontela et al. (1970). A classic example is where an old benchmark input-output table is updated to a later year so as to be in line with the national accounts for that year. Since the national accounts have already been balanced - probably at a much higher level of aggregation - the condition concerning the row and column sums is satisfied. In fact, in this case the row and column sums are exogenous variables whereas they are endogenous variables in the balancing of the national accounts themselves where one starts out with a macro difference between total supply and total use of products.

Contrary to RAS, the Stone algorithm cf. Stone et al. (1942), Byron (1978) is a method of automatic balancing where the row and column totals are endogenous variables. It can therefore be applied to the problem of balancing the national accounts. An elegant and compact formulation is provided in Stone (1984). The advantage of the Stone algorithm is that it is based on the least squares principle and thus in a long and solid tradition in statistics. The chief drawback from a conceptual point of view is that it does not guarantee preservation of sign of the variables. From a practical viewpoint its application has been limited by the fact that it is computationally heavy and because there is seldom any good, objective basis for specifying the variance matrix which is crucial in the minimization of least squares.

5. An Algorithm for Balancing Big Commodity-Flow Systems

In this section we propose an iterative process for balancing supply and use tables of the form illustrated in figure 1.

With reference to the matrices in figure 1 we denote the supply matrix S. As regards the various use matrices we adopt the following notation:

B = use matrix at basic prices
W = use matrix of wholesale trade margins
R = use matrix of retail trade margins
T = use matrix of taxes less subsidies on products excluding VAT
V = use matrix of non-refundable VAT
P = use matrix at purchasers' prices

In the iterative process superscript *t* refers to the t'th iteration. The expression "matrix A after vertical distribution" describes the result of expanding a row vector of border totals into a matrix by means of a matrix of technical coefficients. This is the vertical adjustment of the RAS algorithm. Similarly, the expression "matrix A after horizontal distribution" refers to the result of adjusting each row in a matrix by a factor of proportionality in order for the row sum to attain a certain predetermined value. It is the horizontal adjustment of the RAS algorithm.

At the start of the balancing process there is a difference between total supply of goods and services and total demand or, equivalently, between GDP calculated from the output side, GDP(O), and from the expenditure side GDP(E). Because of the macro imbalance between total supply and total use the RAS algorithm cannot balance the supply and use tables.

In the iterative procedure proposed in the following the macro imbalance between total supply and total uses is gradually eliminated in such a way that both the initial estimates of value added by industry and the initial estimates of final uses are taken into account together with the known technological and other links between products and uses. In addition to the gradual elimination of the macro difference between supply and use of products the cells of the supply and use tables are estimated using a procedure which is essentially the RAS algorithm.

In the algorithm to follow it is assumed that supply of each commodity is determined with certainty. This is not essential and the supply matrix might easily be modified as well as the use matrix. In practice, however, at the level of individual products, supply is generally determined with much higher reliability than uses. The uncertainty regarding GDP from the output side (value added by industry) is therefore conveniently represented in terms of uncertainty concerning intermediate consumption rather than gross output/imports.

The balance between GDP(O) and GDP(E) struck by the algorithm therefore involves modifying the initially estimated GDP(O) by changing intermediate consumption while holding gross output constant at the initially estimated values except in case of clear errors being found on the supply side. Such errors, which are often revealed by the confrontation of supply and demand, are naturally corrected.

To start with the algorithm uses the technological coefficients from the supply and use tables of the preceding year adjusted for changes in relative prices to distribute the initial estimates of intermediate consumption by industry and of final uses by product. The technological coefficients are applied at the level of basic prices. The first step is therefore to split the initial estimates of the border totals of the use matrix at purchasers' prices into border totals at basic prices, wholesale trade margins, retail trade margins, product taxes, net, excluding VAT plus VAT. This is done using the proportions for each category of use between these value levels in the preceding year. The initial estimates of the use matrices of trade margins and taxes on products, net, are calculated using the rates of margins/taxes from the preceding year.

In practice a lot of cells in the use matrix are regarded as being determined with complete certainty and are therefore not modified in the course of the balancing process. These parts of the use matrix are indicated by superscript F for fixed or predetermined values. Examples are exports of goods and services by product which may be known with virtually no margin of error from foreign trade statistics. Other examples are certain types of inventories plus flows of energy products, insurance services etc. which the authorities monitor closely.

In the notation to be presented in the following section the expression "matrix A excluding predetermined cells" signifies the matrix A with all the predetermined cells replaced by zeros.

We adopt the following notation:

$*$ = operator for term-by-term multiplication of two matrices with equal dimensions

$/$ = operator for term-by-term division of two matrices with equal dimensions (TBT-division by zero in a cell yields a zero by definition)

d = superscript denoting diagonalisation of vector in a matrix i.e. a matrix with the arguments of the vector on the diagonal of a matrix with zeros elsewhere

i = unit vector of suitable dimension

m = number of columns in the use table

n = number of products except predetermined commodity-flows

t = superscript indicating the value of a variable in the t'th iteration of an iterative process

ub = superscript indicating that a matrix of trade margins or taxes is unbalanced

b = superscript indicating that a matrix of trade margins or taxes is balanced

λ_j = confidence factor for the j'th use (j'th column in the use matrix), j = 1, ... ,m. $(0 \leq \lambda_j \leq 1)$

λ = 1 x m vector (λ_j)

S = supply matrix at basic prices with two columns for imports of goods respectively services of dimension n x (m + 2)

$B^{v,t}$ = matrix B (basic prices) in the t'th iteration after a "vertical distribution"

$B^{h,t}$ = matrix B (basic prices) in the t'th iteration after a "horizontal distribution"

$B = [b_{i,j}]$ = use matrix at basic prices excluding predetermined cells

$\tilde{B}$ = use matrix at basic prices including predetermined cells

b = iB = vector of column totals in matrix B (1 x m)

B^* = B coefficients matrix where B is divided by the column totals $B^* = B(\beta^d)^{-1}$

B^F = use matrix at basic prices consisting of predetermined values in the use matrix at basic prices

$W = [w_{i,j}]$ = use matrix of wholesale margins excluding predetermined cells

$\tilde{W}$ = use matrix of wholesale margins including predetermined cells,

$(\tilde{w}_j = \sum_{i=1}^{n} \tilde{w}_{i,j})$.

W% = use matrix of wholesale margins as shares of values at basic prices

$(W\% = W/B)$

W^F = use matrix of wholesale trade margins consisting of predetermined values in the use matrix of wholesale margins

w_g = total supply of wholesale trade services i.e. accounting restiction for total wholesale margins (scalar 1x1)

R = use matrix of retail margins excluding predetermined cells

$\tilde{R}$ = use matrix of retail margins including predetermined cells

R% = use matrix of wholesale margins as shares of values at basic prices

$(R\% = R/B)$

R^F = use matrix of retail trade margins consisting of predetermined values in the use matrix of retail margins

r_g = total supply of retail trade services i.e accounting restriction for total retail margins (scalar 1 x 1)

$PT = [pt_{i,j}]$ = use matrix of taxes on products less subsidies on products except VAT excluding predetermined cells

$\tilde{T}$ = use matrix of taxes on products less subsidies on products including

predetermined cells, $(\tilde{t}_j = \sum_{i=1}^{n} \tilde{t}_{i,j})$

pt_g = accounting restriction (tax revenues) on product taxes, net, by product (n x 1)

PT% = use matrix of net product taxes as shares of values at basic prices

$(PT\% = PT/B)$

PT^F = use matrix of predetermined cells of taxes less subsidies on products except VAT

$V = [v_{i,j}]$ = use matrix of non-deductible VAT excluding predetermined cells

$\tilde{V}$ = use matrix of non-deductible VAT including predetermined cells, $(\tilde{v}_j = \sum_{i=1}^{n} \tilde{v}_{i,j})$.

v_g = accounting restriction (revenue) for total VAT on accruals basis (scalar 1 x 1)

V% = use matrix of non-deductible VAT as shares of values at purchasers' prices excluding VAT $(V\% = V/(B + W + R + PT))$

V^F = use matrix of non-deductible VAT consisting of predetermined values in the use matrix of non-deductible VAT

P = use matrix at purchasers' prices excluding predetermined cells

p = use vector consisting of column sums in the use matrix at purchasers' prices excluding predetermined cells

p_j = the j'th column sum in the use vector at purchasers' prices excluding predetermined cells

$\tilde{P}$ = use matrix at purchasers' prices including predetermined cells

$\tilde{p}$ = use vector consisting of column sums in the use matrix at purchasers' prices including predetermined cells

$\tilde{p}_j$ = the j'th column sum in the use vector at purchasers' prices including predetermined cells

$\hat{p}^t$ = "target" column totals in the use matrix t purchasers' prices in the t'th iteration.

p^e = initial (unbalanced) estimates of column totals in the use matrix at purchasers' prices

p_j^e = the j'th column sum in the initial (unbalanced) estimates of column totals at purchasers' prices

p^e stands for the initial estimates of each use category at purchasers' prices included in the initial (unbalanced) estimates of GDP(O) – intermediate consumption – and GDP(E) – final uses. In the first iteration these initial estimates are used as "target totals" for the column totals in the use matrix. In subsequent iterations these "target totals" $\hat{p}^t$ are modified so as to gradually eliminate the macro difference between supply and use of products.

The lambda vector is adjusted in a special way, which calls for a comment. l^0 is the initial confidence factor vector, and l^{t-1} is the lambda vector after the t'th iteration, and the elements in t'th lambda vector are $l^{t-1} = \left(l_1^{t-1},\ l_2^{t-1}, \ldots,\ l_m^{t-1} \right)$. If $l_1^0 = 0.8$ is the first element in the initial confidence factor vector, then $l_1^{4-1} = 0.8^{4-1} = 0.512$ is the first element after the 4th iteration. The values in the lambda vector are thus adjusted with the superscript used as a power. This is special for the lambda vector and does not occur for any of the other vectors or matrices. In all other cases the superscript refers merely to the iteration number.

5.1 Computational steps in the t'th iteration

Step 1: Choose target totals for the use matrix at purchasers' prices.

$$\hat{p}_j^t = p_j^e, \qquad t = 1 \text{ and } j = 1, \ldots, m. \qquad (1.1)$$

$$\hat{p}_j^t = l_j^{t-1}\left[p_j^e - \tilde{p}_j^{t-1}\right] + \tilde{p}_j^{t-1}, \qquad t \geq 2,\ldots ,\text{T; and j = 1,...,m.} \qquad (1.2)$$

Step 2: Calculate corresponding 'target totals' for the column totals of the use matrix at basic prices.

$$b_j^t = b_j^{t-1} \frac{\hat{p}_j^t - \tilde{v}_j^{t-1}\left[\dfrac{\hat{p}_j^t}{\tilde{p}_j^{t-1}}\right] - \tilde{t}_j^{t-1}\left[\dfrac{\hat{p}_j^t}{\tilde{p}_j^{t-1}}\right]}{\tilde{p}_j^{t-1} - \tilde{v}_j^{t-1} - \tilde{t}_j^{t-1}}, \text{ j = 1,....,m.} \qquad (2.1)$$

where b_j^0, $\tilde{p}_j^0$, $\tilde{v}_j^0$ and $\tilde{t}_j^0$,are equal by convention to the j'th column sum of the use matrices (basic prices, purchasers' prices, VAT, product taxes, net) of the preceding year adjusted for relative price changes between the preceding year and the current year.

Step 3: "Vertical distribution in the use matrix at basic prices – the first step of RAS.

$$B^{v,t} = B^{*,t-1}(b^t)^d \qquad (3.1)$$

$$\tilde{B}^{v,t} = B^{v,t} + B^F \qquad (3.1.1)$$

where $B^{*,0}$ is equal by convention to the technical coefficients matrix from the preceding year adjusted for relative price changes between the preceding year and the current year.

Step 4: "Horizontal distribution" in the use matrix at basic prices – second step of RAS.

$$B^{h,t} = \left[(S^t i - B^F i) / B^{v,t} i\right]^d B^{v,t} \qquad (4.1)$$

$$\tilde{B}^{h,t} = B^{h,t} + B^F \qquad (4.1.1)$$

Step 5: Wholesale margins corresponding to the values at basic prices; balancing of wholesale margins, and computation of new *W%* matrix.

$$W^{ub,t} = W\%^{t-1} * B^{h,t} \qquad (5.1)$$

$$\tilde{W}^{ub,t} = W^{ub,t} + W^F \qquad (5.1.1)$$

$$W^{b,t} = W^{ub,t}\left[\frac{w_g - iW^F i}{iW^{ub,t} i}\right] \qquad (5.2)$$

$$\tilde{W}^{b,t} = W^{b,t} + W^F \qquad (5.2.1)$$

$$W\%^t = W^{b,t} / B^{h,t} \qquad (5.3)$$

where $W\%^0$ by convention is equal to the matrix of wholesale margin shares of basic prices from the preceding year.

Step 6: Retail trade margins corresponding to the values at basic prices, balancing of retail trade margins and computation of new R% matrix.

$$R^{ub,t} = R\%^{t-1} * B^{h,t} \qquad (6.1)$$

$$\tilde{R}^{ub,t} = R^{ub,t} + R^F \qquad (6.1.1)$$

$$R^{b,t} = R^{ub,t}\left[\frac{r_g - iR^F i}{iR^{ub,t} i}\right] \qquad (6.2)$$

$$\tilde{R}^{b,t} = R^{b,t} + R^F \qquad (6.2.1)$$

$$R\%^t = R^{b,t} / B^{h,t} \qquad (6.3)$$

where $R\%^0$ by convention is equal to the matrix of retail trade margin shares of basic prices from the preceding year.

Step 7: Taxes on products, net, corresponding to the values at basic prices, balancing of product taxes[1], net, and the computation of new PT% matrix.

$$PT^{ub,t} = PT\%^{t-1} * B^{h,t} \quad (7.1)$$
$$\tilde{T}^{ub,t} = PT^{ub,t} + PT^{F} \quad (7.1.1)$$
$$PT^{b,t} = \left[\frac{(pt_g - PT^F i)}{PT^{ub,t} i}\right]^d PT^{ub,t} \quad (7.2)$$
$$\tilde{T}^{b,t} = PT^{b,t} + PT^{F} \quad (7.2.1)$$
$$PT\%^t = PT^{b,t} / B^{h,t} \quad (7.3)$$

where $PT\%^0$ by convention is equal to the matrix of product taxes, net, as shares of basic price values from the preceding year.

Step 8: VAT corresponding to the values at basic prices; balancing of VAT and computation of new V% matrix.

$$V^{ub,t} = V\%^{t-1} * \left[B^{h,t} + W^{b,t} + R^{b,t} + PT^{b,t}\right] \quad (8.1)$$
$$\tilde{V}^{ub,t} = V^{ub,t} + V^{F} \quad (8.1.1)$$
$$V^{b,t} = V^{ub,t}\left[\frac{v_g - iV^F i}{iV^{ub,t} i}\right] \quad (8.2)$$
$$\tilde{V}^{b,t} = V^{b,t} + V^{F} \quad (8.2.1)$$
$$V\%^t = V^{b,t} / \left[B^{h,t} + W^{b,t} + R^{b,t} + PT^{b,t}\right] \quad (8.3)$$

where $V\%^0$ by convention is equal to the matrix of non-refundable VAT as shares of purchasers' price before VAT values from the preceding year.

Step 9: Computation of the result for the use matrix excluding predetermined cells (9.2) and the border totals of the use matrix including predetermined cells (9.3) at purchasers' prices after the t'th iteration. Equation (9.1) is the computation of the result for the use matrix excluding predetermined cells.

$$P^t = B^{h,t} + W^{b,t} + R^{b,t} + T^{b,t} + V^{b,t} \quad (9.1)$$
$$\tilde{P}^t = \tilde{B}^{h,t} + \tilde{W}^{b,t} + \tilde{R}^{b,t} + \tilde{T}^{b,t} + \tilde{V}^{b,t} \quad (9.2)$$

$$\tilde{p}^t = i\left[B^{h,t} + W^{b,t} + R^{b,t} + PT^{b,t} + V^{b,t} + B^F + W^F + R^F + PT^F + V^F\right] \quad (9.3)$$

Step 10: Check whether convergence has been achieved.

If $(\hat{p}_j^t - \tilde{p}_j^t) \le e_j$ j = 1,... ,m. (10.1)

then stop, where e_j is a convergence threshold value.

Else increase the iteration counter to $t+1$ and repeat steps 1-10.

[1] When balancing taxes on products (equation (7.2)), the proceeds on each product must be known, if that is not the case but only the total proceeds is known, an alternative procedure to balance taxes on products is: $PT^{b,t} = PT^{ub,t}\left[(pt_g - iPT^F i)/iPT^{ub,t} i\right]$. In the numerical example this simplified method is used.

6. Numerical example

The following example shows step by step how the algorithm works on a small-scale commodity-flow system representative of the Danish economy. The structure and proportions in the example resemble the Danish economy.

The task is to balance a very small-scale commodity-flow system. The supply-side figures (domestic gross output and imports by product) are treated as being observed with certainty and are therefore treated as predetermined or fixed values.

To be more precise the data needed to run the algorithm are: technical coefficients, supply-side initial estimates for products by industry, use-side initial estimates of industry and final demand totals, lambda values for intermediate consumption of industries and for final demands plus initial estimates of total wholesale trade margins, retail trade margins, taxes less subsides and VAT.

The economy has 4 industries, agriculture etc., industry, trade and (other) services. The industries produce 4 products, where product 3 is wholesale- and retail trade margins. The 4 products may be used as intermediate consumption in the industries, final consumption, gross capital formation and/or be exported.

Table 1 shows the use matrix forming the basis for the technical coefficients used in the calculations. The export cells in the use matrix are predetermined or fixed as they come from foreign trade statistics and are known with almost certainty. In the tables, rows with predetermined or fixed values are marked with the letter F.

If one rearranges the basic price column in table 1 so that one has the industries in the columns and products in the rows, the basic price column would correspond to the matrix B^0 in equation (4.1). In the same way W^0, R^0, PT^0, V^0 and P^0 can be derived from table 1.

Table 1: Use matrix used for generating technical coefficients.

			Industry	Basic prices	Wholesale trade margins	Retail trade margins	Taxes less subsidies	VAT	Purchasers' prices
Product 1	Intermediate consumption		Agriculture etc	26.0	0.0	0.0	0.0	0.0	26.0
Product 1	Intermediate consumption		Industry	133.0	15.0	1.0	2.0	3.0	154.0
Product 1	Intermediate consumption		Trade	3.0	0.0	0.0	0.0	0.0	3.0
Product 1	Intermediate consumption		Other services	3.0	1.0	0.0	0.0	0.0	4.0
Product 1	Final consumption expenditure			10.0	3.0	6.0	5.0	2.0	26.0
Product 1	Gross fixed formation			3.0	2.0	1.0	1.0	0.0	7.0
Product 1	Exports	F		48.6	4.0	0.0	-1.0	0.0	51.6
Product 2	Intermediate consumption		Agriculture etc.	10.0	0.0	0.0	0.0	0.0	10.0
Product 2	Intermediate consumption		Industry	134.0	15.0	1.0	2.0	3.0	155.0
Product 2	Intermediate consumption		Trade	20.0	1.0	0.0	0.0	1.0	22.0
Product 2	Intermediate consumption		Other services	65.0	3.0	1.0	2.0	5.0	76.0
Product 2	Final consumption expenditure			93.0	9.0	21.0	15.0	16.0	154.0
Product 2	Gross capital formation			136.0	6.0	2.0	3.0	17.0	164.0
Product 2	Exports	F		270.0	13.0	0.0	-4.0	0.0	279.0
Product 4	Intermediate consumption		Agriculture etc.	9.0	0.0	0.0	0.0	0.0	9.0
Product 4	Intermediate consumption		Industry	82.0	9.0	0.0	0.0	0.0	91.0
Product 4	Intermediate consumption		Trade	75.0	2.0	1.0	1.0	1.0	80.0
Product 4	Intermediate consumption		Other services	185.0	8.0	2.0	7.0	13.0	215.0
Product 4	Final consumption expenditure			614.0	13.0	31.0	22.0	51.0	731.0
Product 4	Gross capital formation			32.0	9.0	3.0	4.0	6.0	54.0
Product 4	Exports	F		91.0	20.0	0.0	-6.0	0.0	105.0
Total				**2042.6**	**133.0**	**70.0**	**53.0**	**118.0**	**2416.6**

The total use of product 1 at basic prices is 226.6 ($=26+133+3+3+10+3+48.6$), product 2 is 728.0 and product 4 is 1088.0, cf. table 1.

Table 1b: Use matrix used for generating technical coefficients, aggregated by industries.

	Industry	Basic prices	Whole-sale trade margins	Retail trade margins	Taxes less subsidies	VAT	Purchas-ers' prices
Intermediate consumption	Agriculture etc.	45.0	0.0	0.0	0.0	0.0	45.0
Intermediate consumption	Industry	349.0	39.0	2.0	4.0	6.0	400.0
Intermediate consumption	Trade	98.0	3.0	1.0	1.0	2.0	105.0
Intermediate consumption	Other services	253.0	12.0	3.0	9.0	18.0	295.0
Final consumption expenditure		717.0	25.0	58.0	42.0	69.0	911.0
Gross capital formation		171.0	17.0	6.0	8.0	23.0	225.0
Exports	F	409.6	37.0	0.0	-11.0	0.0	435.6
Total		**2042.6**	**133.0**	**70.0**	**53.0**	**118.0**	**2416.6**

Table 1b presents the use matrix from table 1 aggregated by industries.

In order to balance the total use with the total supply, the supply by products is needed, and the figures can be found in table 2. It is assumed that the supply represents the true values and the demand side has to adjust.

Table 2: Supply by products, initial estimates at basic prices

	Industry	Product 1	Product 2	Product 3	Product 4
Market output	Agriculture etc.	163.0	5.0	0.0	0.0
Market output	Industry	7.0	487.0	0.0	44.0
Market output	Trade	0.0	0.0	202.0	0.0
Market output	Other services	11.0	37.0	0.0	832.0
Imports		51.0	216.0	0.0	251.0
Total		**232.0**	**745.0**	**202.0**	**1127.0**

The total supply of Product 3 at basic prices is 202.0, which by definition in the national accounts has to equal the total value of wholesale and retail trade margins. Table 3 shows the targets for the total supply in the economy, and it can be seen that the "wholesale and retail and trade margins" condition $(135.0+67.0=202.0)$ is fulfilled.

To illustrate the imbalance between the supply and use, tables 1 and 2 shows that the use of product 1 at basic prices is 226.6 and the supply is 232.0.

Table 3 presents the initial estimates of supply at different price levels. The total at basic prices equals the total supply of products 1,2 and 4 $(2104=232+745+1127)$ and the supply of product 3 equals wholesale and retail trade margins $(135+67=202)$. The initial estimates of supply by product are believed to be very reliable especially compared with the initial estimates of demand by product. The small-scale commodity-flow system must therefore be balanced to fit these initial estimates of supply by product by varying intermediate and final demand by product.

Table 3: Initial estimates of supply at different price levels

	Basic prices	Wholesale trade margins	Retail trade margins	Taxes less subsidies	VAT	Purchasers' prices
Total	**2104.0**	**135.0**	**67.0**	**59.0**	**114.0**	**2479.0**

In order to split up industry totals into products, initial shares must be calculated. The result can be seen in table 4. For instance product 1's share of the total intermediate consumption in the agriculture industry at basic prices is 0.58 $(=26/(26+10+9))$, the gross capital formation share of product 4's wholesale trade margins to its basic prices is 0.28 $(=9/32)$ and the VAT share of product 1's final consumption expenditure is 0.08 $(=2/(10+3+6+5))$.

Table 4: The initial technical coefficients.

		Industry	Basic prices, share of total by industry	Wholesale trade margins, share of basic prises	Retail trade margins, share of basic prices	Taxes less subsidies, share of basic prices	VAT, share of purchasers' price before VAT
Product 1	Intermediate consumption	Agriculture etc.	0.58	0.00	0.00	0.00	0.00
Product 2	Intermediate consumption	Agriculture etc.	0.22	0.00	0.00	0.00	0.00
Product 4	Intermediate consumption	Agriculture etc.	0.20	0.00	0.00	0.00	0.00
Product 1	Intermediate consumption	Industry	0.38	0.11	0.01	0.02	0.02
Product 2	Intermediate consumption	Industry	0.38	0.11	0.01	0.01	0.02
Product 4	Intermediate consumption	Industry	0.23	0.11	0.00	0.00	0.00
Product 1	Intermediate consumption	Trade	0.03	0.00	0.00	0.00	0.00
Product 2	Intermediate consumption	Trade	0.20	0.05	0.00	0.00	0.05
Product 4	Intermediate consumption	Trade	0.77	0.03	0.01	0.01	0.01
Product 1	Intermediate consumption	Other services	0.01	0.33	0.00	0.00	0.00
Product 2	Intermediate consumption	Other services	0.26	0.05	0.02	0.03	0.07
Product 4	Intermediate consumption	Other services	0.73	0.04	0.01	0.04	0.06
Product 1	Final consumption expenditure		0.01	0.30	0.60	0.50	0.08
Product 2	Final consumption expenditure		0.13	0.10	0.23	0.16	0.12
Product 4	Final consumption expenditure		0.86	0.02	0.05	0.04	0.08
Product 1	Gross capital formation		0.02	0.67	0.33	0.33	0.00
Product 2	Gross capital formation		0.80	0.04	0.01	0.02	0.12
Product 4	Gross capital formation		0.19	0.28	0.09	0.13	0.13
Product 1	Exports F		0.12	0.08	0.00	-0.02	0.00
Product 2	Exports F		0.66	0.05	0.00	-0.01	0.00
Product 4	Exports F		0.22	0.22	0.00	-0.07	0.00

Note that table 4 is sorted by industry and product, whereas table 1 is sorted by product and industry.

Table 5 illustrates the initial estimates of the different use components at purchasers' prices, the corresponding border totals in the use matrix used for generating technical coefficients and the lambda values. The initial estimates of the different use components are the best available estimates, and the lambda values can be interpreted as indicators of the reliability of these estimates. A lambda value close to 1 indicates a very reliable estimate and a lambda value close to 0 an unreliable estimate. The technical coefficients can be derived from table 1.

Table 5: Initial estimates and lambda values.

	Industry	Use matrix used for generating technical coefficients, purchasers' prises	Initial estimates (by industry), purchasers' prices	Lambda values
Intermediate consumption	Agriculture etc.	45.0	47.0	0.80
Intermediate consumption	Industry	400.0	412.0	0.90
Intermediate consumption	Trade	105.0	106.0	0.90
Intermediate consumption	Other services	295.0	303.0	0.70
Final consumption expenditure		911.0	937.0	0.80
Gross capital formation		225.0	223.0	0.80
Exports F		435.6	435.6	1.00
Total		**2416.6**	**2463.6**	

6.1 The first iteration

Step 1 and step 2: The first step in the algorithm is to calculate the revised intermediate consumption by industry and final demand targets at purchasers' prices. Following equation (1.1) the targets in the first iteration equal the initial estimates. In the next iterations the targets (column totals) at purchasers' prices are revised according to equation (1.2). It is only in the first iteration that the initial estimates are used as targets. The result can be seen in table 6. Once the revised targets at purchasers' prices have been fixed, the revised targets at basic prices can be calculated using equation (2.1). For instance intermediate consumption in Industry is 359.5 at basic prices

$$(= 349.0 * \frac{412 - (6*(412.0/400)) - (4*(412.0/400))}{400-6-4}).$$

Table 6: Revised use targets at basic and purchasers' prices in the 1st iteration.

		Industry	1st Iteration: Initial targets by industry, Basic prices	1st Iteration: Initial targets by industry, Purchasers' prices
Intermediate consumption		Agriculture etc.	47.0	47.4
Intermediate consumption		Industry	359.5	415.1
Intermediate consumption		Trade	98.9	106.8
Intermediate consumption		Other services	259.9	305.3
Final consumption expenditure			737.5	944.1
Gross capital formation			169.5	224.7
Exports	F		409.6	435.6
Total			**2081.8**	**2479.0**

Step 3: The initial targets are split up by products in table 7, following equation (3.1) + (3.1.1) and the shares in table 4. The intermediate consumption of product 1 at basic prices in Industry is 137.0 (= 359.5*0.38). The sum of the intermediate consumption at basic prices in Industry is 359.5 (= 137.0 + 138.0 + 84.5), according to table 7.

Table 7: Use at basic prices in the 1st iteration after "vertical distribution".

		Industry	Product 1, Basic prices	Product 2, Basic prices	Product 4, Basic prices
Intermediate consumption		Agriculture etc.	27.2	10.4	9.4
Intermediate consumption		Industry	137.0	138.0	84.5
Intermediate consumption		Trade	3.0	20.2	75.7
Intermediate consumption		Other services	3.1	66.8	190.0
Final consumption expenditure			10.3	95.7	631.5
Gross capital formation			3.0	134.8	31.7
Exports	F		48.6	270.0	91.0
Total			**232.1**	**735.9**	**1113.8**

6.2 Calculation of the initial matrix

The initial matrix (at purchasers' prices) can be defined as the matrix calculated by using the use matrix at basic prices after vertical distribution (table 7) and adding the unbalanced trade margins and taxes. The unbalanced trade margins and taxes can be found by multiplying the basic price matrix by the respective shares matrices (table 4). Adding trade margins and taxed to the basic price cells one arrives at purchasers' prices. The initial matrix will be unbalanced in the sense that no sums by products add up to the supply targets, but the intermediate consumption by industries and the final use categories are equal to the use side targets (initial estimates). The result of the calculation of the initial matrix is shown in appendix A1.

6.3 The first iteration (continued)

Step 4: The total use at basic prices of products 1,2 and 4 does not add up to the total supply of products 1,2 and 4, cf. tables 7 and 2. Equations (4.1) + (4.1.1) express the mechanism (horizontal distribution) in the algorithm used to eliminate the imbalance between the supply and use at product level and in basic prices. The predetermined values in the use matrix are left unchanged by the mechanism. The result of the horizontal distribution is shown in table 8. The intermediate consumption of product 4 at basic prices in the service industry is 192.5 (=190.0*((1127.0-91.0)/(1113.8-91.0))). The horizontal distribution guarantees that the use and supply of each product at basic prices are balanced, and that the predetermined values are unchanged.

Table 8: Use at basic prices in the 1st iteration after "horizontal distribution".

		Industry	Product 1, Basic prices	Product 2, Basic prices	Product 4, Basic prices
Intermediate consumption		Agriculture etc.	27.1	10.6	9.5
Intermediate consumption		Industry	136.9	140.7	85.5
Intermediate consumption		Trade	3.0	20.6	76.7
Intermediate consumption		Other services	3.1	68.1	192.5
Final consumption expenditure			10.3	97.5	639.7
Gross capital formation			3.0	137.4	32.1
Exports	F		48.6	270.0	91.0
Total			**232.0**	**745.0**	**1127.0**

Step 5 to step 7: New wholesale and retail trade margins and taxes less subsidies can be calculated by multiplying the basic price matrix after horizontal distribution by the respective shares matrices. This corresponds to equations (5.1) to (7.1.1). For instance the wholesale trade margins on intermediate consumption of product 2 in the industry is 15.4 = (140.7*0.11), cf. table 9.

Table 9: Unbalanced distribution of wholesale and retail trade margins and taxes less subsidies.

		Industy	Wholesale trade margins	Retail trade margins	Taxes less subsidies	Wholesale trade margins	Retail trade margins	Taxes less subsidies	Wholesale trade margins	Retail trade margins	Taxes less subsidies
			Product 1			Product 2			Product 4		
Intermediate consumption		Agriculture etc.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Intermediate consumption		Industry	15.4	1.0	2.1	15.8	1.1	2.1	9.4	0.0	0.0
Intermediate consumption		Trade	0.0	0.0	0.0	1.0	0.0	0.0	2.0	1.0	1.0
Intermediate consumption		Other services	1.0	0.0	0.0	3.1	1.0	2.1	8.3	2.1	7.3
Final consumption expenditure			3.1	6.2	5.1	9.4	22.0	15.7	13.5	32.3	22.9
Gross fixed capital formation			2.0	1.0	1.0	6.1	2.0	3.0	9.0	3.0	4.0
Exports	F		4.0	0.0	-1.0	13.0	0.0	-4.0	20.0	0.0	-6.0
Total			**25.5**	**8.2**	**7.2**	**48.4**	**26.1**	**19.0**	**62.3**	**38.4**	**29.2**

The sums of the unbalanced distribution of wholesale and retail trade margins and taxes less subsidies do not add up to the sums given in table 3. The total value of the unbalanced distribution of retail trade margins is 72.7 (=8.2+26.1+38.4) but the unchangeable target is 67.0. Table 10 presents the balanced values[2], calculated by following equation (5.2) to (7.2.1), and e.g. the taxes less subsidies on final consumption expenditure of product 4 is 24.2 (=22.9*[(59.0-(-1.0-4.0-6.0)) / ((7.2+19.0+29.2)-(-1.0-4.0-6.0))]). By adding the matrix at basic prices after horizontal distribution and the balanced distribution of wholesale and retail trade margins and taxes less subsidies one gets the balanced values at purchasers' prices before VAT.

Step 8 and 9: Multiplying the balanced purchasers' prices before VAT by product and industry level with matrix of VAT shares (table 4) gives the unbalanced distribution of VAT. The result of this calculation is shown in table 11 along with the balanced values of VAT and Purchasers' prices. The latter is found by adding balanced VAT with balanced purchasers' prices before VAT.

When calculating balanced VAT, predetermined values are unchanged. The unbalanced value of VAT on intermediate consumption on product 1 in Industry is 3.1 (= (136.9+15.2+0.9+2.2)*0.02), and the equivalent balanced value is 2.9 (=3.1*(114.0/(6.1+43.2+72.6)). This corresponds to equations (8.1) to (8.3).

[2] The balancing of taxes less subsidies does not follow the general balancing method expressed by equation (7.2) but is a more simplified method described in footnote 1.

Table 10: Balanced distribution of wholesale and retail trade margins and taxes less subsidies.

	Industry	Wholesale trade margins	Retail trade margins	Taxes less subsidies	Wholesale trade margins	Retail trade margins	Taxes less subsidies	Wholesale trade margins	Retail trade margins	Taxes less subsidies
		Product 1			Product 2			Product 4		
Intermediate consumption	Agriculture etc.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Intermediate consumption	Industry	15.2	0.9	2.2	15.5	1.0	2.2	9.3	0.0	0.0
Intermediate consumption	Trade	0.0	0.0	0.0	1.0	0.0	0.0	2.0	0.9	1.1
Intermediate consumption	Other services	1.0	0.0	0.0	3.1	1.0	2.2	8.2	1.9	7.7
Final consumption expenditure		3.0	5.7	5.4	9.3	20.3	16.6	13.4	29.7	24.2
Gross capital formation		2.0	0.9	1.0	6.0	1.9	3.2	8.9	2.8	4.2
Exports F		4.0	0.0	-1.0	13.0	0.0	-4.0	20.0	0.0	-6.0
Total		**25.3**	**7.5**	**7.6**	**48.0**	**24.1**	**20.2**	**61.8**	**35.4**	**31.2**

Now balanced values have been calculated at all price levels, and the result of the first iteration is ready. When preparing for the next iteration, new shares must be calculated in a way similar to the procedure used for calculating table 4.

Tabel: 11: Unbalanced and balanced distribution of VAT.

	Industy	VAT, unbalanced	VAT, balanced	Purchasers' prices, balanced	VAT, unbalanced	VAT, balanced	Purchasers' prices, balanced	VAT, unbalanced	VAT, balanced	Purchasers' prices, balanced
		Product 1			Product 2			Product 4		
Intermediate consumption	Agriculture etc.	0.0	0.0	27.1	0.0	0.0	10.6	0.0	0.0	9.5
Intermediate consumption	Industry	3.1	2.9	158.1	3.1	2.9	162.4	0.0	0.0	94.8
Intermediate consumption	Trade	1.0	1.0	22.6	1.0	1.0	81.7	0.0	0.0	3.0
Intermediate consumption	Other Services	0.0	0.0	4.1	5.2	4.9	79.2	13.5	12.7	222.9
Final consumption expenditure		2.0	1.9	26.3	16.7	15.6	159.3	53.0	49.6	756.5
Gross capital formation		0.0	0.0	6.9	17.2	16.1	164.5	6.0	5.6	53.7
Exports F		0.0	0.0	51.6	0.0	0.0	279.0	0.0	0.0	105.0
Total		**6.1**	**5.7**	**296.8**	**43.2**	**40.4**	**936.8**	**72.6**	**67.8**	**1245.5**

6.4 The second iteration

Step 1: The second iteration starts the same way as the first iteration. Revised targets at purchasers' prices are calculated using the balanced purchasers' prices at industry and final demand category level, the initial targets by industries at purchasers' prices and the lambda values. The revised target for intermediate consumption in the trade industry at purchasers' prices is 106.1 ($= 0.9^{2-1}(106.0-107.3)+107.3$) cf. table 12.

Table 12: Revised targets at basic and purchasers' prices.

	Industry	Second iteration: Revised targets by industry, Basic prices	Second iteration: Revised targets by industry, Purchasers' prices	First iteration: Balanced, Purchasers' prices	Initial targets by industries, Purchasers' prices
Intermediate consumption	Agriculture etc.	47.1	47.1	47.3	47.0
Intermediate consumption	Industry	360.5	412.3	415.4	412.0
Intermediate consumption	Trade	99.2	106.1	107.3	106.0
Intermediate consumption	Other services	261.7	304.0	306.3	303.0
Final consumption expenditure		744.2	938.0	942.1	937.0
Gross capital formation		171.3	223.4	225.1	223.0
Exports F		409.6	435.6	435.6	435.6
Total		**2093.5**	**2466.5**	**2479.0**	**2463.6**

The calculation of tables 7 to 12 is repeated until convergence has been achieved. Under normal circumstance convergence will be achieved after 50 iterations.

6.5 Balanced supply and use tables

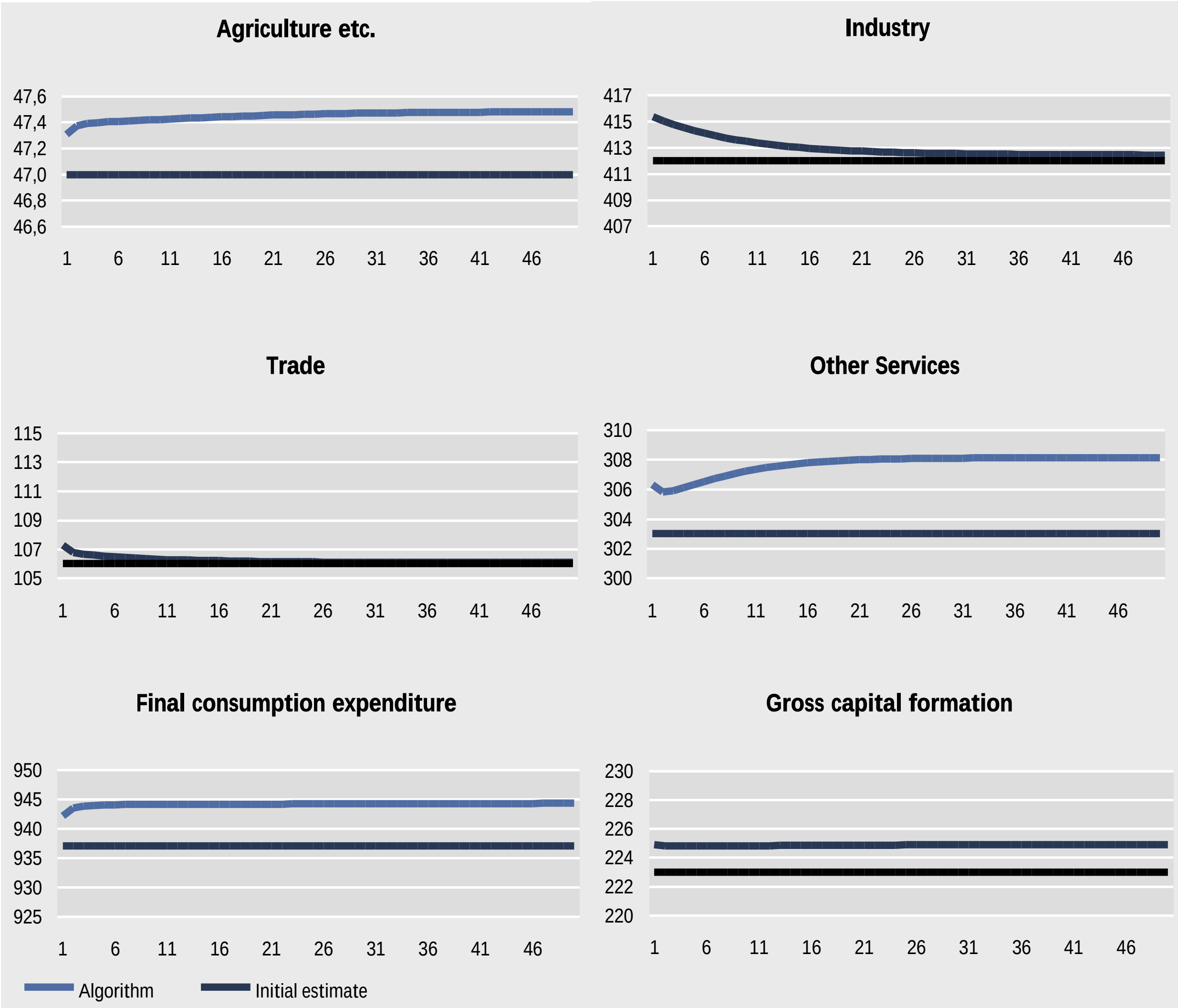
Figure 2 shows the convergence of the estimates of the endogenous border totals in the use matrix at purchasers' prices in the small numerical example.

It is seen that convergence is almost achieved after about 30 iterations and that it is complete after 50. The initial balancing difference between supply and demand in the example is of a realistic magnitude. Other runs have been made with much larger initial differences. Those experiments also indicate that the algorithm has nice convergence properties.

When interpreting the charts in figure 2 it should be kept in mind that the scales on the vertical axis are different. The charts show the status after balancing supply and use in each iteration. It is seen that in this example four out of six endogenous variables follow a monotonous adjustment path to their final balanced values. Intermediate consumption in the industry Other services has a very small movement in the wrong direction in the second and third iterations after which it enters onto a monotonous path to its final balanced value. The adjustment path for investment makes a tiny overshoot of its final value but the deviation is so small that it does not have any significant influence on the overall convergence properties.

Table A2 in appendix A2 show the final result - a balanced supply use table after the 50th iteration.

Figure 2: Convergence, Billion DKK.



7. Running the algorithm on large supply and use tables

The real test of the algorithm proposed in the present paper is of course its ability to handle supply and use tables of the size used in practice by countries compiling such tables as part of their national accounts.

In this section we perform a very interesting experiment namely a balancing of the final national accounts for Denmark for 1998 by means of the algorithm in section 5 which is then compared with the result of the manual balancing of the accounts for the same year resulting in the figures that have been published.

The automatic balancing is performed at a more aggregate level than the manual one. It turns out, however, that the algorithm is so efficient on a modern PC running SAS that it is almost certain that it could handle the full Danish supply and use tables comprising about 2750 products, 130 industries and 72 groups of final household consumption with manageable execution times. This will be tested as part of our further research. As previously mentioned the Danish supply and use tables are unusually large in the product dimension, and the aggregated version used for the experiment is closer to the current international standard regarding the level of detail.

In the experiment reported here the Danish supply and use tables are aggregated in the following manner:

2750 products into 502 products (CPA 4-digit level)
130 industries into 60 industries (NACE/ISIC 2-digit level)
72 consumption groups into 12 consumption groups (COICOP 1-digit level with a breakdown of COICOP 1 into Food and Beverages and tobacco, of COICOP 3 into Housing and Electricity, gas and other fuels, of COICOP 6 into Purchase of vehicles and Other transport and communication, and a separate heading for the Balance of Tourism, net. On the other hand COICOP 8, 9 and 10 (Education; Hotels, cafés and restaurants; Miscellaneous goods and services) have been lumped together in one heading.

Gross fixed capital formation and changes in inventories on the other hand are not aggregated except for a grouping together of residential and non-residential buildings. There are 9 columns in the use table for gross fixed capital formation and 5 columns for changes in inventories. All in all there are 99 intermediate and final uses and hence columns in the use matrix. Each of the use matrices in figure 1 is therefore of dimension 502 x 99.

The algorithm is applied to the initial unbalanced estimates entering the manual balancing process for the final national accounts estimates for 1998. As explained in section 5, the choice of the λ_i is where the algorithm allows the compilers of national accounts to introduce their knowledge or assumptions about the relative reliability of the various column totals in the use matrix. In our standard run of the algorithm we distinguish three different degrees of reliability concerning the column totals of the use matrix at purchasers' prices. These are the same as those more or less explicitly used in the manual balancing process. Firstly, some uses such as intermediate consumption in general government and financial institutions, changes in inventories and exports are assumed to be known with absolute certainty. For those columns in the use matrix λ is set equal to 1 so that the algorithm will realise the initial estimate if that is at all possible. If it is not possible, the algorithm will not converge which will be a sign that the initial estimates are inconsistent with the supply of commodities in the economy. Such a situation has not in fact arisen in our practical application.

Secondly, the column totals for intermediate consumption, which are based on yearly high-quality accounting statistics in almost all cases, are regarded as highly reliable. They consequently get a λ-value of 0.9 which implies that the adjustment rule in equation (1.2) will only reluctantly accept a deviation from the initial estimate and only if it is consistently contradicted by the supply of commodities used for those types of intermediate consumption. Thirdly, the final demand categories which are not known with certainty are also in most cases estimated directly in terms of levels each year based on good primary statistics. However, the grossing-up uncertainty and the uncertainty regarding the breakdown of retail sales by commodity groups imply that, overall, the reliability of the initial estimates of final household consumption and of gross fixed capital formation is lower than is the case for intermediate consumption of industries. Therefore the column totals for household consumption and for gross fixed capital formation get λ-values of 0.7 implying that the initial estimates will get a real chance to influence the final outcome in the first four to five iterations. If they continue to be contradicted by the supply of the products concerned for use in the domestic market, they will then be modified rather quickly and the final outcome will to a larger extent than for intermediate consumption be determined by the balance of initial supply and demand for the products concerned.

The result of this large-scale application of the algorithm is shown in table 13 together with the outcome of the manual balancing which resulted in the national accounts that have been published.

Table 13: Comparison of manual and automatic balancing of the Danish national accounts 1998 (continues)

NACE / COI-COP	Text	λ	Initial estimate of intermediate consumption / final demand (million kr.)	Balanced value after automatic balancing (million kr.)	Balanced value after manuel balancing (published figures) (million kr.)	Absolute difference between automatic and manual balancing (million kr.)	Percentage difference between automatic and manual balancing	Percentage difference between automatic and manual balancing for value added
	Output side							
01	Agriculture, hunting and related service activities	0.9	32 189	32 122	32 187	- 65	-0.20	0.26
02	Forestry, logging and related service activities	0.9	986	964	986	- 21	-2.17	2.15
05	Fishing, operation of fish hatcheries / fish farms	0.9	1 912	1 922	1 912	10	0.54	-0.38
10	Mining of coal and lignite etc.							
11	Extraction of crude petroleum and natural gas etc.	0.9	1 931	1 931	1 931		0.01	0.00
12	Mining of uranium and thorum ores							
13	Mining of metal ores							
14	Other mining and quarrying	0.9	1 174	1 174	1 174		0.01	-0.02
15	Manufacture of food products and beverages	0.9	82 343	82 344	83 786	- 1 442	-1.72	5.50
16	Manufacture of tobacco products	0.9	1 641	1 641	1 641		0.01	-0.01
17	Manufacture of textiles	0.9	4 934	4 934	4 934	1	0.01	-0.02
18	Manufacture of wearing apparel etc.	0.9	3 779	3 779	3 779		0.00	-0.01
19	Manufacture of leather products etc.	0.9	813	813	813		0.01	-0.01
20	Manufacture of wood products except furniture	0.9	7 652	7 654	7 652	2	0.03	-0.04
21	Manufacture of paper and paper products	0.9	6 245	6 246	6 245	1	0.01	-0.02
22	Publishing, printing and reproduction	0.9	18 442	18 444	18 442	2	0.01	-0.01
23	Manufacture of coke, refined petroleum products	0.9	7 681	7 681	7 681	- 1	-0.01	0.14
24	Manufacture of chemicals and chemical products	0.9	23 259	23 261	23 259	2	0.01	-0.01
25	Manufacture of rubber and plastic products	0.9	10 156	10 158	10 156	2	0.02	-0.02
26	Manufacture of other non-metallic min. products	0.9	10 358	10 360	10 358	2	0.02	-0.03
27	Manufacture of basic metals	0.9	7 000	7 002	7 000	2	0.03	-0.05
28	Metal products, except machinery and equipment	0.9	17 058	17 063	17 058	4	0.03	-0.03
29	Manufacture of machinery and equipment n.e.c.	0.9	33 494	33 503	33 494	9	0.03	-0.03
30	Manufacture of office machinery and computers	0.9	960	960	960		0.02	-0.03
31	Electrical machinery and apparatus n.e.c.	0.9	11 018	11 022	11 018	3	0.03	-0.05
32	Radio, television and communication equipment	0.9	8 089	8 091	8 089	2	0.02	-0.04
33	Medical, precision and optical instruments etc.	0.9	5 869	5 870	5 869	1	0.02	-0.02
34	Manufacture of motor vehicles etc.	0.9	4 334	4 335	4 334	1	0.02	-0.03
35	Manufacture of other transport equipment	0.9	7 746	7 747	7 746	2	0.02	-0.05
36	Manufacture of furniture; manufacture n.e.c.	0.9	15 824	15 827	15 824	3	0.02	-0.03
37	Recycling	0.9	692	693	692		0.03	-0.11
40	Electricity, gas, steam and hot water supply	0.9	12 887	12 887	12 787	100	0.78	-0.44
41	Collection, purification and distribution of water	0.9	1 651	1 651	1 651		0.01	-0.01
45	Construction	0.9	89 110	89 137	89 110	28	0.03	-0.05
50	Sale, maintenance and rep. of motor vehicles etc.	0.9	15 734	15 736	15 734	2	0.01	-0.01
51	Wholesale trade except of motor vehicles	0.9	53 200	53 204	53 200	4	0.01	-0.01
52	Retail trade except motor vehicles; repair services	0.9	21 459	21 460	21 459	1	0.01	0.00
55	Hotel and restaurant services	0.9	17 944	17 944	17 944		0.00	0.00
60	Land transport and transport via pipeline services	0.9	23 614	23 615	23 614	1	0.00	0.00
61	Water transport services	0.9	43 964	43 935	43 964	- 29	-0.07	0.25
62	Air transport services	0.9	10 904	10 905	10 904	1	0.01	-0.02
63	Auxiliary transport services, travel agency services	0.9	14 580	14 581	14 580	1	0.01	-0.01
64	Post and telecommunication services	0.9	14 279	14 280	14 279	1	0.01	-0.01
65	Financial intermediation services, except insurance	1.0	17 522	17 522	17 522		0.00	0.00
66	Insurance and pension funding services	1.0	9 735	9 735	9 735		0.00	0.00
67	Services auxiliary to financial intermediation	0.9	2 599	2 599	2 599		0.01	0.00
70	Real estate services	0.9	29 510	29 512	29 510	1	0.00	0.00
71	Renting services of machinery and equipment etc.	0.9	3 441	3 441	3 441		0.01	0.00
72	Computer and related services	0.9	13 390	13 391	13 390	1	0.01	-0.01
73	Research and development services	0.9	2 001	2 001	2 001		0.01	0.00
74	Other business services	0.9	36 794	36 797	36 794	3	0.01	-0.01
75	Public administration and defence services	1.0	34 892	34 892	34 892		0.00	0.00
80	Education services	1.0	16 017	16 017	16 018		0.00	0.00
85	Health and social work services	1.0	34 683	34 683	34 683		0.00	0.00
90	Sewage and refuse disposal services	0.9	8 221	8 222	8 221	1	0.01	-0.01
91	Membership organisation services n.e.c.	1.0	4 250	4 250	4 250		0.00	0.00
92	Recreational, cultural and sporting services	0.9	13 867	13 868	13 867	1	0.01	-0.01
93	Other services	0.9	2 045	2 045	2 045		0.01	0.00
95	Private households with employed persons	0.9						0.00
	FISIM	0.9	32 563	32 547	32 563	- 16	-0.05	-0.05
	Total		908 434	908 396	909 775	- 1 379	-0.15	0.14

Table 13: Comparison of manual and automatic balancing of the Danish national accounts 1998 (continued)

NACE / COI-COP	Text	λ	Initial estimate of intermediate consumption / final demand (million kr.)	Balanced value after automatic balancing (million kr.)	Balanced value after manuel balancing (published figures) (million kr.)	Absolute difference between automatic and manual balancing (million kr.)	Percentage difference between automatic and manual balancing	Percentage difference between automatic and manual balancing for value added
	Expenditure side							
	Final household consumption		575 760	573 782	572 152	1 629	0.28	
1.1	Food	0.7	65 687	65 527	64 771	756	1.17	
1.2	Beverages and tobacco	0.7	36 584	36 419	36 368	51	0.14	
2	Clothing and footwear	0.7	29 512	29 319	29 012	307	1.06	
3.1	Housing, water	0.7	122 362	122 331	122 134	197	0.16	
3.2	Electricity, gas and other fuels	0.7	33 074	33 015	33 646	-630	-1.87	
4	Furnishings, household equipment and routine maintenance of the house	0.7	33 645	33 804	33 222	583	1.75	
5	Health	0.7	14 136	14 176	14 283	-108	-0.75	
6.1	Purchase of vehicles	0.7	36 212	34 997	36 016	-1 018	-2.83	
6.2	Other transport and communication	0.7	53 050	52 969	52 550	419	0.80	
7	Leisure, entertainment and culture	0.7	60 868	61 028	60 532	496	0.82	
8,9,10	Other goods and services	0.7	89 964	89 530	88 953	577	0.65	
99	Balance of tourism, net	0.7	666	666	666	0	-0.04	
	Consumption of NPISH	1.0	9 117	9 117	9 117	0	0.00	
	Government consumption	1.0	300 455	300 452	300 452	0	0.00	
	Gross fixed capital formation		239 903	240 139	240 310	-171	-0.07	
	Machinery and equipment	0.7	69 943	70 811	72 353	-1 541	-2.13	
	Transport equipment	0.7	29 611	28 655	27 414	1 241	4.53	
	Buildings	0.7	88 377	88 608	88 389	219	0.25	
	Civil engineering works	0.7	27 910	27 989	28 091	-102	-0.36	
	Software and large databases	0.7	20 135	20 145	20 135	10	0.05	
	Entertainment, literary and artistic originals	0.7	1 375	1 374	1 375	-1	-0.06	
	Mineral exploration	0.7	444	447	444	3	0.69	
	LIvestock	0.7	92	92	92	0	0.00	
	Valuables	0.7	2 016	2 017	2 016	1	0.04	
	Changes in inventories	1.0	10 094	10 091	10 069	22	0.22	
	Exports	1.0	413 399	413 404	413 404	0	0.00	
	Total final uses		1 548 727	1 546 984	1 545 503	1 481	0.10	
	Imports		390 097	390 097	390 097	0	0.00	
	GDP		1 158 630	1 156 887	1 155 406	1 481	0.13	

The table shows that the outcome of applying the algorithm for automatic balancing with λ-values as indicated above in general is relatively close to the published figures resulting from manual balancing. The two balancing procedures produce balanced GDP figures that differ by only 0.13 per cent.

For the year 1998 the initial estimate of GDP from the output side GDP(O) is 1,156,746 million kr., whereas the initial estimate from the expenditure side GDP(E) is 1,158,630 million kr. The initial discrepancy is thus 1,882 million equivalent to 0.16 per cent of GDP. For the final national accounts the initial discrepancy is normally less than 0.5 per cent of GDP. 0.16 per cent is a small initial discrepancy but in no way an unusual situation. It is seen that the published GDP figure produced by the manual balancing is 1,155,406 and thus actually lower than the initial estimate from the output side. This has to do with the fact that it was decided to disregard the initial estimate for one of the industries based on the evolution in technical coefficients, cf. later. The outcome of the automatic balancing with λ-values of 1.0; 0.9; and 0.7 as explained above is 1,156,887 and thus slightly above the initial estimate of GDP(O) and inside the interval suggested by GDP(O) and GDP(E).

Looking at the third and fourth columns in table 13 it is seen that there are only four differences larger than 1 billion kr. As regards intermediate consumption, it is seen that in the manual balancing it has been decided to deviate substantially from the initial estimates of intermediate consumption by industry in two cases namely industry 15 Manufacture of food products and beverages (+1,443 million) and industry 40 Electricity, gas, steam and hot water supply (-100 million). In all other cases the manual balancing has decided to adjust almost 100 per cent to the initial estimates of intermediate consumption by industry.

In the automatic balancing the high value of λ chosen for intermediate consumption totals by industry result in rather small deviations from the initial estimates of intermediate consumption by industry contained in GDP(O). The largest deviation occurs for agriculture (-65 million) but this is only 0.26 per cent of value added.

Final demand components are modified to a larger extent in both the automatic and the manual balancing. However, the sensitivity analysis reported later in section 8 will show that the split between intermediate and final uses in this case is not very sensitive to the relative values of λ. The overall picture is that total investment is broadly the same (but with a different composition) in the two types of balancing, whereas final household consumption absorbs the slightly higher GDP in the automatic balancing and is 1,629 million or 0.28 per cent higher than in the manually balanced published figures.

There are three differences regarding final demand components which exceed 1 billion kr. namely COICOP 6.1 Purchase of vehicles (-1,018 million), GFCF in Machinery and equipment (-1,541 million) and GFCF in Transport equipment (+1,241 million).

As for the final consumption component Purchase of vehicles and GFCF in Transport equipment, the two deviations are largely two sides of the same coin. The result of the manual balancing is 2.2 billion below the initial estimate of GFCF in Transport equipment, while being only 0.2 billion below the initial estimate of the final household consumption component Purchase of vehicles. The result of the automatic balancing on the other hand is 1.0 billion below the initial estimate of GFCF in transport equipment and 1.2 billion below the initial estimate of the final consumption component Purchase of vehicles. The difference between the results is thus largely explained by the treatment of motor vehicles (passenger cars). Whereas the manual balancing has clearly placed much more weight on the initial estimate of consumers' purchases of vehicles than on the initial estimate of GFCF in transport equipment, the automatic balancing has chosen the middle ground.

The reason for this choice in the manual balancing is clear. The strategic commodities in the balancing are new and used passenger cars for which there is good statistical information from motor vehicle registration statistics. Since COICOP 6.1 is dominated by those two products, whereas investment in transport equipment comprises a large variety of products with less extensive statistical coverage, it is natural to place more weight on the COICOP 6.1 Purchase of vehicles initial estimate than on the initially estimated value of GFCF in transport equipment.

In the automatic balancing in table 13 all types of gross fixed capital formation and all groups of final household consumption expenditure are assigned λ-values of 0.7 that is to say the same relative reliability. Based on the same rationale as in the manual balancing one may assign a higher λ-value to the specific consumption group COICOP 6.1 Purchase of vehicles. If one assigns a λ-value of 0.9 or higher to this consumption group, the result of the automatic balancing will be much more like that of the manual balancing.

As far as the difference of 1.6 billion for GFCF in Machinery and equipment is concerned, it is seen that the outcome of the automatic balancing is much closer to the initial estimate than is the case for the published, manually balanced result. It is likely that the higher level of product detail (2750 products instead of 502) may have informed the manual balancing, and that the link between capital goods plus components and gross fixed capital formation has convinced the national accountants involved in the manual balancing that a big departure from the initial estimate is appropriate in this case. Gross fixed capital formation is undoubtedly that part of the national accounts and supply and use tables benefiting most from a high level of products detail in the commodity-flow system. That is one of the reasons that the Danish national accounts have opted for a high level of detail in the product dimension (700 investment products).

8. Convergence properties and sensitivity analysis

The algorithm presented in sections 5-6 turns out to be efficient. Since it has a core of RAS-like biproportional adjustments, it is only guaranteed to converge under the same conditions where RAS does. However, in practice, we have not encountered a single case of non-convergence. The algorithm usually converges in less than 50 iterations.

Table 14: Convergence properties.

Iteration	Cells changed more than 5%	Cells changed more than 2%	Cells changed more than 1%	Cells changed more than 0.5%	Cells changed more than 0.1%	Cells changed more than 0.01%	Cells changed more than 0.001%
	No. of records changed compared with previous iteration						
2	0	114	468	1856	6358	7949	8126
3	0	7	97	517	4809	7722	8109
4	0	0	36	143	3475	7494	8077
5	0	0	5	58	2397	7077	8001
6	0	0	0	34	1361	6690	7987
7	0	0	0	4	423	6219	7908
8	0	0	0	0	143	5665	7837
9	0	0	0	0	77	4952	7748
10	0	0	0	0	49	4247	7593
11	0	0	0	0	23	3567	7417
12	0	0	0	0	0	3001	7174
13	0	0	0	0	0	2403	6900
14	0	0	0	0	0	1729	6669
15	0	0	0	0	0	1059	6398
16	0	0	0	0	0	548	6077
17	0	0	0	0	0	319	5718
18	0	0	0	0	0	246	5351
19	0	0	0	0	0	187	4999
20	0	0	0	0	0	153	4609
21	0	0	0	0	0	111	4232
22	0	0	0	0	0	28	3844
23	0	0	0	0	0	7	3513
24	0	0	0	0	0	0	3204
25	0	0	0	0	0	0	2913
26	0	0	0	0	0	0	2641
27	0	0	0	0	0	0	2357
28	0	0	0	0	0	0	2043
29	0	0	0	0	0	0	1657
30	0	0	0	0	0	0	1259
31	0	0	0	0	0	0	867
32	0	0	0	0	0	0	572
33	0	0	0	0	0	0	342
34	0	0	0	0	0	0	258
35	0	0	0	0	0	0	234
36	0	0	0	0	0	0	218
37	0	0	0	0	0	0	190
38	0	0	0	0	0	0	171
39	0	0	0	0	0	0	146
40	0	0	0	0	0	0	126
41	0	0	0	0	0	0	112
42	0	0	0	0	0	0	36
43	0	0	0	0	0	0	11
44	0	0	0	0	0	0	1
45	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0

The execution times on a Pentium IV 1.4 GHz processor running SAS version 8.0 are shown in the following table

Run	Minutes
Small numerical example (section 6)	3
National accounts 4-digit CPA (section 7)	27
National accounts 2-digit CPA (section 8)	5

The speed of execution is seen to be so fast that there are no problems running it on a long time series and even if one needs to conduct sensitivity studies with various values of the λ's.The convergence properties of the standard run of the algorithm with λ values of 1.0, 0.9 and 0.7 at the 4-digit CPA level are shown in table 14. The table shows for each iteration the number of cells that have been changed broken down into percentage bands. The total number of non-zero cells is 9452 and 2146 of these are unchangeable (fixed) in the algorithm. It is seen that after the 24th iteration only very small relative changes occur.

For the case of the final national accounts estimates for Denmark for 1998 a sensitivity analysis has been conducted regarding the impact fo r various values of the λs and concerning the level of aggregation in the product dimension.

As regards the choice of the λ_j, the results for various plausible combinations of λ-values are shown in table 15.

Table 15: Sensitivity analysis of the choice of confidence factors

	Manual balancing	Automatic balancing	Automatic balancing 1st alternative	Automatic balancing 2nd alternative	Automatic balancing 3rd alternative	Automatic balancing 4th alternative
	Mill. DKK					
Output side						
Resources	2 455 278	2 455 278	2 455 278	2 455 278	2 455 278	2 455 278
Imports	390 097	390 097	390 097	390 097	390 097	390 097
Taxes less subsides on products	174 781	174 781	174 781	174 781	174 781	174 781
Ouput	1 890 400	1 890 400	1 890 400	1 890 400	1 890 400	1 890 400
Taxes less subsides on products	174 781	174 781	174 781	174 781	174 781	174 781
Intermediate consumption	909 775	908 294	908 247	908 329	908 294	908 300
GDP	**1 155 406**	**1 156 887**	**1 156 934**	**1 156 852**	**1 156 887**	**1 156 881**
Use side						
Intermediate consumption	909 775	908 294	908 247	908 329	908 294	908 300
Private consumption expenditure	572 152	573 782	574 648	574 187	574 227	573 520
Consumption of NPISH	9 117	9 117	9 117	9 117	9 117	9 117
Government consumption	300 452	300 452	300 452	300 452	300 452	300 452
Gross fixed capital formation	240 310	240 139	239 319	239 699	239 694	240 394
Machinery and equipment	72 353	70 811	70 010	70 405	70 405	71 060
Transport equipment	27 414	28 655	28 903	28 734	28 736	28 597
Buildings	88 389	88 608	88 417	88 524	88 520	88 657
Civil engineering works	28 091	27 989	27 923	27 959	27 958	28 009
Software and large databases	20 135	20 145	20 140	20 145	20 144	20 145
Entertainment, literary and artistic originals	1 375	1 374	1 375	1 375	1 375	1 373
Mineral exploration	444	447	445	446	446	446
Livestock	92	92	92	92	92	92
Valuables	2 016	2 017	2 017	2 018	2 018	2 015
Changes in inventories	10 069	10 091	10 091	10 091	10 091	10 091
Exports	413 404	413 404	413 404	413 404	413 404	413 404
Total uses	**2 455 278**	**2 455 278**	**2 455 278**	**2 455 278**	**2 455 278**	**2 455 278**
Lambda values						
Intermediate consumption, agriculture ect, industry		0.9	0.9	0.9	0.9	0.6
Intermediate consumption, trade and other services		0.9 (1	0.9 (1	0.9 (1	0.8 (1	0.6 (1
Final use		0.7 (2	0.9 (2	0.8 (2	0.8 (2	0.6 (2

1) Except general government and financial institutions, where λ = 1.0.

2) Except government and HPISH consumption, export and changes in inventories, where λ=1.0.

The λ-values tried are shown at the bottom of the table. It is seen that the outcome of the automatic balancing is remarkably robust as to the choice of λs. As far as the basic split into intermediate and final uses is concerned, even the 4th alternative where both intermediate consumption totals and final demand totals get the much lower λ-value of 0.6 – and are therefore modified much faster – GDP is virtually the same as in the automatic balancing where intermediate uses get $\lambda = 0.9$ and final uses get $\lambda = 0.7$. The impact of varying the λs shows up mainly in the split of final demand between household consumption and investment. The effect is relatively modest, however, with a maximum of 1.1 billion on total final household consumption and total gross fixed capital formation, corresponding to 0.02 and 0.05 per cent respectively.

A sensitivity analysis was also performed regarding the level of detail in the product dimension. The same runs that were done at the 4-digit CPA level (502 products) were also carried out at the 2-digit CPA level (60 products). It turns out that the impact of disaggregation in the product dimension is quite significant. The impact on GDP is up to 1.4 billion kr. or 0.12 per cent of GDP in the runs performed for the sensitivity analysis in table 15. For the automatic balancing with λ-values of 1,0; 0.9 and 0.7 in table 13 the difference as far as GDP is concerned is 0.9 billion - a difference which is predominantly explained by a different allocation of products between intermediate consumption and gross fixed capital formation.

9. Conclusion

The algorithm for automatic balancing of supply and use tables of SNA93 format presented in this paper has been applied to the Danish supply and use tables for 1998. The algorithm produces results that are economically meaningful and appears to be robust. It also turns out to be efficient in balancing commodity-flow systems. In the application reported in the paper it has been run on a system comprising approximately 500 products and 100 uses of products. The high speed of convergence realised with those dimensions suggests that the algorithm can in fact handle much larger commodity-flow systems with manageable execution times.

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Appendix A1: The initial matrix.

Tabel A1: The Initial Matrix (supply and use table).

			Industry	Basic prices	Wholesale trade margins	Retail trade margins	Taxes less subsidies	VAT	Purchasers' prices
Product 3	Market output		Trade	202.0					
Product 1	Market output		Agriculture etc.	163.0					
Product 1	Market output		Industry	7.0					
Product 1	Market output		Other services	11.0					
Product 1	Imports			51.0					
Product 1	Intermediate consumption		Agriculture etc.	27.2	0.0	0.0	0.0	0.0	27.2
Product 1	Intermediate consumption		Industry	137.0	15.5	1.0	2.1	3.1	158.6
Product 1	Intermediate consumption		Trade	3.0	0.0	0.0	0.0	0.0	3.0
Product 1	Intermediate consumption		Other services	3.1	1.0	0.0	0.0	0.0	4.1
Product 1	Final consumption expenditure			10.3	3.1	6.2	5.1	2.1	26.7
Product 1	Gross capital formation			3.0	2.0	1.0	1.0	0.0	6.9
Product 1	Exports	F		48.6	4.0	0.0	-1.0	0.0	51.6
Product 2	Market output		Agriculture etc.	5.0					
Product 2	Market output		Industry	487.0					
Product 2	Market output		Other services	37.0					
Product 2	Imports			216.0					
Product 2	Intermediate consumption		Agriculture etc.	10.4	0.0	0.0	0.0	0.0	10.4
Product 2	Intermediate consumption		Industry	138.0	15.5	1.0	2.1	3.1	159.7
Product 2	Intermediate consumption		Trade	20.2	1.0	0.0	0.0	1.0	22.2
Product 2	Intermediate consumption		Other services	66.8	3.1	1.0	2.1	5.1	78.1
Product 2	Final consumption expenditure			95.7	9.3	21.6	15.4	16.5	158.4
Product 2	Gross capital formation			134.8	5.9	2.0	3.0	16.8	162.5
Product 2	Exports	F		270.0	13.0	0.0	-4.0	0.0	279.0
Product 4	Market output		Agriculture etc.	0.0					
Product 4	Market output		Industry	44.0					
Product 4	Market output		Other services	832.0					
Product 4	Imports			251.0					
Product 4	Intermediate consumption		Agriculture etc.	9.4	0.0	0.0	0.0	0.0	9.4
Product 4	Intermediate consumption		Industry	84.5	9.3	0.0	0.0	0.0	93.7
Product 4	Intermediate consumption		Trade	75.7	2.0	1.0	1.0	1.0	80.8
Product 4	Intermediate consumption		Other services	190.0	8.2	2.1	7.2	13.4	220.8
Product 4	Final consumption expenditure			631.5	13.4	31.9	22.6	52.5	751.9
Product 4	Gross capital formation			31.7	8.9	3.0	4.0	5.9	53.5
Product 4	Exports	F		91.0	20.0	0.0	-6.0	0.0	105.0
Total	Intermediate consumption		Agriculture etc.	47.0	0.0	0.0	0.0	0.0	47.0
Total	Intermediate consumption		Industry	359.5	40.2	2.1	4.1	6.2	412.0
Total	Intermediate consumption		Trade	98.9	3.0	1.0	1.0	2.0	106.0
Total	Intermediate consumption		Other services	259.9	12.3	3.1	9.2	18.5	303.0
Total	Final consumption expenditure			737.5	25.7	59.7	43.2	71.0	937.0
Total	Gross capital formation			169.5	16.8	5.9	7.9	22.8	223.0
Total	Exports	F		409.6	37.0	0.0	-11.0	0.0	435.6
Total				**2081.8**	**135.1**	**71.8**	**54.5**	**120.5**	**2463.6**
Total	**Supply**			**2306.0**					
Total	**Use**			**2081.8**	**135.1**	**71.8**	**54.5**	**120.5**	**2463.6**

Appendix A2: Balanced supply and use table after the 50th matrix.

Tabel A2: Balanced supply and table after the 50th iteration.

			Industry	Basic prices	Wholesale trade margins	Retail trade margins	Taxes less subsidies	VAT	Purchasers' prices
Product 3	Market output		Trade	202.0					
Product 1	Market output		Agriculture etc.	163.0					
Product 1	Market output		Industry	7.0					
Product 1	Market output		Other services	11.0					
Product 1	Imports			51.0					
Product 1	Intermediate consumption		Agriculture etc.	27.3	0.0	0.0	0.0	0.0	27.3
Product 1	Intermediate consumption		Industry	136.5	15.2	0.9	2.2	2.9	157.7
Product 1	Intermediate consumption		Trade	3.0	0.0	0.0	0.0	0.0	3.0
Product 1	Intermediate consumption		Other services	3.1	1.0	0.0	0.0	0.0	4.2
Product 1	Final consumption expenditure			10.4	3.1	5.7	5.5	7.7	32.4
Product 1	Gross capital formation			3.0	2.0	0.9	1.0	7.4	14.3
Product 1	Exports	F		48.6	4.0	0.0	-1.0	0.0	51.6
Product 2	Market output		Agriculture etc.	5.0					
Product 2	Market output		Industry	487.0					
Product 2	Market output		Other services	37.0					
Product 2	Imports			216.0					
Product 2	Intermediate consumption		Agriculture etc.	10.7	0.0	0.0	0.0	0.0	10.7
Product 2	Intermediate consumption		Industry	139.6	15.4	1.0	2.2	2.9	161.1
Product 2	Intermediate consumption		Trade	20.4	1.0	0.0	0.0	1.0	22.4
Product 2	Intermediate consumption		Other services	68.7	3.1	1.0	2.2	4.9	80.0
Product 2	Final consumption expenditure			98.1	9.4	20.3	16.6	23.5	167.9
Product 2	Gross capital formation			137.6	6.0	1.9	3.2	5.7	154.3
Product 2	Exports	F		270.0	13.0	0.0	-4.0	0.0	279.0
Product 4	Market output		Agriculture etc.	0.0					
Product 4	Market output		Industry	44.0					
Product 4	Market output		Other services	832.0					
Product 4	Imports			251.0					
Product 4	Intermediate consumption		Agriculture etc.	9.5	0.0	0.0	0.0	0.0	9.5
Product 4	Intermediate consumption		Industry	84.5	9.2	0.0	0.0	0.0	93.7
Product 4	Intermediate consumption		Trade	75.8	2.0	0.9	1.1	0.9	80.7
Product 4	Intermediate consumption		Other services	193.4	8.3	1.9	7.7	12.7	224.0
Product 4	Final consumption expenditure			640.8	13.4	29.7	24.1	36.0	744.0
Product 4	Gross capital formation			32.0	8.9	2.8	4.2	8.4	56.3
Product 4	Exports	F		91.0	20.0	0.0	-6.0	0.0	105.0
Total	Intermediate consumption		Agriculture etc.	47.5	0.0	0.0	0.0	0.0	47.5
Total	Intermediate consumption		Industry	360.6	39.8	1.9	4.3	5.8	412.5
Total	Intermediate consumption		Trade	99.2	3.0	0.9	1.1	1.9	106.1
Total	Intermediate consumption		Other services	265.3	12.4	2.9	9.9	17.7	308.1
Total	Final consumption expenditure			749.3	25.9	55.8	46.2	67.2	944.3
Total	Gross capital formation			172.6	16.9	5.5	8.4	21.5	224.9
Total	Exports	F		409.6	37.0	0.0	-11.0	0.0	435.6
Total				**2104.0**	**135.0**	**67.0**	**59.0**	**114.0**	**2479.0**
Total	**Supply**			**2306.0**					
Total	**Use**			**2104.0**	**135.0**	**67.0**	**59.0**	**114.0**	**2479.0**